Study of chiral dependent momentum transport coefficients in a thermal QCD

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Introduction

The heavy-ion collisions in the CERN SPS, RHIC, BNL, and LHC accelerators are thoroughly studied to yield insights about nuclear matter properties under extreme conditions. The predictions of charged hadron elliptic flow from RHIC [1] and their theoretical explanations using dissipative hydrodynamics [2] provided the experimental evidence of existence of the transport processes in the QGP. These transport coefficients are not directly measurable experimentally, rather serve as input parameters in the theoretical modelling of experimental observables such as directed flow, elliptic flow etc. One of the most important transport coefficients in the hydrodynamical description is shear viscosity (η) that governs the rate of momentum transfer in a presence of inhomogeneity of fluid velocity, while the bulk viscosity (ζ) describes the change of local pressure when the fluid element is either expanding or contracting. The relativistic hydrodynamics played one of the most important roles to extract the value of shear viscosity to entropy density ratio (η/s) from the available experimental data [3].

In the present work, we have studied the shear and bulk viscosity in the presence of a weak magnetic field at finite quark chemical potential. The interactions among partons are accounted for by the quasiparticle mass of partons, in which the degeneracy in mass of chiral modes of quarks is lifted in the presence of a weak magnetic field.

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Quasiparticle Model

The interaction among quasiquarks and quasigluons can be incorporated through medium dependent mass of quasiparticles which can be evaluated using oneloop perturbative thermal QCD. Gluons do not interact with magnetic field and hence they will possess the thermally generated mass as [4]

$$m_g^2 = \frac{1}{6} g^2 T^2 \left(C_A + \frac{1}{2} N_f \right).$$

The quark propagator in the presence of weak magnetic field can be expressed in the power series of $(q_f B)$ and hence the one-loop quark self energy upto $\mathcal{O}(q_f B)$ can be determined. Employing the general covariant structure of quark self energy in terms of chiral projection operator and Schwinger-Dyson equation, the effective quark propagator is obtained to be as

$$S^*(P) = \frac{1}{2} \left[P_L \frac{\not{L}}{L^2/2} P_R + P_R \frac{\not{R}}{R^2/2} P_L \right].$$

The static limit ($p_0 = 0, |\mathbf{p}| \to 0$) of $L^2/2$ and $R^2/2$ will give the different medium generated mass for L and R mode thus lifting up the degeneracy in mass which is in contrast to the case of strong magnetic field as

$$m_L^2 = m_{th}^2 + 4g^2 C_F M^2,$$
$$m_R^2 = m_{th}^2 - 4g^2 C_F M^2.$$

Momentum transport coefficients

Assuming the distribution functions to deviate only slightly from equilibrium ($\delta f <<$

f_0), the Boltzmann equation under relaxation approximation in presence of electromagnetic field can be written as

$$p^\mu\partial_\mu f_{eq} = -\frac{p^\mu u_\mu}{\tau}\left[1-\frac{qB\tau b^{\mu\nu}p_\nu}{p^\mu u_\mu}\frac{\partial}{\partial p^\mu}\right]\delta f,$$

with $f_{eq}(\mathbf{p}) = \frac{1}{e^{(\sqrt{\mathbf{p^2}+m^2}-\mu)/T}+1}$ at finite quark chemical potential (μ).

δf can be constitued as linear combination of thermodynamic forces ($Y_{\mu\nu}$) times appropriate tensorial coefficients, resulting in Lorentz scalar, δf as,

$$\delta f = \mathrm{A}Y + \mathrm{B}^\mu Y_\mu + \mathrm{C}^{\mu\nu}Y_{\mu\nu}.$$

The general form of δf for shear viscosity and bulk viscosity can be expressed in terms of fourth rank and second rank projection tensor respectively as

$$\delta f_{shear} = \sum_{r=0}^{4} a_r A^{(r)}_{\mu\nu\alpha\beta}p^\mu p^\nu V^{\alpha\beta},$$
$$\delta f_{bulk} = \sum_{r=1}^{3} a_r A_{(r)\mu\nu}\partial^\mu u^\nu.$$

The magnetic field breaks the anisotropy of the medium resulting in the distinct components of shear and bulk viscosity. Hence, we obtained the five shear ($\eta_0,\eta_1,\eta_2,\eta_3,\eta_4$) and two bulk ($\zeta_1,\zeta_2$) viscous coefficients,

$$\eta_0 = \sum_f \frac{g_f}{15T}\int\frac{d^3p}{(2\pi)^3}\left(\frac{\mathbf{p}^4}{\varepsilon_f^2}\right)(a_1+\bar{a_1}),$$
$$\eta_1 = \sum_f \frac{g_f}{15T}\int\frac{d^3p}{(2\pi)^3}\left(\frac{\mathbf{p}^4}{\varepsilon_f}\right)(u.p)(a_2+\bar{a_2}),$$
$$\eta_2 = \sum_f \frac{g_f}{15T}\int\frac{d^3p}{(2\pi)^3}\left(\frac{\mathbf{p}^4}{\varepsilon_f}\right)\Big[(q_fB\tau_f)a_2+ (q_{\bar f}B\tau_{\bar f})\bar{a}_2\Big],$$
$$\eta_3 = \sum_f \frac{g_f}{15T}\int\frac{d^3p}{(2\pi)^3}\left(\frac{\mathbf{p}^4}{\varepsilon_f}\right)(u.p)(a_3+\bar{a}_3),$$
$$\eta_4 = \sum_f \frac{2g_f}{15T}\int\frac{d^3p}{(2\pi)^3}\left(\frac{\mathbf{p}^4}{\varepsilon_f}\right)\Big[(q_fB\tau_f)a_3+ (q_{\bar f}B\tau_{\bar f})\bar{a}_3\Big],$$
$$\zeta_1 = \zeta_2 = \sum_f \frac{g_f}{3T}\int\frac{d^3p}{(2\pi)^3}\left(\frac{X^2}{\varepsilon_f^2}\right)(a_1+\bar{a_1}),$$

where a_1, a_2, a_3 and X are given as

$$a_1 = f_{eq,f}\,(1-f_{eq,f})\,\tau_f,$$
$$a_2 = \frac{f_{eq,f}\,(1-f_{eq,f})\,\tau_f}{(u.p)^2+(q_fB\tau_f)^2},$$
$$a_3 = \frac{(u.p)\,f_{eq,f}\,(1-f_{eq,f})\,\tau_f}{\Big((u.p)^2+(2q_fB\tau_f)^2\Big)},$$
$$X = (u.p)^2\left(\frac{4}{3}-\lambda'\right)-\frac{1}{3}m^2+ (u.p)\left[(\lambda''-1)\,h-\lambda'''T\right].$$

$\bar{a_1}, \bar{a_2}$ and $\bar{a}_3$ for antiquarks can be defined via $f_{eq}\to\bar{f}_{eq}, \tau_f\to\tau_{\bar f}, q_f\to q_{\bar f}$. $\lambda^{'}, \lambda^{''}$ and $\lambda^{'''}$ are the functions of Bessel functions of second kind.

η_0 is the longitudinal component, η_1,η_3 are the transverse components and η_2,η_4 are the Hall components of shear viscosity. In the absence of magentic field, Hall component vanishes, whereas the transverse component becomes same as longitudinal component. For zero quark chemical potential, the Hall component vanishes, even in the presence of magnetic field. The longitudinal (ζ_1) and transverse (ζ_2) components of bulk viscosity are same and the effect of magnetic field comes through the thermal mass (squared) of quarks and antiquarks with magentic field correction.

References

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