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ABSTRACT

Current hierarchical control quantum communication protocols are mostly limited to linear bidirectional architectures with only two communication agents, lacking cyclic information exchange capabilities and flexible scalability for large-scale scenarios. This limitation impedes the practical deployment of complex quantum communication networks. To address this issue, a novel hierarchical control bidirectional cyclic hybrid quantum information transmission protocol for an arbitrary single-qubit state is proposed. The protocol comprises six participating entities: three communication agents (Alice, Bob, and Charlie) at different levels and three control agents (David, Emma, and Frank) with a clear permission hierarchy. The bidirectional communication tasks of each communication agent require matching control authorization corresponding to their level: high-level communication agents only need David's assistance, mid-level ones require joint control from David and Emma, and low-level ones demand collaborative authorization from all three control agents to complete their quantum teleportation and remote state preparation tasks. Furthermore, the proposed six-party protocol is extended to a 2N-party scenario involving N control agents and N communication agents at different levels, adapting to the needs of large-scale quantum communication networks. This work provides a flexible and scalable theoretical framework for constructing hierarchical quantum communication networks with cyclic information exchange.

I. INTRODUCTION

Quantum entanglement is a distinctive characteristic of quantum mechanics that sets it apart from classical physics, and it serves as the core resource and theoretical cornerstone of quantum information science. This non-local correlation characteristic breaks the limitation in classical physics that "information transmission relies on physical carriers," laying a unique physical foundation for the secure and efficient transmission of quantum information. It directly promotes the formation and development of quantum communication technologies such as quantum key distribution,[1–4] quantum teleportation (QT),[5–8] quantum secure direct communication,[9–13] quantum secret sharing,[14–17] and quantum superdense coding.[18–21]

As one of the important methods in quantum communication, QT utilizes entanglement and classical communication to remotely transfer the unknown quantum state of one particle to another particle. In 1993, a team of six physicists including Bennett, based on in-depth research on the properties of quantum mechanics and the possibility of information transmission, first proposed the QT scheme.[22] In 1997, the feasibility of this theory was verified when QT was first realized in the laboratory.[23] As another key protocol in the field of quantum information, remote state preparation (RSP) proposed in 2000[24] also relies on shared entanglement and classical communication. It enables the sender to assist the receiver in locally preparing their qubit into a specific quantum state, without the sender needing to pre-hold the qubit that carries the target state. Unlike quantum teleportation, which is primarily used to transmit unknown quantum states, in remote state preparation, the sender fully knows the target quantum state to be prepared—a feature that allows it to achieve the remote construction of quantum states with lower classical communication costs. To enhance the security and controllability of communication processes, QT and RSP protocols with one or more control agents have been successively proposed. In early research on such protocols, there was usually only a single control agent,[25–28] and the communication process could only proceed with the consent and assistance of this agent. Later, the emergence of hierarchical control communication protocols[29–34] assigned different numbers of control agents to communication agents with different permissions or identity levels, resulting in protocols with greater flexibility and broader application scenarios. With the rapid development of quantum technology, both QT and RSP protocols based on various multi particle entangled state channels have made rapid progress in both theoretical and experimental fields in recent years.[35–41]

In recent years, significant progress has been made in hybrid quantum communication protocols that integrate QT and RSP.[42–47] In 2017, Fang *et al.*[42] proposed a bidirectional hybrid quantum information transmission (BHQIT) protocol based on a seven-qubit entangled state. In this protocol, with the assistance of a control agent, Alice can teleport an unknown single-qubit state to Bob, while Bob can remotely prepare a known two-qubit state for Alice. In 2018, Ma *et al.*[43] proposed two BHQIT protocols based on six-qubit and nine-qubit entangled states, respectively, which enable Alice to teleport an unknown single-qubit state while Bob can remotely prepare arbitrary single-qubit and two-qubit states for Alice. Later, Huo *et al.*[44] proposed a protocol based on a seven-qubit entangled state. In this protocol, with the assistance of the control agent Charlie, Alice can teleport an arbitrary two-qubit state to Bob, while Bob can remotely prepare a known single-particle state for Alice. In 2019, Jiang *et al.*[45] proposed a cyclic bidirectional hybrid quantum information transmission (CBHQIT) protocol involving three participants, utilizing a quantum channel composed of Bell states and GHZ states. In this scheme, each participant can send an unknown single-qubit state to the other two participants via QT and a known single-qubit state to them via RSP. Additionally, the study analyzed and discussed the fidelity of the proposed protocol under four different noise environments. In 2022, Zhang *et al.*[46] proposed a BHQIT scheme for arbitrary single-qubit states based on butterfly network coding, using five-qubit Brown states as the quantum channel. This research expanded the application scope of quantum network coding and holds good potential for improving communication efficiency in quantum networks. In 2024, Gong *et al.*[47] generalized the quantum states to be cyclically and bidirectionally transmitted in the CBHQIT protocol from arbitrary single-qubit states to arbitrary two-qubit states.

In 2023, Hua *et al.*[48] integrated the hierarchical control mechanism with bidirectional hybrid quantum communication technology and designed a hierarchically controlled bidirectional hybrid quantum information transmission (HCBHQIT) protocol based on a six-qubit entangled state channel. In this protocol, Alice needs to teleport an unknown single-qubit state to Bob under the supervision of the control agent Charlie, while Bob is required to remotely prepare a single-qubit state for Alice under the joint regulation of the control agents Charlie and David. However, this scheme is only applicable to the transmission and preparation of single-qubit states, with certain limitations in its application scope. In 2025, using an eight-qubit entangled state as the quantum channel, Yang[49,50] proposed two HCBHQIT protocols, respectively, one is suitable for the bidirectional transmission of arbitrary unknown single-qubit states and known two-qubit states and the other is suitable for the bidirectional transmission of arbitrary known single-qubit states and unknown two-qubit states. Based on existing HCBHQIT protocols with two communication agents, we propose a six-party HCBCHQIT protocol involving three communication agents and three control agents, which can be further extended to a 2N-party hierarchical communication protocol with N communication agents and N control agents.

The remaining part of the paper is arranged as follows: Sec. II proposes the six-party HCBCHQIT protocol, in which three communication agents with different post levels will complete their respective two-way hybrid communication tasks with the joint assistance of three control agents with different permissions. Section III extends the six-party protocol to the 2N-party scenario. Section IV calculates the intrinsic efficiency of our protocol and conducts a comparative analysis with some existing literature studies. Security analysis is briefly discussed in Sec. V. Finally, Sec. VI includes discussions and conclusions of the research.

II. THE SIX-PARTY HCBCHQIT PROTOCOL

In this section, a six-party HCBCHQIT protocol with three control agents is proposed. The protocol includes three communication agents (Alice, Bob, and Charlie) with different levels and three control agents (David, Emma, and Frank) with a clear permission hierarchy: David (highest authority, master control agent), followed by Emma and Frank (lowest authority). Communication agents act as both information senders and receivers. Figure 1 shows the schematic diagram of the six-party HCBCHQIT protocol.

Specifically, with assistance from David, the high-level agent Alice can teleport an unknown single-qubit state $|\xi_1\rangle_A = a_1|0\rangle + b_1|1\rangle$ to Bob while remotely preparing a known single-qubit state $|\chi_1\rangle = \alpha_1|0\rangle + \beta_1 e^{i\theta_1}|1\rangle$ for Charlie.

The mid-level agent Bob, under the joint control of David and Emma, needs to complete two-way communication tasks: teleporting an unknown single-qubit state $|\xi_2\rangle_B = a_2|0\rangle + b_2|1\rangle$ to Charlie and remotely preparing a known single-qubit state $|\chi_2\rangle = \alpha_2|0\rangle + \beta_2 e^{i\theta_2}|1\rangle$ for Alice.

For the low-level agent Charlie to teleport an unknown single-qubit state $|\xi_3\rangle_C = a_3|0\rangle + b_3|1\rangle$ to Alice and remotely prepare any known single-qubit state $|\chi_3\rangle = \alpha_3|0\rangle + \beta_3 e^{i\theta_3}|1\rangle$ for Bob, joint collaboration and control from all three control agents (David, Emma, and Frank) are required.

The coefficient parameters $a_i, b_i, \alpha_i, \beta_i, \theta_i \quad (i = 1, 2, 3)$ of the quantum states are all real numbers and satisfy the normalization conditions $|a_i|^2 + |b_i|^2 = 1$, $|\alpha_i|^2 + |\beta_i|^2 = 1$, $\theta_i \in [0, 2\pi)$.

During the implementation of the protocol, Alice only knows the coefficient information of the quantum state $|\chi_1\rangle$, not the specific information of other quantum states to be transmitted (including her own $|\xi_1\rangle_A$); similarly, Bob and Charlie only know the coefficient information of $|\chi_2\rangle$ and $|\chi_3\rangle$, respectively, without the knowledge of other states (including their respective $|\xi_2\rangle_B$ and $|\xi_3\rangle_C$). The control agents David, Emma, and Frank have no information about all quantum states.

From the four groups of 16 G-states presented in Ref. 51, we selected the first group of four G-states to construct the 15-qubit

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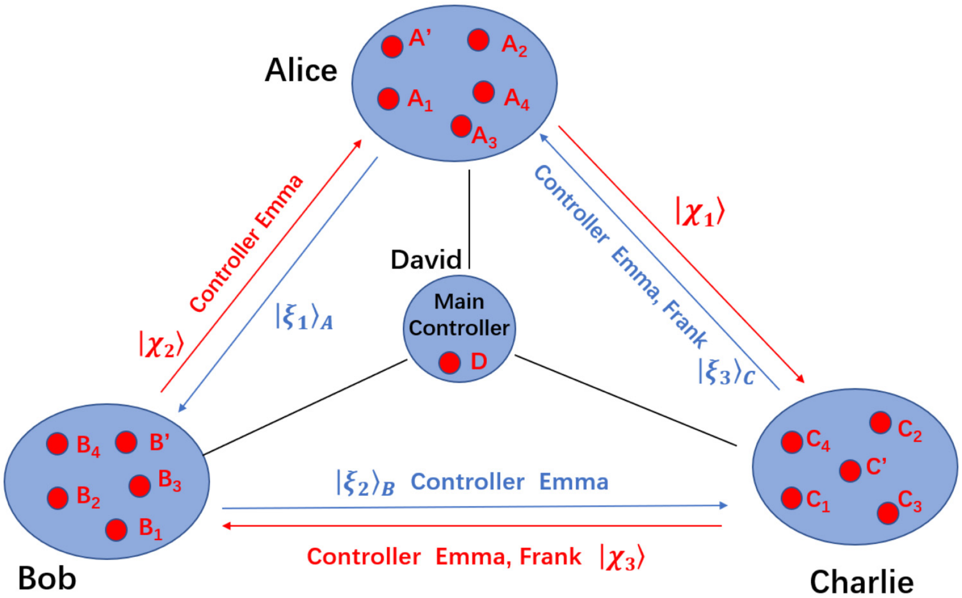

FIG. 1. Schematic diagram of six-party HCBCHQIT.

entangled state quantum channel used in our protocol, which can be expressed as

$$\begin{aligned}|Q_1\rangle_{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15} = {} & [(|g_1\rangle|0\rangle + |g_2\rangle|1\rangle)|g_1\rangle|0\rangle \\ & + (|g_4\rangle|0\rangle + |g_3\rangle|1\rangle)|g_4\rangle|1\rangle]|g_1\rangle|0\rangle \\ & + [(|g_3\rangle|0\rangle + |g_4\rangle|1\rangle)|g_3\rangle|0\rangle \\ & + (|g_2\rangle|0\rangle + |g_1\rangle|1\rangle)|g_2\rangle|1\rangle]|g_2\rangle|1\rangle. \end{aligned} \tag{1}$$

These four G-states are given by

$$\begin{aligned} |g_1\rangle &= \frac{1}{2}(|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle), \\ |g_2\rangle &= \frac{1}{2}(|0000\rangle + |0101\rangle - |1010\rangle - |1111\rangle), \\ |g_3\rangle &= \frac{1}{2}(|0000\rangle - |0101\rangle + |1010\rangle - |1111\rangle), \\ |g_4\rangle &= \frac{1}{2}(|0000\rangle - |0101\rangle - |1010\rangle + |1111\rangle). \end{aligned} \tag{2}$$

Before the initiation of quantum communication, the six agents involved in the process have pre-shared this 15-qubit entangled state $|Q_1\rangle$. This 15-qubit entangled state is prepared by the overall controller David. Specifically, qubits (1, 4, 7, 13) are assigned to Charlie; qubits (3, 6, 9, 12) belong to Bob; qubits (2, 8, 11, 14) are allocated to Alice; and among the three control agents, Frank holds qubit 5, Emma possesses qubit 10, while David himself retains qubit 15. Such an entangled state can be prepared using the quantum circuit method depicted in Fig. 2.

To implement the protocol, the quantum channel needs to be modulated first. Alice, Bob, and Charlie, respectively, introduce auxiliary particles A', B', and C' with the initial state $|0\rangle$. Then, they take qubits 14, 9, and 4 as control qubits, respectively, and perform the CNOT gate operations with $|0\rangle_{A'}$, $|0\rangle_{B'}$, and $|0\rangle_{C'}$ as target qubits.

For the sake of clarity in the narrative, we denote qubit 5 held by Frank as F, qubit 10 held by Emma as E, and qubit 15 held by David as D; the qubits held by Alice, Bob, and Charlie, respectively correspond to the letter notations in Table I.

The overall initial state can be written as

$$\begin{aligned} |\Omega_1\rangle &= |\xi_1\rangle_A \otimes |\xi_2\rangle_B \otimes |\xi_3\rangle_C \otimes |Q_1\rangle_{C_1A_2B_4C_3C'FB_1C_2A_4B_3B'EA_1B_2C_4A_3A'D} \\ &= (a_1|0\rangle + b_1|1\rangle)_A \otimes (a_2|0\rangle + b_2|1\rangle)_B \otimes (a_3|0\rangle + b_3|1\rangle)_C \otimes \\ & \quad \{[(|h_1\rangle|0\rangle + |h_2\rangle|1\rangle)|h_1\rangle|0\rangle + (|h_4\rangle|0\rangle + |h_3\rangle|1\rangle)|h_4\rangle|1\rangle]|h_1\rangle|0\rangle \\ & \quad + [(|h_3\rangle|0\rangle + |h_4\rangle|1\rangle)|h_3\rangle|0\rangle + (|h_2\rangle|0\rangle + |h_1\rangle|1\rangle)|h_2\rangle|1\rangle]|h_2\rangle|1\rangle\}, \end{aligned} \tag{3}$$

Among which,

$$\begin{aligned} |h_1\rangle &= \frac{1}{2}(|00000\rangle + |01011\rangle + |10100\rangle + |11111\rangle), \\ |h_2\rangle &= \frac{1}{2}(|00000\rangle + |01011\rangle - |10100\rangle - |11111\rangle), \\ |h_3\rangle &= \frac{1}{2}(|00000\rangle - |01011\rangle + |10100\rangle - |11111\rangle), \\ |h_4\rangle &= \frac{1}{2}(|00000\rangle - |01011\rangle - |10100\rangle + |11111\rangle). \end{aligned} \tag{4}$$

The following six-step protocol elaborates on the implementation approach of six-party HCBCHQIT protocol. To simplify the description, global phase factors will not be considered during the evolution stage of quantum states in the hybrid quantum communication process. The specific workflow is visualized in Fig. 3,

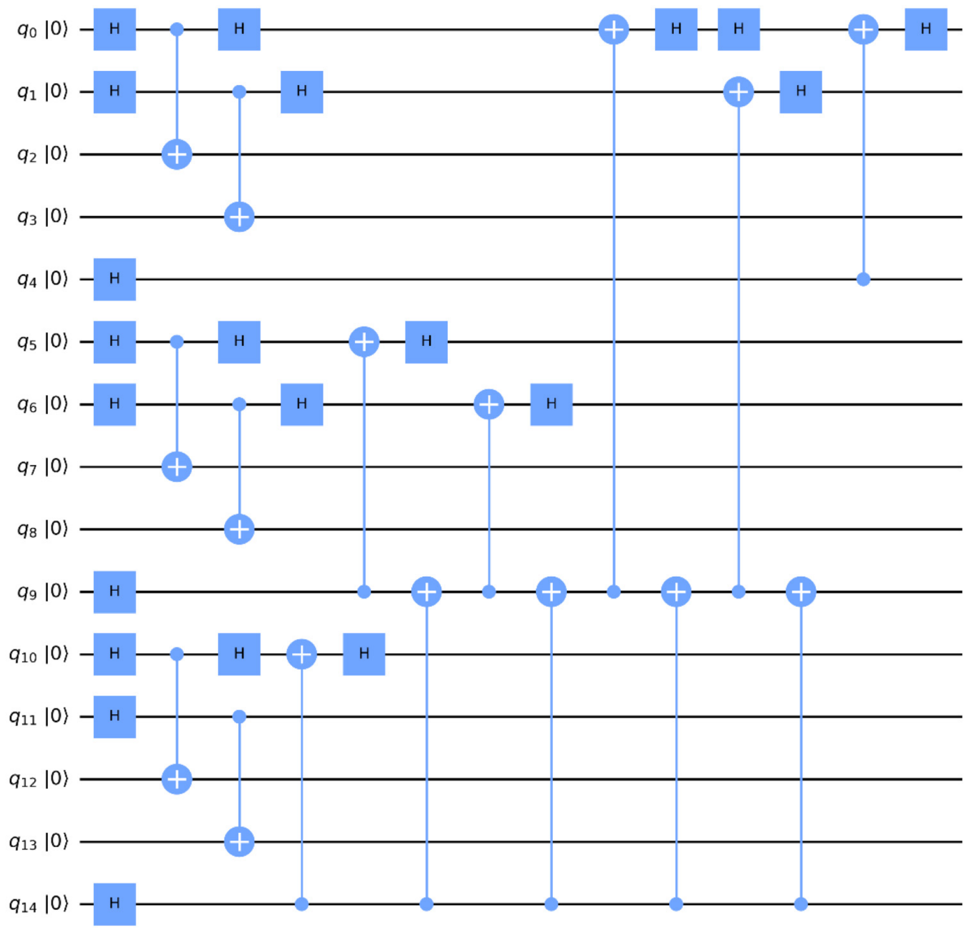

FIG. 2. The circuit diagram of the eight-qubit state of Eq. (1) from state $|0\rangle^{\otimes 15}$.

which clearly illustrates the sequential operations, interactions, and information flows among different agents involved in the protocol. Herein, "BSM" denotes Bell-basis measurement, while "PM" represents projection measurement.

Step 1: Alice conducts a Bell-basis measurement on the qubit pairs (A, A_1). Then, she employs two bits of classical information to reveal her measurement results through a public communication channel. The four Bell measurement bases are

$$\begin{aligned} |\varphi_{00}\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), |\varphi_{01}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle), \\ |\varphi_{10}\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), |\varphi_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle). \end{aligned} \tag{5}$$

TABLE I. Correspondence between qubits, holders, and their symbols.

Alice		Bob		Charlie	
Qubit	Symbol	Qubit	Symbol	Qubit	Symbol
11	A_1	6	B_1	1	C_1
2	A_2	12	B_2	7	C_2
14	A_3	9	B_3	4	C_3
8	A_4	3	B_4	13	C_4
A'	A'	B'	B'	C'	C'

Next, she conducted two single particle projection measurements on particles A' and A_3 in sequence.

First, based on the prior knowledge she has, Alice selects the measurement basis $\{|\varepsilon_1\rangle_{A'}, |\varepsilon_2\rangle_{A'}\}$ to implement the projection strategy on particle A', where

$$\begin{aligned} |\varepsilon_1\rangle &= \alpha_1|0\rangle + \beta_1|1\rangle, \\ |\varepsilon_2\rangle &= \beta_1|0\rangle - \alpha_1|1\rangle. \end{aligned} \tag{6}$$

Then, according to the measurement result of particle A', another set of bases $\{|\lambda_j^{\tau,\theta_l}\rangle_{A_3}; \tau \in \{+, -\}, j \in \{1, 2\}, l = 1\}$ is used to perform projection measurement on particle A_3, and the measurement bases can be written as

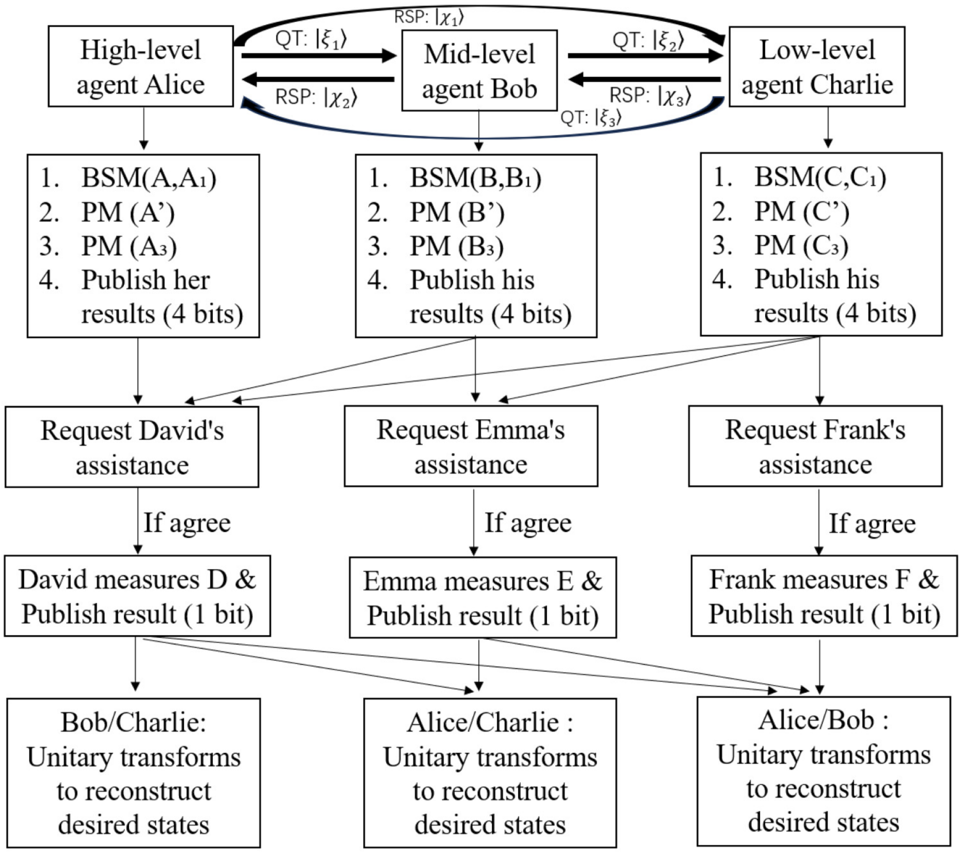

FIG. 3. The flowchart of the six—party HCBCHQIT protocol.

$$\{|\lambda_1^{\tau,\theta_1}\rangle\} = \begin{cases} |\lambda_1^{+,\theta_1}\rangle_{A'} = |0\rangle + e^{-i\theta_1}|1\rangle \\ |\lambda_1^{-,\theta_1}\rangle_{A'} = |0\rangle - e^{-i\theta_1}|1\rangle \end{cases},$$
$$\{|\lambda_2^{\tau,\theta_1}\rangle\} = \begin{cases} |\lambda_2^{+,\theta_1}\rangle_{A'} = e^{-i\theta_1}|0\rangle + |1\rangle \\ |\lambda_2^{-,\theta_1}\rangle_{A'} = e^{-i\theta_1}|0\rangle - |1\rangle \end{cases}. \tag{7}$$

Specifically, if the measurement result of qubit A′ is $|\varepsilon_1\rangle_{A'}$, the measurement basis $\{|\lambda_1^{\tau,\theta_1}\rangle\}$ are used to measure qubit A_3; if the measurement result of qubit A′ is $|\varepsilon_2\rangle_{A'}$, the measurement basis $\{|\lambda_2^{\tau,\theta_1}\rangle\}$ are used to measure qubit A_3.

Assuming that Alice's measurement results are $|\varphi_{00}\rangle_{A,A_1}$, $|\varepsilon_1\rangle_{A'}$, and $|\lambda_1^{+,\theta_1}\rangle_{A_3}$, then the initial state of the system can be rewritten as:

$$\begin{aligned} |\Omega_2\rangle &= |\xi_2\rangle_B \otimes |\xi_3\rangle_C \otimes |Q_2\rangle_{C_1A_2B_4C_3C'FB_1C_2A_4B_3B'EB_2C_4D} \\ &= \frac{1}{4}(a_2|0\rangle + b_2|1\rangle) \otimes (a_3|0\rangle + b_3|1\rangle) \\ &\quad \otimes \{[(|h_1\rangle|0\rangle + |h_2\rangle|1\rangle)|h_1\rangle|0\rangle + (|h_4\rangle|0\rangle + |h_3\rangle|1\rangle)|h_4\rangle|1\rangle] \\ &\quad \otimes (a_1\alpha_1|00\rangle + a_1\beta_1 e^{i\theta_1}|10\rangle + b_1\alpha_1|01\rangle + b_1\beta_1 e^{i\theta_1}|11\rangle) \otimes |0\rangle \\ &\quad + [(|h_3\rangle|0\rangle + |h_4\rangle|1\rangle)|h_3\rangle|0\rangle + (|h_2\rangle|0\rangle + |h_1\rangle|1\rangle)|h_2\rangle|1\rangle] \\ &\quad \otimes (a_1\alpha_1|00\rangle + a_1\beta_1 e^{i\theta_1}|10\rangle - b_1\alpha_1|01\rangle - b_1\beta_1 e^{i\theta_1}|11\rangle) \otimes |1\rangle\}. \end{aligned} \tag{8}$$

Step 2: Next, Alice discloses her measurement outcomes via two bits of classical information and applies to the master control agent David for continuing the communication.

If David agrees to this communication, he will measure qubit 15 in his hand under the $\{|0\rangle_D, |1\rangle_D\}$ basis and discloses his measurement outcome via one-bit of classical information. Assuming that David's measurement result is $|1\rangle_D$, the state of the remaining qubits will collapse to

$$\begin{aligned} |\Omega_3\rangle &= |\xi_2\rangle_B \otimes |\xi_3\rangle_C \otimes |Q_3\rangle_{C_1A_2B_4C_3C'FB_1C_2A_4B_3B'EB_2C_4} \\ &= \frac{1}{4}(a_2|0\rangle + b_2|1\rangle)_B \otimes (a_3|0\rangle + b_3|1\rangle)_C \\ &\quad \otimes [(|h_3\rangle|0\rangle + |h_4\rangle|1\rangle)|h_3\rangle|0\rangle \\ &\quad + (|h_2\rangle|0\rangle + |h_1\rangle|1\rangle)|h_2\rangle|1\rangle]_{C_1A_2B_4C_3C'FB_1C_2A_4B_3B'E} \\ &\quad \otimes big[(a_1|0\rangle - b_1|1\rangle)_{B_2} \otimes (\alpha_1|0\rangle + \beta_1 e^{i\theta_1}|1\rangle)_{C_4}]. \end{aligned} \tag{9}$$

Bob and Charlie, based on the measurement results announced by Alice and David, select an appropriate unitary transformation operation $\sigma^z_{B_2} \otimes I_{C_4}$. In this way, the desired quantum state can be recovered.

In fact, at this point, with only the assistance of the master controller David, the high-level agent Alice has completed her communication tasks. Specifically, through quantum teleportation, the unknown quantum state $|\xi_1\rangle_{B_2}$ to be transmitted is recovered at qubit B_2 owned by Bob. Meanwhile, a known quantum state $|\chi_1\rangle_{C_4}$ is remotely prepared at qubit C_4 of Charlie.

Step 3: The medium-level agent Bob performs measurements on particles B and B_1 using the Bell basis $\{|\varphi_{00}\rangle_{B,B_1}, |\varphi_{01}\rangle_{B,B_1}, |\varphi_{10}\rangle_{B,B_1}, |\varphi_{11}\rangle_{B,B_1}\}$ and employs two bits of classical information to reveal his measurement results through a public communication channel.

Then, he performs a projection measurement on particle B' under the measurement basis $\{|\varepsilon_1\rangle_{B'}, |\varepsilon_2\rangle_{B'}\}$, where

$$\begin{aligned}|\varepsilon_1\rangle_{B'} &= \alpha_2|0\rangle + \beta_2|1\rangle,\\ |\varepsilon_2\rangle_{B'} &= \beta_2|0\rangle - \alpha_2|1\rangle.\end{aligned} \tag{10}$$

According to the measurement result of B', Bob performs a single-particle projection measurement on B_3 under the measurement basis $\{|\lambda_j^{\tau,\theta_l}\rangle_{B_3}; \tau \in \{+,-\},\ j \in \{1,2\},\ l = 2\}$.

After the two single-qubit projection measurements, Bob discloses his measurement results on the public channel via two bits of classical information.

Let's assume that Bob's measurement results are $|\varphi_{00}\rangle_{B,B_1}$, $|\varepsilon_1\rangle_{B'}$, and $|\lambda_1^{+,\theta_2}\rangle_{B_3}$, respectively. Then, the state of the remaining particles collapses to

$$\begin{aligned}|\Omega_4\rangle &= |\xi_3\rangle_C \otimes |Q_4\rangle_{C_1A_2B_4C_3C'FC_2A_4EB_2C_4}\\ &= \frac{1}{4\sqrt{2}}(a_3|0\rangle + b_3|1\rangle)_C \otimes \left[(a_1|0\rangle - b_1|1\rangle)_{B_2} \otimes \left(\alpha_1|0\rangle + \beta_1 e^{i\theta_1}|1\rangle\right)_{C_4}\right]\\ &\quad \otimes \Big[|0\rangle_E \otimes (|h_3\rangle|0\rangle + |h_4\rangle|1\rangle)_{C_1A_2B_4C_3C'F}\\ &\quad \otimes \left(a_2\alpha_2|00\rangle - a_2\beta_2 e^{i\theta_2}|10\rangle + b_2\alpha_2|01\rangle - b_2\beta_2 e^{i\theta_2}|11\rangle\right)_{C_2A_4}\\ &\quad + |1\rangle_E \otimes (|h_2\rangle|0\rangle + |h_1\rangle|1\rangle)_{C_1A_2B_4C_3C'F}\\ &\quad \otimes \left(a_2\alpha_2|00\rangle + a_2\beta_2 e^{i\theta_2}|10\rangle - b_2\alpha_2|01\rangle - b_2\beta_2 e^{i\theta_2}|11\rangle\right)_{C_2A_4}\Big].\end{aligned} \tag{11}$$

Step 4: Then Bob continues to send a communication request to the control agent Emma. If Emma agrees to continue the communication, she will perform a single-particle measurement on her qubit E under the basis $\{|0\rangle_E, |1\rangle_E\}$ and disclose her measurement result via one bit of classical information.

If her measurement result is $|1\rangle_E$, the state of the remaining particles will collapse to

$$\begin{aligned}|\Omega_5\rangle &= |\xi_3\rangle_C \otimes |C\rangle_{C_1A_2B_4C_3C'FC_2A_4B_2C_4}\\ &= \frac{1}{4\sqrt{2}}(a_3|0\rangle + b_3|1\rangle)_C \otimes \left[(a_1|0\rangle - b_1|1\rangle)_{B_2} \otimes \left(\alpha_1|0\rangle + \beta_1 e^{i\theta_1}|1\rangle\right)_{C_4}\right]\\ &\quad \otimes (|h_3\rangle|0\rangle + |h_4\rangle|1\rangle)_{C_1A_2B_4C_3C'F}\\ &\quad \otimes \left(a_2\alpha_2|00\rangle - a_2\beta_2 e^{i\theta_2}|10\rangle + b_2\alpha_2|01\rangle - b_2\beta_2 e^{i\theta_2}|11\rangle\right)_{C_2A_4}\\ &= \frac{1}{4\sqrt{2}}(a_3|0\rangle + b_3|1\rangle)_C \otimes \left[(a_1|0\rangle - b_1|1\rangle)_{B_2} \otimes \left(\alpha_1|0\rangle + \beta_1 e^{i\theta_1}|1\rangle\right)_{C_4}\right]\\ &\quad \otimes (|h_3\rangle|0\rangle + |h_4\rangle|1\rangle)_{C_1A_2B_4C_3C'F}\\ &\quad \otimes \left[(a_2|0\rangle + b_2|1\rangle)_{C_2} \otimes \left(\alpha_2|0\rangle - \beta_2 e^{i\theta_2}|1\rangle\right)_{A_4}\right].\end{aligned} \tag{12}$$

At this time, Alice and Charlie select an appropriate unitary transformation operation $I_{C_2} \otimes \sigma^z_{A_4}$ based on the measurement results announced by Emma, Bob, and David and, thus, can recover the desired quantum state.

In fact, at this point, under the joint control of the master control agent David and the sub-control agent Emma, the medium-level agent Bob has also completed his communication tasks. That is, through quantum teleportation, the unknown quantum state $|\xi_2\rangle_{C_2}$ to be transmitted is recovered at the qubit C_2 owned by Charlie. Meanwhile, a known quantum state $|\chi_2\rangle_{A_4}$ is remotely prepared at the qubit A_4 of Alice.

Step 5: The low-level agent Charlie performs measurements on qubits C and C_1 using the Bell basis $\{|\varphi_{00}\rangle_{C,C_1}, |\varphi_{01}\rangle_{C,C_1}, |\varphi_{10}\rangle_{C,C_1}, |\varphi_{11}\rangle_{C,C_1}\}$. Then, a projection measurement is performed on qubit C' under the measurement basis $\{|\varepsilon_1\rangle_{C'}, |\varepsilon_2\rangle_{C'}\}$, where

$$\begin{aligned}|\varepsilon_1\rangle_{C'} &= \alpha_3|0\rangle + \beta_3|1\rangle,\\ |\varepsilon_2\rangle_{C'} &= \beta_3|0\rangle - \alpha_3|1\rangle.\end{aligned} \tag{13}$$

According to the measurement result of C', a single-qubit projection measurement is performed on C_3 under the measurement basis $\{|\lambda_j^{\tau,\theta_l}\rangle_{B_3}; \tau \in \{+,-\},\ j \in \{1,2\},\ l = 3\}$. Then, she uses two bits of classical information to disclose the measurement results.

Let's assume that Charlie's measurement results are $|\varphi_{00}\rangle_{C,C_1}$, $|\varepsilon_1\rangle_{C'}$, and $|\lambda_1^{+,\theta_3}\rangle_{C_3}$, respectively. Then, the state of the remaining particles collapses to

$$\begin{aligned}|\Omega_6\rangle &= \frac{1}{8}\left[(a_1|0\rangle - b_1|1\rangle)_{B_2} \otimes \left(\alpha_1|0\rangle + \beta_1 e^{i\theta_1}|1\rangle\right)_{C_4}\right]\\ &\quad \otimes \left[(a_2|0\rangle + b_2|1\rangle)_{C_2} \otimes \left(\alpha_2|0\rangle - \beta_2 e^{i\theta_2}|1\rangle\right)_{A_4}\right]\\ &\quad \otimes \Big[\left(a_2\alpha_2|00\rangle - a_2\beta_2 e^{i\theta_2}|10\rangle + b_2\alpha_2|01\rangle - b_2\beta_2 e^{i\theta_2}|11\rangle\right)_{A_2B_4} \otimes |0\rangle_F\\ &\quad + \left(a_2\alpha_2|00\rangle - a_2\beta_2 e^{i\theta_2}|10\rangle - b_2\alpha_2|01\rangle + b_2\beta_2 e^{i\theta_2}|11\rangle\right)_{A_2B_4} \otimes |1\rangle_F\Big].\end{aligned} \tag{14}$$

Step 6: Then, Charlie sends a request for further communication to the control agent Frank. If Frank agrees to continue the communication, he will perform a single-particle measurement on particle *F* under the basis $\{|0\rangle_F, |1\rangle_F\}$ and disclose his measurement result via one bit of classical information.

If Frank's measurement result is $|1\rangle_F$, the remaining qubits' state will collapse into

$$\begin{aligned}|\Omega_7\rangle &= \frac{1}{8}\left[(a_1|0\rangle - b_1|1\rangle)_{B_2} \otimes (\alpha_1|0\rangle + \beta_1 e^{i\theta_1}|1\rangle)_{C_4}\right]\\ &\quad \otimes \left[(a_2|0\rangle + b_2|1\rangle)_{C_2} \otimes (\alpha_2|0\rangle - \beta_2 e^{i\theta_2}|1\rangle)_{A_4}\right]\\ &\quad \otimes \left[(a_3|0\rangle - b_2|1\rangle)_{A_2} \otimes (\alpha_3|0\rangle - \beta_3 e^{i\theta_3}|1\rangle)_{B_4}\right].\end{aligned} \tag{15}$$

Currently, Alice and Bob select appropriate unitary transformation operations $\sigma^z_{A_2} \otimes \sigma^z_{B_4}$ according to the measurement results announced by Charlie, David, Emma, and Frank and, thus, can recover the desired quantum state.

In fact, at this point, under the joint control of the master control agent David and the sub-control agents Emma and Frank, the low-level agent Charlie has also completed his communication tasks. That is, through quantum teleportation, the unknown quantum state $|\xi_3\rangle_{A_2}$ to be transmitted is recovered at the qubit A_2 owned by Alice. Meanwhile, a known quantum state $|\chi_3\rangle_{B_4}$ is remotely prepared at the qubit B_4 of Bob.

From Eq. (15), it can be seen that the multi-agent quantum communication tasks of Alice, Bob, and Charlie have been completed, respectively, with each task falling under the distinct control levels of the agents David, Emma, and Frank.

Consequently, the HCBCHQIT protocol is implemented successfully. Each of these task completions relies on respective unitary operations for Alice, Bob, and Charlie, enabling them to reconstruct the target quantum states according to the publicly announced measurement outcomes.

III. THE 2N-PARTY HCBCHQIT PROTOCOL

This section extends the six-party HCBCHQIT protocol proposed in Sec. II to a 2N-party one. That is, under the respective control of N control agents at different levels, N communication agents conduct bidirectional cyclic hybrid communication with one another.

Specifically, there are N control agents Controller_k $(k = 1, 2, \ldots, N)$ with different control authorities (from high to low) denoted as $M_1 \sim M_N$; the communication agents are $\text{Alice}_k(k = 1, 2, \ldots, N)$ with position levels (from high to low) denoted as $G_1 \sim G_N$. They are both the senders and receivers of quantum information. Precisely because the position levels of each $\text{Alice}_k(k = 1, 2, \ldots, N)$ are different, the number of control agents required for joint and coordinated control in the bidirectional hybrid quantum information transmission tasks executed by $\text{Alice}_k(k = 1, 2, \ldots, N)$ is also different.

Definition 1: $TC_k \triangleq \{\text{Controller}_1, \text{Controller}_2, \ldots, \text{Controller}_k\}$ $(k = 1, 2, \ldots, N)$ is the set of control agents $\{\text{Controller}_1, \text{Controller}_2, \ldots, \text{Controller}_k\}$ with control authorities $\{M_1, M_2, \ldots, M_k\}$, respectively.

Then, the communication task to be achieved in the generalized 2N-party protocol can be described as follows:

Under the joint and cooperative control of the set of control agents $TC_k(k = 1, 2, \ldots, N)$, each $\text{Alice}_k(k = 1, 2, \ldots, N)$ with the post-level G_i performs quantum teleportation of an unknown single-qubit state $|\xi_k\rangle_{A_k} = a_k|0\rangle + b_k|1\rangle$ $(k = 1, 2, \ldots, N)$ to $\text{Alice}_{(k+1)\text{mod}N}(k = 1, 2, \ldots, N)$ with the post-level $G_{(k+1)\text{mod}N}(k = 1, 2, \ldots, N)$; Meanwhile, it transmits a known single-qubit state $|\chi_k\rangle = \alpha_k|0\rangle + \beta_k e^{i\theta_k}|1\rangle$ $(k = 1, 2, \ldots, N)$ to $\text{Alice}_{(k-1)\text{mod}N}(k = 1, 2, \ldots, N)$ with the post-level $G_{(k-1)\text{mod}N}$ $(k=1,2,\ldots,N)$. The parameters $a_k, b_k, \alpha_k, \beta_k, \theta_k$ $(k=1,2,\ldots,N)$ are all real numbers and satisfy the normalization conditions $|a_k|^2+|b_k|^2=1, |\alpha_k|^2+|\beta_k|^2=1, \theta_k\in[0,2\pi)$, $(k=1,2,\ldots,N)$.

In the protocol execution, each participant $\text{Alice}_k(k = 1, 2, \ldots, N)$ solely possesses the coefficient information of $|\chi_k\rangle(k = 1, 2, \ldots, N)$, remaining unaware of other quantum states [including their own $|\xi_k\rangle_{A_k} = a_k|0\rangle + b_k|1\rangle$ $(k=1,2,\ldots,N)$]. Meanwhile, all control agents $\text{Controller}_k(k=1,2,\ldots,N)$ have no knowledge of the specific content of any quantum states. Figure 4 illustrates the HCBCHQIT protocol for 2N parties. Herein, hollow arrows represent the control logic from the controller set $TC_k(k=1,2,\ldots,N)$ (composed of several Controllers) to the corresponding $\text{Alice}_k(k=1,2,\ldots,N)$, while solid arrows characterize the quantum information transmission process between $\text{Alice}_k(k=1,2,\ldots,N)$ nodes, illustrating the complete flow from the collaborative control of multiple Controllers to the transmission of quantum states among Alice nodes.

Definition 2: The set of direct products formed by arbitrarily selecting an N-tuple repeatable permutation from all 16 different G-states $\{|g_1\rangle, |g_2\rangle, \ldots, |g_{16}\rangle\}$ is $PG = \{|PG_1\rangle, |PG_2\rangle, \ldots, |PG_{16^N}\rangle\}$, satisfying $|PG_s\rangle \neq |PG_t\rangle(\forall s \neq t)$. Among them,

$$|PG_j\rangle \triangleq |PG_{j_1,j_2,\ldots,j_N}\rangle = |g_{j_1}\rangle \otimes |g_{j_2}\rangle \otimes \cdots \otimes |g_{j_N}\rangle, \quad \left(j \in \left[1, 16^N\right], j_1, j_2, \ldots, j_N \in [1, 16]\right). \tag{16}$$

Next, the quantum channel required for implementing the 2N-party HCBCHQIT protocol is given. In fact, the quantum channel in Eq. (1) can be rewritten as follows:

$$\begin{aligned}|H_1\rangle &= [(|g_1\rangle|0\rangle + |g_2\rangle|1\rangle)|g_1\rangle|0\rangle + (|g_4\rangle|0\rangle + |g_3\rangle|1\rangle)|g_4\rangle|1\rangle]|g_1\rangle|0\rangle \\ &\quad + [(|g_3\rangle|0\rangle + |g_4\rangle|1\rangle)|g_3\rangle|0\rangle + (|g_2\rangle|0\rangle + |g_1\rangle|1\rangle)|g_2\rangle|1\rangle]|g_2\rangle|1\rangle \\ &= |g_1\rangle|g_1\rangle|g_1\rangle|000\rangle + |g_3\rangle|g_3\rangle|g_2\rangle|001\rangle + |g_4\rangle|g_4\rangle|g_1\rangle|010\rangle \\ &\quad + |g_2\rangle|g_2\rangle|g_2\rangle|011\rangle \\ &\quad + |g_2\rangle|g_1\rangle|g_1\rangle|100\rangle + |g_4\rangle|g_3\rangle|g_2\rangle|101\rangle + |g_3\rangle|g_4\rangle|g_1\rangle|110\rangle \\ &\quad + |g_1\rangle|g_2\rangle|g_2\rangle|111\rangle \\ &= |PG_{111}\rangle|000\rangle + |PG_{332}\rangle|001\rangle + |PG_{441}\rangle|010\rangle + |PG_{222}\rangle|011\rangle \\ &\quad + |PG_{211}\rangle|100\rangle + |PG_{432}\rangle|101\rangle + |PG_{341}\rangle|110\rangle + |PG_{122}\rangle|111\rangle.\end{aligned} \tag{17}$$

Therefore, the quantum channel of the 2N-party HCBCHQIT protocol can be constructed as

$$\begin{aligned}|H_N\rangle &= \sum_{i_1,i_2,\cdots,i_N\in\{0,1\}} |PG_j\rangle|i_1 i_2 \cdots i_N\rangle \\ &= \left(|g_{h_1^0}\rangle_{A_{11}A_{12}A_{13}A_{14}} \otimes |g_{h_2^0}\rangle_{A_{21}A_{22}A_{23}A_{24}} \otimes \cdots \otimes |g_{h_N^0}\rangle_{A_{N1}A_{N2}A_{N3}A_{N4}}\right)|00\ldots0\rangle_{C_1C_2\cdots C_N} \\ &\quad + \left(|g_{h_1^1}\rangle_{A_{11}A_{12}A_{13}A_{14}} \otimes |g_{h_2^1}\rangle_{A_{21}A_{22}A_{23}A_{24}} \otimes \cdots \otimes |g_{h_N^1}\rangle_{A_{N1}A_{N2}A_{N3}A_{N4}}\right)|00\cdots1\rangle_{C_1C_2\cdots C_N} \\ &\quad \cdots\cdots \\ &\quad + \left(|g_{h_1^{2^N-1}}\rangle_{A_{11}A_{12}A_{13}A_{14}} \otimes |g_{h_2^{2^N-1}}\rangle_{A_{21}A_{22}A_{23}A_{24}} \otimes \cdots \otimes |g_{h_N^{2^N-1}}\rangle_{A_{N1}A_{N2}A_{N3}A_{N4}}\right)|11\cdots1\rangle_{C_1C_2\cdots C_N}.\end{aligned} \tag{18}$$

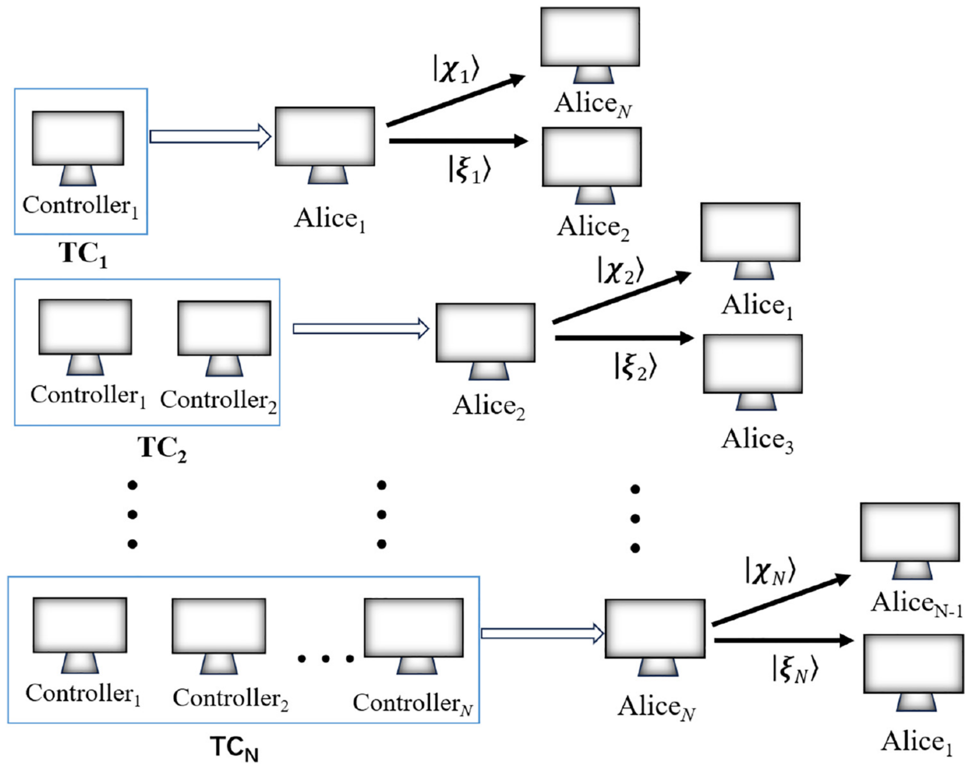

FIG. 4. Schematic diagram of the 2*N*-party HCBCHQIT protocol.

The subscripts $h_1^0, h_2^0, \ldots, h_N^0, h_1^1, h_2^1, \ldots, h_N^1, \ldots, h_1^{2^N-1}, h_2^{2^N-1}, \ldots, h_N^{2^N-1}$ of each selected G-state $|g_i\rangle (i = 1, 2, \ldots, 16)$ are, respectively, composed of N-element repeatable permutations of the distinct set $\{1, 2, \ldots, 16\}$. The qubits $A_{k1}, A_{k2}, A_{k3}, A_{k4} (k = 1, 2, \ldots, N)$, respectively, belong to the communication agents Alice$_k (k = 1, 2, \ldots, N)$; the qubits $C_1, C_2, \ldots, C_N$, respectively, belong to N control agents Controller$_k (k = N, N-1, \ldots, 2, 1)$.

The implementation of the protocol can be divided into the following six steps:

Step 1: Modulate the quantum channel. Each communication agent Alice$_k (k = 1, 2, \ldots, N)$, respectively, introduces an auxiliary particle with the initial state $|0\rangle_{A_{k4'}} (k = 1, 2, \ldots, N)$ and uses the owned qubit $A_{k4} (k = 1, 2, \ldots, N)$ as the control qubit and $|0\rangle_{A_{k4'}} (k = 1, 2, \ldots, N)$ as the target qubit to respectively perform N CNOT gate operations $\mathrm{CNOT}(A_{k4}, A_{k4'}) \quad (k = 1, 2, \ldots, N)$.

Step 2: Each Alice$_k (k = 1, 2, \ldots, N)$ respectively performs a Bell basis measurement on their owned qubit pair (A_k, A_{k1}) $(k = 1, 2, \ldots, N)$ under Bell basis $\{|\varphi_{00}\rangle_{A_k, A_{k1}}, |\varphi_{01}\rangle_{A_k, A_{k1}}, |\varphi_{10}\rangle_{A_k, A_{k1}}, |\varphi_{11}\rangle_{A_k, A_{k1}}\}$, and each, respectively, publishes their measurement results through two-bit classical information.

Step 3: Each Alice$_k (k = 1, 2, \ldots, N)$ successively performs two single-qubit projection measurements on the owned qubits $A_{k4} (k = 1, 2, \ldots, N)$ and $A_{k4'} (k = 1, 2, \ldots, N)$ according to the prior knowledge each grasps.

First, each Alice$_k (k = 1, 2, \ldots, N)$ performs a projection measurement on the qubit $A_{k4} (k = 1, 2, \ldots, N)$ under the measurement basis $\{|\varepsilon_1\rangle_{k4'}, |\varepsilon_2\rangle_{k4'}\}$, where

$$\begin{aligned} |\varepsilon_1\rangle_{k4'} &= \alpha_k|0\rangle + \beta_k|1\rangle, \\ |\varepsilon_2\rangle_{k4'} &= \beta_k|0\rangle - \alpha_k|1\rangle. \end{aligned} \tag{19}$$

Second, according to the measurement result of the qubit $A_{k4'} (k = 1, 2, \ldots, N)$, each Alice$_k (k = 1, 2, \ldots, N)$ selects the measurement basis $\{|\lambda_j^{\tau,\theta_l}\rangle_{B_3}; \tau \in \{+, -\}, j \in \{1, 2\}, l = k\}$ and then performs a projection measurement on the qubit $A_{k4} (k = 1, 2, \cdots, N)$.

After the two measurements, each Alice$_k (k = 1, 2, \ldots, N)$ uses two-bit classical information to announce their measurement results of the qubits $A_{k4} (k = 1, 2, \ldots, N)$ and $A_{k4'} (k = 1, 2, \ldots, N)$.

Step 4: Each control agent Controller$_k (k = 1, 2, \ldots, N)$ successively performs single-qubit measurement on the qubits $C_1, C_2, \ldots, C_N$ in hand under the basis $\{|0\rangle_{C_k}, |1\rangle_{C_k}\}$ $(k = 1, 2, \ldots, N)$.

Step 5: If the communication agent Alice$_k (k = 1, 2, \ldots, N)$ wants to execute her bidirectional hybrid communication tasks. She needs to submit a communication application to the set of control agents $TC_k (k = 1, 2, \ldots, N)$. If all levels of control agents in

$TC_k(k=1,2,\ldots,N)$ agree to this communication, they will, respectively, inform $\text{Alice}_{(k+1)\text{mod}N}(k=1,2,\ldots,N)$ and $\text{Alice}_{(k-1)\text{mod}N}(k=1,2,\ldots,N)$ of their measurement results on $C_1, C_2, \ldots C_k$ separately through the classical channel.

Step 6: $\text{Alice}_{(k+1)\text{mod}N}(k=1,2,\ldots,N)$ and $\text{Alice}_{(k-1)\text{mod}N}$ $(k=1,2,\ldots,N)$ comprehensively consider the measurement results of $TC_k(k=1,2,\ldots,N)$ and $\text{Alice}_k(k=1,2,\ldots,N)$ and select the appropriate unitary transformation operations to reconstruct the quantum state to be transmitted bidirectionally, thus completing the 2N-party HCBCHQIT protocol.

As a matter of fact, in the 2N-party communication protocol, for the $k(k=1,2,\ldots,N)$ level agent, there are four types of Bell measurement results (for QT), four types of single-qubit projection measurements (for RSP), and 2k different outcomes of single-qubit projection measurements performed by the N control agents, that is, there are a total of $4\times4\times2^k=2^{k+4}$ different measurement outcomes. When $k=N$, there are 2^{N+4} possible outcomes in total. However, as discussed earlier, for any measurement outcome, the receiver can select an appropriate unitary transformation to reconstruct the desired quantum state, and all possible scenarios will not be listed one by one here.

IV. EFFICIENCY AND COMPARISON

This section evaluates the inherent efficiency of the proposed scheme. Since there is currently no relevant literature on hierarchical control-based cyclic hybrid communication protocols, we have selected hierarchical control cycle quantum teleportation, as well as cyclic bidirectional hybrid communication protocols under single control[52–55] for comparative analysis, to illustrate the high efficiency of the proposed scheme.

The intrinsic efficiency[56] can be expressed as

$$\eta=\frac{\text{QTN}}{\text{CRN}+\text{QRN}}. \tag{20}$$

Here, QTN represents the total number of qubits transmitted in the protocol, which is the sum of the qubits sent by every communication agent individually. QRN refers to the quantum resource consumption during the implementation of the protocol, for example, "15 + 3" means the sum of the number of qubits in the quantum channel (15) and the number of auxiliary qubits (3). CRN denotes classical resource consumption, mainly including the number of classical bits consumed when all agents publish their measurement results. The information transmission mode (ITM) refers to the specific objects and methods of information transmission in the communication protocol, for example, "QT: $A\overset{D}{\Rightarrow}B\,x|0\rangle+y|1\rangle$" means that under the control of D, A teleports a single-qubit state $x|0\rangle+y|1\rangle$ to B.

Calculated from Eq. (20), the $2N$-party HCBCHQIT protocol we proposed has a fixed intrinsic efficiency (independent of the number of participants N), that is,

$$\eta=\frac{2N}{5N+N+N+4N}=\frac{2}{11}\approx18.18\%. \tag{21}$$

As can be seen from Table II, the authors of Refs. 52–54, respectively, propose three hierarchical control cyclic quantum teleportation schemes for arbitrary single-qubit states. Among them, in the scheme proposed in Sec. III of Ref. 52, the arbitrary single-qubit state of cyclic teleportation is hierarchically controlled by three controllers of different levels. Although this protocol has the same explicit hierarchical control function as our protocol, its intrinsic efficiency is 16.67%, which is lower than the efficiency of our protocol (18.18%). The intrinsic efficiency of Ref. 53 is slightly higher than that of our protocol (18.75% > 18.18%). However, in this protocol, the teleportation of a single-qubit state from Alice to Bob lacks the supervision of a controller, and the step-by-step control capability is not as perfect as that of our protocol. Similarly, in the protocol proposed in Ref. 54, the transmission of quantum information from Alice to Bob is limited by the controller Bob, while the information transmission from Bob to Charlie and from Charlie to Alice is, respectively, controlled by David and Eve and David and Fiva, that is, the controls between the two sub-control agents Eve and Fiva are independent of each other. This leads to the lack of mutual restraint or hierarchical management function in the communication tasks between Bob and Charlie. Reference 55 proposes a four-party and N-party controlled bidirectional cyclic hybrid quantum communication protocol for arbitrary single-particle states. Although this protocol has relatively high communication efficiency and security, the implementation of the protocol only relies on one control agent David and does not have a hierarchical control function. Compared with the protocols proposed in the above-mentioned Refs. 52–54, our protocol not only has more flexible and comprehensive control functions but also maintains relatively high communication efficiency.

V. SECURITY ANALYSIS

In this section, we briefly discuss the security of the proposed scheme. In this protocol, the classical communication involved in the quantum communication process can be transmitted through encryption methods such as symmetric encryption or public-key encryption, and its security is recognized here. In the processes of quantum teleportation and remote state preparation, quantum information is transmitted by virtue of the properties of quantum entanglement rather than through quantum channels. Therefore, in practical applications, attacks targeting quantum channels are more likely to occur during the distribution of quantum states. This means that the security of the protocol mainly lies in whether the legitimate participants have securely pre-shared the entangled quantum states.

First, the protocol can resist eavesdropping attacks.

Suppose that while David was distributing entangled qubits to everyone, an attacker named Eve was preparing to eavesdrop on our protocol. Eve attempted to entangle her qubits with the entangled qubits shared by legitimate agents and steal quantum information through measurement. However, this attempt is futile as she cannot pose a threat to the protocol's implementation nor obtain any communication content.

In fact, take the six-party HCBCHQIT protocol as an example, even if she secretly entangles her auxiliary qubits $|0\rangle_{Eve_1}$, $|0\rangle_{Eve_2}$ with those of a legitimate agent (e.g., Frank) without the knowledge of all legitimate agents, once Alice, Bob, Charlie, and the two controllers David and Emma complete all

TABLE II. Comparison of internal efficiency. ITM, QTN, CRN, and QRN stand for information transmission mode, transmitted qubits, classical resource consumption, and quantum resource consumption, respectively. η represents the intrinsic efficiency of the protocol.

Protocol	Hierarchical controlled (yes/no)	QRN	CRN	QTN	Information transmission mode (ITM)	η(%)
52 (Sec. 3)	Yes	9 + 0	9	3	QT: $A \overset{D}{\Rightarrow} B\, x_0\lvert 0\rangle + x_1\lvert 1\rangle$, QT: $B \overset{D,E}{\Rightarrow} C\, y_0\lvert 0\rangle + y_1\lvert 1\rangle$, QT: $C \overset{D,E,F}{\Rightarrow} A\, z_0\lvert 0\rangle + z_1\lvert 1\rangle$.	16.67
53	Yes	8 + 0	8	3	QT: $A \twoheadrightarrow B\, \alpha_0\lvert 0\rangle + \alpha_1\lvert 1\rangle$, QT: $B \overset{D}{\Rightarrow} C\, \beta_0\lvert 0\rangle + \beta_1\lvert 1\rangle$, QT: $C \overset{D,E}{\Rightarrow} A\, \gamma_0\lvert 0\rangle + \gamma_1\lvert 1\rangle$.	18.75
54 (Sec. 2)	Yes	9 + 0	9	3	QT: $A \overset{D}{\Rightarrow} B\, \alpha_0\lvert 0\rangle + \alpha_1\lvert 1\rangle$, QT: $B \overset{D,E}{\Rightarrow} C\, \beta_0\lvert 0\rangle + \beta_1\lvert 1\rangle$, QT: $C \overset{D,F}{\Rightarrow} A\, \gamma_0\lvert 0\rangle + \gamma_1\lvert 1\rangle$.	16.67
55 (Sec. 4)	No	13 + 3	24	6	$\forall i \in \{1, 2, 3\}$ QT: $A_{i\bmod 3} \overset{D}{\Rightarrow} A_{(i+1)\bmod 3}\, \alpha_i\lvert 0\rangle + \beta_i\lvert 1\rangle$, RSP: $A_{i\bmod 3} \overset{D}{\Rightarrow} A_{(i-1)\bmod 3}\, a_i\lvert 0\rangle + b_i\lvert 1\rangle$.	15.00
Our 2N-party protocol	Yes	5N + N	5N	2N	$\forall k \in \{1, 2, \cdots, N\}$ QT: $A_{k\bmod N} \overset{TC_k}{\Rightarrow} A_{(k+1)\bmod N}\, a_k\lvert 0\rangle + b_k\lvert 1\rangle$, RSP: $A_{k\bmod N} \overset{TC_k}{\Rightarrow} A_{(k-1)\bmod N}\, \alpha_k\lvert 0\rangle + \beta_k e^{i\theta_k}\lvert 1\rangle$.	18.18

measurements, Eq. (14) will become

$$
\begin{aligned}
|\Omega'_6\rangle = \frac{1}{8}&\Big[(a_1|0\rangle - b_1|1\rangle)_{B_2} \otimes (\alpha_1|0\rangle + \beta_1 e^{i\theta_1}|1\rangle)_{C_4}\Big] \\
&\otimes \Big[(a_2|0\rangle + b_2|1\rangle)_{C_2} \otimes (\alpha_2|0\rangle - \beta_2 e^{i\theta_2}|1\rangle)_{A_4}\Big] \\
&\otimes \Big[(a_2\alpha_2|00\rangle - a_2\beta_2 e^{i\theta_2}|10\rangle + b_2\alpha_2|01\rangle - b_2\beta_2 e^{i\theta_2}|11\rangle)_{A_2B_4} \\
&\otimes |000\rangle_{F,Eve_1,Eve_2} + (a_2\alpha_2|00\rangle - a_2\beta_2 e^{i\theta_2}|10\rangle \\
&- b_2\alpha_2|01\rangle + b_2\beta_2 e^{i\theta_2}|11\rangle)_{A_2B_4} \otimes |111\rangle_{F,Eve_1,Eve_2}\Big].
\end{aligned} \tag{22}
$$

When the measurement result of qubit F is $|0\rangle$, the state of the remaining particles will collapse into

$$
\begin{aligned}
|\Omega'_7\rangle = \frac{1}{8}&\Big[(a_1|0\rangle - b_1|1\rangle)_{B_2} \otimes (\alpha_1|0\rangle + \beta_1 e^{i\theta_1}|1\rangle)_{C_4}\Big] \\
&\otimes \Big[(a_2|0\rangle + b_2|1\rangle)_{C_2} \otimes (\alpha_2|0\rangle - \beta_2 e^{i\theta_2}|1\rangle)_{A_4}\Big] \\
&\otimes (a_2\alpha_2|00\rangle - a_2\beta_2 e^{i\theta_2}|10\rangle + b_2\alpha_2|01\rangle - b_2\beta_2 e^{i\theta_2}|11\rangle)_{A_2B_4} \\
&\otimes |00\rangle_{Eve_1,Eve_2}.
\end{aligned} \tag{23}
$$

When the measurement result of qubit F is $|1\rangle$, the state of the remaining particles will collapse into

$$
\begin{aligned}
|\Omega''_7\rangle = \frac{1}{8}&\Big[(a_1|0\rangle - b_1|1\rangle)_{B_2} \otimes (\alpha_1|0\rangle + \beta_1 e^{i\theta_1}|1\rangle)_{C_4}\Big] \\
&\otimes \Big[(a_2|0\rangle + b_2|1\rangle)_{C_2} \otimes (\alpha_2|0\rangle - \beta_2 e^{i\theta_2}|1\rangle)_{A_4}\Big] \\
&\otimes (a_2\alpha_2|00\rangle - a_2\beta_2 e^{i\theta_2}|10\rangle - b_2\alpha_2|01\rangle + b_2\beta_2 e^{i\theta_2}|11\rangle)_{A_2B_4} \\
&\otimes |11\rangle_{Eve_1,Eve_2}].
\end{aligned} \tag{24}
$$

Obviously, the quantum states owned by Frank and Eve will collapse into direct product states—Eve can only measure her own qubits. Even if she gains access to other agents' measurement results, she still cannot retrieve the transmitted quantum information.

Second, the protocol can resist intercept-resend attacks.

To solve this problem, we can draw on the methods used in the existing BB84 quantum key distribution protocol. When distributing qubits, David first randomly prepares multiple single-qubit decoy states using the basis $\{|0\rangle, |1\rangle, |-\rangle, |+\rangle\}$, inserts them into the entangled state that forms the quantum channel (arranged in an order unknown to Eve) and distributes them to all legitimate agents. Suppose Eve intercepts Alice's particles 2, 8, 11, and 14, and additionally prepares a 15-qubit entangled state, sending four of its qubits to Alice. In fact, once all qubits are confirmed to have been successfully received, David will publicly announce the positions of the decoy states and the correct measurement bases via a classical channel. Then, each participating agent measures the decoy states within the qubits they received and reports the results to David. By comparing the accuracy of these results, David can detect the presence of an eavesdropper, thereby eliminating the possibility of Eve carrying out an intercept-resend attack.

Third, the protocol can resist internal collusion attacks.

The collusive attack by control agents is infeasible. For one thing, the control agents have no knowledge of the transmitted quantum state. For another, as can be seen from Eq. (17), the qubits of the control agents are in a direct product state with the remaining particles in the channel. When the communication agents perform entanglement swapping with the qubits in the channel, none of the control agents can obtain the transmitted information. Qubits of the control agents are in an entangled state, which also avoids the occurrence of behavior where lower-level control agents collude to bypass higher-level control agents.

The collusive attack between control agents and communication agents is infeasible. For instance, if Bob and Emma collude with the intention of eavesdropping on the quantum state information sent by Alice to Charlie, as can be seen from Eq. (9), after Alice and David complete their measurements, the quantum state prepared by Alice for Charlie has already been reconstructed on Charlie's qubit C_4. Meanwhile, Bob and Emma's qubits are also in a direct product state with qubit C_4; relying solely on the obtained classical information and measurement results, it is impossible for them to eavesdrop on the information. The same analysis can resist collusive attacks among communication agents.

To conclude, our proposed protocol is capable of readily detecting both internal and external eavesdropping and deception attacks. More importantly, the hierarchical control mechanism embedded in the protocol serves as a critical security enhancement: by requiring collaborative authorization from the group of controlling agents, it fundamentally prevents any single communication agent from unilaterally completing quantum communication tasks. This multi-layered permission structure not only constrains potential malicious behaviors of internal participants through distributed decision-making but also raises the threshold for external attackers to bypass the security system—thereby forming a dual defense line that significantly strengthens the overall security of the protocol.

VI. DISCUSSION AND CONCLUSION

This paper introduces a special method for preparing a 15-qubit entangled quantum state, and based on this entangled state, we propose for the first time a novel hierarchically controlled bidirectional cyclic quantum hybrid communication scheme, namely, the HCBCHQIT protocol. In the six-party communication scheme, there are three communication agents at different levels and three control agents with different permissions. Among them, Alice, as a high-level agent, can successfully complete the transmission tasks of bidirectional arbitrary unknown and known single-qubit states with only the assistance of David, the primary (level-1) control agent. Bob, a medium-level agent, needs to complete his bidirectional communication tasks with the joint assistance of David, the primary control agent, and Emma, the level-2 sub-control agent. As a low-level agent, Charlie must complete quantum communication tasks in both directions under the joint control of three controllers: David, the primary (level-1) control agent; Emma, the sub-control (level-2) agent; and Frank, the sub-control (level-3) agent. Then, we extended this protocol from six parties to 2N parties, enabling it to adapt to larger-scale quantum communication network scenarios and providing a more universal framework for multi-agent hierarchically controlled quantum communication applications. Finally, we calculated the intrinsic efficiency of our protocol and analyzed and compared it with some existing literature studies, demonstrating the efficiency of our protocol.

In the implementation of the protocol, only basic Bell basis measurements, single-qubit projection measurements, and Pauli quantum gate operations required for reconstructing quantum states are utilized. Based on existing quantum experimental technologies, this protocol can be easily reproduced with the aid of simulation platform tools such as IBM Qiskit Aer.

It is worth noting that the protocol we proposed can also meet the needs of diversified combined control for multiple control authorities. Taking the six-party HCBCHQIT protocol as an example, if the qubits E, D, and F in the original channel H_1 in Eq. (1) are only allocated to a single control agent (such as David), then the protocol will degenerate into a conventional controlled cyclic bidirectional hybrid communication protocol with only one controller. At this time, David can perform joint measurement on particles D, E, and F under the measurement basis $\{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\}$, thereby realizing unified and single control over the communication process. In addition, if it is necessary to adjust to a two-level control management mode (for example, to realize individual control over Alice and unified control over Bob and Charlie), two control agents can be set, such as the main control David and the sub-control Emma. At this time, qubit D in the original channel H_1 can be allocated to David and qubits E and F to Emma. David still measures particle D under the $\{|0\rangle, |1\rangle\}$ basis to realize the main control function for the entire communication task; while Emma realizes the sub-control function for the communication tasks of Bob and Charlie by performing joint measurement on particles E and F on the measurement basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. In short, our protocol can not only realize step-by-step individual control but also realize diversified control functions combining partial unified control and partial step-by-step control, providing more flexible and safer application scenario theoretical support for the quantum communication network.

Compared with the hierarchically controlled bidirectional hybrid communication protocols in Refs. 48 and 49, the HCBCHQIT protocol constructed in this paper has significant advantages. In the previously proposed bidirectional hybrid communication protocols involving two parties, the interaction logic is limited to a linear architecture, which can only realize simple interactions with direct associations between two entities. However, our protocol relies on the Alice–Bob–Charlie cyclic link to break through the constraints of linear interaction and integrates the primary control of David and the hierarchical control mechanism of Emma (and Frank). It supports diverse combined control modes, which are not only compatible with complex permission management among multiple entities and can cover the functions of protocols such as those in Refs. 48 and 49 but also realize the innovative logic of "primary-hierarchical coordinated control + cyclic interaction," adapting to complex scenarios involving multiple entities. In addition, the proposed protocol has good scalability and can be extended to 2N-party scenarios, including N communication agents and N control agents belonging to different levels. While retaining the advantages of hierarchical control, it reconstructs information transmission paths through cyclic hybrid communication, breaking through the "point-to-point" linear limitation and forming a more flexible and resilient network topology, which lays the foundation for large-scale quantum communication networking. This design provides a more advanced solution for the construction of quantum communication networks, achieves breakthroughs in control strategy innovation and system scalability, and supports efficient and secure interactions in complex scenarios.

In addition, the protocol proposed has practical application value and can be applied in scenarios such as encrypted transmission of banking information, hierarchical command communication in the workplace, military command collaboration, and logistics scheduling coordination. Next, we take military applications as an example to illustrate the practicality of our proposed protocol. In a combat mission, Alice, a senior communication agent (theater command center), commands overall operational information. Under the sole control of David, the main control agent (senior commander), Alice performs quantum teleportation of the enemy's latest trajectory data to Bob, a mid-level communication agent (air defense brigade), and remotely prepares the quantum state encoded with new attack instructions for Charlie, a junior communication agent (firepower unit). After receiving the enemy's trajectory information, Bob, under the coordinated control of David (senior commander) and Emma (mid-level commander), converts the enemy's trajectory information into specific fire strike coordinates and sends them to Charlie. Meanwhile, he feeds back the evaluation results of Charlie's previous attack to Alice. Upon receiving the specific strike coordinates and preliminary attack instructions, Charlie conducts the fire strike. Subsequently, under the joint control of David, Emma, and Frank, the three-level commanders, he sends the damage rate of this round of fire strike to Bob and synchronizes the enemy's latest activity range with Alice. This information interaction process in the combat scenario fully demonstrates the practical application effectiveness of our proposed protocol in complex hierarchical communication and collaborative control.

In summary, this study integrates bidirectional cyclic hybrid quantum communication with a hierarchical control mechanism, offering theoretical references for the construction of future quantum communication networks featuring explicit hierarchical permission allocation. The main contributions are as follows.

(i) First proposal of the hierarchical control-cyclic bidirectional integrated quantum communication protocol. This work is the first to propose a quantum communication protocol integrating hierarchical control and cyclic bidirectional transmission, which not only fills the technical gap in this combination but also supports diverse application scenarios (e.g., encrypted data transmission and hierarchical command communication).

(ii) 2*N*-party protocol extension with fixed efficiency. The six-party basic protocol is extended to a 2*N*-party scenario involving *N* communication agents and *N* control agents. Notably, the extended protocol maintains an intrinsic transmission efficiency of approximately 18.18%, which is independent of the number of participants (*N*). This effectively addresses the issue of efficiency decay with increasing nodes in traditional multi-party protocols, making it adaptable to the hierarchical deployment of large-scale, complex quantum communication networks.

(iii) General quantum channel construction method. A clear and reproducible general method for quantum channel construction is explicitly proposed. For the six-party scenario, it constructs a 15-particle maximally entangled state composed of partial G-states; for the 2*N*-party scenario, it generates matching multi-particle entangled channels based on the same logic without additional new designs. This method provides referable resource support for channel design in similar multi-agent quantum protocols.

(iv) Strong experimental feasibility. The protocol only involves low-complexity quantum operations, including Pauli gate operations, basic Bell-basis measurements, and single-particle projection measurements, without relying on complex multi-qubit collaborative manipulation. It is compatible with existing quantum simulation and experimental platforms (e.g., IBM Qiskit Aer), significantly reducing the technical threshold for translating theoretical schemes into practical applications and laying a foundation for subsequent experimental verification.

The multi-agent collaborative control model and scalable architecture not only cater to quantum communication scenarios with varied security requirements and scales but also lay a solid theoretical and technical foundation for the hierarchical governance and large-scale deployment of quantum communication networks. In future research, we can further explore the design of protocols under noisy environments and based on non-maximally entangled quantum state channels.

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AUTHOR DECLARATIONS

Conflict of Interest

The author has no conflicts to disclose.

Author Contributions

Benchao Yang: Formal analysis (lead); Methodology (lead); Supervision (lead); Visualization (lead); Writing – original draft (lead); Writing – review & editing (lead).

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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