PROCEEDINGS OF SCIENCE

Precision studies of gluon saturation in DIS at small x: NLO corrections to SIDIS and double inclusive hadron production

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We calculate the next to leading (NLO) corrections to single and double inclusive hadron production in Deep Inelastic Scattering (DIS) at small x using the Color Glass Condensate formalism. It is shown that all UV and soft divergences cancel among various diagrams while the rapidity divergences lead to JIMWLK evolution of dipoles and quadrupoles with rapidity. Finally the collinear divergences are absorbed into parton-hadron fragmentation functions which lead to their DGLAP evolution with Q^2.

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1. Introduction

The Color Glass Condensate (CGC) formalism (see [1] for a review) is an effective theory of QCD at high energy (equivalently at small x). It relies on the experimental fact that gluon distribution function grows very fast as x gets smaller. Such a fast rise naturally leads to a hadron wave function which contains many gluons at fixed transverse size so that a high energy hadron/nucleus can be approximated as a dense state of gluons. In such a scenario the usual description of a hadron/nucleus in terms of quasi free partons is not applicable. Indeed it is more appropriate to describe this state of a hadron/nucleus via semi-classical methods. The small x gluons of a high energy hadron or nucleus are then described as a classical color field generated by the large x partons treated as static sources of color charge. This formalism has been successfully used to understand aspects of scattering of hadronic and nuclear collisions at high energies [2–6]. Nevertheless the most common approach has been to use the CGC formalism at leading order or leading log order which can not be quantitatively reliable. There has been a recent focus on calculating next to leading order corrections to QCD observables at small x using the CGC formalism. Here we describe the NLO corrections to single and double inclusive hadron production in DIS at small x [7].

2. Next to leading order corrections

At small Bjorken x DIS can be described as a two-step process due to the very long life time of the virtual photon; first the virtual photon splits into a quark antiquark and then the quark antiquark system (dipole) scatters on a dense proton or nucleus target and are produced (go on shell). In case one is interested single inclusive hadron production (or the total cross section) one would integrate out one (both) of the final state partons. The leading order production cross section (for one quark flavor) can be written as

$$\begin{aligned}\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, dy_2} \;=\;& \frac{e^2Q^2(z_1z_2)^2N_c}{(2\pi)^7}\delta(1-z_1-z_2)\int d^8\mathbf{x}\,[S_{122'1'}-S_{12}-S_{1'2'}+1] \\ & e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}}\Bigg[4z_1z_2K_0(|\mathbf{x}_{12}|Q_1)K_0(|\mathbf{x}_{1'2'}|Q_1)+ \\ & (z_1^2+z_2^2)\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2'}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2'}|}K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2'}|Q_1)\Bigg]. \qquad (1)\end{aligned}$$

Here $(\mathbf{p}, y_1)$ and $(\mathbf{q}, y_2)$ are the transverse momentum and rapidity of the produced quark and antiquark respectively while Q^2 is the photon virtuality. The first (second) term inside the square bracket gives the contribution of the longitudinally (transversely) polarized photon. All the QCD dynamics is contained in the dipoles and quadrupoles $(S_{122'1}, S_{12}, \cdots)$ that satisfy the JIMWLK evolution equation. The next to leading order corrections arise from radiation of a gluon from the quark or antiquark either before or after crossing the shock wave representing the dense target. As the full expressions are long here we show only a few of the terms (for longitudinally polarized photon) and refer the reader to for full details including the labeling of the terms and our notation.

$$
\begin{aligned}
\frac{d\sigma^L_{1\times 1}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, dy_2} =& \frac{2e^2 g^2 Q^2 N_c^2 z_2^3 (1-z_2)^2 (z_1^2 + (1-z_2)^2)}{(2\pi)^{10} z_1} \int \frac{dz}{z} \int d^{10}x\, K_0(|\mathbf{x}_{12}|Q_2) K_0(|\mathbf{x}_{1'2'}|Q_2) \\
& \Delta^{(3)}_{11'} \left[S_{122'1'} - S_{12} - S_{1'2'} + 1\right] e^{i\mathbf{p}\cdot(\mathbf{x}'_1 - \mathbf{x}_1)} e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} e^{i\frac{z}{z_1}\mathbf{p}\cdot\mathbf{x}_{1'1}} \\
\frac{d\sigma^L_{2\times 2}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, dy_2} =& \frac{2e^2 g^2 Q^2 N_c^2 z_1^3 (1-z_1)^2 (z_2^2 + (1-z_1)^2)}{(2\pi)^{10} z_2} \int \frac{dz}{z} \int d^{10}\mathbf{x}\, K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2'}|Q_1) \\
& \Delta^{(3)}_{22'} \left[S_{122'1'} - S_{12} - S_{1'2'} + 1\right] e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}} e^{i\frac{z}{z_2}\mathbf{q}\cdot\mathbf{x}_{2'2}} \\
& + \cdots\cdots
\end{aligned}
\tag{2}
$$

where the radiation kernel $\Delta^{(3)}_{ij}$ is

$$
\Delta^{(3)}_{ij} = \frac{\mathbf{x}_{3i}\cdot\mathbf{x}_{3j}}{\mathbf{x}^2_{3i}\mathbf{x}^2_{3j}}.
\tag{3}
$$

and z_1, z_2 represent the momentum fraction of the photon carried by the quark and antiquark. In general NLO corrections contain divergences that need to be understood. In our case there are four classes of divergences; Ultraviolet divergences ($k^\mu \to \infty$) in the virtual terms that are cancelled among various diagrams. Soft divergence ($k^\mu \to 0$) exist in both real and virtual corrections but are cancelled between the real and virtual contributions. Rapidity divergences appear when one takes the limit $z \to 0$ (but at finite transverse momentum). Here one introduces a "rapidity factorization scale" z_f to separate the finite contributions $z > z_f$ from the divergent ones $z < z_f$ and then absorbs the divergent contributions into rapidity evolution of dipoles and quadrupoles as given by JIMWLK evolution equation [8]. The last class of divergences are the collinear divergences where angle θ between the radiated gluon and the final state quark (antiquark) goes to zero. Here one introduces a fragmentation function in order to describe hadronization of the parton. It is then shown that collinear divergences are absorbed into DGLAP evolution [9] of the bare fragmentation functions. The resulting expressions are then finite so that the production cross section can be symbolically written as

$$
d\sigma^{\gamma^* A \to h_1 h_2 X} = d\sigma_{LO} \otimes \text{JIMWLK} + d\sigma_{LO} \otimes D_{h_1/q}(z_{h_1}, \mu^2) D_{h_2/\bar{q}}(z_{h_2}, \mu^2) + d\sigma^{\text{finite}}_{NLO}.
\tag{4}
$$

In order to calculate the next to leading order corrections to single inclusive hadron production in DIS (SIDIS) at small x one can start with the above expressions for dihadron production and then integrate out one of the final state partons. The most interesting (dominant ones) are the contributions one gets after integrating out the gluon and antiquark so that the quark is produced which then hadronizes. The leading order cross section is given by

$$
\begin{aligned}
\frac{d\sigma^{\gamma^* A \to \bar{q} X}}{d^2\mathbf{q}\, dy_2} =& \frac{e^2 Q^2 z_2^2 (1-z_2) N_c}{(2\pi)^5} \int d^6\mathbf{x}\, \left[S_{22'} - S_{12} - S_{12'} + 1\right] \\
& e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \Bigg[4z_2(1-z_2) K_0(|\mathbf{x}_{12}|Q_2) K_0(|\mathbf{x}_{12'}|Q_2) + \\
& \left[z_2^2 + (1-z_2)^2\right] \frac{\mathbf{x}_{12}\cdot\mathbf{x}_{12'}}{|\mathbf{x}_{12}||\mathbf{x}_{12'}|} K_1(|\mathbf{x}_{12}|Q_2) K_1(|\mathbf{x}_{12'}|Q_2) \Bigg]
\end{aligned}
\tag{5}
$$

while the next to leader order corrections are (for antiquark production)

$$\frac{d\sigma^L_{2\times 2}}{d^2\mathbf{q}\,dy_2} = \frac{2e^2g^2Q^2N_c^2}{(2\pi)^8 z_2}\int_0^{1-z_2}\frac{dz}{z}(1-z_2-z)^2(z+z_2)^2\left[z_2^2+(z+z_2)^2\right]\int d^8\mathbf{x}$$
$$K_0(|\mathbf{x}_{12}|Q_1)K_0(|\mathbf{x}_{12'}|Q_1)\Delta^{(3)}_{22'}[S_{22'}-S_{12}-S_{12'}+1]e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}}e^{i\frac{z}{z_2}\mathbf{q}\cdot\mathbf{x}_{2'2}}+\cdots\cdots \tag{6}$$

The cancellation or absorption of divergences into evolution of physical quantities proceed very similarly to the case of dihadron production so we will not repeat them here. It is worth emphasizing that one will be able to reach smaller x in SIDIS than in dihadrons so that they may be probed in a wider kinematic region in the proposed Electron Ion Collider (EIC) [10]. Furthermore some effects such as Sudakov suppression which are unavoidable in the back to back dihadron production and may dominate over saturation effects may be avoidable in SIDIS by working in the kinematics where the produced hadron transverse momentum p_t is similar to photon virtuality Q. Nevertheless large Sudakov logs appearing in the kinematic region $Q^2 \gg p_t^2$ in SIDIS can be resummed in a rather straightforward manner [11]. With the full NLO results available for SIDIS and inclusive dihadron production in DIS it is now feasible to apply these results to phenomenological studies of particle production appropriate to EIC kinematics. One will need to use numerical methods as many of the integrals present can not be performed analytically. One will also need initial conditions for rapidity evolution of dipoles and quadrupoles (in case of dihadrons), for which there are various models available in the market. Implementing both saturation and Sudakov effects in SIDIS will be very interesting as one will be able to theoretically disentangle the two effects which will allow us to understand the relevant QCD dynamics appropriate in a given kinematic region. Nevertheless it will also be important to fully account for contribution of large x partons, preliminary steps in this direction are taken in [12].

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