

NAL PROPOSAL # 226

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Coherent K_S Regeneration by Electrons

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June, 1973

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BEAM REQUIREMENTS: Neutral Beam M-4

RUNNING TIME: 30 days (90 shifts)

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ABSTRACT

We propose to extend the use of the E-82 spectrometer in the neutral beam M4 to a determination of the amplitude for regeneration of K_S by electrons. This quantity is directly proportional to the rms neutral kaon charge radius, R^2 . NAL is uniquely suited for such a determination, since the electron contribution to regeneration is ~ 7% at 100 GeV, and grows as the square root of the kaon momentum.

The K^0 is neutral
albeit not quite
it shows its charge radius
to those who are bright.
A photon exchange, when properly done,
Will suddenly turn a K_2 into a K_1 .
The regeneration, while admittedly rare,
Is obviously proportional to the kaon's R^2 .
Where Zel'dovitch sweated, where Rubbia faltered,
the state of the art must clearly be altered.
We ask for no new beams, for no fancy device
Just give us 90 shifts of running
And that will suffice.

COHERENT K_S REGENERATION BY ELECTRONS

In our approved NAL proposal, E-82, we alluded briefly to our intention to study electron regeneration (p.6). In the present proposal we wish to spell out the details of the experiment that we would like to perform. Since we intend to use the apparatus and the beam of E-82 without modifications, the experiment proposed here could be carried out in the near future.

1. Theoretical background.

K^0 and $\bar{K}^0$ have conjugate charge distributions, and will thus scatter from electrons with amplitudes $f^e, \bar{f}^e$ of equal magnitude and opposite sign. Hence there exists a regeneration amplitude

$$\rho_e = 2\pi i NL f_{21}^e / k \quad (1)$$

and f_{21}^e/k is proportional to R^2 , the rms charge radius of neutral kaons, viz. $f_{21}^e/k = -\alpha R^2/3$ (2).

To predict the magnitude of K_S regeneration from electrons, one needs hence an estimate of R^2 ; conversely, a measurement of ρ_e (such as proposed here) serves essentially to determine R^2 .

R^2 can be predicted from the observed rate of $e + \bar{e} \rightarrow K + \bar{K}$ (with some vector dominance and SU(3) assumptions), or from the "algebra of fields" considerations of Kroll, Lee and Zumino ^[1], who give

$$R^2(K^0) = -0.76 \text{ mb.} \quad (3)$$

In either case, we expect regeneration by electrons to be less strong than

predicted in the early work of Zeldovitch [2], corresponding to $R^2 \approx 21$ mb.

An important fact underlying the experiment proposed here is that ρ_e is independent of k , the K_L momentum, whereas the corresponding (and dominant) nuclear amplitude ρ_N is expected to decrease as $k^{-1/2}$. Thus the relative importance of electron regeneration increases with k , making the experiment particularly suited for NAL.

2. Principle of the experiment.

The principle which we propose to adopt for measuring ρ_e is essentially the same as that used by the Aachen-CERN-Torino group in their experiment directed at the same goal [3] (for facilitating the reviewer's task, we enclose a copy of this reference). There are, however, certain important differences -- of method rather than of principle -- between our approach and that of Ref. 3; these will be discussed later on (see Section 3)

The so-called "transmission regeneration" amplitude ρ at the exit of a slab of number density N and thickness L (thin compared to the mean K_S decay length, $\Lambda(k)$) is given (per K_L) by

$$\rho = \rho_N + \rho_e = 2\pi i NL [f_{21}^N(0) + Z f_{21}^e(0)] / k, \quad (4)$$

The intensity of transmission-regenerated K_S^0 is

$$N^{tr} = |\rho|^2 = \left(\frac{2\pi NL}{k}\right)^2 |f_{21}^N(0) + Z f_{21}^e(0)|^2, \quad (5)$$

where we have neglected attenuation in the slab, and CP-interference effects behind it. Since terms quadratic in f_{21}^e will always be negligible in practice, we can make the approximation

$$N^{tr} \approx (2\pi N L)^2 \left| \frac{f_{21}^N(0)}{k} \right|^2 \left[1 + 2 \operatorname{Re} \frac{Z f_{21}^e(0)}{f_{21}^N(0)} \right]. \quad (6)$$

The essential point is that $|f_{21}^N(0)/k|$ can be determined independently from the so-called diffraction regeneration extrapolated to $\gamma = 0$. When momentum is

transferred to the scattering centers, then these act incoherently among each other and lead (again neglecting attenuation and CP-effects) to the re-generated K_S intensity.

$$\frac{dN^{\text{diff}}}{d\Omega}(\mathcal{J}) = NL \left\{ |f_{21}^N(\mathcal{J})|^2 + Z^2 |f_{21}^e(\mathcal{J})|^2 \right\} \\ \approx NL |f_{21}(\mathcal{J})|^2, \quad (7)$$

again neglecting the electron term. In a more convenient notation, one has

$$\frac{dN^{\text{diff}}}{dt}(t) = \pi NL \left| \frac{f_{21}^N(t)}{k} \right|^2 \quad (8)$$

Extrapolation to $t = 0$ (see Fig. 1) yields $dN/dt(0)$, and one can rewrite (6) in terms of observed quantities

$$\frac{1}{4\pi NL} \frac{N^{\text{tr}}}{(dN/dt(0))\Delta t} = 1 + 2 \operatorname{Re} \frac{Z f_{21}^e(0)}{f_{21}^N(0)} \quad (9)$$

or $R_{\text{obs}}/4\pi NL = 1 + \epsilon$;

where $R_{\text{obs}} = N^{\text{tr}}/\Delta t [dN^{\text{diff}}/dt(0)]$ is the well-known "Good ratio" (for a thin slab), and ϵ the quantity to be determined. Thus the experiment, in its idealized version, consists in measuring R_{obs} .

At this point it is interesting to estimate numerically

$$\epsilon = 2 \operatorname{Re} \frac{Z f_{21}^e(0)/k}{f_{21}^N(0)/k} \quad (10)$$

For Pb, $(f_e - \bar{f}_e)/k = 82 \times 3.65 \times 10^{-3} = 0.30$ mb according to Eqn.

(2,3) (note that this quantity is real and momentum-independent); on

the other hand, $(f^n - \bar{f}^n)/k$ has at 4 GeV a modulus of 32 mb, and a phase of -135° [3]. Thus

$$\epsilon(\text{Pb}, 4 \text{ GeV}) = -\frac{0.30 \sqrt{2}}{32} = -1.4\% \quad (11)$$

It is anticipated that (see our proposal for E-82) the nucleon amplitudes $(f-\bar{f})/k$ fall as $k^{-1/2}$, and we assume that the nuclear regeneration exhibits the same momentum dependence (this assumption was borne out by detailed optical model calculations). Thus ϵ is expected to grow as $k^{1/2}$, yielding typically

$$\epsilon(\text{Pb}, 100 \text{ GeV}) \approx 57\%,$$

a large effect.

3. The actual experimental method.

In the idealized experiment described in the previous section, it is the departure of the observed Good ratio R_{obs} from its "theoretical" value which yields ϵ . In actual practice, some of the simplifying assumptions used in the derivation above do not hold, i.e.

(a) the slab thickness L may not be small in terms of $\Lambda(k)$, the mean decay length of K_S^0 's;

(b) L may not be small in terms of $\mu = 1/N \sigma_T$; the mean free path for interactions in the regenerator;

(c) Instead of the regenerated K_S intensity, one has to consider the corresponding amplitude and CP interference effects downstream of the regenerator.

Of these, (a) is least relevant at our energies. $\Lambda(100 \text{ GeV})$ is 500 cm, far greater than L for any practical regenerator.

Assumption (b) is most serious. Aside from attenuation (which is the same for K_L and K_S !) multiple scattering effects arise in a "thick" regenerator, affecting the Good ratio drastically. Piccioni and his collaborators [4] have presented an analytic approximation for the "modified" Good ratio, while Foeth et al. [3] have treated the same problem by Monte

Carlo techniques. In either case, the result depends on the nuclear parameters σ_T , σ_{22} and $d\sigma_{21}/dt$, and these must be well-known to make an adequately accurate prediction of the Good ratio possible.

Corrections due to CP interference effects are comparatively easy to handle. In a sense they are complementary to the multiple scattering effects, since they are easiest to apply to "strong", i.e. thick, regenerators.

Note that in the case of the Aachen-CERN-Torino experiment [3], for which Eqn. (11) predicts a net effect $\epsilon \approx -1.4\%$, the various corrections to the "ideal" Good ratio amounted to about 30%.

Our approach is to decouple ourselves as much as possible from the dependence of R_∞ (the Good ratio calculated for the convergent series corresponding to infinitely many possible scattering in the regenerator) on the nuclear parameters in question. In other words, we seek that "magic length" L_0 for which R_∞ exhibits an extremum respect to these parameters.

It is convenient to measure the regenerator length in units of the interaction mean free path $\mu = 1/N \sigma_T$, i.e. $L/\mu = LN\sigma_T = x$. We find that the magic length is $x_0 \approx 2$. Note that since N_S , the intensity of coherently (transmission) regenerated K_S 's, goes (for $L/\lambda \ll 1$) as $x^2 e^{-x}$, $x_0 = 2$ also corresponds to the maximum possible K_S intensity (per incident K_L).

To obtain this result, we first consider R_∞ in the absence of CP-violating interference effects (for $L/\lambda \ll 1$) as given by Ref. 4.

$$R_0/R_\infty = \left[\sum_{n=1}^{\infty} \frac{(NL \sigma_{22})^n}{(n)!} G_n(G=0) \left(\frac{n}{|f_{22}(0)|^2} + (NL)^2 + \frac{n(n-1)}{|f_{22}(0)|^2} \right) - \frac{n NL \sigma_T}{|f_{22}(0)|^2} \right] / NL^2 \quad (12)$$

where $G_n(=0) = \frac{|f_{22}(0)|^2}{n \sigma_{22}}$, $\lambda = \frac{2\pi}{K}$, and $R_0 = 4\pi NL(1+\epsilon)$ is the Good ratio in the absence of multiple scattering and CP interference.

This formula can be simplified by noting that $f_{22} = (f + \bar{f})/2$ is essentially purely imaginary at the energies of interest here. Thus

$$\sigma_T = \frac{4\pi}{k} \text{Im } f_{22}(0) = \frac{4\pi}{k} f_{22}(0). \quad (13)$$

σ_{22} , the elastic scattering cross section, is given by $\int d\Omega |f_{22}|^2$, and it is useful to introduce the variable $\alpha \equiv \sigma_{22}/\sigma_T$. With these notations one has

$$R_0/R_\infty = \sum_{n=1}^{\infty} \frac{(\alpha x)^{n-1}}{(n-1)!} \left(1 - \frac{x}{2n}\right)^2 \quad (14)$$

Using the optical model and the measured charged kaon-nuclear total cross sections (extrapolated to 100 GeV) as input parameters, we have computed the relevant amplitudes and cross sections for Pb; our results are given in Table I. Since the charge averaged cross sections are, for $k > 20$ GeV/c, anticipated (at the 1% level) to be momentum independent, α is also expected to be so.

Fig. 2 shows a plot of R_∞/R_0 as a function of x , calculated with $\alpha = 0.319$ (see Table I). Since α is k -independent, this curve is "universal". With $\mu = 13.1$ cm, L_0 is 26.2 cm and $\ell \equiv L/\Lambda = 26.2/500 = 0.052$ for $k = 100$ GeV/c. As $\ell \ll 1$, the thin slab approximation is valid, and excludes any additional momentum dependence of L_0 .

The prime question of interest is the sensitivity of R_∞ to the nuclear parameters σ_T and σ_{22} for $x = x_0$, since these are not too well known a priori. We find that R_∞ changes by 2.0% for a 1% change in σ_{22}

and by 0.5% for a 1% change in σ_T . Before accepting the regenerator of "magic thickness", we have to remove the last simplifying assumption made above, i.e. we have to allow for CP interference effects downstream of the regenerator. As is well-known, interference between transmission regenerated K_S 's and K_L decays yields the 2π -distribution:

$$I_{2\pi}^{\text{trans}} = C |\rho|^2 \left\{ e^{-t/\tau_S} + |\eta_{+-}/\rho|^2 e^{-t/\tau_L} + 2|\eta_{+-}/\rho| e^{-t/2\tau_S} \cos(\Delta t - \phi) \right\} \quad (15)$$

which can be fitted to yield $|\rho/\eta_{+-}|$. ($\phi = \phi_\rho - \phi_{+-} \cong 90^\circ$) Similarly, the diffraction regenerated component has a distribution:

$$\frac{dI_{22}^{\text{diff}}(t)/dt}{(C|\rho|^2/R_0)} = \sum_{n=1}^{\infty} \frac{(\alpha x)^{n-1}}{(n-1)!} \left\{ e^{-t/\tau_S} \left(\frac{x}{2n} - 1 \right)^2 + |\eta_{+-}/\rho|^2 \frac{1}{n^2} e^{-t/\tau_L} + 2 \left(\frac{x}{2n} - 1 \right) |\eta_{+-}/\rho| \frac{1}{n} e^{-t/2\tau_S} \cos(\Delta t - \phi') \right\}$$

where $\rho = f_{21}(0)/f_{22}(0)$ and $\phi' = \phi_{21} - \phi_{22} - \phi_{+-} = 90^\circ$. Interestingly enough $\rho' = \rho$ for the "magic" thickness $x_0 = 2$! Integration over m K_S proper lifetimes yields (with $m \tau_S \ll \tau_L$)

$$\frac{R_\infty}{R_0} = \frac{1 - e^{-m} + |\eta_{+-}/\rho|^2 m + 2 |\eta_{+-}/\rho| \left[1 - e^{-m/2} (\sin \frac{m}{2} + \cos \frac{m}{2}) \right]}{\sum_{n=1}^{\infty} \frac{(\alpha x)^{n-1}}{(n-1)!} \left\{ \left(\frac{x}{2n} - 1 \right)^2 (1 - e^{-m}) + m |\eta_{+-}/\rho|^2 \frac{1}{n^2} - 2 \left(\frac{x}{2n} - 1 \right) \frac{|\eta_{+-}/\rho|}{n} \left[1 - e^{-m/2} (\sin \frac{m}{2} + \cos \frac{m}{2}) \right] \right\}}$$

As seen in Fig. 3, if we integrate over a fixed distance in the lab, R_∞/R_0 is practically momentum independent.

Thus, including CP-effects, $x_0 \approx 2$ remains that "magic" slab thickness for which the predicted Good ratio R_∞ is particularly insensitive to nuclear parameters. We shall describe the requisite ancillary measurements of the latter in the next section.

There is one effect which we have not mentioned so far. In addition to K_S regeneration by transmission and by elastic diffraction, also inelastic regeneration can occur, i.e. the process $K_L + \text{nucleus} \rightarrow K_S + \text{nucleus} + \text{hadrons}$ (or $\rightarrow \text{nuclear fragments} + \text{hadrons}$). This process which is due to the inelastic scattering of $K^0, \bar{K}^0$ on nucleons, has a vastly different t -dependence from that of ordinary diffraction regeneration [$\sigma_{21}(t)$ is

governed by the nuclear size!] but nevertheless constitutes a background to the latter. Thus $dN^{\text{diff}}/dt(0)$ is "contaminated" by inelastic regeneration. This "contamination" is hard to calculate theoretically, but could amount to ~10% [5]. Wanting to determine the Good ratio to 1%, it will hence probably be sufficient to determine this background experimentally to 10% accuracy; such a determination can be done (a) by exploiting the different t -dependence, (b) by suitable co-incidence or anticoincidence requirements [5] involving the unwanted secondaries.

It should also be pointed out that the transmission and diffraction regenerated events should have the same momentum (k) distribution (to the extent that the Good ratio is momentum independent), thus providing an easy check for the contamination in question.

4. Ancillary Measurements

a) Measurement of σ_T

The attenuation of K_L mesons is easily measured by standard "good geometry" transmission techniques, i.e. by inserting material in the K_S beam far upstream of the usual regenerator position. The transmitted K_L 's can be detected either via their copious K_{L3} modes, or by using a thick regenerator as a "converter" (to $K_S \rightarrow 2\pi$), or even via the rare $K_L \rightarrow \pi^+ \pi^-$ mode. In either case, 1% accuracy in σ_T is easily obtained. The "magic thickness" of the regenerator actually to be used in the experiment proper is also readily checked by transmission.

b) Measurement of σ_{22}

We recall that $\sigma_{22} = \int d\Omega |f_{22}(\Omega)|^2$, and that $f_{22}(0)$ is essentially pure imaginary. Thus $|f_{22}(0)|$ is given directly by σ_T , while the angular (or t) dependence is most easily obtained from a measurement of σ_{21} and an appeal to the optical model. Table II, based on detailed

calculations, shows that while σ_{21} and the differential cross section $d\sigma_{22}/dt$ vary with the nuclear shape parameters, ~~very with the nuclear shape parameters,~~ the integral σ_{22} is very insensitive to the latter (which are, in the case of Pb, already rather well known^[5]). Thus a measurement of σ_{21} basically constitutes a determination of σ_{22} .

There are also direct methods for determining σ_{22} , e.g. studying the diffraction of K_L 's (detected via the K_{L3} mode) or of K_S 's from a short-lived beam (such as built for E-8).

(c) Measurement of σ_{21}

This is readily done by measuring the diffraction regeneration $d\sigma_{21}/dt$ from a moderately thin regenerator (to keep multiple scattering effects down).

(d) Measurement of inelastic regeneration.

This can be done in a number of ways. The simplest is to see (with a moderately thin regenerator) how the minima of the diffraction regeneration distribution ($\sim d\sigma_{21}/dt$) get "filled in" by the inelastic events, and in fact to follow that distribution out to large t 's where the elastic events should practically vanish. A more direct way is to trigger only on inelastic events, requiring a charged secondary from the regenerator in coincidence with the usual 2π -trigger. Finally, one can study regeneration free of inelastic events by a suitable anti-coincidence requirement. This requires a special run at very low neutral beam rates, since the anticoincidence would otherwise be constantly "on".

5. Organization of experiment and running times.

Since we expect a 7% effect, ϵ , at 100 GeV, we set ourselves the

goal to measure R_{obs} to 1% at this energy and to be able to predict R_{∞} to 1.5%, yielding a combined error of $\sim 1.8\%$ in ϵ . This prediction of R_{∞} requires a determination of σ_T to 0.5% and of σ_{22} to 0.75%.

Of course, 100 GeV is just an arbitrarily chosen reference energy, since all $K_L > 30$ GeV are studied simultaneously. Fig. 4 shows the K_L spectrum predicted for our beam line, as well as the number of K_S 's regenerated by a Pb block of "magic" thickness $L_0 \approx 26$ cm and accepted by our spectrometer.

Assume that we require 10^5 regenerated 100 GeV events to measure R_{obs} to 1% (Note that in Ref. 3 80K events were collected to determine R_{obs} to 1%). At the time of writing, 0.5×10^{12} interacting protons/burst appear to be a reasonable estimate. This yields ~ 1 regenerated and detected $K_S \pi^+ \pi^-$ events at $(100 \pm 10 \text{ GeV})/\text{burst}$, or $\sim 18\text{K}$ such events/day. Thus, we could, in principle, collect the main body of data in ~ 5 days (15 shifts) of running, achieving our stated statistical goals at 100 GeV and exceeding them at lower momenta. In practice, we may need more time. Under the running conditions stated above, we predict a total rate of detected 2π -events of $\sim 26/\text{burst}$. Whether we can handle such a large rate, depends on the number of unwanted triggers (neutron stars, unrejected leptonic decays, etc.), the length of the spill, the time-structure of the beam etc. It is clear that with a core-memory wire-spark chamber spectrometer such as currently operated by us and as described in our proposal (E-82), and a spill of 300 msec, one would be limited to ≤ 50 total events/burst, corresponding to perhaps 5 "good" events/burst. Thus one would have to run with a 5 times lower beam intensity (10 interacting protons/burst) for 25 days (75 shifts).

The ancillary measurements discussed in Section 4 would require, even with such a reduced beam, at most another 5 days (15 shifts).

Since our proposal E-82 was submitted (August, 1970), we have greatly improved our spectrometer. Hodoscopes and MWPC's have been added to act as roadmaking devices, and an elaborate on-line processing system (involving two Supernovas in addition to our standard processor, the ASI-6040), has been assembled. This is however not all: in view of the present (June 1973) status of the structure and length of the beam spill at NAL, we are currently building a full complement of MWPC's sufficient to replace all the wire-spark chambers in our initially proposed setup. These MWPC's will have been built and tested by the fall of 1973, and they should certainly be operational well before the experiment proposed here could be scheduled. The construction of these chambers and of the associated electronics is being greatly helped by the expertise gained by our supporting staff in connection with other NAL projects (e.g. E-98, Mo-p scattering).

References

- [1] Kroll, Lee, and Zumino, P. R. 157, 1376 (1967).
- [2] Y. B. Zel'dovich, JETP 36(9), 984 (1959).
- [3] Foeth et al., Phys. Letters, 30B, 276 (1969).
- [4] R. M. Good et al., Phys. Rev. 124, 1223 (1961).
- [5] Foeth et al., Phys. Letters, 31B, 544 (1970)

TABLE I. Optical Model Calculations for Pb

<u>Inputs:</u>	$\sigma_T(K^+p) = 18.2 \text{ mb}^{[a]}$	$\sigma_T(K^+n) = 18.7 \text{ mb}^{[a]}$	55 GeV/c
	$\sigma_T(K^-p) = 20.5 \text{ mb}^{[b]}$	$\sigma_T(K^-n) = 19.5 \text{ mb}^{[b]}$	
	$f(K^-N)$ assumed pure imaginary.		
	$f(K^+N)$ have real parts chosen to yield $\phi_{21} = -135^\circ$.		
	Extrapolation to 100 GeV:		
	$\sigma_T(K^+p) = 18.5 \text{ mb}$	$\sigma_T(K^+n) = 18.8 \text{ mb}$	
	$\sigma_T(K^-p) = 20.2 \text{ mb}$	$\sigma_T(K^-n) = 19.4 \text{ mb}$	
	Same assumptions for amplitudes.		
	Nuclear model: $\rho_p = \rho_n = C [1 + e^{(r - 6.6)/0.5}]^{-1}$ (r in fm.)		
<u>Output:</u>	55 GeV/c: $(f - \bar{f})/k = 9.98 \text{ mb}$	$\sigma_T = 2329 \text{ mb}^{[c]}$	
	$\sigma_{22} = 742 \text{ mb}$	$\sigma_{21} = 0.631 \text{ mb}$	
	100 GeV/c: $(f - \bar{f})/k = 7.41 \text{ mb}$	$\sigma_T = 2329 \text{ mb}$	
	$\sigma_{22} = 742 \text{ mb}$	$\sigma_{21} = 0.348 \text{ mb}$	
	$\mu = 1/N\sigma_T = 13.1 \text{ cm}$	$\alpha = \frac{\sigma_{22}}{\sigma_T} = 0.319$	

[a] Phys. Lett. 36B, 415 (1971).

[b] Denisov et al., contributed paper No. 924 at the 1972 NAL Conference.

[c] At 9 GeV, Lakin et al. measure $\sigma_T = 2307 (55) \text{ mb}$.

TABLE II. Dependence of Optical Model Results for Pb on Nuclear Parameters

(Input at 55 GeV/c as in Table I)

Nuclear Density Distribution $\rho_{p,n} = C \left[1 + e^{(r-r_{p,n})/a_{p,n}} \right]^{-1}$

r_p (fm)	r_n (fm)	a_p (fm)	a_n (fm)	σ_T (mb)	σ_{22} (mb)	σ_{21} (mb)	t_1^* (GeV) ²	t_2^* (GeV) ²
6.60	6.60	0.50	0.05	2329	742	0.631	-.0098	-.0378
6.60	6.60	0.50	0.68	2464	747	0.761	-.0092	-.0354
6.60	7.29	0.50	0.50	2378	736	0.678	-.0090	-.0333

* t_1, t_2 are the positions of the first and second diffraction minima in $d\sigma_{22}/dt$

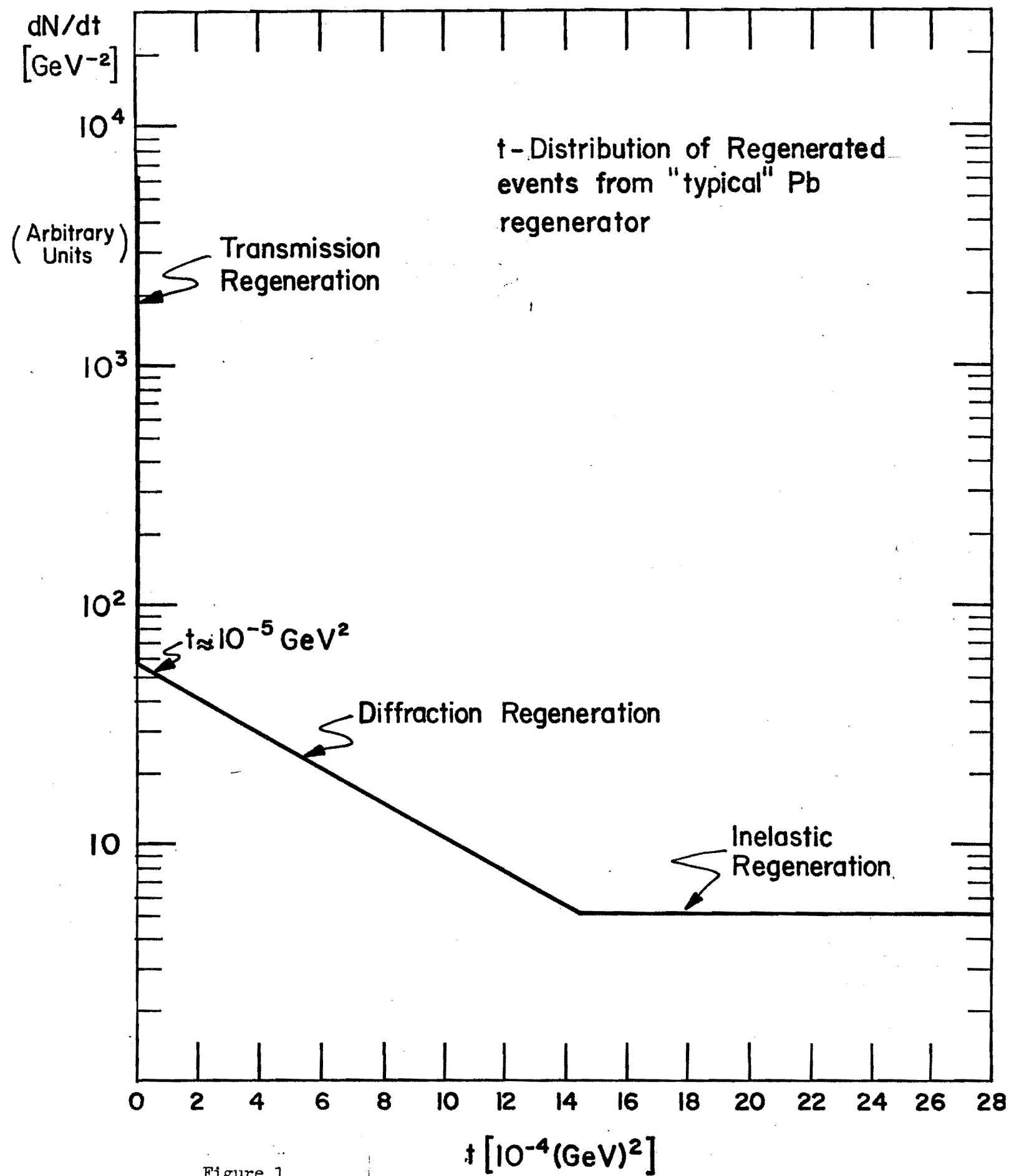
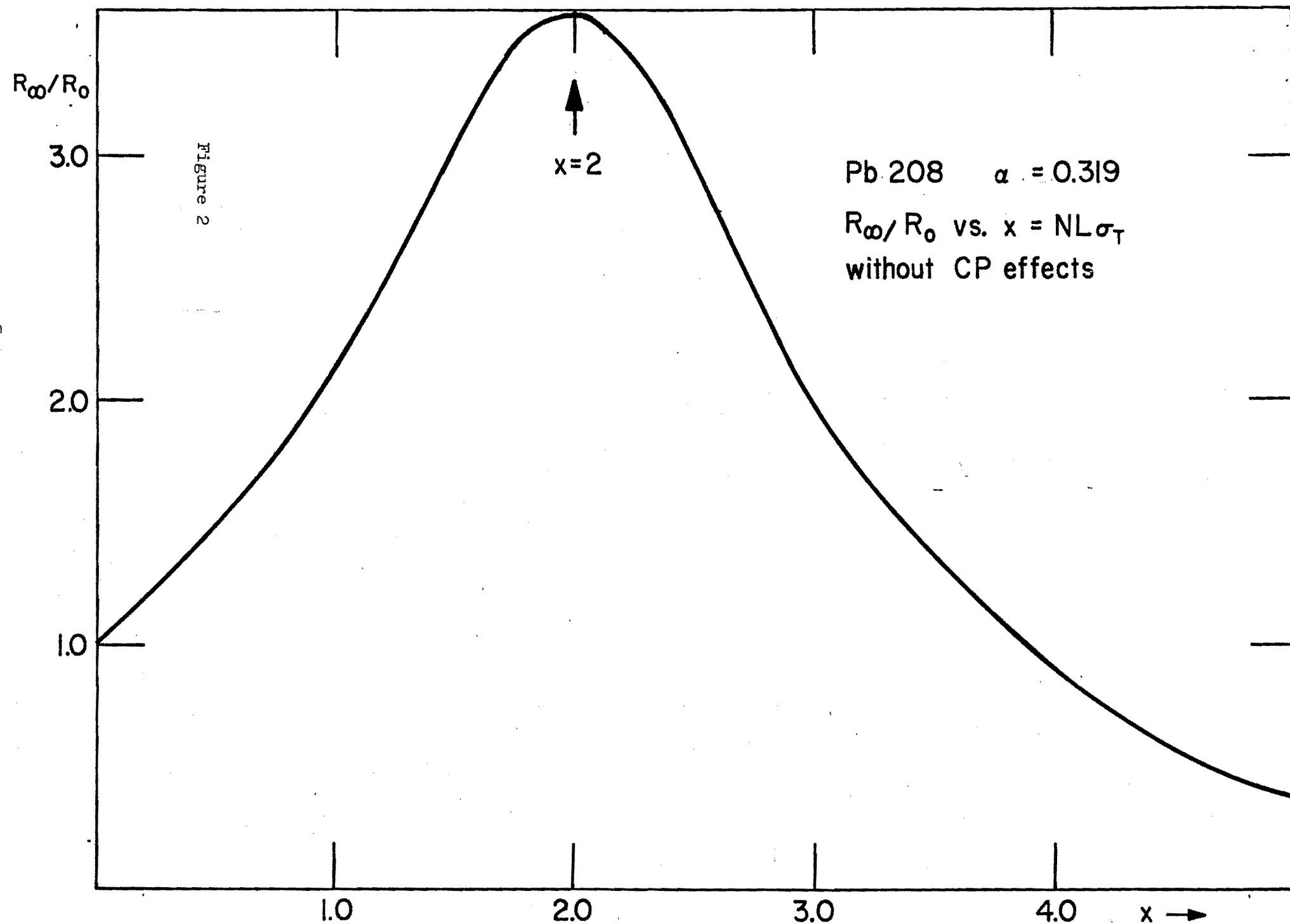
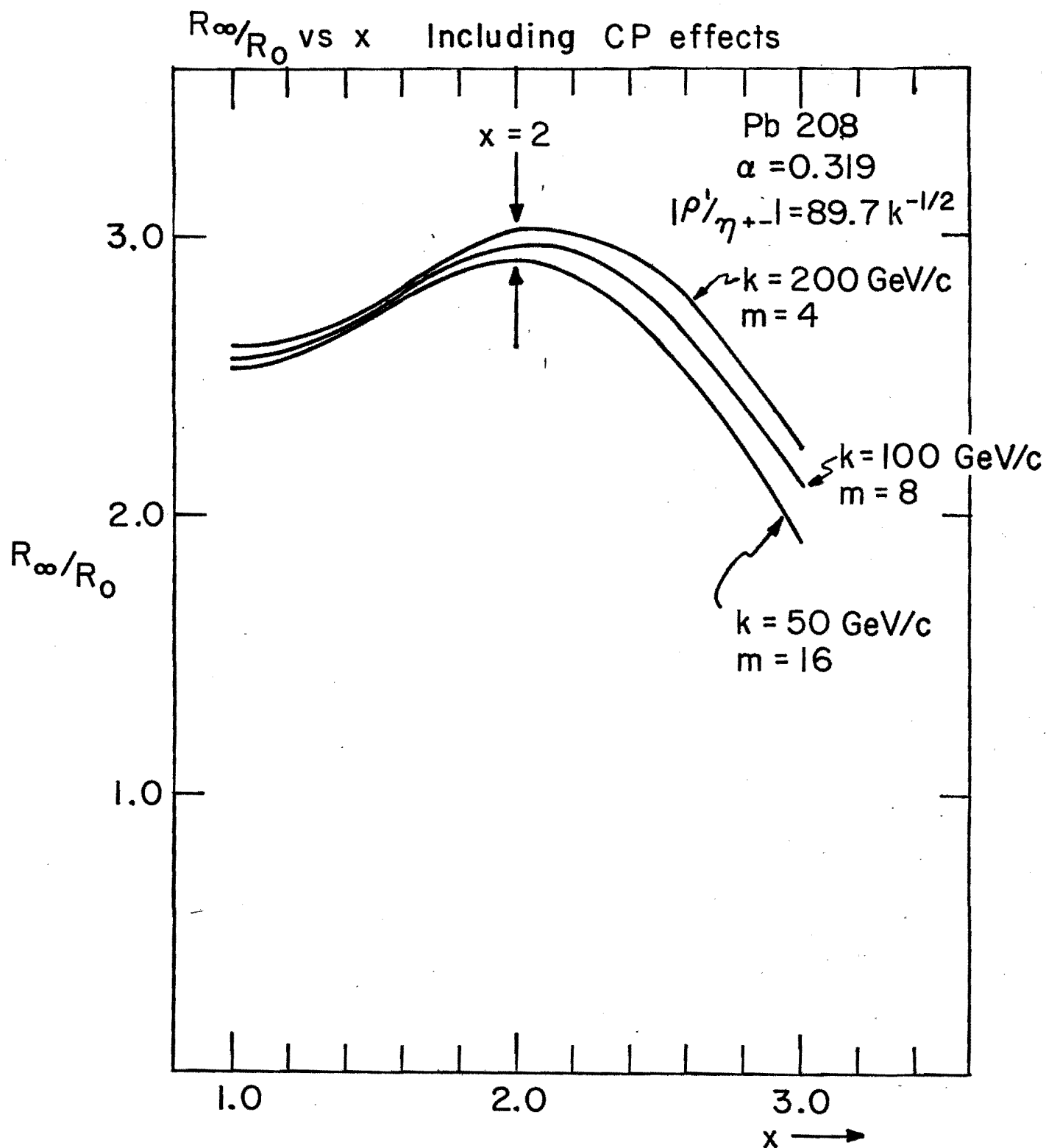


Figure 1





Note: We integrate over fixed $z \simeq 40$ meters; thus

$$m \simeq \frac{40 \times 10^2 \text{ cm}}{k/m_k c \tau_S} = \frac{800}{k} \text{ where } k \text{ is in GeV/c}$$

Figure 3

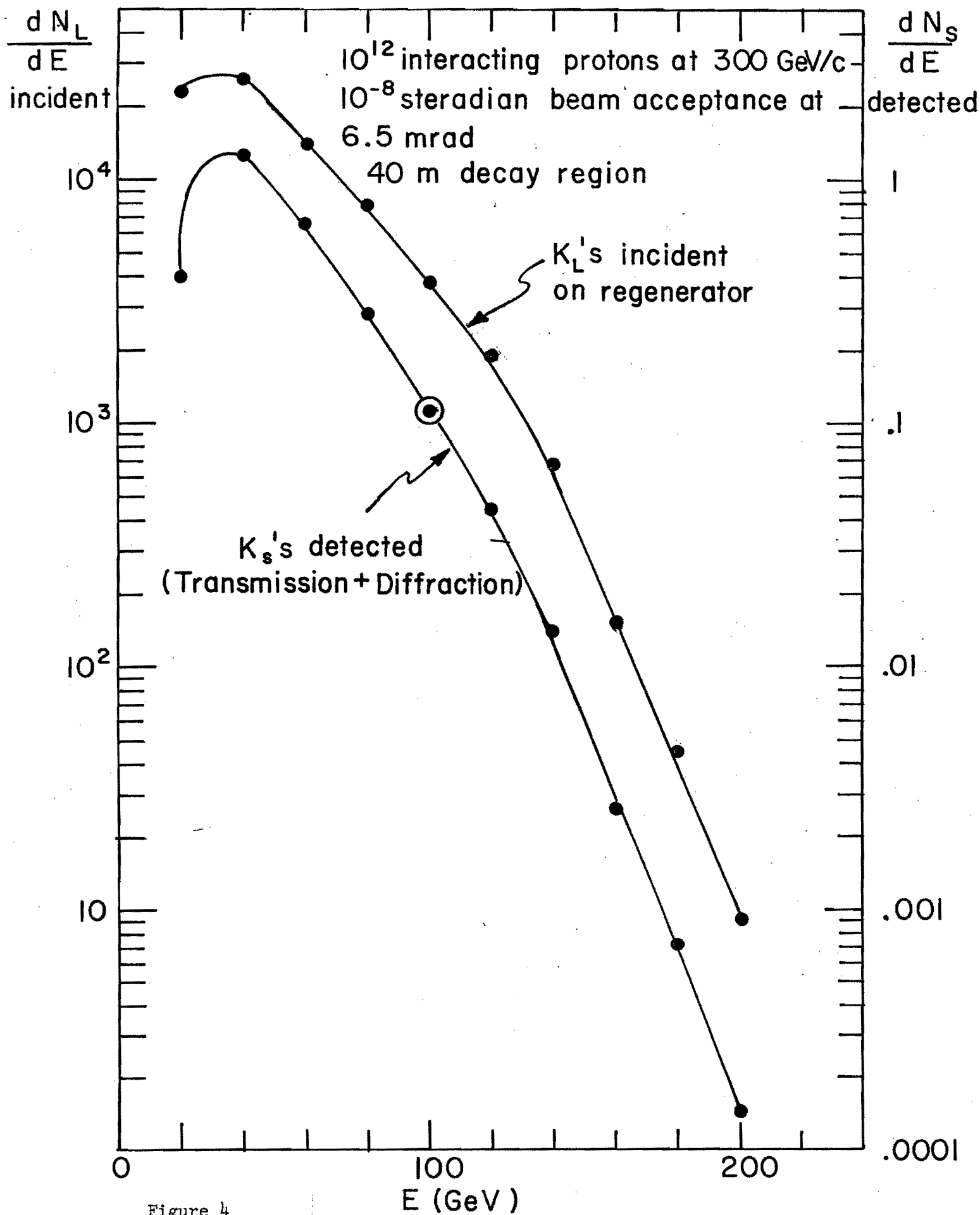


Figure 4

SEARCH FOR COHERENT REGENERATION FROM ELECTRONS THE K_0 CHARGE RADIUS

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Received 8 September 1969

We have searched for coherent interactions of a long-lived neutral kaon with the electrons of lead and copper plates. Results are compatible with the absence of such effects. In terms of the kaon r.m.s. charge radius we find $\sqrt{\langle R^2 \rangle} = 0.22^{+0.20}_{-0.22}$ fm.

The transformation of a long-lived neutral kaon in interactions with matter into a coherent superposition of K_L and K_S states is usually attributed to differences in the nuclear interactions of their linear combinations of definite strangeness, K_0 and $\bar{K}_0$ [1]. Interactions with electrons could also contribute appreciably, since it is possible [2] for electromagnetic interactions to have opposite signs for K_0 and $\bar{K}_0$. In the K_L - K_S representation such an interaction does not have diagonal elements by virtue of the true neutrality of K_L and K_S states. Non-diagonal elements, capable of producing K_L - K_S transitions are, however, possible [3]. In particular for an incident K_L , the outgoing scattered wave will be a pure K_S current.

This electromagnetic K_0 interaction (linear in the electromagnetic field) is proportional [2] to $\text{div}(E)$, that is to the charge density at the point of the K_0 ($\bar{K}_0$) state. Hence if a neutral K meson passes through a region of field, in absence of charges there will be no K_L - K_S transitions.

The electromagnetic interaction is expected to be the main form of interaction between electrons and neutral kaons. For nucleons, since K_0 and $\bar{K}_0$ interact strongly, these effects are probably only very small corrections.

Let us consider a K_L wave traversing a block

of matter (the regenerator). Interactions with electrons can produce both single regenerative scatterings and coherent regeneration effects. Single scattering events are sharply peaked forwards, the maximum scattering angle θ_{max} being smaller than $(m_e/m_K) = 10^{-3}$ radians. They cannot be resolved from the forward direction with our present detector. Furthermore, even in the most optimistic assumptions [3] the electromagnetic scattering cross-section is negligibly small when compared to the corresponding nuclear regenerative process. However, in the exact forward direction, where the coherent transmission regeneration dominates, this interaction leads to effects linear in the scattering amplitude from electrons, because of interference with the nuclear amplitude. Zel'dovich [3] and more recently Placchi and Zavattini [4] have remarked that this effect could be substantially large.

The differential cross-section for K_0 electron scattering in the centre-of-mass system [2] is

$$\frac{d\sigma}{d\Omega}(\theta)|_{\text{c.m.}} = \frac{e^4 \lambda^2}{8\pi^2} \frac{1}{(E_e + E_K)^2} \times \\ \times [2k^2 E_e E_K + 2E_K^2 E_e^2 - m_K^2 (E_K^2 - m_K^2) + \\ + \cos \theta (k^4 + k^2 E_K^2 + 2k^2 E_e E_K)] \quad (1)$$

where k is the centre-of-mass momentum, E_e, E_K are the total energies, m_K is the K_0 mass, λ is the K_0 form factor:

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$$F(q^2) = 0 + \lambda q^2 + \text{other higher order terms.}$$

The parameter λ can be related to the r.m.s. charge radius of the K_0 :

$$\lambda = -\frac{1}{6} \langle R^2 \rangle.$$

Adding $\hbar$ and c whenever necessary and transforming formula (1) in the laboratory system, we get the differential cross-section in the forward direction and zero momentum transfer:

$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{lab}} = 4\alpha^2 \lambda^2 \left(\frac{m_K c}{\hbar} \right)^2 \gamma_K^2 \quad (2)$$

where γ_K is the usual relativistic term for the incident K meson. The forward regeneration amplitude is real and has the simple form

$$f(0) - \bar{f}(0) = 4\alpha \lambda \left(\frac{m_K c}{\hbar} \right) \gamma_K \quad (3)$$

For an atom the coherent amplitude will be Z times formula (3). Since the nuclear forward regeneration amplitude varies approximately like $A^{1/3}$, the effects of electron regeneration are expected to be larger for heavier nuclei.

Following Zel'dovich [3], we have searched for electron coherent regeneration comparing the angular distribution of the nuclear regenerative scattering and the transmission regeneration after thin copper and lead plates. It has been shown by Good [1] that the K_S transmission regenerated intensity is proportional to the differential intensity for regenerative scattering in the forward direction:

$$Z^2 = \frac{dV}{d\Omega} (0) \frac{N \Lambda \lambda^2}{\delta^2 + \frac{1}{4}} \frac{1 + \exp(-l) - 2 \exp(-\frac{1}{2}l) \cos(\delta l)}{1 - \exp(-l)} \quad (4)$$

where $l = d/\Lambda$ is the regenerator thickness in units of K_S decay length Λ , δ is the K_L - K_S mass difference in lifetime units, N is the number of atoms per unit of volume and λ is the kaon wavelength.

Therefore it is possible to compute the transmission regeneration due solely to nuclear collisions provided that the corresponding differential cross-section is known in the forward direction. Scattering events on electrons are confined to angles smaller than θ_{max} . Outside this very narrow forward cone all the observed regenerative cross-section must be of nuclear origin. Since the angular distribution of the latter is very much wider, it is easy to bridge the gap between θ_{max} and the exact forward direction extrapolating to zero angle the quantity $(dV/d\Omega)(\theta)$, (observed at angles larger than θ_{max}). Let $|\rho_{\text{exp}}|^2$ and $|\rho_{\text{extrap}}|^2$ be, respectively, the observed and extrapolated transmis-

sion regeneration intensities. Since the observed regeneration amplitude is the sum of the amplitudes due to electrons and nucleons, it is easy to prove that for small contributions from electrons:

$$Z = 1 - \frac{|\rho_{\text{extrap}}|^2}{|\rho_{\text{exp}}|^2} \approx 2 \operatorname{Re} \frac{[f(0) - \bar{f}(0)]_{\text{electrons}}}{[f(0) - \bar{f}(0)]_{\text{exp}}}$$

where $[f(0) - \bar{f}(0)]_{\text{exp}}$ is the experimentally observed regeneration amplitude. This quantity has been extensively studied and its modulus and phase are well known [5, 6]. Hence the quantity Z can be directly related to $\langle R^2 \rangle$, the r.m.s. charge radius of the K_0 .

It is relevant to remark that the regenerator is electrically neutral. One might think that since electromagnetic interactions with protons are of opposite sign, they cancel the electron interactions. Actually electromagnetic effects due to protons are indistinguishable from nuclear effects. Since the nuclear amplitude is taken from experiment, it includes also all electromagnetic corrections. Therefore, if there were electromagnetic interactions between K_0 and protons but not with electrons, $Z = 0$.

The experiment has been performed in a long-lived K_L beam derived at 140 mrad from an external target of the CERN Proton Synchrotron.

Momenta of charged decay particles are measured with a magnetic spectrometer and wire chambers. The triggering logic requires one charged particle at each side of the beam in the front and after the magnet, and a veto condition in the anticounter at the exit face of the regenerator plate. Electrons and muons are identified by a gas Cerenkov counter and by penetration in a 80 cm thick iron shield. For a more complete description of the experimental set-up we refer to refs. 6 and 7.

The basic philosophy of the experiment is to compare the transmission regeneration intensity and the extrapolation to forward direction of the elastic regenerative scattering cross-section. Regeneration is identified by looking at $\pi^+\pi^-$ decays (mostly $K_S \rightarrow \pi^+\pi^-$) occurring immediately after the regenerator plate. The scattering angle is determined from the reconstructed momentum of the $\pi^+\pi^-$ pair and the (a priori) known direction of flight of the incident K_L beam.

Two important effects have to be accounted for:

i) The CP violating decay $K_L \rightarrow \pi^+\pi^-$. This decay mode interferes with $K_S \rightarrow \pi^+\pi^-$ in the transmission regeneration and, to a smaller extent, in the regenerative scattering. The correction to

the transmission regeneration has been directly measured. The idea we follow is that regenerators of same material, of equal lengths but of different densities have a regeneration amplitude proportional to density (i.e., they have the same regeneration phase). Let d be the relative density of the regenerator plate. Then the number of 2π decays over a given decay volume is of the form:

$$N(d) = Ad^2 + Bd + N(0), \quad (6)$$

where the first term is due to $K_S \rightarrow \pi^+\pi^-$, the last one to $K_L \rightarrow \pi^+\pi^-$ and the second one to interference between the two decays. The quantity A , the number of 2π decays corrected for CP violating effects for unit relative density ($d=1$) is easily obtained fitting the expression (6) with data at different densities.

The correction due to CP violating effects in the scattered events is much smaller since both K_L and K_S amplitudes now have to be scattered. It amounts in the present experiment to about 2%, computed from the known relevant parameters.

ii) Multiple scattering in the scattered events. As well known, the main effect consists of destructive interference between regenerative scattering and coherent transmission regeneration. An analytic formula for the correction factor has been given by Good et al. [8] with the as-

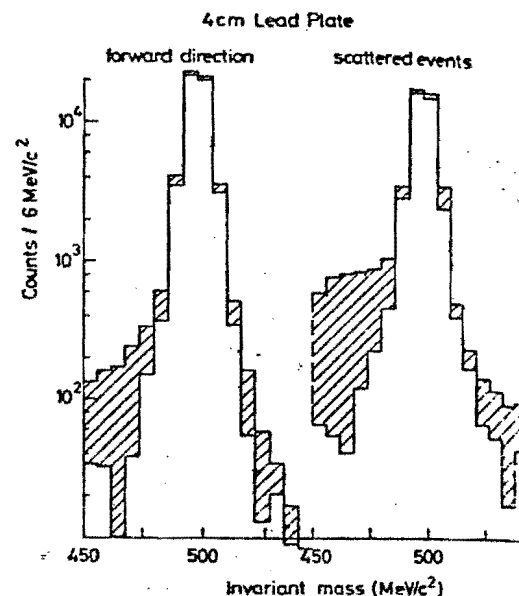


Fig. 1. Invariant mass distribution of transmission and scattered events. Dashed regions indicate background subtractions.

sumption that the K_0 and $\bar{K}_0$ scattering amplitudes have the same Gaussian angular distribution and that their sum is close to being imaginary. We have made an exact calculation by Monte Carlo

Table 1
List of experimental conditions and of correction factors.

Regenerator element	Length (cm)	Average density (g/cm ³)	Number of observed $\pi^+\pi^-$ decays		Inelastic events contamination to $\frac{d\sigma}{d\Omega}(0)$	Multiple scatterings contribution to $\frac{d\sigma}{d\Omega}(0)$	Remarks
			Transmission regeneration	Regeneration scattering			
Lead 1	4.007	11.323	35999	43761	0.0259 ± 0.0011	0.190 ± 0.008	Main regenerator
Lead 2	4.000	5.645	2169	4449	0.0166 ± 0.0025	0.09 ± 0.007	to correct for $K_L \rightarrow \pi^+\pi^-$ decays, transmission regeneration
Lead 3	4.000	2.830	1014	2814	0.0087 ± 0.0023	0.057 ± 0.006	
Lead 4	3.007	11.179	6718	4121	0.0443 ± 0.0045	0.371 ± 0.010	
Lead 5	12.027	11.319	17638	6816	0.0716 ± 0.0054	0.493 ± 0.014	to correct for multiple scattering in regenerative scattering
Copper 1	3.000	8.890	12399	17206	0.0208 ± 0.0018	0.160 ± 0.007	Main regenerator
Copper 2	3.000	4.456	1077	2363	0.0109 ± 0.0038	0.034 ± 0.007	to correct for $K_L \rightarrow \pi^+\pi^-$ decays, transmission regeneration
Copper 3	3.000	2.944	583	1566	0.0099 ± 0.0048	0.055 ± 0.006	
Copper 4	6.002	8.888	3453	2465	0.0060 ± 0.0038	0.301 ± 0.009	
Copper 5	8.904	8.876	3619	1795	0.0379 ± 0.0083	0.417 ± 0.010	to correct for multiple scattering in regenerative scattering
Copper 6	11.945	8.888	3709	1329	0.0322 ± 0.0107	0.529 ± 0.130	

states propagating an incident K_L state with complete scattering and regeneration matrices.

The relevant nuclear parameters have been determined with the help of experimental data for thicker regenerators where these corrections are much more important and with the observed angular distributions of the scattered events [9].

A list of experimental conditions is summarized in table 1. The thickness of the main regenerators has been chosen as a compromise between multiple scattering and CP -violation corrections. The reduced density regenerators are used to make the CP -violation corrections with the help of formula (6). They consist of several closely-spaced thin plates.

Thicker regenerators are used in order to determine the multiple scattering correction. Finally some data have been taken with no regenerator in the beam for background subtractions and measurement of the quantity $N(0)$ of formula (6).

Several cuts have been applied to the data in order to obtain a sample of 2π events practically free of any background:

- i) The kaon momentum is restricted to $2.5 \text{ GeV}/c < p_K < 6.5 \text{ GeV}/c$, to minimize errors in the averaging of momentum dependent parameters. Only 9% of the events are rejected.
- ii) Events with an identified decay electron (30%) or a muon (15%) are removed.

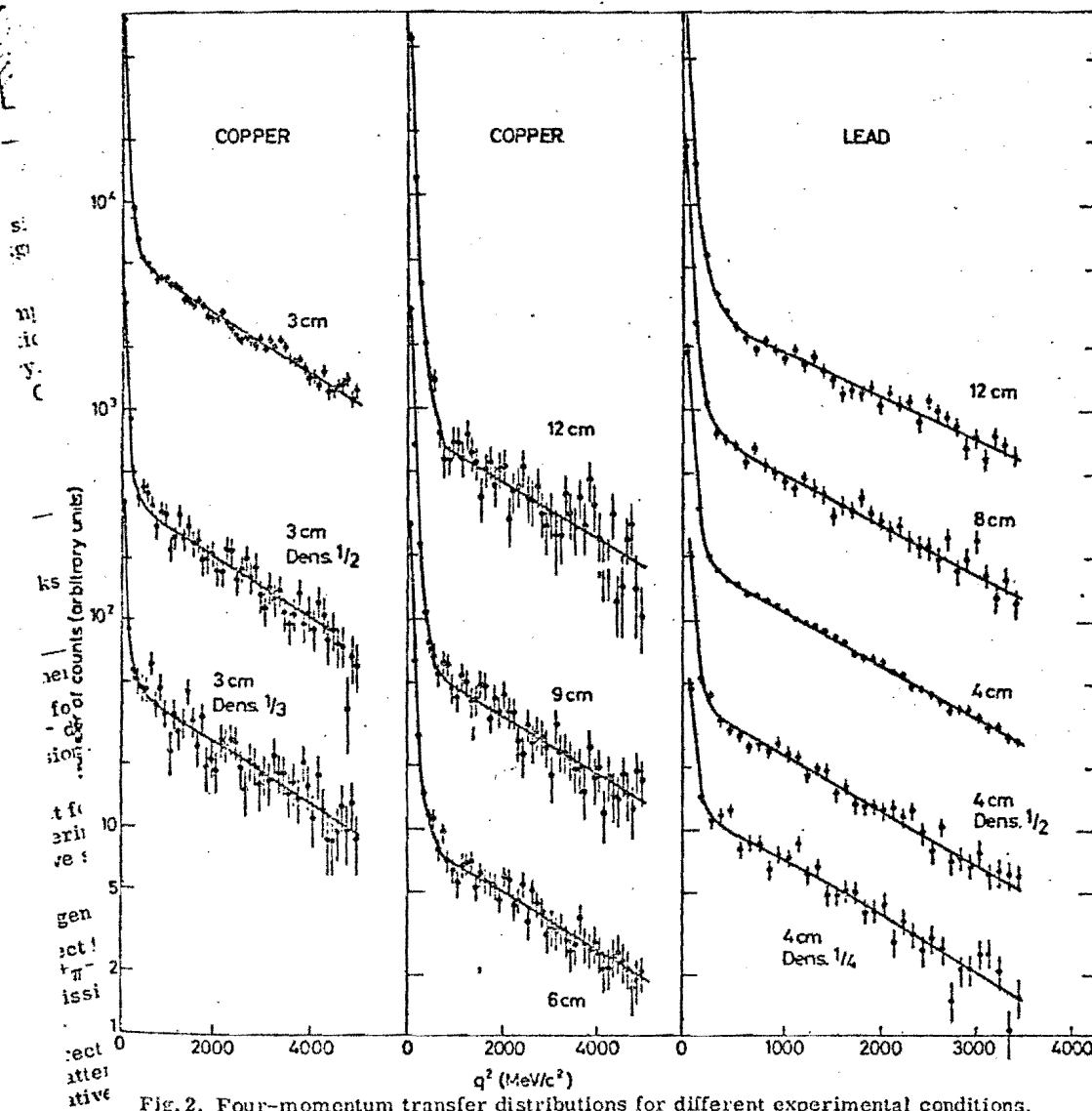


Fig. 2. Four-momentum transfer distributions for different experimental conditions.

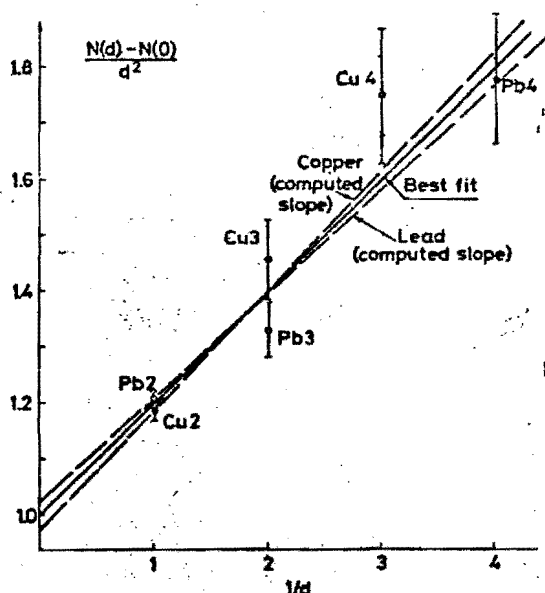


Fig. 3. Determination of the correction due to CP-violation effects in the transmission regeneration, according to formula (6). Dashed lines show the correction factor as determined from the time dependence of the 2π decays. The quantity d is the average regenerator density normalized to full density.

iii) Fiducial volume cuts are applied for the decay vertex as well as for the two outgoing particles in the chambers, magnet gap and trigger counters. Furthermore, only events which decay between 0.1 and 2.0 Kg lifetimes from the downstream edge of the regenerator are retained. The lower bound is to avoid the sudden drop in the detection efficiency at the veto counter and the upper one is a compromise between event rate and corrections due to CP violation.

The final event sample, after subtraction of backgrounds measured without a regenerator, is exceedingly clean. This is apparent in fig. 1, where the invariant mass distribution for transmission regenerated and regenerative scatterings are shown. The only background correction which still needs to be done is for inelastic regenerative scattering events. These events can easily be subtracted out, since they have a flat angular distribution extending to much larger angles than the elastic ones. This angular distribution has been experimentally determined in a separate measurement in which the veto counter in front of the regenerator has been switched into a coincidence requirement [9]. The contamination of inelastic events is finally extracted from each set of data from the large angle behaviour. Correction factors are listed in table 1.

Four-momentum transfer distribution for

Table 2
Table of results for $\frac{|\rho_{\text{exp}}|^2}{|\rho_{\text{extrap}}|^2}$

	3-cm copper plate	4-cm lead plate
Measured initial value	1.366 ± 0.024	1.492 ± 0.017
Correction for CP-violation effects in the forward direction		
a) from formula (6)	0.813 ± 0.026	0.847 ± 0.022
b) from the time dependence of 2π decay intensity	0.787 ± 0.031	0.855 ± 0.022
c) combined value of a) and b)	0.807 ± 0.020	0.851 ± 0.015
Correction for CP-violation effects in the scattering events	0.979 ± 0.003	0.979 ± 0.003
Multiple scattering correction	0.870 ± 0.017	0.805 ± 0.008
Subtraction of inelastic events	1.021 ± 0.002	1.026 ± 0.001
Final results [a) and b) combined]	0.962 ± 0.036	1.026 ± 0.023

various regenerators are shown in fig. 2. In the extrapolation procedure, four-momentum transfers are limited to $0.035 (\text{GeV}/c)^2$ for lead and $0.060 (\text{GeV}/c)^2$ for copper. Widely different upper cuts can be used without affecting the results appreciably. Fig. 3 shows the extrapolation procedure according to formula (6). The slope parameter B can either be fitted from the data in fig. 3 or computed from the time dependence of the 2π rate and the known CP-violating parameters. The two independent procedures give consistent results (see table 2). The actual correction factors applied to the data are listed in table 2. The final result is compatible with absence of electron regeneration. In order to relate the results to an r.m.s. K_0 radius we have taken for the regeneration amplitude at our average momenta the measured values $|f(0) - \bar{f}(0)|/k = (31.2 \pm 1.0) \times 10^{-27} \text{ cm}^2$ for lead [9] and $(19.8 \pm 1.0) \times 10^{-27} \text{ cm}^2$ for copper [6], and the regeneration phases [5, 6, 9] $\arg[i\{f(0) - \bar{f}(0)\}] = -(43.8 \pm 7.2)^\circ$ and $-(44.7 \pm 4.6)^\circ$, respectively.

With the help of formulae (5) and (3) we get

$$\langle R^2 \rangle = -(0.5 \pm 1.3) \times 10^{-27} \text{ cm}^2$$

to be compared with the prediction of the Vector Dominance Model [10] $\langle R^2 \rangle = 0.76 \times 10^{-27} \text{ cm}^2$

higher estimates of Zel'dovich [3] gave the considerably larger value, $\langle R^2 \rangle = 21.0 \times 10^{-27} \text{ cm}^2$, in disagreement with our result.

We would like to thank Professors J. Bell, H. Faissner, L. A. Okun and J. Sandweiss for very stimulating discussions and remarks. The author group wishes to thank the Bundesministerium für Wissenschaft und Forschung for support.

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Addendum to Fermilab Proposal #226

November, 1974

Since submitting the original proposal, we have refined the original estimates of the requisite running times. The major change is that the actual K_L fluxes in the M4 beam line, as determined in E-82, are significantly lower than those anticipated from a Hagedorn-Ranft type calculation (see Fig. 1). In addition, we improperly calculated the running time by scaling the number of transmitted events of Foeth et al., but in actuality it is the diffracted events which dominate the statistical accuracy of the experiment. These changes have led us to rethink our approach to determining the effect of the K_S regeneration off electrons.

As outlined in our proposal, we have set ourselves the goal of measuring the observed "M.L. Good" ratio to 1% for kaon momenta of 100 ± 10 GeV/c. Since the ratio of transmission regenerated events to diffraction regenerated events in a small momentum transfer interval ($\Delta t \sim 200 \text{ MeV}^2/c^2$) is $\gg 1$, the statistical error in the "Good ratio" is determined by the number of diffraction regenerated events in this t interval. The minimum running time to collect a given number of such events occurs for $x=1$, i.e. a 1 interaction length regenerator. Fortunately, the dependence on the nuclear parameters σ_{22} and σ_{total} , as discussed in our proposal, is about the same as for the originally proposed regenerator, $x=2$. (This is because the increased dependence on x is compensated for by a decreased dependence on α - See Equation 14 of Proposal #226.)

Even with this rate optimization, the loss of flux leads to prohibitively long running times in M4, the 7.5 mrad neutral beam. It is, however, now experimentally known that in M3 (1 mrad beam) a much larger K_L flux is available, particularly around 100 GeV/c. Using the M3 beam, we propose to attain the goals of our proposal in 1600 hours. (We assume the use of 50cm of Al to decrease the n/K ratio in M3. This yields a factor of 10 more K_L flux at 100 GeV/c than presently available in M4, if we use the results of Longo et al.) An alternative scheme, which we have not yet fully explored, is to increase the solid angle acceptance of the M4 beam line.

The critical reader will now raise the point whether our spectrometer can stand the increased trigger rate ($\sim 300/\text{pulse}$) in M3. The answer is affirmative, based on the fact that we have been working on the full conversion to MWPC's for the past 2 years. The system, 5 planes (8000 wires), is about ready for testing and should be available for operation by the time we are to move into M3.

Many aspects of this experiment do not require the high flux of M3 and can be explored with the present flux in M4. These include determination of the nuclear parameters σ_T and σ_{22} , optimization of anticounter configurations near the regenerator used to suppress inelastic K_S regeneration and neutron induced events, and the study of our ability to clearly identify the diffraction regeneration events from a lead regenerator. The measurement of σ_T in M4 is expected to take about 200 hours with our present apparatus.

In summary, we would like to perform the experiment in two steps:

Phase I: Preliminary measurements and determination of some nuclear parameters, ~ 500 hours in beam M4.

Phase II: Electron regeneration proper, 1600 hours in beam M3.

We note that we can begin Phase I with our present setup before the long hydrogen target needed to complete E-82 (our current effort) becomes available.

