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Chiral Symmetry and Nuclear Interactions

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Abstract About 3 decades ago, Steven Weinberg came up with an idea of using the effective chiral Lagrangian to describe nuclear interactions, which has had a long-lasting impact on nuclear physics. Here, I will reflect on what has been learned since that time about the role of chiral symmetry in this context and discuss achievements and challenges in advancing chiral EFT into a precision tool for light nuclei.

1 Introduction

I am incredibly honored to have been awarded the 2021 Faddeev Medal. Ludvig Dmitrievich Faddeev was a giant of theoretical physics and mathematics. The famous work on the quantum-mechanical three-body problem [1] he did in the early stages of his career laid out the foundations of few-body physics, being just one of his numerous and fundamental contributions to mathematical physics. It is an even greater honor to have my name alongside the inaugural prize winners Vitaly Efimov and Rudolf Grimm, whose seminal work opened up a new frontier in few-body physics.

My research direction is in some sense complementary to Efimov physics, aiming at understanding *non-universal* properties of nuclear systems beyond those dictated by the large two-nucleon (NN) S-wave scattering lengths. The physics principle that makes it possible is the spontaneously broken chiral symmetry of QCD. In this short contribution, I describe some efforts of my collaborators and myself towards gaining a better understanding of the role of chiral symmetry in low-energy nuclear dynamics and unfolding its full power. This paper is, however, neither intended to provide an introduction [2] nor a review [3–6] of this topic.

2 30 Years Ago: Chiral EFT in its Infancy

My personal acquaintance with effective field theory (EFT) and chiral perturbation theory started during my student days in the mid-nineties when I joined the group of Walter Glöckle at the Ruhr University Bochum. This was the time when nuclear chiral EFT was still in its infancy—just a few years after Weinberg's seminal papers [7,8], followed by the first quantitative application of his ideas to the NN system by Carlos Ordóñez and collaborators [9]. These new developments offered the prospect of a deeper and more systematic understanding of nuclear interactions. Naturally, Walter got very enthusiastic and excited about these ideas, and he suggested to me to work on the derivation of the two-nucleon potential from chiral EFT as a diploma project. Around the same time, we got in contact with Ulf Meißner, and this has evolved into a long-standing and fruitful collaboration.

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It was an interesting time—full of new ideas, unexpected findings, controversies and intense discussions in the emerging nuclear EFT community on how to organize the chiral expansion for nuclear systems in the most efficient and consistent way [10], some of which are still ongoing [11]. In this contribution I will however only briefly touch upon some of these issues, focusing on the question that was driving Weinberg's famous work. In his own words [8]:

> *"The purpose of this work is not to improve our detailed picture of nuclear forces, which it is hardly likely could be accomplished with these methods, but rather to take a first step toward identifying those aspects of nuclear forces and pion-nucleon interactions that can be derived from the symmetry properties of quantum chromodynamics."*

So, perhaps, it is a good time to reflect on where we stand in identifying the role of chiral symmetry after more than 3 decades of intense research along this line. This is what I am going to address below.

3 Two Pion Exchange: Unveiling the Chiral Symmetry

The emergence of nuclear physics as a science is usually attributed to the prediction of a pion as the field quantum mediating the strong nuclear force by Hideki Yukawa [12]. After the experimental discovery of pions in the late 1940s, a robust qualitative picture of the nuclear force has emerged in terms of the relevant length scales [13]:

- The long-range interaction at internucleon distances of $r \gtrsim 2$ fm is dominated by the one-pion exchange (OPE);
- At intermediate distances of about 1–2 fm, the interaction mediated by two-pion exchange (TPE) is expected to become important;
- At even shorter distances, nuclear forces are expected to be governed by other mechanisms involving three-pion exchange, exchange of heavier mesons, nucleon excitations, etc..

Accordingly, considerable efforts have been devoted in the second half of the 20th century towards understanding the intermediate- and short-range part of the nuclear force in the framework of the so-called meson field theory based on phenomenological meson-baryon Lagrangians. These attempts could not be considered entirely successful for a variety of reasons, one of them being the lack of a systematic expansion scheme. Indeed, the pseudoscalar pion-nucleon coupling constant $g_{\pi \mathrm{N}}^2/(4\pi) \simeq 14$, where $g_{\pi \mathrm{N}}$ is defined through the residue of the induced pseudoscalar form factor of the nucleon at the pion pole, is large and does not support a perturbative description of multi-pion exchange interactions between the nucleons. But most importantly, meson field theory had no obvious relationship to QCD, the underlying theory of the strong interaction.

The two key ingredients that were missing at that time and have lead to a true revival of the field in the past decades are the concepts of an EFT and chiral symmetry. The first one provides a resolution-dependent framework for systematically exploiting hierarchies of scales in quantum field theory. This idea is, of course, not entirely new and has been widely used in nuclear physics in the form of zero-range theory or the effective range expansion [14]. The modern formulation and extension of this method is known as pionless EFT and provides an appropriate tool to study halo-nuclei, strongly clustered systems and nuclear dynamics at extremely low energies [15]. The second key ingredient establishes a connection between nuclear physics and the underlying theory of the strong interaction through the spontaneously broken approximate chiral symmetry of QCD. This symmetry and its breaking pattern has far reaching implications by strongly constraining the form of multi-pion interactions and ensuring their derivative nature. This last feature is at the heart of chiral perturbation theory and allows one to compute multi-pion and pion-nucleon (πN) scattering amplitudes as well as processes involving external electroweak sources by performing a controlled perturbative expansion in powers of momenta and quark masses, commonly referred to as the chiral expansion [16]. These amplitudes are, in fact, all that is needed to determine the long-distance behavior of the nuclear forces and currents. Long-distance interactions thus come out as parameter-free predictions in chiral EFT, provided all relevant low-energy constants (LECs) from the pion and pion-nucleon Lagrangians are fixed from experimental data or lattice-QCD simulations.

At the two-nucleon (NN) level, the most interesting place to apply these ideas and test the predictive power of chiral EFT is the TPE. In the middle of 1990s, the expressions for the leading and subleading contributions to the chiral TPE potential were independently derived and cross checked using different methods [17–19]. All these calculations employed dimensional regularization or equivalent methods to evaluate divergent loop integrals. In momentum space, the resulting TPE potentials are unambiguously determined up to a polynomial, which is at most quadratic in momenta or pion mass. The ambiguous polynomial terms contribute to the interaction at

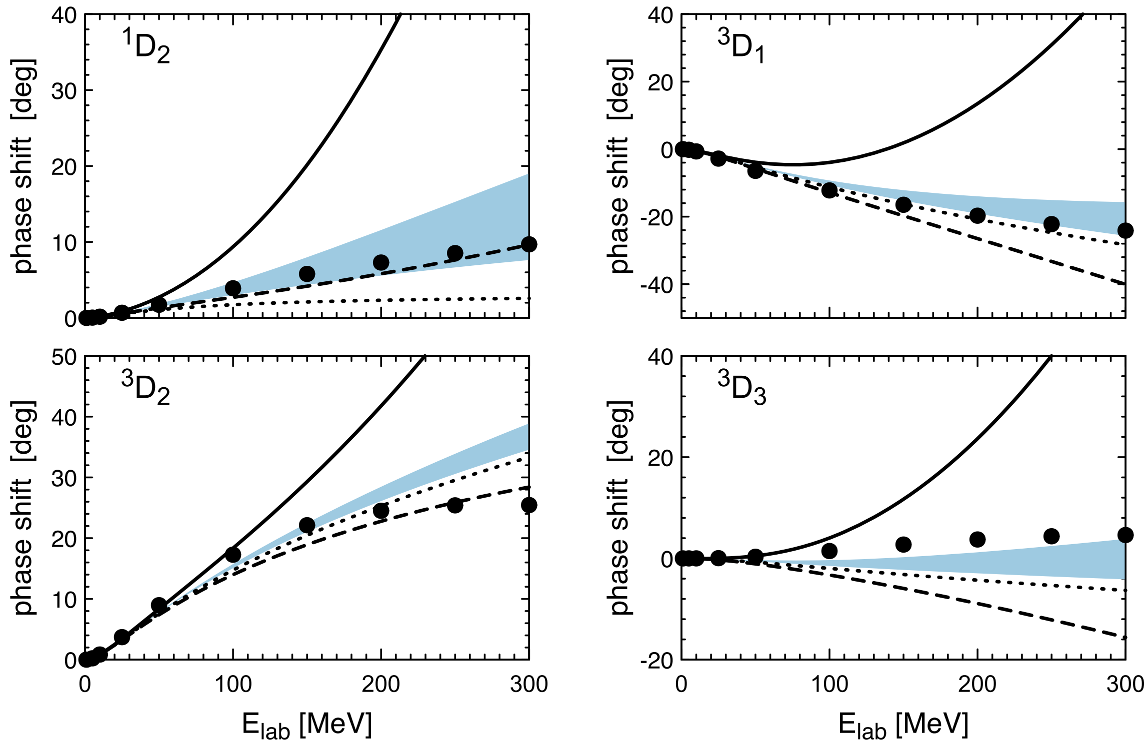

Fig. 1 Neutron–proton D-wave phase shifts at different orders in chiral EFT. Dotted lines show the LO predictions based solely on the OPE potential, while dashed and solid lines also take into account the chiral TPE potential at NLO and N^{2}LO, respectively. Light-shaded blue bands show the N^{2}LO predictions using the sharp cutoff $\Lambda = 500 \ldots 800$ MeV in the spectral representation of the TPE potential. All calculations are performed using the Born approximation and K-matrix unitarization. Filled circles are results of the Nijmegen partial wave analysis [20]. Figure adapted from Ref. [21]

short distances but do not affect NN scattering in D- and higher partial waves. Thanks to the centrifugal barrier that acts as filter for long-range interactions, peripheral NN scattering may be considered as a testing ground for chiral dynamics. These ideas have been used by the Munich group to test the parameter-free expressions for the chiral TPE [18]. Surprisingly, their results indicated poor convergence of the chiral expansion in D-waves, see black lines in Fig. 1, and even in more peripheral F-waves.

The results indicated an unphysically strong attraction generated by the central part of the subleading TPE potential, which was however not entirely unexpected. Indeed, the analytical expressions for the subleading TPE at next-to-next-to-leading order (N^{2}LO) show an enhancement due to a large numerical prefactor. An additional enhancement is generated by the large value of the πN LEC c_3 entering the central potential, which is largely driven by the intermediate $\Delta(1232)$ excitation [22].

The findings of the Munich group motivated my collaborators and me to take a closer look at the origin of the unphysical attraction of the subleading TPE [21]. Using the dispersive representation of the TPE potential

$$V_{2\pi}(q) = \frac{2}{\pi} \int_{2M_\pi}^{\infty} \mu d\mu \, \frac{\rho(\mu)}{q^2 + \mu^2} \; + \; \ldots , \tag{1}$$

where $q \equiv |\mathbf{q}|$ is the nucleon momentum transfer, $\rho(\mu) = \Im\big[V_{2\pi}(q)\big]_{q=0^+ - i\mu}$ is the spectral function and ellipses refer to subtraction terms polynomial in momenta, we have identified the origin of the unphysical attraction with short-range admixtures in $V_{2\pi}(q)$ stemming from the spectral components with masses above the breakdown scale Λ_b of chiral EFT, $\mu > \Lambda_b$. This is visualized by the light-shaded blue bands in Fig. 1, which show the N^{2}LO predictions using the TPE with the spectral integral in Eq. (1) being cut off at $\Lambda = 500 \ldots 800$ MeV. In chiral EFT, the spectral function $\rho(\mu)$ is computed in terms of an expansion in powers of μ/Λ_b and M_π/Λ_b that is inapplicable for $\mu \gtrsim \Lambda_b$. The expressions for $V(q)$ obtained using dimensional regularization involve spectral components with $2M_\pi \leq \mu < \infty$ and thus also short-range contributions that

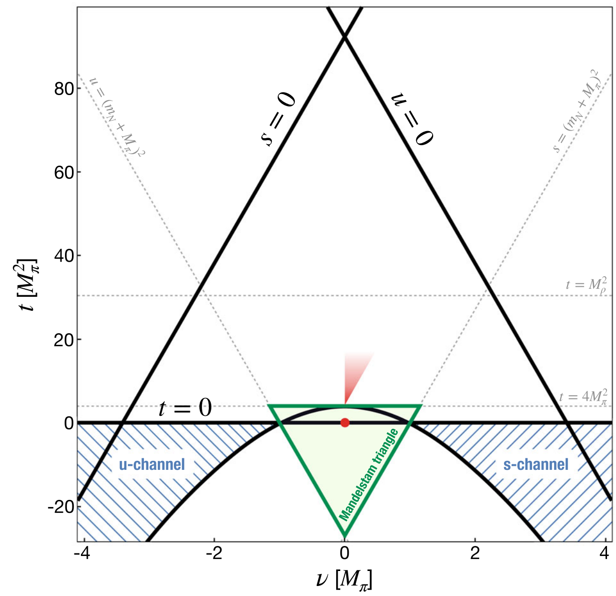

Fig. 2 Mandelstam plane for pion-nucleon scattering. Blue hatched regions show physical regions, while the green light-shaded area corresponds to the Mandelstam triangle. The red solid dot is the subthreshold kinematical point. The red light-shaded triangle area shows the t- and Re(ν)-values of the πN scattering amplitude that enter the calculation of the long-range part of the TPEP as explained in the text

cannot be calculated reliably in chiral EFT. Keeping this uncontrolled short-distance dynamics in the TPE would require a promotion of higher-order contact interactions to compensate for the unphysical attraction in D- and F-waves, thereby decreasing the predictive power of chiral EFT. On the other hand, a simple spectral function regularization with $\Lambda \sim \Lambda_b$ [21] removes the spurious short-range physics and results in a more efficient formulation. These insights played an important role in the development of realistic NN potentials in subsequent years.

Another milestone on the route to high-precision NN chiral interactions has been reached by Norbert Kaiser, who derived analytic expressions for the two- and three-pion exchange potentials at fourth order (i.e., N^{3}LO) in the chiral expansion [23–27]. Instead of directly evaluating highly complicated two-loop integrals, he applied the Cutkosky cutting technique to extract the corresponding spectral functions from integrals over the pion phase space involving the on-shell πN scattering amplitudes, analytically continued to complex values of the Mandelstam variables s and u. These analytical results have been incorporated in the (first generation of) N^{3}LO NN potentials [28,29].

The next essential development I would like to briefly touch upon was the work by the Bonn group on πN scattering [30]. A reliable determination of the TPE requires a precise knowledge of the πN LECs, which can in principle be obtained from the analysis of the empirical πN phase shifts or scattering data in chiral perturbation theory. This however has two important drawbacks. First, only a limited amount of low-energy πN scattering data is available, and the existing partial wave analyses may suffer from uncertainties that are difficult to quantify. Secondly, the physical region of πN scattering corresponds to $s \geq (m_N + M_\pi)^2$ and negative values of t, with t and s being the Mandelstam variables, which is located somewhat far away from the kinematics probed by the TPE spectral function, see Fig. 2. This may result in significant uncertainties from the interpolation/analytic continuation of the πN scattering amplitude. These issues have been addressed by the Bonn group, who performed a dispersive analysis of πN scattering in the framework of Roy-Steiner equations. Using the available experimental information, they obtained accurate results for the πN amplitude with quantified uncertainties, which fulfill constraints from analyticity, unitarity and crossing symmetry. The πN LECs have then been determined by matching chiral perturbation theory to the solution of Roy-Steiner equations at the experimentally not directly accessible subthreshold point with $t = \nu \equiv (s - u)/(4m_N) = 0$. The latter is located inside the Mandelstam triangle, where the amplitude is purely real and the chiral expansion is expected to converge best [31]. This allowed for a reliable determination of the πN LECs.

To really benefit from this extensive work on the accurate determination of the TPE potential, it was important to ensure that its long-distance behavior, ultimately predicted by chiral EFT, is not affected by the regulator. This motivated us to replace the non-local regulator for long-range interactions used, e.g., in [28,29] by a local one [32,33]. In its final form [34], regularization of the OPE and TPE is carried out in momentum space via

$$
\begin{aligned}
V_{1\pi}(q) \to V_{1\pi}^{\Lambda}(q) &= V_{1\pi}(q)\, e^{-\frac{q^2+M_\pi^2}{\Lambda^2}} + \ldots, \\
V_{2\pi}(q) \to V_{2\pi}^{\Lambda}(q) &= \frac{2}{\pi}\int_{2M_\pi}^{\infty} \mu\, d\mu\, \frac{\rho(\mu)}{q^2+\mu^2}\, e^{-\frac{q^2+\mu^2}{2\Lambda^2}} + \ldots,
\end{aligned} \tag{2}
$$

where the ellipses refer to subtraction terms polynomial in q^2, which are chosen in such a way that the corresponding r-space potentials and their derivatives vanish at the origin. This convention ensures that the regularized OPE and TPE potentials involve only long-range interactions. Most importantly, the employed regulator does not affect the analytic structure of the amplitude, i.e. the residue of $V_{1\pi}(q)$ at the pion pole and the discontinuity along the left-hand t-channel cut of $V_{2\pi}(q)$. This is in contrast to the angle-independent nonlocal regulators of Refs. [28,29,35], which induce long-range artifacts that may potentially decrease the predictive power of chiral EFT.

In Fig. 3, I show the chiral expansion of the two strongest components of the long- and intermediate-range NN potential. These results highlight the power of chiral EFT to provide parameter-free predictions for long-range nuclear forces, which rely solely on the chiral symmetry of QCD (and its breaking pattern) and the empirical information on the πN system entering through the values of the corresponding LECs. The isovector tensor potential in the left panel is dominated by the OPE at large distances, but receives significant contributions from TPE at intermediate distances below $r \sim 2$ fm. This is of course fully in line with the qualitative picture suggested in Ref. [13] and mentioned at the beginning of this section. The right panel shows the isoscalar central potential which receives its first contribution at N^{2}LO. Phenomenologically, this TPE component is often modeled in terms of the σ-meson exchange and known to be strongly attractive. Here, we confirm the strong intermediate-range attraction in the isoscalar-scalar channel of the NN force but using a systematic and theoretically well-founded framework of chiral EFT. Figure 3 also gives an idea about the convergence pattern of the chiral expansion. At large distances of $r \gtrsim M_\pi^{-1}$, the TPE potential $V_{2\pi}(r)$ is dominated by low-energy components of the πN amplitude (i.e., the part of the red light-shaded area in Fig. 2 that is closest to the subthreshold point), and the chiral expansion shows a convergence pattern consistent with the breakdown scale of $\Lambda_b \sim 600$–700 MeV [32,36,37]. On the other hand, the chiral expansion for $V_{2\pi}(r)$ is not under control at short distances $r \lesssim \Lambda_b^{-1}$, featuring a highly singular short-range behavior (i.e., $V_{2\pi}(r) \sim r^{-n}$ with $n \geq 5$ and increasing with the order of the expansion) when using dimensional regularization. This unphysical part of the of $V_{2\pi}(r)$ is removed by the regulator defined in Eq. (2) without affecting its large-distance behavior. The short-range part of the nuclear force is then modeled in a general and systematic way in terms of contact interactions, whose coefficients are adjusted to reproduce NN scattering data. This program was carried out in Ref. [34] up to fifth order (i.e., N^{4}LO) using the regularization scheme defined in Eq. (2) in combination with a nonlocal Gaussian-type cutoff for contact interactions, yielding encouraging and robust results. In particular, we observed a systematic order-by-order improvement of the description of NN data as expected from a converged expansion, with all determined LECs coming out of a natural size. Moreover, the resulting description of NN scattering observables at the highest considered order showed almost no residual Λ-dependence in the range of $\Lambda = 400 \ldots 550$ MeV.[1] By comparing the description of NN data at the subsequent orders NLO/N^{2}LO and N^{3}LO/N^{4}LO, we found a clear evidence of the important role played by the chiral symmetry. Indeed, the only additional (isospin-invariant) contributions to the NN potential at N^{2}LO and N^{4}LO stem from the TPE predicted by chiral EFT. These parameter-free contributions were demonstrated to lead to a very significant improvement in the description of the data, see [32,33] for similar earlier findings. For example, using $\Lambda = 450$ MeV, the χ^2/datum for the description of neutron-proton data up to $E_{\rm lab} = 300$ MeV reduced from ~ 14 at NLO to 4.2 at N^{2}LO when including the subleading TPE potential. At the highest considered order, a nearly perfect description of NN scattering data was achieved by tuning 27 LECs as compared to $\sim 40 \ldots 50$ adjustable parameters typically employed in phenomenological high-precision potentials. This reduction in the number of parameters stems from the inclusion of the TPE

[1] The somewhat narrow available cutoff range is caused by the non-perturbative treatment of the TPE potential that is not required by the power counting. It can probably be increased using a partly perturbative scheme formulated in Refs. [38,39], which has been *explicitly* demonstrated to yield a properly renormalized NN amplitude.

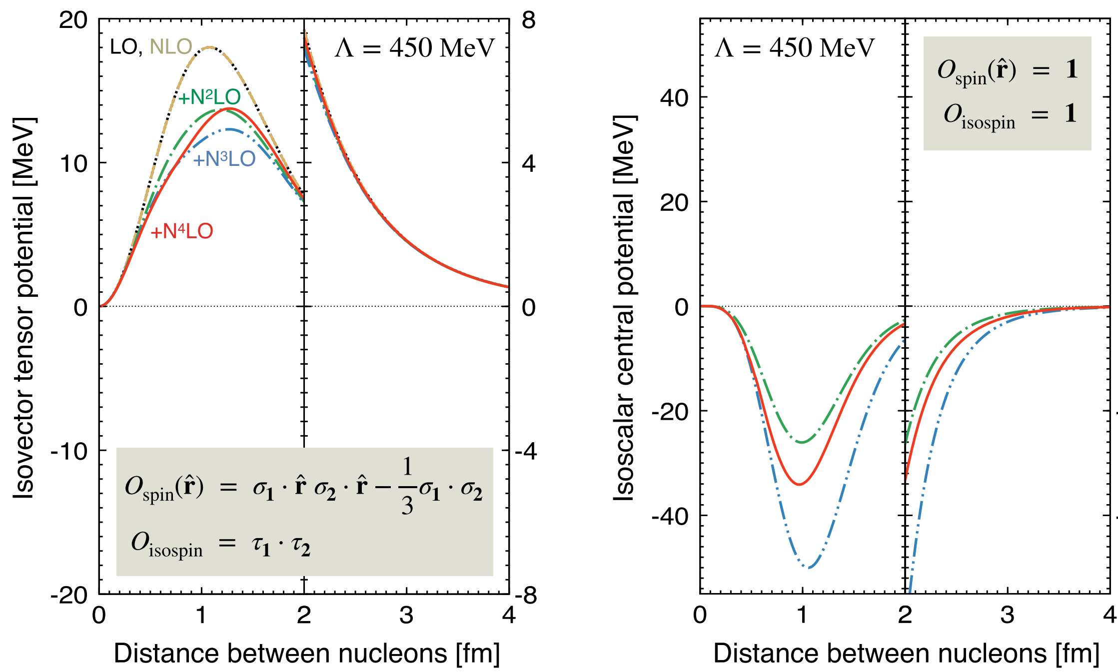

Fig. 3 Chiral expansion of the isovector tensor (left) and isoscalar central (right) NN potentials due to one- and two-pion exchange. The OPE potential is taken in the static limit. The potentials are obtained using the local regulator specified in Eq. (2) with $\Lambda = 450$ MeV

potential and provides yet another evidence of the important role played by the chiral symmetry that governs the long- and intermediate-range NN interaction.

4 Entering Precision Era

The developments outlined above, along with the newly established Bayesian methods to quantify truncation errors in EFT [36], opened the way for pushing chiral EFT in the two-nucleon sector into a precision tool. I just mention two examples of precision studies carried out in our group:

- In [40], the NN potentials of [34] have been updated to include isospin-breaking contributions up through $\mathrm{N^4LO}$ and used to perform a precision determination of the πN coupling constants from NN scattering data, yielding the value $g^2_{\pi \mathrm{N}}/(4\pi) = 13.92 \pm 0.09$ in a good agreement with the most recent determination from pionic hydrogen [41]. As a part of this project, we have also performed a full-fledged partial wave analysis of NN scattering data up to pion production threshold (including selection of mutually consistent data sets) in the framework of chiral EFT. The achieved theoretical accuracy is exemplified in the left panel of Fig. 4 for tensor analyzing power P in proton-proton scattering at different orders in chiral EFT using the Bayesian approach of [36,37] for estimating truncation errors.
- The second example is the high-accuracy calculation of the deuteron structure radius, $r_{\mathrm{str}} = 1.9729^{+0.0015}_{-0.0012}$ fm, and the quadrupole moment, $Q_d = 0.2854^{+0.0038}_{-0.0017}$ fm^2, at $\mathrm{N^4LO}$ in [42]. The calculated quadrupole moment is in agreement with the very precise experimental datum $Q_d^{\mathrm{exo}} = 0.285699(15)(18)$ fm^2 [43], while the predicted structure radius was used to extract the neutron charge radius from isotope-shift data on the deuteron−proton charge radii difference, yielding the value of $r_n^2 = -0.105^{+0.005}_{-0.006}$ fm^2.

5 Three-Body Forces

Three-nucleon forces (3NF) have been an important frontier in nuclear physics since its inception. Contrary to the two-nucleon sector, a precise and statistically satisfactory description of the nucleon-deuteron scattering

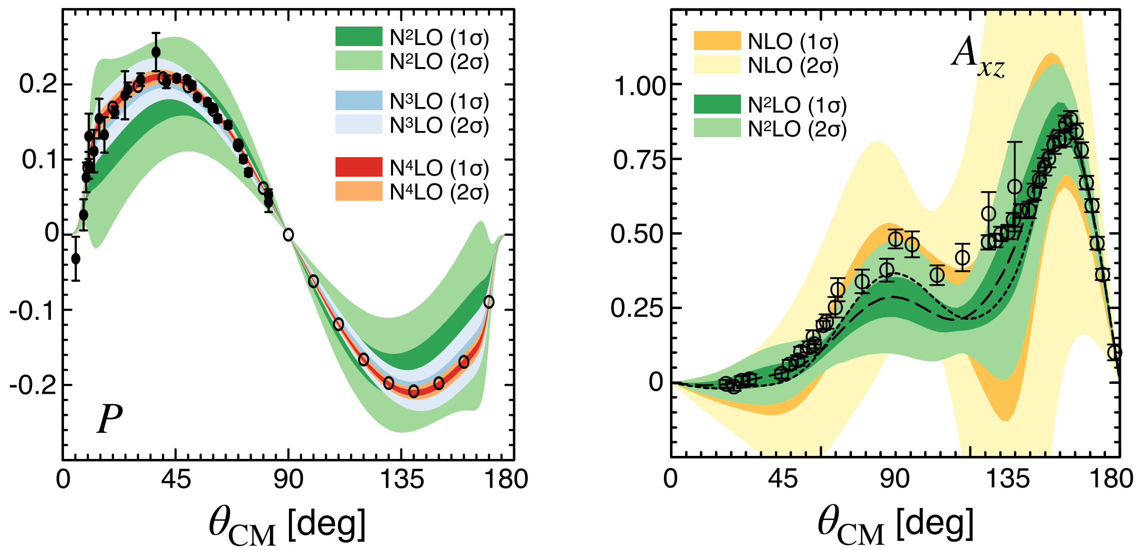

Fig. 4 Representative examples of chiral EFT state-of-the-art results for proton-proton scattering at $E_{\text{lab}} = 142$ MeV (left) and nucleon-deuteron scattering at $E_{\text{lab}} = 135$ MeV (right). Shaded bands show the 68% (1σ) and 95% (2σ) DoB truncation errors at different EFT orders estimated using the Bayesian approach. Left and right panels show the vector and tensor analyzing powers P and A_{xz}, respectively. Open circles are the predictions of the Nijmegen partial wave analysis [20]. Experimental data for P (A_{xz}) are taken from Ref. [44] (from Ref. [45])

data could not be achieved so far [46], indicating that the simplest nuclear system beyond the 2N one remains poorly understood. Apart from the substantial computational cost of solving the Faddeev equations, the main hurdle in modelling the 3NF is the high complexity of its spin-isospin-momentum structure and the shear amount of information needed to parametrize its most general form [47]. Here, chiral EFT may be expected to offer a decisive advantage over more phenomenological approaches by determining the long- and intermediate-range behavior of the 3NF in a model-independent and parameter-free way and providing an importance hierarchy of short-range operators. So, where do we stand in the journey towards high-precision 3NF from chiral EFT?

The dominant contributions to the 3NF from tree-level diagrams are simple and have been known since decades [48,49]. Lots of calculations that include these 3NFs are available, and I only show in the right panel of Fig. 4 a representative example from the work by the LENPIC Collaboration [50]. While promising, this example shows that a quantitative description of 3N scattering data would require the chiral expansion to be pushed beyond N^{2}LO (as expected from the experience gained in the NN sector). Corrections to the 3NF at N^{3}LO and N^{4}LO have been worked out using dimensional regularization to evaluate loop integrals [51–56]. However, a consistent cutoff regularization of the derived 3NF turned out to be a highly nontrivial task, and a simultaneous usage of dimensional and cutoff regularizations was shown to violate chiral symmetry [5]. Thus, 3NF contributions beyond the leading one need to be re-derived using a symmetry-preserving cutoff regulator instead of dimensional regularization. The same issue affects the four-body forces [57] and contributions to exchange currents beyond N^{2}LO [58]. First steps along this ambitious program were made in [59,60] by employing the gradient flow regularization method, which preserves the chiral and gauge symmetries and is extensively used in lattice-QCD simulations. Interestingly, at the OPE level, the gradient flow regulator reduces to the one shown in Eq. 2. These new developments open the avenue for the development of accurate and precise 3NF in harmony with the chiral symmetry of QCD.

6 Conclusion

I think it is fair to say that the pivotal role of chiral symmetry in describing nuclear interactions at large and intermediate distances has been convincingly demonstrated in the two-nucleon sector. In fact, chiral EFT seems to work even better than anticipated by Weinberg by being capable of providing not just qualitative but also quantitative insights into low-energy nuclear dynamics. The method was shown to be remarkably successful in describing the phenomenology of NN interactions. The next great opportunity (and challenge) for chiral EFT lies in the description of three-nucleon forces. To leverage the full predictive power of the method beyond the NN system, it will however be necessary to employ a rigorous symmetry-preserving regularization scheme, a task that goes far beyond Eq. (2). First steps along these lines have already been taken in [59,60]. To conclude, I think we are still at the beginning of our journey to explore manifestations of chiral symmetry in nuclear physics, and I look forward to seeing where it will bring us.

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Author Contributions E.E. wrote the manuscript.

Data Availability No datasets were generated or analysed during the current study.

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Declarations

Conflict of interest The authors declare no conflict of interest.

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