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https://doi.org/10.3390/sym15122193

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The Horizons in Circular Accelerated Motions and Its Consequences

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Abstract: In this work, we study the existence of horizons in circular accelerated motions and its consequences. One particular case is the existence of two horizons in any uniform circular motion. The radiation of the Poincaré invariant vacuum is related to the spontaneous breakdown of the conformal symmetry in Quantum Field Theory The main consequence of the existence of these horizons is the Unruh radiation coming from such horizons. This consequence allows us to study the possible experimental detection of the Unruh radiation in such motions. The radiation of the Poincaré invariant vacuum is related to the spontaneous breakdown of the conformal symmetry in Quantum Field Theory. This radiation is associated with an effective temperature that can be detected using an Unruh–DeWitt detector. In fact, this effective temperature at the relativistic limit depends linearly with respect to the proper acceleration. However, in general, this dependence is not linear, contrary of what happens in the classical Unruh effect. In the relativistic limit and high density case, the uniform circular motion becomes a rotating black hole. This allows for future studies of pre-black hole configurations.

Keywords: Unruh radiation; circular motions; quantum fluctuations; uncertainty principle

PACS: 04.60. m Quantum gravity; 04.62.+v Quantum fields in curved space-time; 04.70.Dy Quantum aspects of black holes; evaporation; thermodynamics

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Citation: Giné, J. The Horizons in Circular Accelerated Motions and Its Consequences. *Symmetry* **2023**, *15*, 2193. https://doi.org/10.3390/sym15122193

Academic Editors: Stefano Profum and Sergei D. Odintsov

Received: 22 September 2023
Revised: 21 November 2023
Accepted: 23 November 2023
Published: 12 December 2023

1. Introduction

An observer in an accelerated frame sees quantum fluctuations, which results in the so-called Unruh effect predicted in [1] using the quantum field theory. Indeed, the Unruh effect was first described by Fulling [2] and Davies [3]. In order to describe the Unruh effect different approaches have been developed. The original one is based on Bogolyubov transformations. In this approach the field quantization in the Rindler space is considered, see [2,4]. Using the concept of Unruh-DeWitt detector another approach study the response of these accelerating detectors under the quantum fluctuations of the fields. In the present work we consider this operational approach. Another one use operator algebra in Modular Theory. In this approach the concept of KMS (Kubo-Martin-Schwinger) takes a dominant role, see [5–7]. Finally the last one use the Thermalization Theorem [8,9]. This last approach uses the Quantum Field Theory (QFT) in curved space time via the path integral. Using this Thermalization Theorem approach a restoration of the symmetry can be analyzed. The internal continuous symmetry is spontaneously breaking by an accelerating observer. Indeed, the destabilization of the Poincaré vacuum provokes the radiation due to special conformal transformations associated to the accelerations, see [10]. The special conformal transformations are given by

$$x^\mu \to x'^\mu = \frac{x^\mu + k^\mu x^2}{1 + 2kx + k^2 x^2} \tag{1}$$

which are transitions to accelerated observers systems with acceleration $a = 2k$ and the conformal accelerations $k^\mu \in \mathbb{R}^4$. More specifically, the Poincaré vacuum is described as a coherent state of conformal zero modes. Under special conformal transformations the Poincaré vacuum is unstable, unlike for inertial observers it is undetectable.

The main idea is that the quantum vacuum is not really empty for all observers. In reality, the quantum vacuum is filled with zero-point quantum field fluctuations. In fact, other non-zero vacuum expectation values, lead to observable consequences. For instance the zero-point energy is the responsible of the Casimir effect. The behavior of the Universe on cosmological scales is also affected by the zero point energy. In fact, vacuum energy contributes to the cosmological constant and, consequently, the expansion of the universe. It has recently been detected that the universe is expanding at an accelerated rate and dark energy is the widely accepted proposal to explain such observations, see [11] and references therein.

The Unruh effect can be theoretically detected using what is known as Unruh–DeWitt detector. This Unruh–DeWitt detector in its motion must be weakly coupled to the quantum scalar field, see [4,12,13]. In such works, the interaction between a quantum field and an accelerating particle detector is analyzed with respect to an inertial observer. It is shown in detail how the absorption of a Rindler particle corresponds to the emission of a Minkowski particle. Under a uniform proper linear acceleration a inside a Minkowski space, the Unruh–DeWitt detector perceives an Unruh temperature

$$T_{\text{U}} = \frac{\hbar a}{2\pi k_{\text{B}} c}, \tag{2}$$

where $\hbar$ is the reduced Planck constant, c is the speed of light, and k_{B} is the Boltzmann constant. The experimental verification of the Unruh effect is extremely difficult using Equation (2) since to produce a temperature of $T \sim 1$ K, a linear acceleration of $a \sim 10^{20}$ m/s^2 is required [14]. Consequently a confirmation of the Unruh effect has remained elusive because of the high magnitude of the acceleration required [15]. The experimental detection of the Unruh effect has an intrinsic interest: first, because it is proof of the existence of the quantum fluctuations, and second, because the connections with the Hawking effect [16], the early universe, and the quantum effects are responsible for the origin and structure of the present Universe [17,18].

The Unruh radiation appears due to the existence of a horizon in the Rindler spacetime associated with the accelerated observer. In fact, via the equivalence principle Unruh radiation, the Hawking radiation of the black holes [16] is connected, as given by

$$T_{\text{H}} = \frac{\hbar c^3}{8\pi k_{\text{B}} G M}, \tag{3}$$

where G is the gravitational constant and M is the mass of the black hole. In fact, in Schwarzschild coordinates (t, r, θ, ϕ), the line element for proper time has the form

$$-c^2\, d\tau^2 = -\left(1 - \frac{r_{\text{s}}}{r}\right) c^2\, dt^2 + \left(1 - \frac{r_{\text{s}}}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{4}$$

where $d\Omega^2 = \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$, whose singularity gives the location of the horizon, and the Schwarzschild radius of the massive body r_s is given by $r_s = 2GM/c^2$.

In this work, we focus one the case of a uniform circular motion having constant angular speed Ω. This case is interesting from both theoretical and experimental points of view. The uniform circular motion can give experimental verification of the Unruh radiation as is described in [19,20]. For a linear acceleration, it is hard to achieve accelerations of order 10^{20} m/s^2. However, for circular accelerations, this is possible. For instance, storage rings at LEP can have $a \sim 10^{23}$ m/s^2, giving Unruh temperatures $T \sim 1000$ K [14].

In the case of uniform circular classical motion, the tangential speed $v = \Omega\rho$ increases when the radius ρ increases. The classical centripetal acceleration is given by $a_c = v^2/\rho$. However, as we allow tangential speeds close to the speed of light c, we use the Special Relativity applied to the case of circular motion, and, in this case, the proper acceleration is $a_c = \gamma^2\rho\Omega^2$, where $\gamma = (1 - v^2/c^2)^{-1/2}$.

It is expected that the Unruh temperature associated with the uniform circular motion will be expressed in terms of the proper acceleration in the same way as for the uniformly linear acceleration case. Indeed, this is the case as we recall in the next section, see the complete development in [4,21]. However, as we will see, it is not true that for uniform circular motion, Formula (2) would directly predict the Unruh temperature. More specifically, it is not true that the Unruh temperature for uniform circular motion is given by

$$T_c = \frac{\hbar\gamma^2\rho\Omega^2}{2\pi k_B c}. \tag{5}$$

The Unruh effect, and its analog using the equivalence principle, as well as the Hawking effect in black holes, are often explained in terms of the geometric notion of an event horizon and the quantum fluctuations near it, see [1,16]. However, it was believed that there are no horizons for motions with non-uniform accelerations or finite-time accelerations, because in such movements, the acceleration changes direction or does not exist after a period. Although the Unruh effect is a kinematic effect, the line element associated with each movement can have singularities that can become event horizons.

In this work, we prove the existence of two horizons for the case of the uniform circular motion, and, in general, for any accelerated circular motion, and we analyze its consequences.

One important difference between the classical Unruh effect associated with the uniform proper linear acceleration and the one associated with non-uniform motions is that, in the last case, the detector senses a non-thermal radiation, see [22,23]. Nonthermality increases the further we deviate from a uniform linear acceleration. It is important to emphasize that a detector that is not uniformly linear accelerated (in circular or other oscillatory motions) belongs to a non-equilibrium state. In such cases, the detector registers non-thermal radiations, which becomes thermal in a limit condition such as linear uniform acceleration.

The difference between linear and circular uniform motion lies in the difference between linear acceleration and angular acceleration. In the first case, the proper acceleration is fundamental. In the second case, the angular acceleration and the radius of the orbit, of which is equivalent to the cross-radial velocity, are fundamental. For the linear uniform motion, the velocity asymptotically approaches the speed of light, implying the appearance of an event horizon, see [22,23]. For the uniform circular motion, the velocity direction changes but its magnitude remains constant, and it seems that there is no event horizon. However, as we will see, there appears an effective Unruh temperature which implies that the quantum fluctuations are coming from an event horizon. This event horizon appears because for large values of the radius orbit, the cross-radial velocity tends to the speed of light. The true existence of the event horizon can be heuristically explained using the relativity principle. Assume that the detector is in a circular uniform motion. Then, in the comoving frame with the detector is all the universe that is rotating in the opposite direction. However, far away from the detector is the cross-radial velocity that tends to the speed of light, which implies that outside of this limit, there is a region not connected to the detector (the region outside of the event horizon).

In the next section, we review the Unruh–DeWitt detector and its application to the uniform circular motion. Later, we study the existence of two horizons for the case of the uniform circular motion and we analyze its consequences. Finally, a conclusion and discussion section will be given in the last section of the work. In the next section, we use units in which $\hbar = c = k_B = 1$ for simplicity.

2. Unruh–DeWitt Detector Method Applied to Uniform Circular Motion

In this section, we revisited the Unruh–Dewitt detector method for the computation of the Unruh temperature in the case of circular motion, following the general setting presented in [4,12,13]. We assume a detector passing through a region permeated via a quantum scalar field $\phi(x^\mu(\tau))$, where $x^\mu(\tau)$ is the trajectory along the Minkowski spacetime with a proper time τ. We also assume that the detector is coupled with the scalar field via the coupling $g\mu(\tau)\phi(x^\mu(\tau))$, where g is the coupling constant and $\mu(\tau)$ is defined as always as $\mu(\tau) = e^{iH_0\tau}\mu(0)e^{-iH_0\tau}$.

Then, when the detector accelerates, it will measure the energy transition from the energy E_0 of the ground state to the high energy E. The transition probability rate per unit proper time is defined as

$$\mathcal{P}(E) = g^2 \sum_E |\langle E|\mu(0)|E\rangle|^2 \mathcal{F}(E), \tag{6}$$

where the response function $\mathcal{F}$ is

$$\mathcal{F}(E) = \int_{\infty}^{-\infty} e^{-i(E-E_0)\Delta\tau}\, \mathcal{G}^+(\Delta\tau)\, \mathrm{d}(\Delta\tau), \tag{7}$$

where $\mathcal{G}^+(\Delta\tau) = \mathcal{G}^+(x(\tau), x'(\tau)) = \langle 0|\phi(x)\phi(x')|0\rangle$ is the so-called positive frequency Wightman function, where $\Delta\tau$ is given by $\Delta\tau = \tau - \tau'$. The information about the particular spacetime trajectory is embedded in $\mathcal{G}^+(x(\tau), x'(\tau))$.

2.1. Detector in a Linear Uniform Acceleration

For a detector along a linear acceleration a, the trajectory in Rindler coordinates is

$$x^\mu(\tau) = \left(\frac{1}{a}\sinh(a\tau), \frac{1}{a}\cosh(a\tau), 0, 0\right), \tag{8}$$

where we take $\bar{Y} = \bar{Z} = 0$ for simplicity, and the Wightman function is

$$\mathcal{G}_L^+(\Delta\tau) \propto \frac{a^2}{\sinh^2(\frac{a}{2}(\Delta\tau - i\epsilon))}, \tag{9}$$

which leads to the response function

$$\mathcal{F}_L^+(E) \propto \left(1 - \exp\left(\frac{2\pi\Delta E}{a}\right)\right)^{-1}, \tag{10}$$

where $\Delta E = E - E_0$. Hence, the Unruh temperature can be deduced from the comparative of the previous expression with the Planck distribution, see [4,12,13], and we obtain the well-known result $T_{\text{U}} = a/(2\pi)$.

2.2. Detector in a Uniform Circular Motion

Now, we compute the temperature measured using an Unruh–DeWitt detector in a circular motion. We assume that the detector is rotating in a uniform circular motion of radius ρ around the z-axis and with a finite constant angular velocity Ω. The trajectory is given by

$$x^\mu(\tau) = (t, x, y, z) = (\gamma\tau, \rho\cos(\gamma\Omega\tau), \rho\sin(\gamma\Omega\tau), 0). \tag{11}$$

Now, by taking cylindrical coordinates (t, ρ, φ, z), the line element

$$\mathrm{d}s^2 = -(1 - \Omega^2\rho^2)\mathrm{d}t^2 + 2\Omega\rho^2\mathrm{d}\varphi\mathrm{d}t + \mathrm{d}\rho^2 + \rho^2\mathrm{d}\varphi^2 + \mathrm{d}z^2, \tag{12}$$

describes the rotating frame. This line element has a singularity which corresponds to the maximum radius of ρ given by the value $\rho_{max} = 1/\Omega$ [24]. Indeed, the proper acceleration

$a_c = \gamma^2\rho\Omega^2$ vanishes for $v = 0$ that corresponds to $\rho = 0$, and tends to infinity when $v \to 1$ that corresponds to the radius $\rho \to \rho_{max}$. The explicit form of the Wightman function is

$$\begin{aligned} \mathcal{G}_C^+(\Delta\tau) &\propto \frac{1}{\gamma^2(\Delta\tau - i\epsilon)^2 - 4\rho^2\sin^2(\frac{\gamma\Omega}{2}\Delta\tau)} \\ &\approx \frac{1}{\gamma^2(\Delta\tau - i\epsilon)^2}\Big(1 + \frac{1}{12}(a_c\Delta r)^2 \\ &- \frac{1}{360v^2}(a_c\Delta r)^4 + \cdots\Big)^{-1}, \end{aligned} \tag{13}$$

which is the expansion of $\mathcal{G}_C^+$ up to $\mathcal{O}(a_c^4)$. However, it is non-trivial to construct the response function rate which is obtained in [25–27] using slightly different ways. Surprisingly, in [25], and using the appropriate definition of a rotating vacuum state where the region beyond the light cylinder is circumvented, the detector fails to respond and no Unruh radiation is detected. Nevertheless, in the relativistic limit [19,20], and taking $\mathcal{G}_C^+$ up to $\mathcal{O}(a_c^2)$, the response function is

$$\mathcal{F}_C^+(E) \propto a e^{-2\sqrt{3}\frac{\Delta E}{a}}, \tag{14}$$

and although it is not Planckian, one can define the effective temperature for the circular motion

$$T_C = \frac{a}{2\sqrt{3}}. \tag{15}$$

Following [21], there are two important differences between the Unruh effect for linear accelerations and for circular accelerations. The first is that the spectrum is not exactly thermal for the circular motion. The second is that for the circular case, the temperature does not have a simple expression as (2), and in the relativistic limit, one could define an effective temperature given by (15). This implies that $T_C/T_U = \pi/\sqrt{3} \approx 1.8$. In the recent work [23], analytic results for T_C for a massless scalar field in $3+1$ and $2+1$ spacetime dimensions in several asymptotic regions of the parameter space are given.

Note that for large values of the radius in the circular motion, the motion looks like the case of a uniform linear acceleration, but with a speed perpendicular to the acceleration direction. This case is investigated in [28–30] and the temperature for the drifted Rindler motion is obtained and compared with the temperature for the linear acceleration case.

3. The Horizons in the Uniform Circular Motion

In the classical Unruh effect for a linear uniform acceleration, it is well known that a horizon exists in the Rindler spacetime at distance c^2/a to any trajectory given a fixed value of a, see Figure 1. The horizon is a barrier for all the radiation coming from behind the horizon, and the pair of quantum fluctuations near the horizon produce the so-called Unruh radiation.

In the uniform circular motion, the acceleration has a constant modulus but the direction changes instantly according to the angular velocity Ω. The trajectories are at distance $\rho = a_c/(\gamma^2\Omega^2)$, where ρ is the radius of the circumference described by the circular motion. The shape of the horizons is obtained by taking Rindler coordinates. The horizons are defined by the Rindler coordinates and their relation with the local coordinates (t, x, y, z) that define the circular motion of radius ρ, with centripetal acceleration a_c.

The Rindler coordinates are defined by

$$\bar{T} = x\sinh(\alpha t),\ \bar{X} = x\cosh(\alpha t),\ \bar{Y} = y,\ \bar{Z} = z, \tag{16}$$

where the proper acceleration is $\alpha = a_c = \gamma^2\rho\Omega^2$ and the circular motion in local coordinates is described by

$$x = \rho cos(\gamma\Omega\tau), \qquad y = \rho sin(\gamma\Omega\tau), \tag{17}$$

where $x^2 + y^2 = \rho^2$.

In the classical Unruh effect for a linear uniform acceleration, from the Rindler coordinates (16), we have $x^2 = \bar{X}^2 - \bar{T}^2$ and the Rindler horizon corresponds to the locus $x = 0$, that is, $\bar{X}^2 = \bar{T}^2$, which consists of two null half-planes, ruled by a null geodesic congruence, see Figure 1.

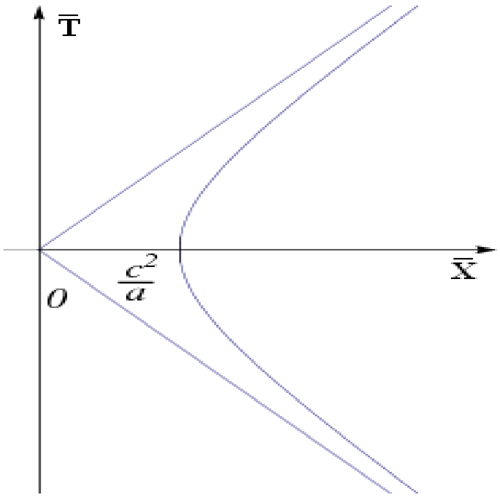

Figure 1. The Rindler horizon to its left (at a distance c^2/a away).

In the case of circular movement from (16) and (17), we have

$$\rho^2 = x^2 + y^2 = \bar{X}^2 + \bar{Y}^2 - \bar{T}^2. \tag{18}$$

Taking (18) into account, the case $\rho = 0$, which corresponds to $x = y = 0$ and $\alpha = 0$, gives the infinite conus

$$\bar{X}^2 + \bar{Y}^2 = \bar{T}^2, \tag{19}$$

which is the equation of one horizon. Figure 2 corresponds with the infinite conus passing through the origin. However, the important case is the case when ρ takes the maximum values $\rho_{max} = 1/\Omega$, that corresponds to a proper acceleration a_c tending to infinite for $v \to 1$. In this case, we have the horizon given by

$$\rho_{max}^2 = \bar{X}^2 + \bar{Y}^2 - \bar{T}^2. \tag{20}$$

which is a one-sheeted hyperboloid and is the second horizon. Figure 1 corresponds to the exterior hyperboloid. Figure 3 is the Rindler chart taking $\bar{Y} = 0$ in Figure 2. If we consider hyperboloids $\rho^2 = \bar{X}^2 + \bar{Y}^2 - \bar{T}^2$ inside the two horizons, that is, for ρ satisfying $0 \leq \rho \leq \rho_{max}$, none of the points of such hyperboloids can ever receive light signals from events outside from the two horizons. Hence, an accelerating observer in one hyperboloid allowed us to see a radiation coming from these two horizons (19) and (20).

We can heuristically deduce the effective temperature following the same reasonings made in [31,32]. The simple derivation is based on the uncertain principle. The uncertainty in the position of a particle of the Unruh radiation captured using the Unruh–DeWitt detector is given by the unique information that is coming from the horizon (19) The maximum distance to the horizon is $\rho_{max} = c/\Omega$. However, we have the contribution of the other horizon (20). Therefore, we assume that

$$\Delta x = \kappa \frac{\pi c}{\Omega}. \tag{21}$$

where κ is an arbitrary constant. Taking into account that $\rho = a_c/(\gamma^2\Omega^2)$, the corresponding proper acceleration for ρ_{max} is $a_{c\,max} = c\Omega\gamma^2$ Hence, the uncertainty principle takes the form

$$\Delta x \Delta p = \kappa \frac{\pi c^2 \gamma^2}{a_c} \Delta p \simeq \hbar/2. \tag{22}$$

From here we have $\Delta p \simeq (\hbar a_c)/(2\kappa\pi c^2\gamma^2)$. Since the energy of the photon is given by $E = pc$, we have $\Delta E = c\Delta p$ and

$$\Delta E \simeq \frac{\hbar a_c}{2\kappa\pi c}, \tag{23}$$

for $v \ll c$, which implies $\gamma \approx 1$. The radiation in the circular motion is not thermalized but, at first approximation, we can assume, in order to compute the effective temperature, that $E = k_B T$, where k_B is the Boltzmann constant and Equation (23) becomes

$$\Delta T \simeq \frac{\hbar a_c}{2\kappa\pi c\, k_B}. \tag{24}$$

Then, comparing Equations (15) with (24), we obtain that the value of κ in Equation (24) is $\kappa = \sqrt{3}/\pi \approx 0.55$. In fact, the temperature that sees an observed is greater than in the linear movement. Recall that in the previous section we have obtained $T_c/T_U = \pi/\sqrt{3} \approx 1.8$.

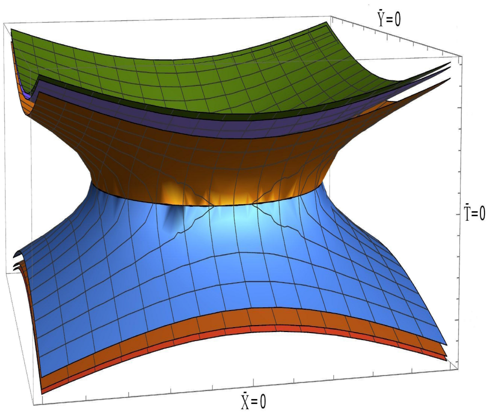

Figure 2. Two hyperboloids and the infinite conus.

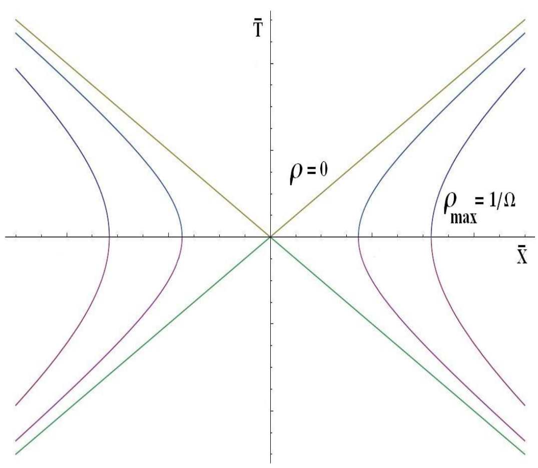

Figure 3. Rindler chart taking $\bar{Y} = 0$ in Figure 1.

4. The Uniformly Accelerated Circular Motion

We consider now the uniformly accelerated circular motion. In this case, the angular velocity is not constant and is indeed $\Omega = a_\varphi \tau$, where a_φ is a constant angular acceleration.

This implies a centripetal acceleration given by $a_c = \gamma^2 v^2/\rho = \gamma^2\Omega^2\rho$ and also a constant tangential acceleration $a_t = a_\varphi \rho$. The total acceleration is $a = \sqrt{a_t^2 + a_c^2}$ that does not have a constant modulus and the direction also changes instantly. The circular motion is also described by

$$x = \rho cos(\gamma\Omega\tau), \qquad y = \rho sin(\gamma\Omega\tau), \tag{25}$$

where $x^2 + y^2 = \rho^2$. We have taken the initial angular velocity $\Omega_0 = 0$. The trajectory is also given by (11). We also have the same line element in cylindrical coordinates that undergoes a uniform circular motion. Therefore, the singularity also corresponds to the maximum radius $\rho_{max} = c/\Omega = c/(a_\varphi \tau)$. However, we have a moving singularity because it depends on τ. Hence, taking the Rindler coordinates defined by (16), where the proper acceleration is $\alpha = a$, we obtain the same results as in the previous case. However, when τ increases the cross-radial velocity, it tends to the seep of light, that is, the event horizon is getting closer and closer to the trajectory. The event horizon is moving until it collapses with the trajectory in the limit when $v \to c$.

5. The Oscillating Motions

In this section we revisited the results for the detectors moving along oscillating movements. Accelerated detectors under these regimes will also reproduce Unruh behaviors with associate effective temperatures. However, it is difficult here describe the associate event horizons.

We consider the sinusoidal motion, which indeed is a projection of a circular motion before described onto the $t - z$ subspace. The trajectory is given by

$$x^\mu(\tau) = (\gamma t, 0, 0, -\rho\cos(\gamma\Omega t)). \tag{26}$$

where ρ is the oscillation amplitude, and Ω is the oscillation frequency in coordinate time. The restriction $\rho\gamma\Omega < 1$ is necessary in order for the motion to remain time-like. The can compute explicitly the proper time as a function of the coordinate time t using the elliptic integral of the second kind $\tau = (\gamma\Omega)^{-1} E(\gamma\Omega t, \rho^2\gamma^2\Omega^2)$, from where we can find numerically $t(\tau)$. The directional proper acceleration is $a = \rho\gamma^2\Omega^2\cos(\gamma\Omega t)(1 - (\rho\gamma\Omega)^2\sin^2(\gamma\Omega t))^{-3/2}$. Hence the proper acceleration $\alpha = |a|$. The period of oscillations is $T = 2\pi/(\gamma\Omega)$ and in the proper-time is given by $\tau_p = (\gamma\Omega)^{-1}E(2\pi, \rho^2\gamma^2\Omega^2)$. The time averaged proper acceleration, i.e., the acceleration over one period of oscillation is expressed as

$$\bar{a} = \frac{\gamma\Omega\tanh^{-1}(\rho\gamma\Omega)}{E(2\pi, \rho^2\gamma^2\Omega^2)}. \tag{27}$$

Then by analogy with the case of linear uniform acceleration one expects that the effective temperature at late times recorded by an oscillating detector satisfies

$$\bar{T}_{\text{eff}} = \frac{\hbar\bar{a}}{2\pi c k_B}. \tag{28}$$

However such formula does not always work even at late times as it is proved in [22].

6. Conclusions and Discussion

We have reviewed the computation of the effective temperature for the circular motion. From a theoretical point of view, the Unruh temperature in a circular motion increases from the zero temperature at $\rho = 0$ that corresponds to $a_c = 0$ up to a finite value for $\rho_{max} = c/\Omega$. However, this value of $\rho = 0$ is not allowed because it defines one of the horizons of the uniform circular movement. Moreover, in the case $\rho_{max} = c/\Omega$, that correspond to a proper acceleration a_c tending to infinite for $v \to c$, the temperature is finite and close to zero, see [28]. Therefore, when the proper acceleration goes to infinity, the Unruh temperature remains finite as long as the velocity approaches the speed of light.

Indeed, the Killing vector is time-like inside the hyperboloid surface, space-like beyond the hyperboloid surface, and null at the hyperboloid surface which corresponds to $\rho = \rho_{max}$, see [33]. Consequently, since there is no object that travels faster than the speed of light, then no object can be at rest with respect to the rotating frame beyond the hyperboloid surface $\rho = \rho_{max}$.

Such a region is beyond the hyperboloid surface and $\rho = \rho_{max}$ has a similar behavior to the region inside the ergosphere of a rotating black hole [34]. We recall here that in a rotating black hole, the particle creation can happens in two regions, close to the event horizon and also in the ergosphere, see [35,36]. Therefore, for a detector undergoing a circular motion, the computed Unruh effect can have a close relation with the ergoregion effect of a rotating black hole [37,38], where it should be possible to also define a temperature associated with this ergosphere region. Indeed, in the appendix of [38], it is explicitly shown how the metric of the accelerated frame of a planar motion is identical to the limiting form of the Kerr metric for points close to the equator of the rotating black hole when the mass of the hole tends to infinity.

Moreover, in the case of a rotating black hole, at limit case $\rho_{max} \to r_s$, we have that $r_s \sim \rho \sim \rho_{max}$ and the speed v tends to the light speed. The two horizons collapse into a classical horizon of a black hole. Indeed, in Rindler coordinates, the horizon at $\rho = 0$ is unrealistic for a black hole present. The analysis made before is now the following. The uncertainty in the position is now $\Delta x = 2\pi r_s$ and then

$$\Delta x \Delta p = 2\pi r_s \Delta p \simeq \hbar/2. \tag{29}$$

Since the energy of the photon is given by $E = pc$, we have $\Delta E = c\Delta p$ and

$$\Delta E \simeq \frac{\hbar c}{4\pi r_s}. \tag{30}$$

Finally, taking into account that in the limit we have thermalized radiation with $E = k_B T$, Equation (30) becomes

$$\Delta T \simeq \frac{\hbar c^3}{8\pi k_B G M}. \tag{31}$$

and we obtain the Hawking temperature of a black hole given by Equation (3).

Moreover, while it does not exist in nature, a Schwarzschild black hole is theoretically an object. This is because the mass accretion for constituting any future black hole is formed by mass with some initial speed that is captured in a circular or elliptical movement falling down to the central mass, producing a rotating black hole, or what is call a Kerr black hole, with a privileged accretion plane. Then, the study of the radiation around a circular motion and other accelerated motions will be the basis of further studies about the origin of black holes and their evolution.

Funding: The author is partially supported by the Agencia Estatal de Investigación grant PID2020-113758GB-I00 and an AGAUR (Generalitat de Catalunya grant number 2021SGR 01618.

Data Availability Statement: Data are contained within the article.

Acknowledgments: The author is grateful to the referees for their valuable comments and suggestions to improve this paper.

Conflicts of Interest: The author declares no conflict of interest.

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