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An approach to the Second Quantum Revolution for secondary school students: the case of the random walk algorithm

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Abstract. The teaching of quantum technologies has now become a leading topic and is at the heart of numerous international programs (e.g., quantum flagship, National Quantum Initiative, UK national quantum technology program) with the aim of widening the workforce and preparing the next generations of experts. In the present contribution, we present an approach for teaching the Second Quantum Revolution to secondary school students that we developed in recent years. The approach aims to emphasize the ongoing revolution as first of all a cultural revolution and its intrinsic interdisciplinary character. As an emblematic case of some aspects of the approach, we present an activity on the classical and quantum random walk algorithms whose main aims are to recontextualize some basic quantum concepts (quantum state, state manipulation/evolution, measurement, and entanglement) in the case of the algorithm and to reflect on the main differences between the classical and quantum case in terms of logic behind its functioning as well as from an epistemological perspective.

1. Introduction

Quantum technologies are currently among the most promising technological developments. In the last few years, the Second Quantum Revolution caught on and has marked new trends in research and enterprises. Many strategic plans have been presented by various countries that invest in the research and development of these new technologies: the European Union with the Quantum Flagship has started an investment of €1 billion in 2018; China, after several investments in the past years, declared quantum technologies one of the new high techs in its 14th Five-Year Plan (2021-2025); the U.S. with the National Quantum Initiative Act of 2018 has allocated more than 1 billion; the UK launched the UK national quantum technology program investing similar figures. From these first years, today the investments have become much greater. Many large companies (IBM, Google, Microsoft, and Intel) and start-ups (e.g., Algorithmq), born on project funds, have been focusing on quantum technologies and have already achieved significant results. The contemporary challenges do not concern only the research and development of these new technologies, but also education. For example, there is the need to increase the workforce and involve non-physics students, aligning universities and schools to orient the young generation toward STEM careers and promoting quantum literacy and citizenship awareness [1,2]. Therefore, quantum technologies represent promising opportunities in a wide range of research fields, and, at the same time, they are challenging our society. [3].

From an educational perspective, teaching and learning the scope of the Second Quantum Revolution and the emergent technologies are very meaningful since they are a window into some contemporary societal challenges and new ways of doing research [4]. Furthermore, being a STEM (Science, Technology, Engineering, Mathematics) topic, they require an interdisciplinary approach to their teaching that involves a kind of learning "that crosses subject boundaries to facilitate a better learning experience", fostering "students to explore and integrate multiple perspectives from different subject disciplines, sub-disciplines and areas of expertise" [5] Interdisciplinarity and the development of interdisciplinary skills are also considered one of the goals set by the OECD learning compass [6]. From this perspective, quantum computation and information are a particularly relevant and rich topic, on the boundaries between physics, mathematics, and computer science, disciplines with which students, at least in scientific schools, have to deal.

Some algorithms and protocols such as the quantum random walk algorithms, quantum cryptography, and quantum teleportation protocols are proven to be within the reach of secondary school students [4,7] and effective also to introduce some basic and key quantum concepts like the quantum state, the state's manipulation/evolution, the measurement, and the entanglement [8,9]. In recent years the University of Bologna, to contribute to the challenges posed to science education, has developed a course on the Second Quantum Revolution implemented both with Secondary school students in different contexts (PLS – Piano Lauree Scientifiche - extracurricular courses and intensive PCTO - Pathway for Transversal Competences for Orientation- summer/spring schools in collaboration with the university of Como and Pavia) and with prospective and in-service teachers. In this contribution, we present the approach to the Second Quantum Revolution developed with a focus on the interdisciplinary aspects that characterize it. In particular, we aim to contribute to the following research question: Which principles and teaching approach can be designed to value the Second Quantum Revolution as a cultural revolution? After the presentation of the approach to the Second Quantum Revolution we present the emblematic case of the activity on the classical and quantum random walk.

2. The teaching approach

The approach is rooted in the quite long group's tradition of learning and teaching quantum physics. This tradition is based on the idea that "the teaching/learning processes are meaningful for the students when they productively make three complex systems resonate: society, the system of physics disciplinary knowledge, and the cognitive system of learners" [10]. This assumption shapes the learning environment as properly complex territory. This notion includes some forms of productive complexity (multi-perspectivenes, multi-dimensionality, and longitudinality) that should be implemented in the design to pursue the general goal of enabling students to find out their ways to understand the content and make them able to interpret what is happening around, at societal level and act as an aware citizen [10].

2.1. Principles to value the cultural dimension of the Second Quantum Revolution

As soon as we started to deal with quantum technologies, we noticed that, in popular scientific journals, these technologies were mainly described as more powerful technologies than the classical ones, with a higher speed of calculation. Popular articles usually talk about the race for achieving quantum supremacy; scientists and politics to increase and train the workforce. The standard popular vocabulary, with which we refer to quantum technologies, belongs mainly to the rhetoric of war or the muscle. Starting from the idea of properly complex territory developed by the group, we explored the Second Quantum Revolution and designed a teaching approach to enhance it as, first of all, a cultural revolution.

Attending popular seminars and conferences we noticed an asymmetry. A popular conference on classical computation would rarely start by explaining the physical laws according to which hardware and logic gates are realized and operate. On the contrary, many public seminars and conferences on quantum computation usually include an introduction to the new unit base and its features (superposition principle and entanglement) and a discussion about, for example, the better techniques to create qubits and manipulate them [7]. In the actual collective imaginery, *quantum computers are experiments* (first epistemological knot). In Preskill's words "Information, after all, is something that is encoded in the state of a physical system; a computation is something that can be carried out on an actual physically realizable device. So, the study of information and computation should be linked to the study of the underlying physical processes." [11]. The awareness that physical processes can be considered forms of computation sheds light on the inextricable link between computation (computer science as a discipline) and physics [12,13]. This association creates a direct link between the "physical information" of a system and the "computational information": the physical information can be encoded in terms of bits and qubits, which embody, at the same time, a logical and "material" nature [14]. In this perspective, the unit bases of computation (the bit and the qubit) do not only allow us to compare the classical and the quantum nature of the physical objects but also compare them in terms of the logic at the basis of the functioning of their hardware (second epistemological knot).

The principles that guided the development of the module are presented and deeply discussed in [7,15]. The principles that aim to value the Second Quantum Revolution as a cultural revolution and that inspire the specific activity on the classical and quantum random walk are:

- Design principle #1: to foster a close comparison between classical and quantum computers through an analysis of the different logic underlined in the basic mechanisms on which the hardware is built.
- Design principle #2: to reconceptualize the foundational experiments in terms of computation, so as to discuss why and how experiments can be considered as "simulators" or devices to process information (circuits). This means, operationally, re-reading the three main phases of an experiment - *state preparation, state evolution/manipulation, and measurement* - in terms of *input - processing - output information*.

2.2. Principles to value the intrinsic interdisciplinarity of quantum technologies

The design principles introduced in the previous section not only highlight some key cultural aspects of the Second Quantum Revolution but also embody the approach developed within the IDENTITIES project. It is an Erasmus+ project that started in 2019 and ended in 2022, coordinated by the University of Bologna. The project aimed to develop novel teaching approaches to interdisciplinarity in science and mathematics to innovate pre-service teacher education and explored both emergent advanced STEM themes (i.e., covid pandemic, nanotechnologies, quantum technologies) and curricular interdisciplinary topics (parabola and parabolic motion, cryptography, modelling) as contexts to investigate inter-multi-trans-disciplinary forms of knowledge organization, designing classroom activities and new models of co-teaching. The heart of the approach lies in the nature of the word interdisciplinarity. It contains the word "discipline" whose Latin root, 'discere', means learning. Disciplines are a body of knowledge and skills that ground their roots in the educational necessity to re-organize knowledge for teaching, learning, and communicating it [16]. The re-organization has to be such that students, in building their knowledge, can also develop epistemic skills, such as problem-solving, modeling, representing, arguing, explaining, testing, and sharing [17]. The suffix inter- emphasizes a process of "integrating, interacting, linking, and focusing" different disciplinary domains, and different epistemic cores [18]. Considering this stance toward interdisciplinarity, we integrated two different frameworks that allow us to deepen the identities of the disciplines (in this case, physics, mathematics, and computer science) and consider their integration as a dynamic process that involves boundarycrossing mechanisms. In this respect, the Family Resemblance Approach (FRA), first developed by Irzik and Nola [19] and then by Erduran and Dagher [20], is considered to deepen and discuss the disciplinary aspects while the boundary crossing and boundary objects framework [21] was chosen to reflect on the intertwining, the mechanisms of crossing the boundaries between the different disciplinary domains.

The FRA is based on investigating how the sciences are similar or dissimilar, "building up from scratch polythetic sets of characteristics for each individual science" [19]. So Irzik and Nola, with the idea that "there is no fixed set of necessary and sufficient conditions which determine the meaning of [science]" [19], argue that each discipline resembles some family disciplines in some aspects and other disciplines in other aspects, just like in a family. The potential of this approach is to avoid defining what science is and provide an overall picture of the many aspects that characterize sciences [19,20]. Following [20], namely the revised version of FRA for science education, each scientific discipline is composed of a cognitive-epistemological system and a socio-institutional one. In our case, particularly interesting is the first system through which the discipline can be characterized in terms of aims and values, scientific practices, methods and methodological rules, and knowledge [20]. The boundary crossing and boundary objects framework outlines a vocabulary to characterize the intertwining of different disciplines. In particular, the authors introduce the idea of boundary objects as ideas, concepts, and models that can trigger boundary crossing by addressing and articulating meanings from different perspectives. These objects are characterized by an intrinsic ambiguity: they "belong to both one world and another" and, at the same time, "they belong to neither one nor the other world." The authors also identify four mechanisms of boundary crossing to characterize the kind of intertwining and dialogue: identification, coordination, reflection, and transformation [21].

Starting from these ideas, two other design principles were carried out to emphasize the interdisciplinary nature of these new technologies [15]:

- to scaffold a comparison between the disciplines that intertwine in a STEM context both in terms of the epistemic-cognitive nucleus, that is shedding light on aims and values, scientific practices, methods and methodological rules, and knowledge and in terms of the social institutional systems (at least as an integrated domain).
- to trigger boundary crossing mechanism by focusing on boundary objects, on how they are exchanged, what kind of knowledge they embody, and what kind of knowledge they carry before, during, and after the intertwining.

The activity of classical and quantum random walks is designed starting from these ideas. It revolves around the concept of probability in the classical and the quantum case, gradually filling with meaning through three different perspectives: the mathematical, the physical, and the computer science ones. The integration of the different perspectives dealing with the algorithm allows also us to reach the epistemological core of the concept in the two different cases and to highlight the deep aspects both of the First and the Second Quantum Revolution. Before going into the details of the activity we briefly describe the module of the Second Quantum Revolution and the implementations.

3. The educational choices and the design of the course

The course on the Second Quantum Revolution was first implemented in 2019 and, following the grounded theory processes [22], was revised and refined many times and it has finally the following structure. Table 1: Structure of the Second Quantum Revolution course.

Table 1.: Structure of the Second Quantum Revolution course

Day	Conceptual-epistemological activities (2h)	Future-oriented/citizenship education activities (1h)
1	Introduction to the Second Quantum Revolution The history of classical computers (introduction to classical computers' structure, binary logic, classical logic gates and circuits, computers' evolution from the first to the fourth generations)	"Quantum Technologies & …": teamwork activity to reflect on the impact of QT on the societal, political, economic, environmental, educational (etc.) dimensions.
2	Introduction to the physics of quantum computers_Part 1: Introduction through a simplified spin-first approach of the concepts of state, state manipulation/evolution, and measurement Passage to information: encoding information in Qubits (input information), information processing (one-qubit logic gates and circuits), reading information (output information)	Delivery of the teamwork activity "Quantum Technologies & …"
3	Introduction to the physics of quantum computers_Part 2: Introduction of entanglement (two-qubits systems) Quantum cryptography protocol (BB84)	The Eve city_part 1: In the role of policymakers. The teamwork activity involves students taking on the role of mayor and deciding whether to invest in the new QT by discussing the different

		perspectives.
4	Teleportation protocol and the future quantum internet	The Eve city_part 2: reflections about the concept of scenario. Teamwork activity in which students reflect on if and how QT is changing the relationship between nature, humans, and technology.
5	Classical and quantum random walk	Future-oriented activity: in groups, students think about a problem that they would like to be solved in 2040 (foresight) and try to figure out, as active members, the possible steps that can lead to solving it (backcasting). Teamwork in preparation for the workshop.
6	Delivery and discussion of the teamwork or creative workshop (e.g., Reflection on how scientific and technological revolution in the Great History impacted arts, literature… and vice versa)	

The course lasts 18h, 3h hours per meeting one or twice a week, and is composed of two different kinds of activities. The conceptual-epistemological activities aim to introduce students to the basic concepts of quantum physics necessary and sufficient to deal with quantum technologies at an introductory level as well as to some pivotal technologies to which contemporary research revolves around. Furthermore, these activities, designed following the principles, aim to introduce students to the revolutionary aspects of the Second Quantum Revolution in comparison with the First one and to highlight the differences between the classical and the quantum case exploiting, in particular, the logic with which the classical and quantum hardware works. The second kind of activity, the futureoriented/citizenship education activities, aims to make students reflect and explore the impact of the Second Quantum Revolution on contemporary society and our future. In the last implementations, we dedicated part of these hours to reflect on new narratives, languages, and aesthetics that can be designed to talk about the quantum revolution in light of a problem perceived from the very first implementation, namely quantum technologies are not yet part of the collective imaginery. Therefore, secondary school students usually proved to lack a deep capacity to project themselves into the future and reflect on how quantum technologies could impact and change the society we live in. A deep description of the activities can be found in [15].

4. The emblematic activity of the classical and quantum random walk

4.1. Principles to value the cultural dimension of the Second Quantum Revolution

The classical and the quantum random walk activity, like the others ones, is designed and revised following the Model of Educational Reconstruction, MER, [23,24] that identifies three components of the process of content reconstruction: analysis and clarifications of science content, analysis/research on teaching and learning with an emphasis on students' perspectives, and instruction design. The first component consists of "hermeneutic-analytical research on subject matter clarification and analysis of the educational significance of particular science content. The interconnected set of core ideas of a particular content domain is detected (in the aforementioned sense) from the perspective of key aims of science instruction" [23]. The second component comprises "studies on students' conceptions, i.e., investigations into students' pre-instructional conceptions and their development towards the intended science view" [23]. The third component refers to the need to strictly link academic research with school practices and, indeed, "comprises development and evaluation of pilot instructional modules rather early in the process of educational reconstruction" [23].

In the case of the random walk activity, the analysis and clarification of scientific content involved research and analysis of scientific papers. From this analysis, we identified a few papers epistemologically relevant that shed light on the epistemological structure of the algorithm [25, 26, 27].

Particularly meaningful for the reconstruction is [27], since the author discusses quantum random walks both from physical and computer science perspectives, with the aim of "trying to develop an intuition". She, after providing a general flavor of the phenomenon in a physical setting, formalizes both the discrete and the continuous models of quantum random walks.

The second phase, carried out in the design of all activities, takes into account the literature on students' conceptions and difficulties in learning quantum physics to transform the contents to bring them within the reach of secondary students.

As for the third phase (instruction design and pilot investigation), the activities and materials we developed were implemented four times completely in collaboration with Piano Lauree Scientifiche (PLS) project, and other four times partially in collaboration with the University of Como and the University of Pavia. Implementation after implementation we refined the activities of the module based on students' difficulties and reactions, and our perceptions of what happened in class. A deep analysis of the three components is reported in [15].

4.2. The classical and quantum random walk activity

This activity has the learning objectives of i. recontextualizing the key quantum physics concepts (quantum state, state manipulation/evolution, measurement, and entanglement) in the context of the random walk algorithm, ii. reflecting on the main differences between the classical and the quantum logic at the basis of the implementation of the algorithm, iii. shed light on the intrinsic interdisciplinarity of quantum computation and iv. show some important applications of the quantum random walk and some examples of problems in which it can be used [15]. The activity is structured as the following:

- teamwork activity about the drunkard's walk and collective discussion;
- introduction to the model of the random walk to scaffold the comparison between the classical and the quantum cases;
- exploration of the classical and quantum random walk through the interdisciplinary approaches (mathematic, physics, computer science);
- introduction of some application.

We start from the teamwork activity and the problem we pose is: "Charlie, after a long evening of vices and extravagances outside the city, returns, a little staggering, to the city of Eve. As soon as he crosses the city gates, a problem arises: Charlie no longer remembers where he lives or the way back. He then begins to walk between the blocks, proceeding randomly and never going back, hoping to find the right way. What is the probability that Charlie will reach his house (green square) at random? Is the probability that Charlie will reach, at random, his friend Bob's house (yellow square) the same? How would you model the problem?" We attach to the problem figure 1.

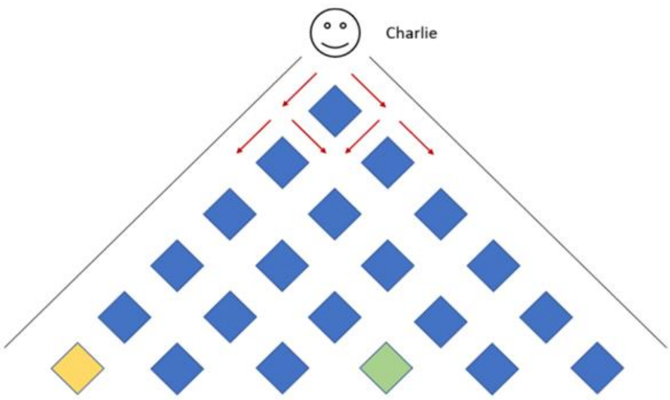


Figure 1. Problem representation to guide students to solve the problem.

Furthermore, we ask students not only to solve the drunkard's walk problem but also to "put themselves in the shoes of mathematicians, physicists or computer scientists" and reflect on what are the disciplines' peculiarities in their approach to the problem, what kind of knowledge, which tools, practices, methods, aims and values, and knowledge they can put into play in solving the problem from the perspective of physics, mathematics or computer science. After a collective discussion about both the results obtained by the students and the disciplinary identities, we open to the quantum model by asking: What can happen if Charlie follows the law of quantum mechanics?

Students are then asked to think about how to model the drunkard's walk, in particular, how to model the "proceeding randomly" of Charlie and its way of moving "never going back". We then model the city of Charly like in figure 2, the "proceeding randomly" can be simulated with a coin and the way of moving by setting a shift operator. The random model can be simulated by, step by step, flipping the coin and applying the shift operator.

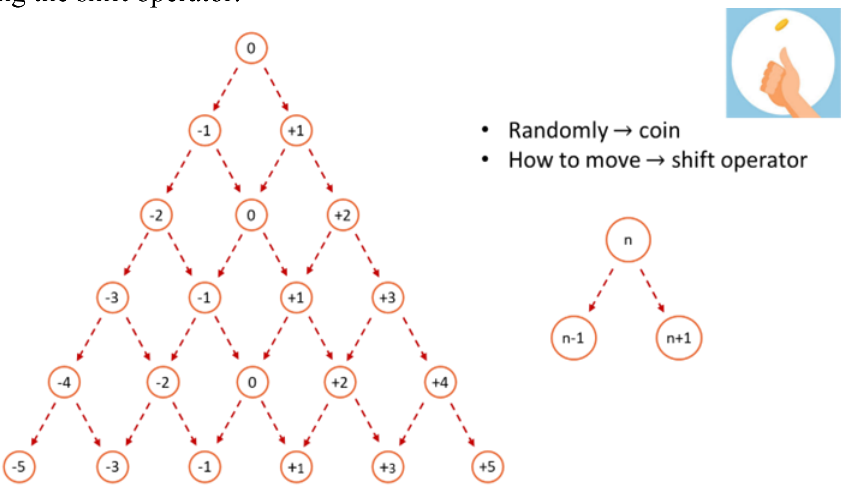
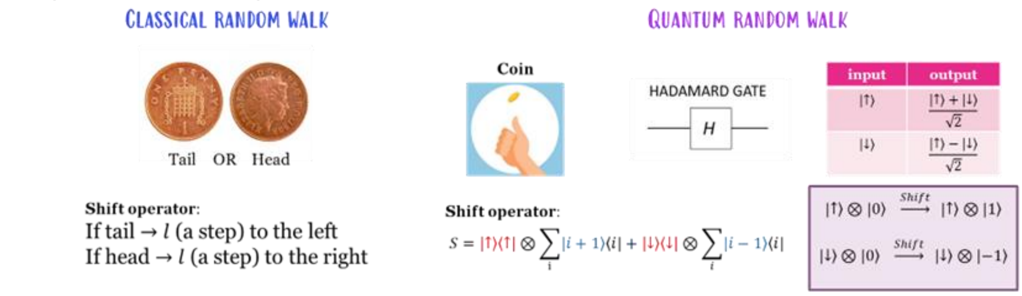


Figure 2. The random walk model.

We start to scaffold the comparison between classical and quantum random walks by stressing the different logic of the coin (figure 3).



Shit operator		Head	Tail
	Right	X	
	Left		X
	Prob.	½	½

Shit operator		$\lvert\uparrow\rangle$	$\lvert\downarrow\rangle$
	Right	X	
	Left		X
	Prob.	½	½

Figure 3. Comparison between the classical and the quantum model.

While the shift operator in the classical and quantum case behaves in the same way, the difference lies in the coin: from the flipping of the classical coin, the coin can assume only the "state of head or cross", determining if the drunkard turns left or right while the quantum coin, in our case the Hadamard logic gate, transforms the initial state into a superposition state.

We pass to the formal dimensions of mathematics by engaging students in the step-by-step calculation, starting from the state $|\uparrow>\otimes|0>$ and then comparing the different probabilities of the drunkard's walk problem (figure 4).

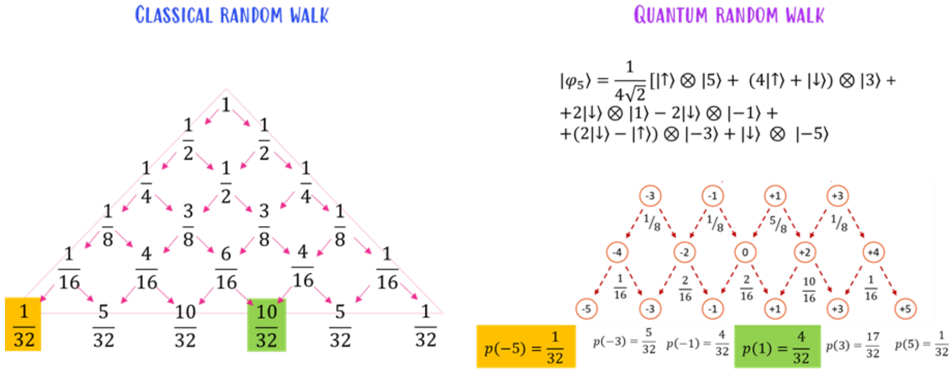


Figure 4. comparison between classical and quantum random walk in terms of probability calculation.

We then compare the model from the physical perspective (figure 5) by, for the classical case, recalling the Galton board and discussing what it means to build it, how experimentally you can get that distribution, what are the conditions to get that distribution, etc.

For the quantum case, we show a small part of an instructional video developed by Sapienza, University of Rome.

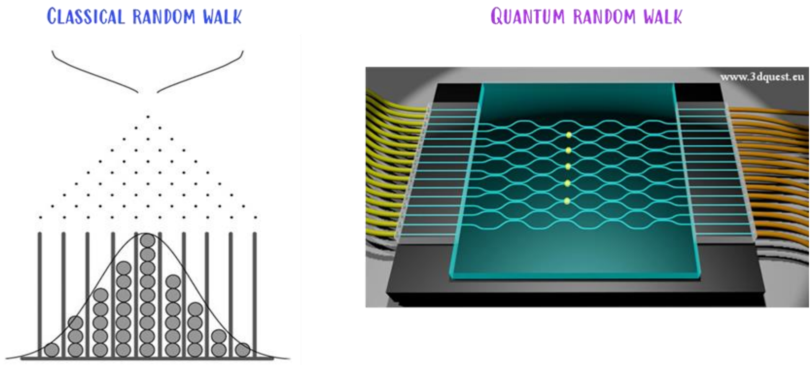


Figure 5. Comparison from a physics perspective.

The experiment that the video reproduces is a boson sampling experiment, which is an approach to quantum artificial intelligence. It is a quite complicated experiment, so we use the video only to carefully visualize the idea of sampling the information space and to stress the scope, in this case, of the superposition principle, and of the possibilities to isolate and manipulate a single quantum object.

We finally compare the classical and the quantum random walk from a computational perspective (figure 6).

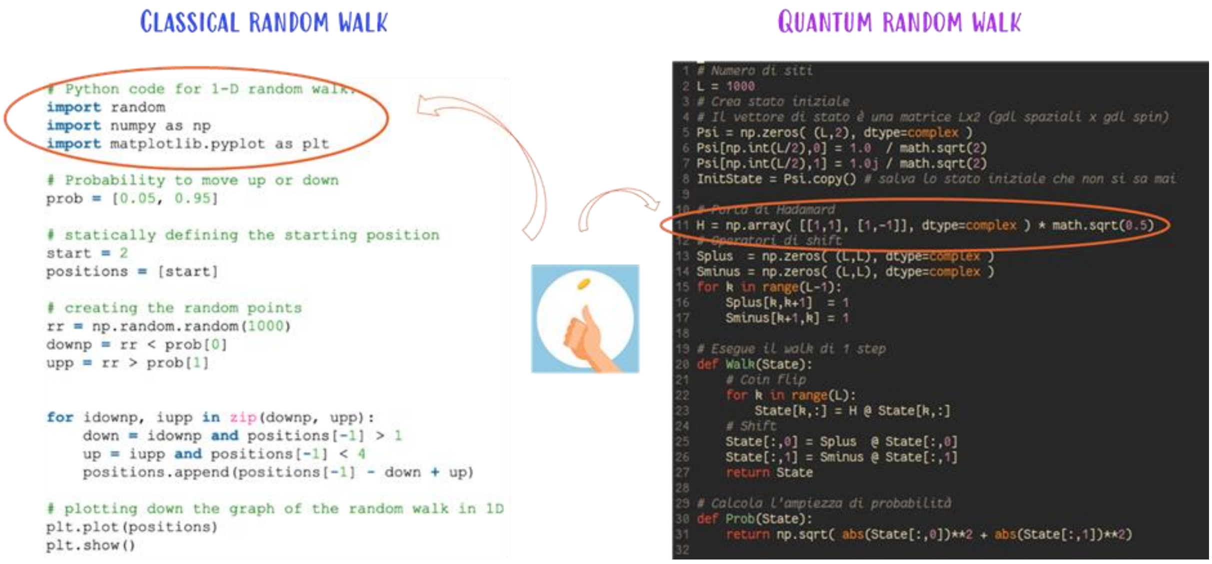


Figure 6. Comparison from a computational perspective

In the classical case, we introduce students to the problem of random number generation and to the impossibility of generating “truly” random numbers with classical computers. In the quantum case, instead, the randomness is intrinsic in the nature of the coin, presenting the Stern and Gerlach experiment as a true random numbers’ generator. This allows us to pave the way to introduce an important philosophical and epistemological debate. In the classical case, chance does not have an ontological status, so we talk about epistemic probability and apparent randomness. In the quantum case, chance has an ontological status, and we talk about ontological probability, and intrinsic randomness [28].

Finally, we show students a program in python that simultaneously implements a classical and quantum random walk, showing on the same screen the evolution of the distribution step by step. Discussing the differences, we stress that the quantum random walk "samples" the "position space" of the particle faster than the classic case.

We conclude the activity by showing some application fields of the random walk algorithm, such as research algorithms, decision-making algorithms, optimization problems, econophysics, and art.

5. Conclusion

We developed an approach for secondary school students on the Second Quantum Revolution that aimed to value the Second Quantum Revolution as a cultural revolution. We identified two main cultural aspects on which our design principles were formulated. In the present contribution, we focused on the random walk activity as an emblematic case of some aspects of the approach. The activity was designed in order to emphasize the different logic behind the functioning of the classical and quantum algorithms and the intrinsic inter-disciplinarity of the topic. The concept of probability was treated, following the XX approach, as a boundary object that, integrating the different perspectives, becomes a probability distribution, the interference effects in the random walk algorithm’s evolution allowing to touch very deep debates such as the problem of generating causal numbers, the differences between determinism and indeterminism, the differences between epistemological and ontological probability. The last phase of the MER, which consists of the implementation of the course, data collection and analysis, is described in [23]. For now, we collected data about students’ perceptions of the interdisciplinary approach. The students, in general, found the approach interesting and useful emphasizing that an integrated STEM approach can help “*to better understand*”, to “*have a more comprehensive view*” as well as to see how disciplines dialogue with each other (e.g., "*The fusion of subjects with the aim of achieving a better understanding of reality, integrating the most effective parts of them*"; "*It opens your mind! In the sense that one discipline can explain something that you don’t see in another and vice versa. This way you have a global view of a phenomenon and you can understand it much better. It’s nice to know that you can look for the answer to a question elsewhere and that the answer is still valid*."). In the next implementations, we are going to investigate how and to what extent this approach can impact students’ understanding of the key concepts as well as how the concept of probability as a boundary object can trigger boundary crossing mechanisms.

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