

Alpha decay of thermally excited nuclei

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Abstract

One of the prominent decay modes of heavy nuclei which are produced in astrophysical environments at temperatures of the order of 10^9 K is the α (^4He) decay. Thermally enhanced α decay rates are evaluated within the standard scheme of a tunneling decay where the α particle tunnels through the potential barrier formed by its interaction with the daughter nucleus. Following the observation that there exists several excited levels with the possibility of an α decay when the daughter nucleus is at a shell closure, we focus in particular on decays producing daughter nuclei with the neutron number, $N = 126$. Within a statistical approach we find that the half-lives, $t_{1/2}(T)$, for temperatures ranging from $T = 0$ to 2.4 GK can decrease by 1–2 orders of magnitude with the exception of the decay of ^{212}Po which decays to the doubly magic daughter ^{208}Pb , where $t_{1/2}(T)$ decreases by 5 orders of magnitude. The effect of these thermally enhanced α decays on the r -process nucleosynthesis can be significant in view of the mass build-up at the waiting point nuclei with closed neutron shells.

Keywords: alpha decay, nucleosynthesis, r -process, nuclear excited states, kilonova

(Some figures may appear in colour only in the online journal)

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1. Introduction

The unprecedented observational data from compact object mergers in recent years have confirmed the general model of the synthesis of heavy elements via rapid-neutron captures [1–4]. However, details required for the interpretation of the correlated ultraviolet, optical and infrared electromagnetic emissions, also known as kilonova (for a review see [5]), are still unclear. For example, accurate kilonova modeling relies on the knowledge of the fraction of mass ejected during the event, its opacity, and the radioactive decay of freshly produced lanthanide and actinide nuclei [6, 7].

Nucleosynthesis involving the production of heavy and super heavy nuclei is expected to take place at very high temperatures of the order of a few Giga Kelvin (GK). The *transuranic* elements in particular are considered to be produced in the *r*-process occurring in environments where the neutron captures are faster than the β decays. The conditions during the *r*-process, involving densities, $\rho(n) \sim 10^{20} \text{ cm}^{-3}$ and temperatures $T \sim 10^9 \text{ K}$, are explosive conditions [8]. The possible sites for the *r*-process are considered to be [8] (a) neutronized atmosphere above the proto-neutron star in a Type II supernova, (b) neutron-rich jets from supernovae or neutron star mergers, (c) inhomogeneous Big Bang, (d) He/C zones in Type II supernovae, (e) red giant He flash, (f) spallation neutrons in He zone (g) neutrino driven wind from freshly born neutron stars and (h) outflows from black hole accretion discs originated in compact object mergers or collapsars (for recent reviews of the possible *r*-process sites see Cowan *et al* [9] and Côté *et al* [10]). The abundance of elements is found through a network [11, 12] of coupled differential equations involving nuclear reaction rates at elevated temperatures. Theory and models play an important role in determining the latter for neutron-rich nuclei which cannot be measured in terrestrial laboratories.

The *r*-process nucleosynthesis path is along highly unstable, exotic, and neutron-rich nuclei that in principle does not involve alpha emitters. However, once heavy neutron-rich nuclei in the region with $Z > 82$ are formed, and with the depletion of further neutron captures (i.e. after *r*-process freeze out), those nuclei decay by different modes (e.g. beta, alpha, fission). Some decay modes would compete. This process can even lead to the formation of nuclei in the actinide region. The nuclei studied in this work are part of the mass region ($A > 208$) where several alpha emitters are found [13, 14]. Thus, it is not only the photodissociation and neutron capture cross sections but also fission (spontaneous and induced) and the decay rates which are important for the abundance evolution. The explosive conditions in supernovae and neutron star mergers [15–17] leading to considerably high temperatures could result in nuclei existing in excited states. Though the possible influence of these nuclear thermal excitations is taken into account in the production reactions as well as in their reverse reactions, with libraries publicly available for the scientific community (e.g. [18–20]), the same is not true in the case of α decay. These decay rates, entering as an input to the network calculations, are taken to be the ground state (or terrestrial) half-lives [11]. However, one must note that for high ambient temperatures, the population factor for the excited energy levels of a nucleus is large. These are thermal excitations and one must take into account the possibility of α decay of thermally excited nuclei. This is in particular quite important for the *r*-process nucleosynthesis where the closed neutron shells present waiting points due to the fact that it takes a long time for the successive β decays, which are slow at the shell closures, to occur and allow progression through higher N nuclei. In section 4, we will see that the α decays of parent nuclei producing daughter nuclei at the shell closures display a stronger temperature dependence with increased decay rates at higher temperatures.

Apart from the paper of Perrone and Clayton [21], published in 1970, there is indeed no estimate of the possible effects of temperature on the α decay half-lives. However, given the

fact that about 50 years ago, the data on excited levels of nuclei was scarce, calculations were performed assuming a continuum of states described by the available density of states. The latter assumption as we will see leads to a very large overestimate of the enhancement in the decay rate due to temperature.

In the present work, we investigate the temperature dependence of the α decay rates relevant for the r -process nucleosynthesis. The calculations are performed within two different approaches: (i) a statistical model which makes use of the experimentally measured excited levels and an empirical decay law (fitted to data) in the absence of available data on the half-lives and (ii) a theoretical model which treats the α decay as a semiclassical tunneling of the α particle through the barrier created by the interaction of the α and the daughter nucleus which exist inside the parent in the form of a preformed cluster. The latter calculation is performed using a density dependent folding model which has been reasonably successful in reproducing the measured α decay half-lives [22, 23]. The formalism is presented in sections 2, and 3. In section 4 we connect to shell closures, and in section 5 we discuss the results. Finally, we summarize our findings in section 6.

2. Alpha decay formalism

One of the most successful achievements of the quantum theory is the explanation of the α -decay of radioactive nuclei as a tunneling problem. This approach was developed independently by Gamow [24] and by Gurney and Condon [25] in the late twenties. Though α -decay has been studied since then within different quantum mechanical approaches [26], semiclassical approaches based on the tunneling of an α particle through the potential barrier created by its interaction with the daughter nucleus produced in the decay are some of the most popular and widely used methods for calculating half-lives. The interaction potential between the α (${}^4\text{He}$ nucleus) and the daughter nucleus, and the Q -value, which is usually taken to be the energy of the tunneling α , play the main role in determining the tunneling probability and hence the half-life. Using the JWKB approximation [27], different semiclassical approaches lead to the same expression for the α -decay width [22]

$$\Gamma = P_\alpha \frac{\hbar^2}{2\mu} \left[\int_{r_1}^{r_2} \frac{dr}{\kappa(r)} \right]^{-1} \exp \left[-2 \int_{r_2}^{r_3} \kappa(r) dr \right], \quad (1)$$

with the so-called wave number $\kappa(r) = \sqrt{\frac{2\mu}{\hbar^2} |V(r) - Q|}$ and μ the reduced mass of the daughter- α system. The classical turning points r_1 , r_2 and r_3 are obtained by solving the equation $V(r) = Q$ where Q is the energy of the tunneling α -particle. The factor in front of the exponential arises due to the normalization of the bound state wave function in the region between r_1 and r_2 . The exponential factor is the penetration probability. The α -decay half-life of an isotope is evaluated as

$$t_{1/2} = \frac{\hbar \ln 2}{\Gamma}. \quad (2)$$

Since the tunneling decay assumes the existence of a preformed cluster of the ${}^4\text{He}$ and daughter nucleus inside the decaying parent nucleus, one must include a preformation probability P_α in the expression for half-life. This factor, in principle, can be expressed as an overlap between the wave functions of the parent nucleus and the decaying-state wave function describing the α -particle coupled to the daughter nucleus. Such a microscopic undertaking is still considered a difficult task and a phenomenological way to determine P_α is

simply taking the ratio of the theoretical and experimental half-lives, such that, $P_\alpha = t_{1/2}^{\text{theory}}/t_{1/2}^{\text{exp}}$, where, $t_{1/2}^{\text{theory}}$ is evaluated using Γ from equation (1) but with $P_\alpha = 1$.

The total potential between the α and the daughter nucleus is typically written as a function of the distance between their centers of mass as,

$$V(r) = V_n(r) + V_C(r) + \frac{\hbar^2(l + 1/2)^2}{2\mu r^2}, \quad (3)$$

where $V_n(r)$ and $V_C(r)$ are the nuclear and Coulomb potentials, respectively. The last term in equation (3) represents the Langer-modified centrifugal potential [28] which must be used while using the JWKB approximation. Some of the calculations presented in this work will be performed within the density dependent double folding model (DFM) which is based on realistic nucleon-nucleon interactions and has been reasonably successful in reproducing the experimental half-lives. The details of this potential can be found in [23, 29]. Here we describe it briefly. In the DFM, the nucleus-nucleus interaction is related to the NN interaction by folding an effective NN interaction over the density distribution of the two nuclei. The folded nuclear potential is written as

$$V_n(\mathbf{r}) = \lambda \int d\mathbf{r}_1 d\mathbf{r}_2 \rho_\alpha(\mathbf{r}_1) v_N(|\mathbf{s}| = |\mathbf{r} + \mathbf{r}_2 - \mathbf{r}_1|) \rho_d(\mathbf{r}_2), \quad (4)$$

where ρ_i ($i = d, \alpha$) are the densities of the alpha and the daughter nucleus in decay, and $v_N(|\mathbf{s}|)$ is the nucleon-nucleon (NN) interaction (see [29] for the figure with details). The matter density distribution of the heavy daughter is calculated as

$$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R}{a}\right)}, \quad (5)$$

where ρ_0 is obtained by normalizing $\rho(r)$ to the mass number, $\int \rho(\mathbf{r}) d\mathbf{r} = A$, and the constants are given as $R = 1.07A^{1/3}$ fm and $a = 0.54$ fm. The alpha or ${}^4\text{He}$ density distribution is given using a standard Gaussian form [30], namely,

$$\rho_\alpha(r) = 0.4229 \exp(-0.7024 r^2). \quad (6)$$

We use the popular choice of the effective NN interaction which is based on the M3Y-Reid-type soft core potential,

$$v_N(|\mathbf{s}|) = 7999 \frac{\exp(-4|\mathbf{s}|)}{4|\mathbf{s}|} - 2134 \frac{\exp(-2.5|\mathbf{s}|)}{2.5|\mathbf{s}|} + J_{00} \delta(\mathbf{s}), \quad (7)$$

where $|\mathbf{s}| = |\mathbf{r} + \mathbf{r}_2 - \mathbf{r}_1|$ is the distance between a nucleon in the daughter nucleus and a nucleon in the alpha. The above NN interaction consists of a short-ranged repulsive part and a long-ranged attractive one, in addition to the zero-range contribution $J_{00} \delta(\mathbf{s})$ with $J_{00} = -276(1 - 0.005 E/A_e)$. The latter is the so-called knock-on exchange term which takes into account the antisymmetrization of identical nucleons in the alpha and the daughter nucleus. It represents a kind of nonlocality in the DFM potential and in order to avoid double counting, is usually not included in the calculation if one uses nonlocal nuclear potentials [29]. The strength of the nuclear potential, λ , is deduced by requiring the Bohr-Sommerfeld quantization condition to be satisfied [23]. The Coulomb potential, $V_C(r)$, is obtained in a similar way with the matter densities of the alpha and the daughter replaced by their charge densities (which have the same form as above but are normalized to the number of protons).

The angular momentum, l , carried by the alpha particle must satisfy the following spin-parity selection rules,

Table 1. Comparison of the alpha decay half-lives evaluated using the double folding model (DFM) and a universal decay law (UDL) (9), with experiments. The phenomenological preformation factors, $P_\alpha = t_{1/2}^{\text{DFM}}/t_{1/2}^{\text{exp}}$ using the DFM half-lives are also shown. Here, $t_{1/2}^{\text{DFM}}$ is evaluated using Γ from equation (1) but with $P_\alpha = 1$.

Isotope	Q -Value [MeV]	$t_{1/2}^{\text{exp}}$ [s]	$t_{1/2}^{\text{DFM}}$ [s]	P_α [DFM]	$t_{1/2}^{\text{UDL}}$ [s]
$^{212}_{84}\text{Po}$	8.954	2.9×10^{-7}	6.49×10^{-8}	0.22	1.57×10^{-7}
$^{214}_{86}\text{Rn}$	9.208	2.7×10^{-7}	7.91×10^{-8}	0.29	2.06×10^{-7}
$^{215}_{87}\text{Fr}$	9.540	8.6×10^{-8}	2.9×10^{-8}	0.33	7.10×10^{-8}
$^{216}_{88}\text{Ra}$	9.526	1.8×10^{-7}	6.86×10^{-8}	0.37	1.86×10^{-7}
$^{217}_{89}\text{Ac}$	9.832	6.9×10^{-8}	2.98×10^{-8}	0.43	7.67×10^{-8}

$$|J_p - J_d| \leq l \leq |J_p + J_d| \quad \text{and} \quad \pi_d = \pi_p(-1)^l, \quad (8)$$

where (J_p, π_p) and (J_d, π_d) are the (spin, parity) of the parent and daughter nuclei, respectively.

In table 1 we present the half-lives for some nuclei using the DFM. We examine transitions for which the alpha particle has the minimum angular momentum value, l_{\min} , satisfying equations (8). For the decays considered in table 1, $l_{\min} = 0$. The experimental half-lives and the corresponding preformation factors are also listed in table 1. The theoretical values obtained are close to some others found in the literature [31].

The DFM calculations can in principle be improved with the inclusion of deformation and nonlocalities in the interaction potential [23, 29]. However, the objective of the present work is to perform a comparative study of approaches for half-lives measured on earth and in a hot astrophysical environment and hence it suffices to perform calculations within a model which can reproduce alpha decay half-lives reasonably well. Indeed, we shall also use an empirical formula (a universal decay law (UDL) for α and cluster decay, obtained from fits to extensive data) for the half-lives calculated within the statistical approach (to be explained in the next section) since (i) the UDL gives the right order of magnitude estimate of half-lives and (ii) it would be quite a tedious undertaking to evaluate the half-lives of several excited states within the DFM without any significant advantage. Such UDLs are usually obtained by starting with an analytical expression [32] which is based on the assumption of a rectangular well for the nuclear potential and a point-like Coulomb potential between the decay products of the radioactive nucleus. The constants appearing in such an expression are then assumed to be free parameters and fitted to an extensive set of data. The latter compensates for the simplistic assumptions made in the derivation of the empirical formula and provides a useful expression depending on the number of nucleons and the Q -value of the decay. We use the following UDL obtained in [33, 34].

$$\log_{10} t_{1/2} = 2aZ_d \sqrt{\frac{A}{Q}} + b\sqrt{4AZ_d(A_d^{1/3} + 4^{1/3})} + c + d\sqrt{4AZ_d(A_d^{1/3} + 4^{1/3})} \sqrt{l(l+1)}, \quad (9)$$

where $A = \frac{4A_d}{A_d + 4}$, with A_d, Z_d the mass and proton numbers of the daughter nucleus respectively, and the constants are given by $a = 0.439\,206\,0$, $b = -0.394\,41\,74$, $c = -27.064\,87\,30$ and $d = 0.005\,18\,25$ such that Q is taken in MeV resulting in the half-life, $t_{1/2}$, in seconds.

In the next section, we shall also describe the effective Q -value approach to evaluate the temperature-dependent half-lives using the DFM and the UDL. Comparing it with the statistical approach gives us an idea of the usefulness of this approach in the context of α decay half-lives in an astrophysical environment. We shall also discuss one of the earliest attempts to evaluate the thermally enhanced α -decay rates in connection with the s -process nucleosynthesis [21]. The authors in [21] predicted a decrease in the half-lives by about 20–60 orders of magnitude depending on the nucleus for a temperature around 2 GK. We do not find such spectacular effects of temperature in our calculations. The reason for this difference will become evident in the next sections.

3. Temperature-dependent half-lives

Nucleosynthesis in the later stages of stellar evolution, especially through the r -process is considered to take place at considerably high temperatures of the order of 10^9 K. The calculation of the abundance of heavy elements depends on a precise determination of the nuclear reaction rates of the processes which produce the elements as well as the processes which destroy the newly formed nuclei. Though the neutron capture cross sections and their reverse reaction rates at elevated temperatures are carefully taken into account, the network codes usually rely on the laboratory values of the half-lives of α decays from the ground states of nuclei. Perrone and Clayton [21] investigated the effect of thermally excited states in the alpha decay of some nuclei and its application in the s -process nucleosynthesis. However, such effects are not included in the r -process simulations.

In what follows, we present a statistical calculation of thermally enhanced α -decay rates that includes experimentally observed excited levels for some nuclei. We also propose a model that uses an effective Q -value approach and which makes use of an average excitation energy at a given temperature. Finally, we discuss the approach of Perrone and Clayton briefly for completeness.

3.1. Statistical calculation

The temperature-dependent half-life, $t_{1/2}(T) = \ln 2/\lambda(T)$, can be evaluated within the standard statistical approach [35] by defining the temperature-dependent decay constant as follows:

$$\lambda(T) = \sum_i p_i \sum_j \lambda_{ij}. \quad (10)$$

Here the sums over i and j are over the parent and daughter states respectively. Thus, λ_{ij} is the decay constant for the decay of the i^{th} level in the parent to the j^{th} level in the daughter such that

$$\lambda_{ij} = \frac{\ln(2)}{t_{1/2}^{ij}}. \quad (11)$$

The population probability, p_i , is given with a Boltzmann factor as [36]

$$p_i = \frac{(2J_i + 1)e^{(-E_i/k_B T)}}{\sum_l (2J_l + 1)e^{(-E_l/k_B T)}}, \quad (12)$$

where J_i and E_i are the spin and the excitation energy of the state i , respectively. Inserting (12) in (10)

$$\lambda(T) = \frac{\ln(2)}{\sum_i (2J_i + 1) e^{-E_i/k_B T}} \sum_{i,j} \frac{(2J_i + 1) e^{-E_i/k_B T}}{t_{1/2}^i} (BR)_{ij} \quad (13)$$

where $(BR)_{ij}$ is the branching fraction for the decay from the i^{th} level of the parent nucleus to the j^{th} level in the daughter nucleus. The detailed decay schemes and the percentage decay to a particular channel, i.e. $I = [\lambda_{ij}/\lambda_{\text{tot}}] * 100\%$ can be found at the website in [37]. The branching fraction, $(BR)_{ij} = \lambda_{ij}/\lambda_{\text{tot}}$, can thus be obtained from the data tables. To evaluate the temperature-dependent half-life in the statistical approach, we shall use equation (13) with the input half-lives, $t_{1/2}^i$ and $(BR)_{ij}$ taken from the experiment. If the experimental half-life of a level is not known, it is calculated using the UDL at an effective Q -value given by $Q + E_i$, where E_i is the energy of the excited level. In such cases, even if the experimental branching ratio is known, it is not used but taken to be 100% since the UDL per definition is formulated only for the alpha decay channel.

3.2. Effective Q -value model

Alpha decay half-lives are very sensitive to the penetrability factor which at the same time means that they are very sensitive to the Q -values too. For a tunneling decay of an α particle taking place in a very hot surrounding, one can model the effect of temperature by an increase in the energy, Q , with which the α tunnels through the Coulomb barrier. A higher Q -value would clearly reduce the area under the integral in the exponential term in equation (1) and hence lead to an increase in the decay probability (and therefore a decrease in the half-life). This increase in the Q -value can be modelled by adding an average excitation energy of the nucleus at a given temperature. Such an effective Q -value of an α -decay can be expressed as

$$Q_{\text{eff}} = Q + \bar{\epsilon}(A, Z, T) \quad (14)$$

where $\bar{\epsilon}(A, Z, T)$ is given by the standard definition of the average excitation energy [38] in statistical physics as,

$$\bar{\epsilon}(A, Z, T) = -\frac{\partial}{\partial \beta} \ln Z(A, Z, T) \quad (15)$$

with the canonical partition function Z given by

$$Z(A, Z, T) = \sum_i^n g_i \exp(-\beta E_i) + \int_{E_n}^{E_{\text{max}}} D(E) \exp(-\beta E) dE \quad (16)$$

where, $\beta = 1/(k_B T)$, $g_i = 2J_i + 1$ and J_i is the spin of the i^{th} level and $D(E)$ is the nuclear level density for which we choose the following form [39]:

$$D(E) = \frac{\sqrt{\pi}}{12} \frac{e^{2(aE)^{1/2}}}{a^{1/4} E^{5/4}}. \quad (17)$$

The level density parameter, a , is taken to be $A/9$ where A is the mass number of the parent nucleus. If we consider the discrete levels as well as the continuum, the average excitation energy, using the above equations can be expressed as

$$\bar{\epsilon}(A, Z, T) = \frac{\sum_i^n g_i E_i \exp(-\beta E_i) + \int_{E_n}^{E_{\text{max}}} E \times D(E) \exp(-\beta E) dE}{\sum_i^n g_i \exp(-\beta E_i) + \int_{E_n}^{E_{\text{max}}} D(E) \exp(-\beta E) dE}. \quad (18)$$

The temperature, T , appearing in the above expression (through $\beta = 1/(k_B T)$), is in principle the *nuclear temperature*. Modification of nuclear properties at finite temperatures is relevant both for applications in astrophysics [40–42] and for models of finite nuclei and nuclear

Table 2. Average excitation energies evaluated using equation (18) in conjunction with (17).

	Q [MeV]	$\bar{\epsilon}(A, Z, T)$ [MeV]				
		0.8 GK	1.2 GK	1.6 GK	2 GK	2.4 GK
$^{212}_{84}\text{Po}$	8.954	0.000096	0.0034	0.0231	0.081 42	0.2004
$^{214}_{86}\text{Rn}$	9.208	0.000147	0.004 38	0.025 20	0.0754	0.1609
$^{215}_{87}\text{Fr}$	9.540	0.00096	0.027 38	0.1349	0.3031	0.4646
$^{216}_{88}\text{Ra}$	9.526	0.00020	0.004 55	0.0252	0.0729	0.1510
$^{217}_{89}\text{Ac}$	9.832	0.001126	0.02741	0.1233	0.2684	0.4129

matter at high excitation energy [43, 44]. The internal excitations of nuclei can play an important role in regulating their abundance. The excitations form an important ingredient in multifragmentation studies of hot nuclei. For example, the authors in [45], working within a statistical multifragmentation model find significant temperature-dependent modifications relevant for stellar dynamics and nucleosynthesis. They perform calculations for supernova matter by assuming that the nuclei have the same internal temperature as the surrounding medium. In the present work, we shall also assume a dynamical equilibrium such that the temperature, T , above can be assumed to be the surrounding temperature.

Having defined the effective Q -value in this manner, the temperature-dependent half-life within the density dependent DFM can be evaluated using equation (1) with Q replaced by Q_{eff} . With a similar replacement we can also estimate the temperature dependence using the UDL in equation (9). In table 2 we list the average excitation energies at different temperatures for some heavy nuclei which will be studied in this work.

3.3. Perrone and Clayton approach

The unique attempt in literature for the evaluation of the thermally enhanced α decay half-lives of nuclei in stellar environments can be found in [21] by Perrone and Clayton. They evaluated the α decay rates of several nuclei formed in the s -process nucleosynthesis to find that the half-lives are greatly enhanced if one considered the contributions from thermally excited nuclei at elevated temperatures. The temperature-dependent half-life in [21] was given by

$$[t_{1/2}^{\text{PC}}(Z, A, T)]^{-1} = \int_0^\infty \sum_J \frac{F(Z, A, E, J, T)D(Z, A, E, J)dE}{t_{1/2}(Z, A, E, J)}. \quad (19)$$

where $t_{1/2}(Z, A, E, J)$, the temperature-independent half-life for the decay of the parent nucleus to the daughter particle ground state is weighed by the nuclear density of states $D(Z, A, E, J)$ and the occupation probability $F(E, J, T)$, which for an excited nucleus with energy E_i and spin J_i is given by

$$F(E_i, J_i, T) = \frac{(2J_i + 1)e^{-E_i/k_B T}}{\sum_i (2J_i + 1)e^{-E_i/k_B T}} \approx \frac{(2J_i + 1)e^{-E_i/k_B T}}{2J_0 + 1}, \quad (20)$$

with J_0 the spin of the ground state and k_B the Boltzmann constant. The last approximation was justified by mentioning that the nuclear ground state dominates the sum over states for temperatures below 2 GK. The nuclear level density used was taken from [46]. The

temperature-independent half-life in the denominator of equation (19) was calculated using the standard semiclassical approach for tunneling decay where the penetration probability depends on the Q -value of α decay. The tunneling α is assumed to have an energy equal to the Q -value plus the excitation energy (which is indeed the dummy variable of the integral in 19). Since the penetration factor is evaluated within a simple model for the potential, the calculation of the temperature-independent half-life appearing inside the integral in equation (19) essentially resembles the half-life as found in textbooks [32] but evaluated at an effective Q -value. With such a half-life as an input, the temperature-dependent half-life was evaluated as an integral over energies from 0 to ∞ .

It is important to note that although the nuclear energy levels are discrete, the half-life $t_{1/2}^{\text{PC}}(Z, A, T)$ defined by (19) in [21] treats the nucleus as having a continuum of excited states with the density of states given by $D(Z, A, E, J)$. The authors mention the need for explicitly taking the discrete levels into account if one wished to use the calculations in context with astrophysics. However, given the meagre data available in 1970, the authors found their approach appropriate for an initial survey of the problem. The present work takes into account this omission made by Perrone and Clayton due to the lack of data and finds that even if the decrease in half-lives is not as spectacular as that in [21], it is surely significant and possibly relevant for nucleosynthesis calculations.

4. Excited nuclear levels, Q-values and shell closures

A naive expectation for the decay of thermally excited nuclei would be that for a nucleus which decays by α decay in its ground state, there must exist some excited levels which decay by emitting an α too. However, experimental results show that this is, in general, not true. A careful examination of the nuclear data tables reveals that the α decay occurs in heavy nuclei mostly in the ground state. An excited parent nucleus often decays by emitting a photon (γ -decay). In fact, the excited nucleus undergoes several successive γ -decays before reaching its ground state. If this were true for all nuclei decaying by α decay, an undertaking as in the present work would not make much sense. However, based on a conjecture in an earlier work [47], we did find exceptions.

In [47], the authors noted an interesting phenomenon while performing a calculation of the tunneling times in α decay. The amount of time spent by an α in front of the barrier before tunneling (the so-called transmission dwell time), reaches a minimum at $N = 128$ of the parent nucleus in the region from $N = 116$ to $N = 132$ which was investigated. $N = 128$ of the parent however corresponds to $N = 126$ of the daughter, implying that the α spends the least amount of time with the magic daughter. The latter essentially corresponded to the shortest half-life or the highest decay rate for a nucleus with $N = 128$. Putting it differently, one can say that a parent nucleus decaying by emitting an α does it readily when the daughter happens to be at the shell closure of $N = 126$. Motivated by this finding in [47], we examined the list of excited levels of nuclei with $N = 128$ and with the possibility of an α decay in the ground state. Not much to our surprise, indeed, we found that the nuclei, ${}_{84}^{212}\text{Po}$, ${}_{86}^{214}\text{Rn}$, ${}_{87}^{215}\text{Fr}$, ${}_{88}^{216}\text{Ra}$ and ${}_{89}^{217}\text{Ac}$ had several experimentally observed α decays from excited levels. These nuclei are formed in the r -process nucleosynthesis and will be studied in the present work.

The above phenomenon of a larger number of excited levels decaying by α decay should in principle happen at the other N as well as Z shell closures too. Inspecting the parent nuclei near N or $Z = 84$ we find that they do display some such excited states, but the effect is either not so pronounced or the data is scarce. In the range of the medium heavy nuclei, with daughters near the shell closure of $Z = 50$, there are hardly any nuclei decaying by α decay

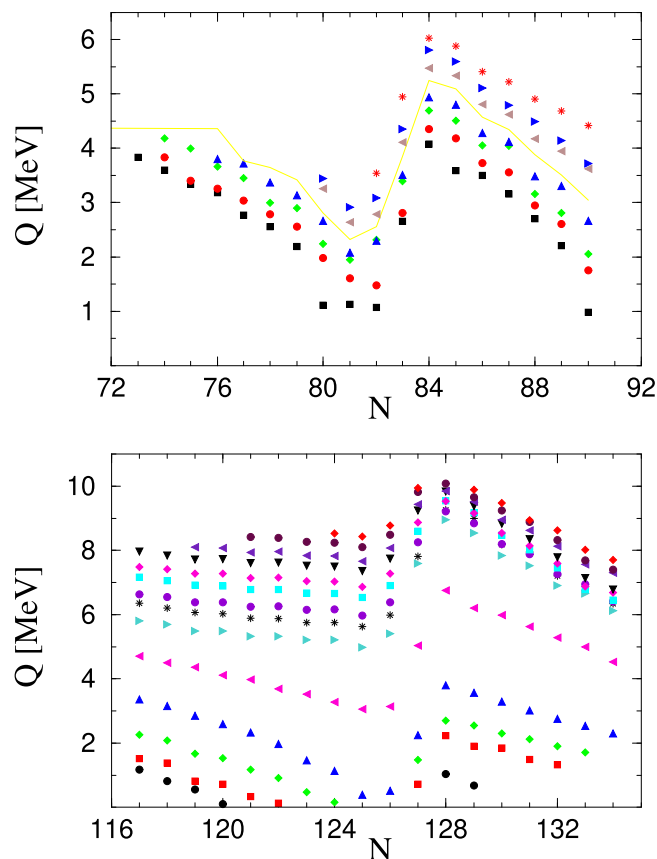


Figure 1. Q-values in α decay as a function of the neutron number of the parent nuclei. Similar symbols indicate nuclei with the same number of protons. Note the steep rise at $N = 84$ and $N = 128$ corresponding to the shell closures of 82 and 126 in the daughter nuclei.

and near $N = 50$, none. We have an interesting case however at the lowest magic number of 2. ${}^8\text{Be}$ decays to two α 's, i.e. ${}^4\text{He}$ nuclei and hence both the daughters in the decay have N as well as $Z = 2$. The number of excited levels which decay by α decay are 13 and one would expect a strong effect in the thermally enhanced decay rates. However, one does not observe a big change in the decay rate due to temperature since the spacing between levels is much larger than those in the heavy nuclei. For example, the first excited state in ${}^8\text{Be}$ is around 3 MeV and ${}^{212}\text{Po}$ has about 50 excited levels between 0 and 3 MeV. The high density of excited states in heavy nuclei is expected and was explained a long time ago by Bethe [48].

The peculiar observations mentioned above are in fact a reflection of the behaviour of the Q -values in α decay as a function of the neutron and proton numbers, N and Z in the decaying nuclei. In figure 1 we see that close to the neutron numbers of $N^m + 2$ with N^m being the magic numbers 82 and 126 the Q -values rise sharply. Higher Q -values imply a large tunneling probability for the α and hence a bigger decay rate. Though not shown in the figure such a steep rise happens also as a function of the proton number, Z for $Z^m + 2$ with Z^m being magic. However, there are not many α decays in the vicinity of $Z = 52$ and 84. When the parent nucleus has a neutron number of $N^m + 2$, we expect the decay probability to be large.

The Q -value as such is a difference of the masses of nuclei and hence is a function of the binding energies of nuclei. The cluster preformation probability was shown in [49] to be directly proportional to the intrinsic energy of the cluster which in turn depends on the difference of binding energies of the nuclei involved. A larger clustering leads to a larger probability of populating excited states which decay by emitting an α [50–52]. The latter is due to the fact that increasing the energy increases the probability of tunneling of an already formed α cluster.

Looking back at the heavy nuclei studied by Perrone and Clayton (for which data was most likely not available in 1970), namely, ^{144}Nd , ^{148}Sm , ^{150}Sm , ^{152}Sm , ^{158}Dy , ^{174}Hf and ^{176}Hf , we do not find experimental evidence of excited levels decaying by α emission in any of these nuclei. This is essentially due to the fact that the Q -values in these decays are relatively small. These nuclides have long half-lives (of the order of 10^{15} years) or are stable. From a potential barrier perspective, their probability of transmission is too small due to the fact that the Q -value is small. One could consider the possibility of a γ delayed α decay of a nucleus due to the thermally excited levels. However, the γ decay half-lives of the excited levels are much smaller than the ground state α decay half-lives and the delay would be negligible.

5. Results and discussions

In view of the physics discussed in the previous section, we shall present calculations for the temperature dependence of half-lives of nuclei with the neutron number, $N = 128$. The daughter nucleus in this case is at the shell closure of $N = 126$. To be specific, we shall consider the following nuclei: $^{212}_{84}\text{Po}$, $^{214}_{86}\text{Rn}$, $^{215}_{87}\text{Fr}$, $^{216}_{88}\text{Ra}$ and $^{217}_{89}\text{Ac}$. Each of these nuclei decays to a daughter nucleus with $N = 126$. $^{212}_{84}\text{Po}$ decays to the doubly magic nucleus $^{208}_{82}\text{Pb}$.

In the case of the statistical approach involving a sum over all excited states, we perform the calculation using the information provided in table 3. In the absence of data on the half-lives, we use a UDL at a shifted Q -value, namely, $Q_i = Q + E_i$, where E_i is the energy of the i^{th} excited state of the parent nucleus. For the UDL calculation, we use the minimum allowed value of the orbital angular momentum quantum number l for each excited level decay and have listed it in table 3. Higher values of l could be included, however, the nature of the UDL is such that even if larger values of l were taken into account, the final result would not change much. Note that the difference between the b and d parameters of the second and third term in equation (9), respectively, is of two orders of magnitude (the third term being the smallest). The ratio of these two terms grows roughly linearly as a function of angular momentum $1/10 \times \sqrt{l(l+1)} \sim l/10$, therefore only very high values of angular momentum would make the third term comparable to the second. Those large momenta would happen for large values of J_p and in such cases l_{min} is also large providing a good approximation to the half-life. For the half-life using the UDL, we do not include any preformation factor, i.e. we assume it to be unity. The factor is usually calculated phenomenologically from the ratio of the theoretical and experimental half-lives. Performing such a calculation for each individual excited level is a formidable task and out of the scope of the present work. Besides, since the contribution of this factor will not vary exponentially, it appeared to us a reasonable assumption to take it to be a constant. We are interested in the relative decrease in half-lives at elevated temperatures as compared to the terrestrial ones and expect that the error introduced due to this omission is not large.

We provide two sets of results for the half-lives of nuclei with neutron number $N = 128$ evaluated using two prescriptions. In the first set, table 4, we calculate half-lives using the double folding approach and we compare them with the half-lives obtained by utilizing

Table 3. Energy level, spin, parity, branching ratio and measured half-lives of levels which decay by alpha emission. If the experimental half-life of a level is not known, it is calculated using the UDL at an effective Q -value given by $Q + E_i$, where E_i is the energy of the excited level. In such cases, even if the experimental branching ratio is known, it is not used but taken to be 100% since the UDL per definition is formulated only for the alpha decay channel. l_{\min} is the minimum value of the orbital angular momentum quantum number, allowed by the selection rules.

Parent	E_i [MeV]	J_p^π	J_d^π	l_{\min}	% B.R. (α)	$t_{1/2}^{\dagger}$ [s]
$^{212}_{84}\text{Po}$	0	0+	0+	0	100	2.9×10^{-7}
—	0.727	2+	0+	2	0.033	1.42×10^{-11}
—	1.132	4+	0+	4	0.5	1.12×10^{-8} (UDL)
—	1.249	0+	0+	0	100	1.52×10^{-10} (UDL)
—	1.355	6+	0+	6	3	7.60×10^{-10}
—	1.476	8+	0+	8	3	1.46×10^{-8}
—	1.547	0+	0+	0	100	3.49×10^{-11} (UDL)
—	1.578	0+	0+	0	100	3.01×10^{-11} (UDL)
—	1.612	0+	0+	0	100	2.55×10^{-11} (UDL)
—	1.657	0+	0+	0	100	2.06×10^{-11} (UDL)
—	1.679	2+	0+	2	0.3	5.40×10^{-13}
—	1.800	0+	0+	0	26	1.05×10^{-11} (UDL)
—	1.805	2+	0+	2	1.6	7.85×10^{-11} (UDL)
—	2.930	18+	0+	18	99.93 (96.83)	45.1
—	2.930	18+	3—	15	99.93(1)	45.1
—	2.930	18+	5—	13	99.93(2.05)	45.1
$^{214}_{86}\text{Rn}$	0	0+	0+	0	100	2.7×10^{-7}
—	1.442	6+	0+	6	2 ^a	6.9×10^{-8}
—	1.625	8+	0+	8	4.1	6.5×10^{-9}
$^{215}_{87}\text{Fr}$	0	9/2—	9/2—	0	100	8.6×10^{-8}
—	0.835	13/2+	9/2—	3	4.3	1.43×10^{-8} (UDL)
—	1.121	17/2—	9/2—	4	0.9	8.07×10^{-9} (UDL)
—	1.149	15/2—	9/2—	4	0.9	7.04×10^{-9} (UDL)
—	1.440	19/2—	9/2—	6	4.7	4.0×10^{-9}
—	1.573	23/2—	9/2—	8	4.1	3.5×10^{-9}
$^{216}_{88}\text{Ra}$	0	0+	0+	0	100	1.82×10^{-7}
—	1.164	4+	0+	4	0.23	1.61×10^{-8} (UDL)
—	1.507	6+	0+	6	0.58	2.0×10^{-10}
—	1.711	8+	0+	8	1.86	1.42×10^{-9}
—	2.026	10+	0+	10	0.12	6.0×10^{-10}
$^{217}_{89}\text{Ac}$	0	9/2—	9/2—	0	100	6.9×10^{-8}
—	1.498	19/2—	9/2—	6	0.46	9.81×10^{-9} (UDL)
—	1.528	21/2—	9/2—	6	0.46	1.00×10^{-8}
—	2.012	29/2+	9/2—	11	4.51(4.1)	7.40×10^{-7}
—	2.012	29/2+	7/2—	11	4.51(0.32)	7.40×10^{-7}
—	2.012	29/2+	13/2+	8	4.51(0.122)	7.40×10^{-7}

Note.

^a since the listed value in the data tables is $\% \alpha > 0$, we choose 2% for an estimate.

Table 4. Half-lives within the effective Q -value approach with the first row displaying the half-lives using the UDL at $Q_{\text{eff}} = Q + \bar{\epsilon}$ (with $\bar{\epsilon}$ given by (18) and listed in table 2). The second row displays the half-lives obtained by using DFM and Q_{eff} . Quantities in brackets show the percentage decrease in the half-life due to temperature.

	Q	$t_{1/2}(T)[\text{s}]$					
		0 GK	0.8 GK	1.2 GK	1.6 GK	2 GK	2.4 GK
^{212}Po [UDL]	8.954	1.572×10^{-7}	1.57×10^{-7} (0.06)	1.54×10^{-7} (2.09)	1.36×10^{-7} (13.2)	9.57×10^{-8} (39.1)	5.69×10^{-8} (70.1)
^{212}Po [DFM]		6.49×10^{-8}	6.48×10^{-8} (0.15)	6.36×10^{-8} (1.92)	6.45×10^{-8} (12.1)	4.12×10^{-8} (36.4)	2.16×10^{-8} (66.7)
^{214}Rn [UDL]	9.208	2.07×10^{-7}	2.06×10^{-7} (0.09)	2.01×10^{-7} (2.60)	1.77×10^{-7} (14.0)	1.31×10^{-7} (36.4)	7.49×10^{-8} (61.6)
^{214}Rn [DFM]		7.92×10^{-8}	7.91×10^{-8} (0.08)	7.73×10^{-8} (2.39)	6.89×10^{-8} (12.98)	5.23×10^{-8} (33.9)	3.31×10^{-8} (58.2)
^{215}Fr [UDL]	9.540	7.11×10^{-8}	7.07×10^{-8} (0.55)	6.07×10^{-8} (14.6)	3.28×10^{-8} (53.8)	1.28×10^{-8} (81.9)	5.32×10^{-9} (92.5)
^{215}Fr [DFM]		2.92×10^{-8}	2.90×10^{-8} (0.50)	2.53×10^{-8} (13.4)	1.45×10^{-8} (50.4)	6.19×10^{-9} (78.8)	2.79×10^{-9} (90.5)
^{216}Ra [UDL]	9.526	1.862×10^{-7}	1.86×10^{-7} (0.93)	1.81×10^{-7} (2.63)	1.60×10^{-7} (13.7)	1.21×10^{-7} (34.6)	7.76×10^{-8} (58.3)
^{216}Ra [DFM]		6.86×10^{-8}	6.86×10^{-8} (0.09)	6.70×10^{-8} (2.40)	6.00×10^{-8} (12.6)	4.66×10^{-8} (32.1)	3.1×10^{-8} (54.9)
^{217}Ac [UDL]	9.832	7.67×10^{-8}	7.61×10^{-8} (0.63)	6.56×10^{-8} (14.3)	3.84×10^{-8} (49.8)	1.73×10^{-8} (77.4)	7.96×10^{-9} (89.6)
^{217}Ac [DFM]		2.98×10^{-8}	2.96×10^{-8} (0.56)	2.59×10^{-8} (13.0)	1.59×10^{-8} (46.5)	7.75×10^{-9} (74.0)	3.89×10^{-9} (87.1)

Table 5. Alpha decay half-lives at different temperatures evaluated using (10) within the statistical approach. The half-lives in the rows marked [A] are evaluated using (10), including the entire set of tabulated levels. Rows marked [B] take into account only those levels which are experimentally known to decay by the emission of an α .

Isotope	Q	$t_{1/2}(T)[s]$					
		0 GK	0.8 GK	1.2 GK	1.6 GK	2 GK	2.4 GK
$^{212}\text{Po}[A]$	8.954	2.99×10^{-7}	2.913×10^{-7}	3.502×10^{-9}	6.27×10^{-11}	5.86×10^{-12}	1.31×10^{-12}
$^{212}\text{Po}[B]$		2.99×10^{-7}	2.916×10^{-7}	3.611×10^{-9}	6.45×10^{-11}	6.01×10^{-12}	1.32×10^{-12}
$^{214}\text{Rn}[A]$	9.208	2.7×10^{-7}	2.59×10^{-7}	1.22×10^{-7}	3.36×10^{-8}	7.67×10^{-9}	1.04×10^{-9}
$^{214}\text{Rn}[B]$		2.7×10^{-7}	2.69×10^{-7}	2.68×10^{-7}	2.67×10^{-7}	2.51×10^{-7}	2.0×10^{-7}
$^{215}\text{Fr}[A]$	9.540	8.6×10^{-8}	8.53×10^{-8}	7.09×10^{-8}	3.68×10^{-8}	1.22×10^{-8}	2.60×10^{-9}
$^{215}\text{Fr}[B]$		8.6×10^{-8}	8.59×10^{-8}	8.58×10^{-8}	8.44×10^{-8}	8.03×10^{-8}	7.36×10^{-8}
$^{216}\text{Ra}[A]$	9.526	1.82×10^{-7}	1.79×10^{-7}	1.29×10^{-7}	4.39×10^{-8}	9.88×10^{-9}	1.4×10^{-9}
$^{216}\text{Ra}[B]$		1.82×10^{-7}	1.81×10^{-7}	1.69×10^{-7}	7.68×10^{-8}	1.99×10^{-8}	6.52×10^{-9}
$^{217}\text{Ac}[A]$	9.832	6.9×10^{-8}	6.85×10^{-8}	5.87×10^{-8}	3.19×10^{-8}	1.20×10^{-8}	4.41×10^{-9}
$^{217}\text{Ac}[B]$		6.9×10^{-8}	6.89×10^{-8}	6.88×10^{-8}	6.87×10^{-8}	6.86×10^{-8}	6.82×10^{-8}

equation (9). We take $Q \rightarrow Q_{\text{eff}}$, with the Q_{eff} approach of section 3.2, in which the extra energy is taken as in equation (18). In our second set, table 5, we use the statistical method where the decay constant and, in turn the half-life, are given by equation (13). We compare two approaches in table 5: one in which all available listed levels are included and another one in which only levels experimentally found to decay by alpha emission are used. For those levels (in both approaches) with unknown experimental half-lives, the UDL (equation (9)) is invoked to estimate $t_{1/2}$ of those levels. The results in table 4 ensure that the estimate obtained from equation (9) and used as an input in the statistical approach (for the experimentally unknown half-lives) is reasonable. In table 4, a comparison of the half-lives using the UDL expression and the DFM is given for a range of temperatures from 0 to 2.4 GK. The half-life at a given temperature is determined using an effective Q-value given by equation (14), namely, $Q_{\text{eff}} = Q + \bar{\epsilon}(A, Z, T)$ where $\bar{\epsilon}(A, Z, T)$ is the average excitation energy at a given temperature. The latter is evaluated considering all excited levels of a given nucleus. For the sake of comparison we shall choose the preformation factor, $P_{\alpha} = 1$ in equation (1).

From table 4, we see that the increase in temperature in general decreases the half-lives with the decrease being at the most an order of magnitude from $T = 0$ to 2.4 GK. Though the half-lives evaluated using the UDL are not exactly the same as those in the more realistic DFM, the percentage decrease in both cases is roughly the same. The percentage decrease is calculated as

$$pd = \frac{t_{1/2}^{T=0} - t_{1/2}^T}{t_{1/2}^{T=0}} \times 100. \quad (21)$$

This fact allows us with some reliability to use the UDL given by (9) in the statistical approach for the calculation of the missing half-lives in the available data for excited levels.

The temperature-dependent half-lives for several isotopes using the statistical approach are displayed in table 5. As the temperature increases the half-lives are reduced and the reduction is larger than found in the effective Q-value approach. The calculations are done using the experimentally listed half-lives. For these particular isotopes, several excited levels have been observed to decay by alpha decay, however, the half-lives of many of these excited states have not been measured. For the cases with no experimental information, we use the UDL to evaluate the half-lives to be used in (10). The two rows labelled [A] and [B] in the table display the calculations including half-lives of all listed levels in (10) and only those which decay by α decay, respectively. Particularly interesting in this table is the case of ^{212}Po . This nucleus decays to the doubly magic nucleus ^{208}Pb and an α which could possibly be the reason (as argued in earlier sections) that ^{212}Po has many more excited levels which decay by emitting an α as compared to the other nuclei in the table.

As mentioned above, the calculation labeled as [A] includes decay from all parent states. The results are, in most cases, very similar to those when only the levels experimentally known to decay by emitting an alpha (labeled as [B]) are used. This is a good indication that a general simpler formulation would be appropriate and would permit an extension to more nuclei, thus facilitating nucleosynthesis calculations.

A small note on the comparison of the two approaches, namely, the effective Q-value approach and the statistical approach is in order here. With the aim of providing temperature-dependent half-lives for nucleosynthesis applications, we began by formulating an approach where the effective Q-value would enter in the UDL with the advantage of avoiding the task of performing calculations of half-lives for many individual levels populated at very high temperatures, the use of a density level model; thus, making the proposed effective Q-value model feasible in a network calculation. However, this advantage comes at the expense of

missing information about the variation in the half-lives of different levels as well as the population probability of the excited states. Performing an average over the excitation energies and calculating for just one effective Q -value is equivalent to considering decay from one excited level which occurs at an effective excitation energy. A more realistic description is provided in the statistical approach, which would be model-independent as long as the half-lives of the excited levels and the branching ratios for alpha decay are known. Our introduction of the UDL for the unknown half-lives gives a pathway to extend the calculations to a larger number of alpha emitters. In this work, we use the known UDL from literature and evaluate the half-life of an unknown level with energy E_i , by replacing the Q -value in the UDL by $Q + E_i$. However, formulating a UDL for excited levels using all available data on the alpha decay of excited parent and daughter nuclei is a task which we plan for the future.

Finally, in passing we mention that there also exists the possibility that the system can transit from the excited state to the lower state and then decay by alpha. However, given the exponential nature of the population probability, namely, $p_i \propto \exp(-E_i/kT)$ (with E_i being the energy of the excited state), at a given temperature, the likelihood of a higher level being populated and decaying from a lower level to which it transits will surely be smaller than the lower level itself getting populated and decaying by emitting an alpha. Furthermore, in the cases where experimental information exists, the branching fraction, BR_{ij} , accounts for the effect.

Alpha decay plays a role, in competition with beta decay and fission, in powering and shaping the light curves of kilonovae [53–55]. It is customary in nucleosynthesis calculations to consider alpha emission only from the ground state, i.e at zero temperature. In view of the reduction of the α -decay half-lives in hot environments found above, it seems appropriate to replace these inputs by temperature-dependent ones. Such a detailed calculation, though necessary is out of scope of the present work.

6. Summary

In this work, we have explored the role of temperature in alpha emission from nuclear excited states. We used a statistical approach and proposed a model that can potentially be extended for several nuclei. We particularly focused on nuclei that can be produced in r -process, motivated by the impact that alpha decay has on the heating of light curves of kilonovae. Thermally enhanced alpha decay rates were calculated for nuclei with the neutron number $N = 128$ decaying to a daughter nucleus at the shell closure with $N = 126$. The latter choice was made due to the occurrence of more excited levels decaying by α emission as compared to other nuclei. The calculation performed within the statistical approach is in principle model-independent. It requires the experimental input of the energies, spins and half-lives (as well as their branching fractions for α decay) of the excited levels. However, sometimes the experimental data are incomplete (e.g. even if it is known that an excited level decays via α decay, its half-life is not known). In such a case we supply the missing information by calculating the half-life using a UDL for the half-life. The latter introduces some uncertainty in the results, however, we do not expect the results to change drastically since the temperature dependence of the half-lives using the UDL is in good agreement with the predictions of the more elaborate DFM for tunneling decay.

We found that temperatures of the order of GK can increase the half-lives of the nuclides studied here by at least a factor of ten. Particularly, for the case of ^{212}Po and depending on the model, the change can be by orders of magnitude.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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