

## Research Article

# Dark Energy as a Cosmological Consequence of Existence of the Dirac Scalar Field in Nature

**O. V. Babourova and B. N. Frolov**

*Faculty of Physics and Informational Technologies, Moscow Pedagogical State University, M. Pirogovskaya Ulitsa 29, Moscow 119992, Russia*

Correspondence should be addressed to O. V. Babourova; babourova@orc.ru

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The solution of the field equations of the conformal theory of gravitation with Dirac scalar field in Cartan-Weyl spacetime at the very early Universe is obtained. In this theory dark energy (described by an effective cosmological constant) is a function of the Dirac scalar field  $\beta$ . This solution describes the exponential decreasing of  $\beta$  at the inflation stage and has a limit to a constant value of the dark energy at large time. This can give a way to solving the fundamental cosmological constant problem as a consequence of the fields dynamics in the early Universe.

## 1. Introduction

The Poincaré-Weyl gauge theory of gravitation (PWTG) has been developed in [1]. This theory is invariant concerning both the Poincaré subgroup and the Weyl subgroup—extensions and compressions (dilatations) of spacetime. Dilatations are equivalent in the mathematical sense to the transformations of the group of length calibre changes, which is the gauge group of the Weyl theory developed in 1918 [2]. The gauge field introduced by the subgroup of dilatations is named as dilatation field. Its vector potential is the Weyl vector, and its strength is Weyl's segmental curvature tensor arising in the geometrical interpretation of the theory together with the curvature and torsion tensors. The dilatation gauge field does not coincide with electromagnetic field (that has been asserted by Weyl in his basic work [2]) but represents a field of another type [3]. In particular, quanta of this field can have nonzero rest masses.

As it has been shown in [1], an additional scalar field  $\beta(x)$  is introduced in PWTG as an essential geometrical addendum to the metric tensor. The properties of this field coincide with those of the scalar field introduced by Dirac in his well-known article [4] and also by Deser in [5]. We will name this field as Dirac scalar field. The Dirac scalar field plays an important role in construction of the gravitation

Lagrangian, some members of which have structure of the Higgs Lagrangian and after spontaneous breaking of dilatational invariance can cause an appearance of nonzero rest masses of particles [6].

On the basis of the observational data, it is accepted in modern cosmology that the dark energy (described by the cosmological constant) is of dominant importance in dynamics of the Universe. In this connection the major unsolved problem of modern fundamental physics is very large difference of around 120 orders of magnitude between a very small value of Einstein cosmological constant  $\Lambda$ , which can be estimated on the basis of modern observations in cosmology, and the value of the cosmological constant in the early Universe, which has been estimated by theoretical calculations in quantum field theory of quantum fluctuation contributions to the vacuum energy [7–10]. In the present work we try to understand the cosmological constant problem as the effect of the gravitational field and the Dirac scalar field dynamics in the Cartan-Weyl spacetime in the early Universe.

## 2. Gravitational Lagrangian

Spacetime in PWTG has the geometrical structure of the Cartan-Weyl space with a curvature 2-form  $\mathcal{R}^a_b$ , a torsion

2-form  $\mathcal{T}^a$ , and a nonmetricity 1-form  $\mathcal{Q}_{ab}$  of the Weyl type,  $\mathcal{Q}_{ab} = (1/4)g_{ab}\mathcal{Q}$ , where  $\mathcal{Q}$  is a Weyl 1-form.

On the basis of PWTG, a conformal theory of gravitation in Cartan-Weyl spacetime with the Dirac scalar field has been developed in the external form formalism with the Lagrangian density 4-form  $\mathcal{L}$  [11-13]:

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_m + \beta^4 \Lambda^{ab} \wedge \left( \mathcal{Q}_{ab} - \frac{1}{4} g_{ab} \mathcal{Q} \right), \quad (1)$$

where  $\mathcal{L}_m$  is the matter Lagrangian density 4-form and  $\Lambda^{ab}$  are the Lagrange multipliers and the gravitational field Lagrangian density 4-form  $\mathcal{L}_G$  reads:

$$\begin{aligned} \mathcal{L}_G &= 2f_0 \left[ \frac{1}{2} \beta^2 \mathcal{R}^a_b \wedge \eta_a^b + \beta^4 \Lambda \eta \right. \\ &\quad + \frac{1}{4} \lambda \mathcal{R}^a_a \wedge * \mathcal{R}^b_b \\ &\quad + \tau_1 \mathcal{R}^{[a}_b] \wedge * \mathcal{R}^b_a \\ &\quad + \tau_2 (\mathcal{R}^{[ab]} \wedge \theta_a) \wedge * (\mathcal{R}^{[c]}_b \wedge \theta_c) \\ &\quad + \tau_3 (\mathcal{R}^{[ab]} \wedge \theta_c) \wedge * (\mathcal{R}^{[c]}_b \wedge \theta_a) \\ &\quad + \tau_4 (\mathcal{R}^{[a}_b] \wedge \theta_a \wedge \theta^b) \wedge * (\mathcal{R}^{[c]}_d \wedge \theta_c \wedge \theta^d) \\ &\quad + \tau_5 (\mathcal{R}^{[a}_b] \wedge \theta_a \wedge \theta^d) \wedge * (\mathcal{R}^{[c]}_d \wedge \theta_c \wedge \theta^b) \\ &\quad + \tau_6 (\mathcal{R}^a_b \wedge \theta_c \wedge \theta^d) \wedge * (\mathcal{R}^c_d \wedge \theta_a \wedge \theta^b) \\ &\quad + \rho_1 \beta^2 \mathcal{T}^a \wedge * \mathcal{T}_a \\ &\quad + \rho_2 \beta^2 (\mathcal{T}^a \wedge \theta_b) \wedge * (\mathcal{T}^b \wedge \theta_a) \\ &\quad + \rho_3 \beta^2 (\mathcal{T}^a \wedge \theta_a) \wedge * (\mathcal{T}^b \wedge \theta_b) \\ &\quad + \xi \beta^2 \mathcal{Q} \wedge * \mathcal{Q} + \zeta \beta^2 \mathcal{Q} \wedge \theta^a \wedge * \mathcal{T}_a \\ &\quad + l_1 d\beta \wedge * d\beta + l_2 \beta d\beta \wedge \theta^a \wedge * \mathcal{T}_a \\ &\quad \left. + l_3 \beta d\beta \wedge * \mathcal{Q} \right]. \end{aligned} \quad (2)$$

Here  $\wedge$  is the exterior product sign,  $d$  is the exterior derivative operator, and  $*$  is the Hodge dual conjugation. The second term in (2) is the effective cosmological constant which is interpreted as the dark energy density ( $\Lambda$  is the Einstein cosmological constant).

Variational field equations in the Cartan-Weyl spacetime have been derived from  $\mathcal{L}$  by exterior form variational formalism [14]. Independent variables are basis 1-form  $\theta^a$ , a nonholonomic connection 1-form  $\Gamma^a_b$ , the scalar field  $\beta$ , and Lagrange multipliers  $\Lambda^{ab}$ . As a result we have  $\Gamma$ -,  $\theta$ -, and  $\beta$ -equations, which have the following forms in vacuum ( $\mathcal{L}_m \approx 0$ ) [11-13]:

$\Gamma$ -equation:

$$\begin{aligned} 2f_0 \left[ \beta^2 \left( -\frac{1}{4} \mathcal{Q} \wedge \eta_a^b + \frac{1}{2} \mathcal{T}^c \wedge \eta_a^b \right. \right. \\ \left. \left. + \frac{1}{2} \eta_{ae} \wedge \mathcal{Q}^{be} + d \ln \beta \wedge \eta_a^b \right) \right. \\ \left. + \lambda \frac{1}{2} \mathcal{D} (\delta_a^b * \mathcal{R}^c_c) + \tau_1 2\mathcal{D} (* \mathcal{R}^{[b]}_a) \right. \\ \left. + \tau_2 \mathcal{D} (2\delta_a^{[d} \delta_{c]}^b \theta_d \wedge * (\mathcal{R}^{[fc]} \wedge \theta_f)) \right. \\ \left. + \tau_3 \mathcal{D} (2\delta_a^{[d} \delta_{c]}^b \theta_f \wedge * (\mathcal{R}^{[fc]} \wedge \theta_d)) \right. \\ \left. + \tau_4 2\mathcal{D} (* (\mathcal{R}^c_d \wedge \theta_c \wedge \theta^d) \theta_a \wedge \theta^b) \right. \\ \left. + \tau_5 2\mathcal{D} (* (\mathcal{R}^{[c]}_d \wedge \theta_c \wedge \theta^{[b]} \theta_{a]} \wedge \theta^d) \right. \\ \left. + \tau_6 2\mathcal{D} (* (\mathcal{R}^c_d \wedge \theta_a \wedge \theta^b) \theta_c \wedge \theta^d) \right. \\ \left. + \rho_1 2\beta^2 \theta^b \wedge * \mathcal{T}_a \right. \\ \left. + \rho_2 2\beta^2 \theta^b \wedge \theta_c \wedge * (\mathcal{T}^c \wedge \theta_a) \right. \\ \left. + \rho_3 2\beta^2 \theta^b \wedge \theta_a \wedge * (\mathcal{T}^c \wedge \theta_c) \right. \\ \left. + \xi 4\beta^2 \delta_a^b * \mathcal{Q} \right. \\ \left. + \zeta \beta^2 (2\delta_a^b \theta^c \wedge * \mathcal{T}_c + \theta^b \wedge * (\mathcal{Q} \wedge \theta_a)) \right. \\ \left. + l_2 \beta \theta^b \wedge * (d\beta \wedge \theta_a) \right. \\ \left. + l_3 2\beta \delta_a^b * d\beta \right] - 2\beta^4 \Lambda_a^b = 0. \end{aligned} \quad (3)$$

$\theta$ -equation:

$$\begin{aligned} 2f_0 \left[ \beta^2 \left( \frac{1}{2} \mathcal{R}^b_c \wedge \eta_b^c \right) + \beta^4 \Lambda \eta_a \right. \\ \left. + \lambda \left( \frac{1}{4} \mathcal{R}^c_c \wedge * (\mathcal{R}^b_b \wedge \theta_a) \right. \right. \\ \left. \left. + \frac{1}{4} * (* \mathcal{R}^b_b \wedge \theta_a) \wedge * \mathcal{R}^c_c \right) \right. \\ \left. + \tau_1 (\mathcal{R}^{[a]}_b] \wedge * (\mathcal{R}^b_a \wedge \theta_c) \right. \\ \left. + * (* \mathcal{R}^b_a \wedge \theta_c) \wedge * \mathcal{R}^{[a]}_b] \right. \\ \left. + \tau_2 (2\mathcal{R}_{[ab]} \wedge * (\mathcal{R}^{[cb]} \wedge \theta_c) \right. \\ \left. - * (\mathcal{R}^{[b]}_c] \wedge \theta_b \wedge \theta_a) \mathcal{R}^{[dc]} \wedge \theta_d \right. \\ \left. - * (* (\mathcal{R}^{[b]}_c] \wedge \theta_b) \wedge \theta_a) \wedge * (\mathcal{R}^{[dc]} \wedge \theta_d)) \right. \\ \left. + \tau_3 (2\mathcal{R}^{[b]}_c] \wedge * (\mathcal{R}^{[c]}_a \wedge \theta_b) \right. \\ \left. - * (\mathcal{R}^{[b]}_c] \wedge \theta_d \wedge \theta_a) \wedge \mathcal{R}^{[dc]} \wedge \theta_b \right. \end{aligned}$$

$$\begin{aligned}
 & - * (* (\mathcal{R}^{[b]}_{c]} \wedge \theta_d) \wedge \theta_a) \\
 & \wedge * (\mathcal{R}^{[dc]} \wedge \theta_b)) \\
 + \tau_4 & (* (\mathcal{R}^{[b]}_{c]} \wedge \theta_b \wedge \theta^c) \\
 & \cdot (4\mathcal{R}_{[af]} \wedge \theta^f + * (\mathcal{R}^{[e]}_{f]} \wedge \theta_e \wedge \theta^f) \eta_a)) \\
 + \tau_5 & (* (\mathcal{R}^{[b]}_{c]} \wedge \theta_b \wedge \theta^d) \\
 & \cdot (2\mathcal{R}_{[ad]} \wedge \theta^c - 2\delta_a^c \mathcal{R}_{[fd]} \wedge \theta^f \\
 & + * (\mathcal{R}^{[f]}_{d]} \wedge \theta_f \wedge \theta^c) \eta_a)) \\
 + \tau_6 & (* (\mathcal{R}^{[b]}_{c]} \wedge \theta_e \wedge \theta^f) \\
 & \cdot (* (\mathcal{R}^e_f \wedge \theta_b \wedge \theta^c) \eta_a \\
 & + 2g_{ab} \mathcal{R}^e_f \wedge \theta^c - 2\delta_a^c \mathcal{R}^e_f \wedge \theta_b)) \\
 + \rho_1 \beta^2 & (2\mathcal{D} (* \mathcal{T}_a) + \mathcal{T}^c \wedge * (\mathcal{T}_c \wedge \theta_a) \\
 & + * (* \mathcal{T}_c \wedge \theta_a) \wedge * \mathcal{T}^c \\
 & + 4d \ln \beta \wedge * \mathcal{T}_a) \\
 + \rho_2 \beta^2 & (2\mathcal{T}^d \wedge * (\mathcal{T}_a \wedge \theta_d) \\
 & + 2\mathcal{D} (\theta_b \wedge * (\mathcal{T}^b \wedge \theta_a)) \\
 & + 4d \ln \beta \wedge \theta_b \wedge * (\mathcal{T}^b \wedge \theta_a) \\
 & - * (* (\mathcal{T}^c \wedge \theta_d) \wedge \theta_a) \\
 & \wedge * (\mathcal{T}^d \wedge \theta_c) \\
 & - * (\mathcal{T}^b \wedge \theta_c \wedge \theta_a) (\mathcal{T}^c \wedge \theta_b)) \\
 + \rho_3 \beta^2 & (2\mathcal{D} (\theta_a \wedge * (\mathcal{T}^b \wedge \theta_b)) \\
 & + 2\mathcal{T}_a \wedge * (\mathcal{T}^b \wedge \theta_b) \\
 & - * (\mathcal{T}^b \wedge \theta_b \wedge \theta_a) (\mathcal{T}^c \wedge \theta_c) \\
 & - * (* (\mathcal{T}^b \wedge \theta_b) \wedge \theta_a) \wedge * (\mathcal{T}^c \wedge \theta_c) \\
 & + 4d \ln \beta \wedge \theta_a \wedge * (\mathcal{T}^b \wedge \theta_b)) \\
 + \xi \beta^2 & (-\mathcal{Q} \wedge * (\mathcal{Q} \wedge \theta_a) - * (* \mathcal{Q} \wedge \theta_a) * \mathcal{Q}) \\
 + \zeta \beta^2 & (\mathcal{D} * (\mathcal{Q} \wedge \theta_a) - \mathcal{Q} \wedge * \mathcal{T}_a \\
 & + \mathcal{Q} \wedge \theta^b \wedge * (\mathcal{T}_b \wedge \theta_a) \\
 & + 2d \ln \beta \wedge * (\mathcal{Q} \wedge \theta_a) \\
 & + * (* \mathcal{T}_b \wedge \theta_a) \wedge * (\mathcal{Q} \wedge \theta^b)) \\
 + l_1 & (-d\beta \wedge * (d\beta \wedge \theta_a)
 \end{aligned}$$

$$\begin{aligned}
 & - * (* d\beta \wedge \theta_a) \wedge * d\beta) \\
 + l_2 & (\beta (\mathcal{D} * (d\beta \wedge \theta_a) + d\beta \wedge \theta^b \wedge * (\mathcal{T}_b \wedge \theta_a) \\
 & - d\beta \wedge * \mathcal{T}_a \\
 & + * (* \mathcal{T}_b \wedge \theta_a) \wedge * (d\beta \wedge \theta^b)) \\
 + d\beta & \wedge * (d\beta \wedge \theta_a)) \\
 + l_3 & \beta (-d\beta \wedge * (\mathcal{Q} \wedge \theta_a) - * (* \mathcal{Q} \wedge \theta_a) * d\beta) \Big] = 0.
 \end{aligned} \tag{4}$$

$\beta$ -equation:

$$\begin{aligned}
 2f_0 & [\beta \mathcal{R}^a_b \wedge \eta^b - 4\beta^3 \Lambda \eta \\
 & + 2\rho_1 \beta \mathcal{T}^a \wedge * \mathcal{T}_a \\
 & + 2\rho_2 \beta (\mathcal{T}^a \wedge \theta_b) \wedge * (\mathcal{T}^b \wedge \theta_a) \\
 & + 2\rho_3 \beta (\mathcal{T}^a \wedge \theta_a) \wedge * (\mathcal{T}^b \wedge \theta_b) \\
 & + 2\xi \beta \mathcal{Q} \wedge * \mathcal{Q} + 2\zeta \beta \mathcal{Q} \wedge \theta^a \wedge * \mathcal{T}_a \\
 & + l_1 (-2d * d\beta) + l_2 (-\beta d (\theta^a \wedge * \mathcal{T}_a)) \\
 & + l_3 (-\beta d * \mathcal{Q})] \\
 + 4\beta^3 \Lambda^{ab} & \wedge \left( \mathcal{Q}_{ab} - \frac{1}{4} g_{ab} \mathcal{Q} \right) = 0.
 \end{aligned} \tag{5}$$

The variation with respect to the Lagrange multipliers  $\Lambda^{ab}$  gives the Weyl condition for the nonmetricity 1-form  $\mathcal{Q}_{ab}$ :

$$\mathcal{Q}_{ab} - \frac{1}{4} g_{ab} \mathcal{Q} = 0. \tag{6}$$

### 3. Solutions of the Field Equations at Ultra-Early Universe

We will solve the field equations for the scale factor  $a(t)$  and the scalar Dirac field  $\beta$  at the very early stage of evolution of Universe, when a matter density has been very small,  $\mathcal{L}_m \approx 0$ . We will omit the terms quadratic in curvature for simplicity.

In homogeneous and isotropic spacetime the conditions  $\mathcal{T}^a = (1/3)\mathcal{T} \wedge \theta^a$  are valid, and we will find, as the consequence of the field equations, the torsion and nonmetricity in the forms,  $\mathcal{T}_\mu = \chi_T d \ln \beta$ ,  $\mathcal{Q}_\mu = \chi_Q d \ln \beta$ , where the coefficients  $\chi_T$ ,  $\chi_Q$  are expressed by couple constants of Lagrangian density (2) (see the Appendix).

We consider the spatially flat Friedmann-Robertson-Walker (FRW) metric:

$$ds^2 = dt^2 - a^2(t) (dx^2 + dy^2 + dz^2). \tag{7}$$

Taking into account that  $\mathcal{L}_m \approx 0$ , we obtain from the  $\theta$ -equation together with the  $\beta$ -equation the following system of equations [11–13]:

$$(0, 0): \quad 3\frac{\dot{a}^2}{a^2} + 6\frac{\dot{a}\dot{\beta}}{a\beta} + 3B_3\left(\frac{\dot{\beta}}{\beta}\right)^2 = \Lambda\beta^2, \quad (8)$$

$$(1, 1): \quad 2\frac{\ddot{a}}{a} + 2\frac{\ddot{\beta}}{\beta} + 4\frac{\dot{a}\dot{\beta}}{a\beta} + \left(\frac{\dot{a}}{a}\right)^2 + (B_2 - 2)\left(\frac{\dot{\beta}}{\beta}\right)^2 = \Lambda\beta^2, \quad (9)$$

$$\beta: \quad A\left(\frac{\ddot{\beta}}{\beta} + 3\frac{\dot{a}\dot{\beta}}{a\beta}\right) + (B - A)\left(\frac{\dot{\beta}}{\beta}\right)^2 = 0, \quad (10)$$

where the constants  $A, B, B_3 = (1/3)(2B_1 + B_2), B_1$ , and  $B_2$  are expressed through the parameters of Lagrangian density (2), the components (2, 2) and (3, 3) being equal to the component (1, 1).

The system of (8)–(10) is inconsistent, because we have three equations for two unknown functions  $a(t)$  and  $\beta(t)$ . Let us put in this system  $B_1 = B_2 = B_3 = 1$  and also  $u = \ln a$ ,  $v = \ln \beta$ . Then subtract (8) from (10). As a result we obtain the following system of equations:

$$(\dot{u})^2 + 2\dot{u}\dot{v} + (\dot{v})^2 = \frac{\Lambda}{3}e^{2v}, \quad (11)$$

$$\ddot{u} + \ddot{v} - \dot{u}\dot{v} - (\dot{v})^2 = 0, \quad (12)$$

$$\dot{v} + 3\dot{u}\dot{v} + \frac{B}{A}(\dot{v})^2 = 0. \quad (13)$$

Equation (11) is equivalent to the equation

$$\dot{u} + \dot{v} = \pm \frac{\lambda}{3}e^v, \quad \lambda = \sqrt{3\Lambda}. \quad (14)$$

It is easy to check that (12) is fulfilled identically as a consequence of (14). Therefore we have only 2 equations (13) and (14) for 2 unknown functions  $a(t), \beta(t)$ , and this system of equations is consistent. In what follows we choose the sign “+” in (14).

Let us find  $\dot{u}$  from (14) and put it in (13). We obtain the equation

$$\dot{v} + \lambda e^v \dot{v} + \omega(\dot{v})^2 = 0, \quad \omega = \frac{B}{A} - 3. \quad (15)$$

The first integral of this equation is the following:

$$\dot{v} = \lambda_0 e^{-\omega v} - \frac{\lambda}{1 + \omega} e^v, \quad (16)$$

where  $\lambda_0$  is a constant of integration.

The system of (14), (16) has a large variety of integrable solutions parameterized by  $\omega$  and  $\lambda_0$ . Let us obtain the solution for the case  $\omega = 0$ . If we put in (16)  $\lambda_0 = \lambda$ , then this equation reads,  $\dot{v} = \lambda(1 - e^v)$ , and we have a solution [13]

$$\beta(t) = e^{v(t)} = \frac{1}{1 - e^{-\lambda(t+t_0)}}, \quad (17)$$

$$a(t) = a_0 e^{(\lambda/3)(t+t_0)} (1 - e^{-\lambda(t+t_0)})^{4/3}.$$

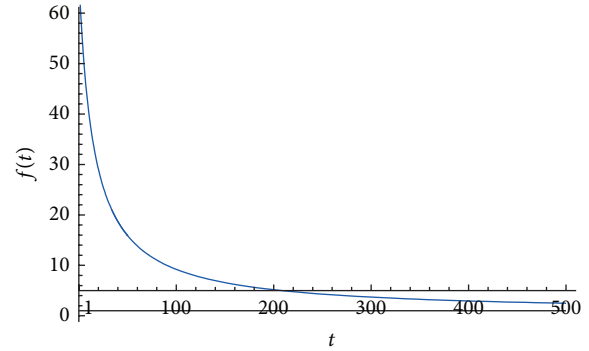


FIGURE 1: Solution (17) for the Dirac scalar field in the early Universe.

We assume that the value of  $\beta$  is very large, when  $t = 0$ . Therefore the constant of integration  $t_0$  should be very small ( $0 < t_0 \ll \lambda^{-1}$ ). Then from (17) under  $t \gg t_0$  one has approximately

$$\beta(t) = (\beta_{01})^{\exp(-\lambda t)}, \quad a(t) = (a_{01}) e^{(\lambda/3)t}. \quad (18)$$

These solutions realize exponential diminution of a field  $\beta$  (see Figure 1 for function (17)), and thus sharp exponential decrease of physical vacuum energy (dark energy) by many orders. We have  $\beta \rightarrow 1, \Lambda_{\text{eff}} = \beta^2 \Lambda \rightarrow \Lambda$  in a limit at  $t \rightarrow \infty$ . Thus, the effective cosmological constant can slightly differ already by the end of inflation from the limiting value equal to its modern size  $\Lambda$  that provides the subsequent transition from the Friedmann epoch to the epoch of the accelerated expansion in accordance with the modern observed cosmological data.

Our solutions are realized, if the conditions are valid,  $B = 3A, B_1 = 1$ , and  $B_2 = 1$ . These conditions are determined in rather complicated manner by the 16 coupling constants of gravitational Lagrangian density (2) and can be easily fulfilled (see the Appendix).

#### 4. Discussion and Final Remarks

Here the field equations for the spatially flat homogeneous and isotropic ultra-early Universe are investigated and the solution for the Dirac scalar field (and therefore for the effective cosmological constant) is found. This solution for the scalar field has the behavior of the very extensive diminishing exponent, whose limits are the modern value of the Einstein cosmological constant. Solution (17) could be realized at the very beginning of the Universe evolution, when the cosmological constant  $\Lambda_0$  estimated by quantum field theory was equal to  $\Lambda_0/\Lambda = \beta_0^2 \sim 10^{120}$ , and the number  $\beta_0 \sim 10^{60}$  was very large.

Thus our result can explain the exponential decrease in time at very early Universe of the dark energy (the energy of physical vacuum), described by the effective cosmological constant. This can give a way to solving the problem of cosmological constant as a consequence of fields dynamics at the early Universe. It is well known that this problem is one of

the fundamental problems of the modern theoretical physics [7–10].

We point out that the ultra-rapid decrease of the energy of physical vacuum according to law (17) occurs only prior to the Friedmann era evolution of the Universe. Further evolution of the Universe is determined not only by a scalar field only but also by the born ultra-relativistic matter and the radiation interacting with it.

The Dirac scalar field condenses near massive objects. As a consequence of this, the authors formulate the hypothesis that *the Dirac scalar field is realized itself not only as the “dark energy” but also as one of the components of the “dark matter”* [13].

## Appendix

Let us show how the conditions  $B_1 = B_2 = 1$  can be fulfilled.

As a consequence of the  $\Gamma$ -equation, taking into account that  $d\mathcal{Q} = 0$  for metric (7) (see [15]), we have the following equalities:

$$\begin{aligned} 2(1 - \rho_1 + 2\rho_2)\mathcal{F} - 3\left(\frac{1}{4} + \zeta\right)\mathcal{Q} + 3(2 - l_2)d\ln\beta &= 0, \\ (2\rho_1 - 4\rho_2 + 8\zeta)\mathcal{F} + (16\xi + 3\zeta)\mathcal{Q} \\ + (3l_2 + 8l_3)d\ln\beta &= 0. \end{aligned} \tag{A.1}$$

We can put  $\mathcal{F} = \chi_{\mathcal{F}}d\ln\beta$ ,  $\mathcal{Q} = \chi_{\mathcal{Q}}d\ln\beta$ , as a consequence of (A.1).

Let us choose for the coupling constants of Lagrangian density (2) the values  $l_2 = 2$ ,  $l_3 = 0$ , and  $\zeta = 0$  for simplicity. Then we have from equalities (A.1)

$$\begin{aligned} \chi_{\mathcal{F}} &= \frac{3}{8(1 + \rho_1 - 2\rho_2)}\chi_{\mathcal{Q}}, \\ \chi_{\mathcal{Q}} &= -6\left(16\xi + \frac{3(\rho_1 - 2\rho_2)}{8(1 - \rho_1 + 2\rho_2)}\right)^{-1}. \end{aligned} \tag{A.2}$$

Now let us put the results obtained into the  $\theta$ - and  $\beta$ -equations and get the system of (8)–(10) with

$$\begin{aligned} B_1 &= \frac{1}{4}\chi_{\mathcal{F}}\chi_{\mathcal{Q}} + \frac{3}{32}\chi_{\mathcal{Q}}^2 - \frac{3}{4}\chi_{\mathcal{Q}} - 2l_1, \\ B_1 - 6B_2 &= \left(\frac{3}{16} - 2\xi\right)\chi_{\mathcal{Q}}^2 - \frac{5}{4}\chi_{\mathcal{Q}} - \frac{32}{3}\chi_{\mathcal{F}} - 4l_1. \end{aligned} \tag{A.3}$$

Let us put the value  $\chi_{\mathcal{Q}} = -8$ . Then we have

$$\frac{\rho_1 - 2\rho_2}{-1 + \rho_1 - 2\rho_2} = \frac{128\xi + 6}{3}, \quad \chi_{\mathcal{F}} = 128\xi + 3. \tag{A.4}$$

As a consequence, the following system of equations is obtained from (A.3):

$$\begin{aligned} B_1 &= -2\chi_{\mathcal{F}} - 2l_1 + 12, \\ B_1 - 6B_2 &= -\left(\frac{32}{3}\right)\chi_{\mathcal{F}} - 4l_1 - 128\xi + 22. \end{aligned} \tag{A.5}$$

If we put now  $B_1 = B_2 = 1$ , then we obtain

$$l_1 \approx 4.5 > 0, \quad \xi \approx -0.01 < 0. \tag{A.6}$$

Therefore we can conclude from (2) that the Dirac scalar field  $\beta$  is the physical field, but the Weyl 1-form  $\mathcal{Q}$  is tachyonic.

It is also easy to satisfy the condition  $\omega = B/A - 3 = 0$ . We have

$$\begin{aligned} A &= 2(3 - l_1), \\ B &= 4(3 - l_1) + \chi_{\mathcal{F}}\chi_{\mathcal{Q}}\left(-\frac{5}{6} + \frac{1}{3}\rho_1 - \frac{1}{2}\rho_2\right) \\ &\quad - \chi_{\mathcal{Q}}^2\left(\frac{3}{4} + \frac{1}{3}\rho_1\right) \\ &\quad + \chi_{\mathcal{F}}\left(-\frac{11}{4} - 4\rho_1 - 8\rho_2\right) - \chi_{\mathcal{Q}}\left(\frac{3}{2} + 6\xi\right). \end{aligned} \tag{A.7}$$

If we put now  $\rho_1 = 0$  for simplicity, then we obtain  $\rho_2 \approx -0.5$ , with the values of the coupling constants chosen and calculated before.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

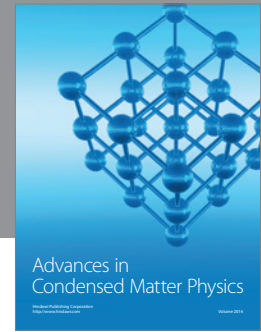
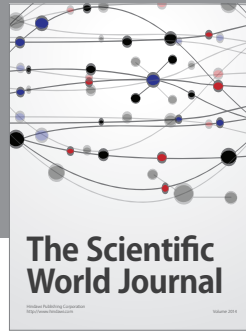
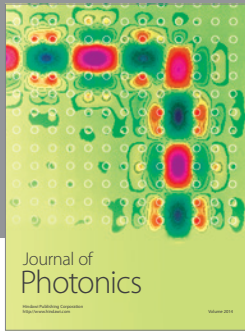
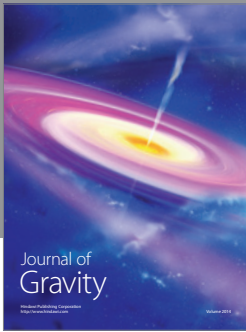
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