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Coupled Channel Approach to $(S,I) = (-2, 0)$ Baryon–Baryon Interactions from Lattice QCD

Abstract We investigate baryon–baryon interactions with strangeness $S = -2$ and isospin $I = 0$ system from Lattice QCD. In order to solve this system, we prepare three types of baryon–baryon operators ($\Lambda\Lambda$, $N\Xi$ and $\Sigma\Sigma$) for the sink and construct three source operators diagonalizing the 3×3 correlation matrix. Combining of the prepared sink operators with the diagonalized source operators, we obtain nine effective Nambu–Bethe–Salpeter (NBS) wave functions. The 3×3 potential matrix is calculated by solving the coupled-channel Schrödinger equation. The flavor $SU(3)$ breaking effects of the potential matrix are also discussed by comparing with the results of the $SU(3)$ limit calculation. Our numerical results are obtained from three sets of $2 + 1$ flavor QCD gauge configurations provided by the CP-PACS/JLQCD Collaborations.

1 Introduction

Completion of the knowledge of the generalized nuclear force, which includes not only the nucleon–nucleon (NN) interaction but also hyperon–nucleon (YN) and hyperon–hyperon (YY) interactions, brought the deeper understanding of atomic nuclei, structure of neutron stars and supernova explosions. However it is hard to know the properties of the YN and YY interactions because their scattering data in free-space are scarce.

Recently, a method to extract the *NN* potential through the NBS wave function from lattice QCD simulations has been proposed in [1]. The obtained potential is found to have desirable features, such as attractive well at long and medium distances, and the central repulsive core at short distance [1; 2]. Further applications have been done in Refs. [3; 4; 5; 6; 7].

In this work, we focus on the $S = -2, I = 0$ B–B system to seek the $\Lambda\Lambda$ interaction and to see the $SU(3)_f$ breaking effects of B–B interaction from lattice QCD simulation.

2 Formulation

The $(S,I) = (-2,0)$ baryon–baryon state consists of the $\Lambda\Lambda$, $N\Xi$ and $\Sigma\Sigma$ components in terms of low-lying baryons. Mass differences of these components are quite small, and it causes the contamination of NBS wave function from excited states. In such situation the source operator should be optimized to extract the energy eigen states through the variational method [8; 9].

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Fig. 1 Transition potentials in the SU(3) IR basis. *Box* (*red*), *diamond* (*blue*) and *circle* (*green*) symbol, respectively, correspond to the result of Set1, Set2 and Set3. The result of the SU(3) symmetric configuration is also plotted with an *X* (*brown*) symbol

The equal-time NBS wave function $\psi^{B_1 B_2}(\mathbf{r}, E)$ for an energy eigen state with E is extracted from the four point function,

$$W^{B_1 B_2}(t - t_0, \mathbf{r}) = \sum_{\mathbf{x}} \langle 0 \mid B_1(t, \mathbf{x} + \mathbf{r}) B_2(t, \mathbf{x}) \, \bar{\mathscr{I}}_E(t_0) \mid 0 \rangle,$$

where $\mathscr{I}_E$ is diagonalized wall-source operator. The transition potential matrix of 3-states coupled channel equation can be acquired in a particle basis or a SU(3) irreducible representation (IR) basis. They are connected by unitary trandformation (see in Appendix B in Ref. [7]). The non-diagonal part of potential matrix in IR basis is a good measure of the SU(3) breaking effect.

3 Numerical Setup

In this calculation we employ the $2 + 1$-flavor full QCD gauge configurations of Japan Lattice Data Grid (JLDG)/International Lattice Data Grid (ILDG). They are generated by the CP-PACS and JLQCD Collaborations with a renormalization-group improved gauge action and a non-perturbatively $O(a)$ improved clover quark action at $\beta = 6/g^2 = 1.83$, corresponding to lattice spacings of $a = 0.1209$ fm [10]. We choose three ensembles of the $L^3 \times T = 16^3 \times 32$ lattice which means the spatial volume of about $(2.0 \text{fm})^3$. Quark propagators are calculated from the spatial wall source at t_0 with the Dirichlet boundary condition in temporal direction at $t - t_0 = 16$. The numerical computation is carried out at KEK supercomputer system, Blue Gene/L. The hadron masses are shown in Table 1.

4 Result

In Fig. 1 we compare the results of potential matrix in the IR basis calculated in different configuration sets. We found the growth of repulstive core in the V^{27} potential with decreasing the light quark mass. The V_{1-27} and V_{8-27} transition potential are consistent with zero within error bar. On the other hand, it is noteworthy that the V_{1-8} transition potential which is not allowed in the SU(3) symmetric world is strengthen as the $SU(3)_f$ breaking gets larger.

5 Conclusion

We have investigated the $(S, I) = (-2, 0)$ BB state, which is known as the $\Lambda\Lambda$, $N\Xi$ and $\Sigma\Sigma$ coupled state, from lattice QCD. We have found a small transition potential between the singlet and octet state in terms of the SU(3) IR basis. Such transition can not be allowed in the SU(3) symmetric world. This method could greatly assist us to complete the knowledge of not only the generalized nuclear force but also the interaction of hadrons including mesons, baryons and quarks.

Table 1 Hadron masses in unit of [MeV] are listed

	κ_{ud}	m_π	m_K	m_N	m_Λ	m_Σ	m_Ξ
Set 1	0.13760	875(1)	916(1)	1806(3)	1835(3)	1841(3)	1867(2)
Set 2	0.13800	749(1)	828(1)	1616(3)	1671(2)	1685(2)	1734(2)
Set 3	0.13825	661(1)	768(1)	1482(3)	1557(3)	1576(3)	1640(3)

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