

LATEST UPDATES ON JSPEC - AN IBS AND ELECTRON COOLING SIMULATION PROGRAM*

H. Zhang[†], Y. Zhang, S. Benson, Jefferson Lab, Newport News, VA 23606, USA
F. Lin, Oak Ridge National Laboratory, Oak Ridge, TN 37830, USA

Abstract

JSPEC (JLab Simulation Package on Electron Cooling) is an open-source C++ program developed at Jefferson Lab, which simulates the evolution of the ion beam under the influence of both IBS and electron cooling effects. In this paper, we will report the latest updates to JSPEC. Firstly, we have added theoretical and numerical models that simulate the effect of the electron beam dispersion on non-magnetized cooling. Secondly, the cooler can now be treated as an element with length, rather than a thin lens. This change will impact the modeling of the ions and the electrons in the cooling rate calculation for both magnetized and non-magnetized cooling. Numerical results will be provided to demonstrate the performance of the new models.

INTRODUCTION

Introbeam scattering (IBS) [1] is an effect that may reduce the quality of a high-intensity beam and impair the luminosity of a collider, while electron cooling [2] is a traditionally effective method to mitigate the IBS effect. JSPEC (JLab Simulation Package on Electron Cooling) is an open-source program [3] developed at Jefferson Lab, which simulates the evolution of the ion beam under the influence of both IBS and electron cooling effects. Originally developed to support the Electron Ion Collider (EIC) design at Jefferson Lab [4], our goal is now to provide a convenient toolkit for the IBS and electron cooling effect simulation to the wider accelerator community. JSPEC includes the most frequently used formulas for the friction forces and variant models of electron/ion beams. The main code is developed in C++ with the consideration given to both the validity of the physical models and the efficiency of computation. Most modules in JSPEC supports parallel computing in shared-memory structure using OPENMP. A Python wrapper, pyJSPEC, gives users access to most JSPEC functions in Python 3.0 environment allowing JSPEC to work collaboratively with other simulation tools that have a Python interface. [5,6]

In this report, we will introduce the latest updates to JSPEC. One update is to allow users to set electron beam dispersion at the cooler, which helps to distribute cooling effect from the longitudinal direction to the transverse directions. In our previous simulations on the JLab EIC cooling scheme, we often observed that cooling overwhelms IBS in the longitudinal direction but succumbs to IBS in

the horizontal direction. This is because the IBS effect, affected by the dispersion over the ion ring, is usually more significant in the horizontal direction than the other two directions. If we increase the cooling barely enough to compensate the horizontal IBS, it will be excessive in the other two directions. We will be able to propose more efficient cooling plan if we could control the distribution of the IBS and cooling effect in different directions. Introducing transverse coupling could transfer horizontal IBS effect into vertical direction and take usage of the cooling in that direction. Introducing dispersion for both the ion beam and the electron beam could also transfer the cooling effect from longitudinal direction to transverse directions. Previously, JSPEC had supported ion beam dispersion in cooling rate calculation. Now we also include the electron beam dispersion in our calculation. Another update is to simulate the cooler as an element with length. JSPEC has been treating the cooler as a thin lens, which simply adds a kick to each ion and does not change its position. However, the strong cooling required by new machines often needs coolers longer than 100 meters. Now JSPEC can simulate the motion of the ions and the change of the electron beam property in the cooler and take these changes into account in the cooling rate calculation.

COOLING WITH ELECTRON BEAM DISPERSION

Assuming the electron beam has a Gaussian distribution when no dispersion exists, after introducing a horizontal dispersion D , the density function and the velocity distribution function in the lab frame will change to [7]

$$n_e(\mathbf{r}) = n_{e0} \exp \left[-\frac{x^2}{2(\sigma_x^2 + D^2\sigma_p^2)} - \frac{y^2}{2\sigma_y^2} - \frac{s_0^2}{2\sigma_s^2} \right],$$

$$f_v(u_x, u_y, u_z) = \frac{1}{(2\pi)^{3/2} \sigma_1 \sigma_2 \sigma_3 \sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} \left[\frac{(u_x - \bar{u}_x)^2}{\sigma_1^2} + \frac{(u_z - \bar{u}_z)^2}{\sigma_3^2} - 2\rho \frac{(u_x - \bar{u}_x)(u_z - \bar{u}_z)}{\sigma_1 \sigma_3} \right] - \frac{(u_y - \bar{u}_y)^2}{2\sigma_2^2} \right\},$$

with

$$\bar{u}_x = -\frac{x\alpha_x\epsilon_x}{\epsilon_x\beta_x + D^2\sigma_p^2}, \quad \bar{u}_y = -\frac{y\alpha_y}{\beta_y}, \quad \bar{u}_z = \frac{Dx\sigma_p^2}{\epsilon_x\beta_x + D^2\sigma_p^2},$$

$$\sigma_1^2 = \frac{\epsilon_x}{\beta_x} \left(1 + \frac{\alpha_x^2 D^2 \sigma_p^2}{\epsilon_x \beta_x + D^2 \sigma_p^2} \right), \quad \sigma_2^2 = \frac{\epsilon_y}{\beta_y}, \quad \sigma_3^2 = \frac{\sigma_p^2 \epsilon_x^2 \beta_x}{\epsilon_x \beta_x + D^2 \sigma_p^2}$$

$$\rho = \frac{\alpha_x D \sigma_p}{\sqrt{\epsilon_x \beta_x + D^2 \sigma_p^2 (1 + \alpha_x^2)}},$$

where n_{e0} is the maximum density, $\sigma_{x,y,s}$ the RMS bunch size/length in the respective direction, $\sigma_{1,2,3}$ the RMS value

* Work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under contract DE-AC05-06OR23177 and Office of Basic Energy Sciences under contract DE-AC05-00OR22725.

[†] hezhang@jlab.org

of the velocity in the respective direction, σ_p the momentum spread, $\alpha_{x,y}$, $\beta_{x,y}$ and $\epsilon_{x,y}$ the TWISS parameter and the emittance in the horizontal or vertical direction. The friction force of non-magnetized cooling can be calculated in the beam frame as an integration over electron velocity

$$\mathbf{F}(\mathbf{r}, \mathbf{u}_i) = -\frac{4\pi Z^2 e^4 n_e(\mathbf{r})}{m_e} \int d\mathbf{u}^3 L_C(\mathbf{u}_i) \frac{\mathbf{u}_i - \mathbf{u}}{|\mathbf{u}_i - \mathbf{u}|^3} f_v(\mathbf{u}),$$

where Z is the ion atomic number, $\mathbf{u}_i$ is the ion velocity, e is the unit charge, m_e is the electron mass, L_C is the Coulomb logarithm. From the perspective of computation, the change of electron density will change $n_e(\mathbf{r})$ in the non-magnetized friction force formula. This change is easy to carry out. The change of the electron velocity distribution makes the friction force computation slightly more complicated. Previously, all the friction force formulas in JSPEC assumed that the transverse velocity distribution of the electron beam had axial symmetry, allowing the distribution function to be determined with a unique parameter, the transverse temperature. The friction force calculation involves a numerical integration over the distribution function in cylindrical coordinates. However, the effect of the horizontal dispersion breaks the axial symmetry. But we can simplify the case by assuming the TWISS function β is constant or $\alpha = 0$ at the cooler and $\sigma_1 = \sigma_2 = \sigma$. In this case, the velocity function reduces to

$$f_v(\mathbf{u}) = \frac{1}{(2\pi)^{3/2} \sigma^2 \sigma_3} \exp \left[-\frac{u_r^2}{2\sigma^2} - \frac{(u_z - \bar{u}_z)^2}{2\sigma_3^2} \right],$$

with $\bar{u}_x = 0$, $\bar{u}_y = 0$, $\bar{u}_z = \frac{Dx\sigma_p^2}{\epsilon_x\beta_x + D^2\sigma_p^2}$, $\sigma_1^2 = \frac{\epsilon_x}{\beta_x}$, $\sigma_2^2 = \frac{\epsilon_y}{\beta_y}$, and $\sigma_3^2 = \frac{\sigma_p^2 \epsilon_x \beta_x}{\epsilon_x \beta_x + D^2 \sigma_p^2}$. In the simplified case, the horizontal

dispersion only affects the longitudinal cooling, leading to a longitudinal velocity shift and a reduction of the longitudinal temperature. The axial symmetry is maintained. After these adjustments are made, all the asymptotic formulas and numerical formulas for non-magnetized friction force can be used. This formula may be used when we model a short cooler as a thin lens. However, in a long cooler, the β function of a non-magnetized electron beam will change and the α function will not be zero everywhere, which results in average velocity drifts and temperature change in all three directions as well as a cross term that couples the horizontal direction and the longitudinal direction inside the exponential function of the velocity distribution. To take all the effects into account in the friction force calculation, JSPEC performs 3D numerical integration on a user-defined grid using either the Cartesian coordinates over the velocity v_x , v_y , and v_z or the cylindrical coordinates cover the velocity v_r , v_z and the angle ϕ . The cooling rates obtained in these two ways are in reasonable agreement.

A numerical model without the assumption of initial Gaussian distribution is also developed, which is based on the user-defined arbitrary electron beam model [8] in JSPEC. This model does not assume the Gaussian distribution of the electron beam and in principle could work with arbitrary density distribution and velocity distribution. Using this model, we first generate a set of particles that represent the electron beam distribution without dispersion. The 6D coordinates of each particle will change when the

dispersion exists, which can be calculated from the old coordinates and the dispersion value. Then we implement a tree-based algorithm to group the particles according to their location and density. All the particles are enclosed inside a large cubic box, which was then divided into eight small boxes of equal size. The process is repeated until the number of particles in each box is less than a predetermined number. Thus smaller/larger boxes will be generated where there is higher/lower electron density. The relation between the boxes can be represented as a partial tree after removing all the empty boxes. A 2D example is illustrated in Fig. 1. To calculate the friction force, we need to identify the box in which the ion locates. The local electron density can be calculated from the number of particles inside the box and the volume of the box. The velocities of the particles inside the box will be used as the velocity samples to calculate the numerical integration for friction force. The total particle number and the maximum particle number in each leaf box are crucial in this model. If these numbers are too large, the algorithm may suffer from low efficiency. If they are too small, we may not have enough sampling to obtain an accurate result of the numerical integration. We need more testing to better understand the numerical property of this model.

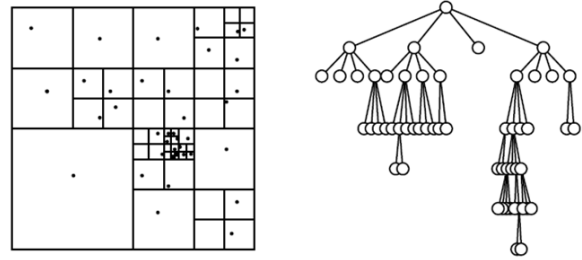


Figure 1: Tree structure of boxes.

COOLER WITH LENGTH

The TWISS function α plays an important role when electron beam dispersion is present. To model this effect, it is necessary to consider the changes of β and α as the electron beam passes the cooler. JSPEC has a thin lens model of the cooler. It assumes the electron beam remains constant during the cooling process, while each ion receives a kick due to the friction force, changing its momentum but not its position. In the new model, the cooler is treated as a lengthy element, which is divided into n slides. It may be tempting to make a model of n sequential coolers each having $1/n$ of the total length. However, this approach is incorrect because the length of the cooler affects the value of the Coulomb logarithm, which, in turn, affects the friction force. Thus, the full length of the cooler should always be used to calculate the friction force. For ions, the cooler is treated as a drift. After passing a slide of the cooler, the velocity of each ion remains constant but the position changes. Regarding non-magnetized electron beam, it is not necessary to consider the motion of individual electrons but rather the properties of the whole electron beam. It is necessary to calculate the TWISS function β and α at different locations and then compute the new bunch size, density function, and velocity distribution function. In

magnetized cooling, electrons always travel a helical trajectory along the magnetic line. The "matched" β function is constant in the cooler, resulting in the constant bunch size and the elimination of alpha function. Therefore, the electron beam can still be considered constant in the cooler for magnetized cooling.

The model for a lengthy cooler can be summarized into the following steps: (1) Divide the cooler into n slides; (2) For non-magnetized cooling, calculate the TWISS functions, density function, and velocity distribution function; (3) Calculate the friction force on each ion and apply the kick; (4) Move all the ions to the center of the next slide; and (5) repeat from step (2) till the last slide.

NUMERICAL EXAMPLES

As numerical examples, we will present calculations for a dual energy storage ring cooler [9]. This cooler consists of two storage rings connected by an energy recovery linac, which provides the energy difference. The higher energy one has a 500 MeV damping section for the electron beam and the lower energy ring drives a cooling section running from 22 MeV to 150 MeV. Strong magnetized cooling with 2T magnetic field was originally proposed to cool the 275 GeV proton beam in EIC [10]. We are studying the feasibility of non-magnetized cooling. The key parameters of the proton and electron beams are listed in Table 1. The original electron beam proposed for the strong magnetized cooling does not provide enough cooling in non-magnetized cooling process. Preliminary simulation results suggest that we should increase the electron number per bunch by a factor of four to 2.8×10^{11} and reduce the transverse emittance from 622/31 μm to 7.6/3 μm . In the following, we present examples where the new models are used.

Table 1: Proton & Electron Beam Parameters

	Electron beam	Proton beam
Lorentz factor γ	293.1	293.1
Particle number	2.8×10^{11}	6.9×10^{10}
Norm. emit. (μm)	7.6/3.0	2.8/0.45
Bunch length (cm)	6.0	6.0
Momentum spread	7.4×10^{-4}	6.8×10^{-4}
β cooler center (m)	66/26	180/180

Table 2: Cooling Rate for Various Number of Slides

N	$R_x (10^{-5}\text{s}^{-1})$	$R_y (10^{-5}\text{s}^{-1})$	$R_z (10^{-5}\text{s}^{-1})$
1	2.444	2.884	17.41
2	2.129	2.554	14.96
4	2.127	2.487	14.82
6	2.101	2.518	14.78
8	2.094	2.430	14.74
10	2.093	2.419	14.78
100	2.094	2.485	14.75

To fully consider the effect of electron beam dispersion, we utilize the lengthy cooler model. Table 2 displays the cooling rates for different number of slides used in the cooler model, ranging from one to 100. The results in Table 2 assume zero electron dispersion and proton dispersion. Generally, the convergence of the results as the slide number increases depends on how significant the beam changes are in the cooler. For this particular case, when the slide number is six or greater, the results become relatively stable. For all subsequent examples, we use six slides to model the cooler.

When both beams have zero dispersion, we observe overcooling in the longitudinal direction and insufficient cooling in the horizontal direction. To address this, we can adjust the dispersion of both beams to convert longitudinal cooling into transverse cooling. In the following example, we set the proton beam dispersion to 2.2 m and vary the electron beam dispersion from 0 to 2 m. As shown in Fig. 2, increasing the electron beam dispersion reduces longitudinal cooling and enhances horizontal cooling. With a dispersion of 2 m, we achieve cooling in all three directions. Table 3 presents the IBS expansion rate and cooling rate for this case.

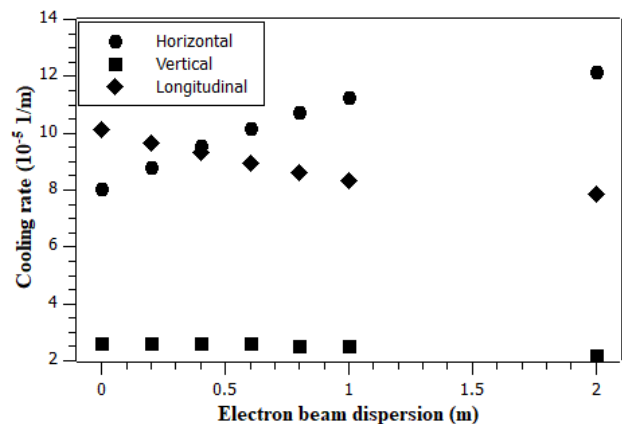


Figure 2: Cooling rate changes with electron dispersion.

Table 3: IBS and Electron Cooling Rate

	$R_x (10^{-5}\text{s}^{-1})$	$R_y (10^{-5}\text{s}^{-1})$	$R_z (10^{-5}\text{s}^{-1})$
IBS	11.81	0.097	6.810
Cooling	-12.15	-2.191	-7.843
Total	-0.034	-2.094	-1.033

CONCLUSION AND FUTURE WORK

JSPEC has been updated to include electron beam dispersion in cooling simulations. A simplified theoretical model assuming axial symmetrical transverse velocity, a complete theoretical model, and a numerical model have been provided. We will perform further testing on the numerical model to ensure its numerical properties. The cooler model has also been updated from a thin lens to a lengthy element. We will port the new models to the Python wrapper, pyJSPEC. Additionally, we are working on a portal to Fortran that aims to call JSPEC functions in BMAD to simulate electron cooling together with the space charge effect. BMAD will be responsible for ion

tracking, including the space charge effect, while JSPEC will carry out the electron cooling effect simulation.

REFERENCES

- [1] A. Piwinski, “Intra-beam-scattering”, in *Proc. 9th Int. Conf. on High Energy Accelerators*, Stanford, CA, USA, May 1974, pp. 405-409. doi: 10.5170/CERN-1992-001.226
- [2] Ya. Derbenev, "Theory of electron cooling", arXiv:1703.09735. doi:10.18429/10.48550/arXiv.1703.09735
- [3] JSPEC, <https://github.com/zhanghe9704/electroncooling/>
- [4] S. Abeyratne *et al.*, “MEIC Design Summary”, arXiv:1504.07961. doi:10.48550/arXiv.1504.07961
- [5] pyJSPEC, <https://github.com/zhanghe9704/jspec2-python/>
- [6] H. Zhang *et al.*, “pyJSPEC – A Python module for IBS and electron cooling”, in *Proc. 5th North American Particle Accelerator Conf. (NAPAC2022)*, Albuquerque, NM, USA, Aug. 2022, pp. 672-674.
doi: 10.18429/JACoW-NAPAC2022-WEPA24
- [7] M. Blaskiewicz, “Dispersion and electron cooling”, Brookhaven National Lab, Upton, NY, USA, Rep. BNL-210932-2019-TECH, 2019
- [8] H. Zhang *et al.*, “The latest code development progress of JSPEC”, in *Proc. 4th North American Particle Accelerator Conf. (NAPAC2019)*, Lansing, MI, USA, Sep. 2019, pp. 539-541. doi: 10.18429/JACoW-NAPAC2019-TUPLO04
- [9] B. Dhital *et al.*, “Cooling performance in a dual energy storage ring cooler”, in *Proc. 13th Int. Particle Accelerator Conf. (IPAC2022)*, Bangkok, Thailand, Jun. 2022, pp. 568-570. doi:10.18429/JACoW-IPAC2022-MOPOTK047
- [10] C. Montag *et al.*, “Electron-Ion Collider design status”, in *Proc. 13th Int. Particle Accelerator Conf. (IPAC2022)*, Bangkok, Thailand, Jun. 2022, pp. 1954-1957. doi: 10.18429/JACoW-IPAC2022-WEPOPT044