THEORETICAL FOUNDATIONS OF TRANSITION RADIATION*

by

G. M. Garibian

Yerevan Physics Institute
Yerevan, Soviet Armenia, USSR

INTRODUCTION

In 1946, Ginzburg and Frank[1] predicted that a uniformly moving charged particle can emit radiation when passing from one medium to another. This theory established the existence of a new type of radiation, referred to as transition radiation, which can be generated at any given velocity by a moving charged particle. A significant property of this radiation lies in the fact that if one of the media is vacuum, the transition radiation carries an "impression" of the particle field in vacuum, and in this way a dependence of this field on the particle energy is obtained. It is of great significance that the above dependence of the transition radiation on the particle's energy, unlike the one in Cerenkov radiation, does not saturate at large particle energies. As noted in Frank's Nobel Lecture[2], this property is very attractive to particle physicists for high energy particles, and in recent years it has given rise to a large number of theoretical and experimental studies in transition radiation.

EXPERIMENTAL INVESTIGATIONS

No matter how surprising it may seem, the transition radiation appears to have been experimentally observed as early as 1919 by Lilienfeld[3], when a metal surface was bombarded by electrons with the energy of a few KeV. However, the mechanism of radiation production remained unrevealed at that time and those studies would have been entirely forgotten if no attention had been given to them in reference 4.

Sometime after the theory[1] was published, transition radiation was experimentally observed almost simultaneously by Chudakov in the Soviet Union (cf. reference 2) and by Jelley, Elliot and Goldsmith in Great Britain (see also refs. 2 and 5). The first published work belongs to Goldsmith and Jelley[6] who observed transition radiation generated by slow protons through metal foils. In 1957-1961 Mikhaliak[7] investigated transition radiation generated by electrons having a few KeV energy.

On the other hand, in the early 60's, two experimental studies [8, 9] appeared dealing with electromagnetic radiation generated by electrons transmitted through thin silver films. These studies were initiated by the theoretical work of Ferrell[10] according to which electrons are expected to produce in metal foils certain longitudinal waves (plasmons) later to be transformed in vacuum into transverse electromagnetic waves. However, Silin and Fetisov [11, 12] showed that only transition radiation was observed in the experimental conditions given by references 8, 9 (see also ref. 4). In the work that followed by Ritchie and Eldridge [13], the Ferrell theory was found to be a particular case of transition radiation theory (see also refs. 14, 15, 15a).

Later on a great number of experiments were performed [4, 16-29] where optical transition radiation, generated by electrons having energies up to about 100 KeV, was observed. It followed from these investigations that theory and experiment were in good agreement except for an anomaly occurring for an incident charged particle at a shallow angle on silver surfaces [30-34]. These questions were elucidated fairly well in the review by Frank[35]. They were also pointed out in work by the same author[36] dealing with incident non-normal charged particles. Concerning the question of electron incidence at a shallow angle, it should be noted that the large value of the radiation intensity obtained in this case [30-32] has been attributed [30, 31] to excitation of surface plasma waves which radiate into vacuum only on the condition that the silver surface is not an ideal plane[37-39]. But the application of new boundary conditions [40], taking into account plasma waves in metals, cannot explain the phenomena observed in incidence at a shallow angle. On the other hand, in some studies [33, 34], where the results of a theoretical investigation[41] are used, it is stated that all the radiation observed in incidence at a shallow angle can be interpreted in terms of conventional transition and bremsstrahlung radiation. Thus, this question still remains unresolved (see ref. 40 and references stated therein).

Now let us turn to experiments on transition radiation with ultrarelativistic particles. Radiation in the optical portion of the spectrum was investigated by Yuan with co-workers [42, 43], where a logarithmic increase in radiation intensity with the particle energy was shown (see also refs. 44, 44a). At the Yerevan Physics Institute [45], a method of angular discrimination in the optical range of frequencies was suggested, allowing a stronger dependence of radiation intensity on particle energies. This method was later verified experimentally [46, 47] (see also ref. 48), and the possibility was discussed [49] to make a detector of high energy particles using the optical region transition radiation (see ref. 50).

The feasibility of experimentally detecting x-ray transition radiation and its utilization in high energy physics was suggested by Alikhanian in 1960 and a proposed experimental setup was also given [51]. Later, x-ray transition radiation produced by ultrarelativistic particles was observed [52, 53], according to the earlier suggested method [51]. X-ray transition radiation was detected by means of a germanium detector[54], and also by means of CsI crystals[55]. Alikhanian, Lorikian et al. observed x-ray transition radiation using a streamer chamber [56] where the dependence of the number of quanta on the particle's energy proved to be linear.

II. THEORETICAL CONSIDERATION

We now turn to the solution of the transition radiation problem (see Fig. 1). The charge fields in each of

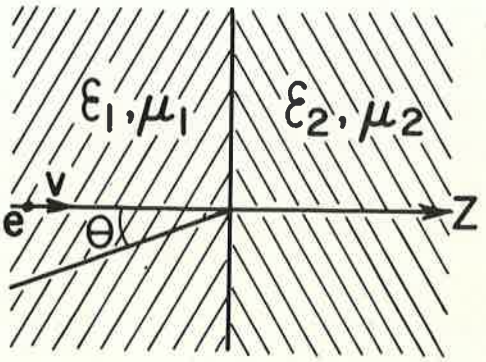

Figure 1

the media are known. These are the solutions of the Maxwell equations for a point charge moving at a constant velocity v. For example, the three-dimensional Fourier integral expansion of the charge electric field in the first and second media (indices 1 and 2) is of the form:

$$\vec{E}_{1,2}(\vec{r},t) = \int \vec{E}_{1,2}(\vec{k}) e^{i(\vec{k}\vec{r}-\omega t)} d\vec{k},$$

$$\vec{E}_{1,2}(\vec{k}) = \frac{ei}{2\pi^2} \cdot \frac{1}{\epsilon_{1,2}} \frac{\frac{\omega}{c^2} \chi_{1,2} \vec{v} - \vec{k}}{k^2 - \frac{\omega^2}{c^2} \chi_{1,2}}, \qquad (1)$$

where $\omega = \vec{k}\vec{v} = k_z \cdot v$, $d\vec{k} = d\vec{\kappa}\, dk$, $k^2 = \kappa^2 + \frac{\omega^2}{2}$, $\chi_{1,2} = \epsilon_{1,2} \cdot \mu_{1,2}$, c is the velocity of light, $\epsilon_{1,2}(\omega)$ and $\mu_{1,2}(\omega)$ are

*As the review literature on transition radiation is not available at this time, ADVENTURES IN EXPERIMENTAL PHYSICS presents this brief summary on the current theoretical and experimental situation with the complete list of references. Simple derivations are given for the cases of transition radiation generated from a single interface, a plate and a stack of plates, including the region of x-ray quanta emission.

the dielectric and magnetic permeability of the first and the second media and the particle is assumed to be moving along the z axis.

It is readily seen that at the interface the electric fields in Eq. (1) and their respective magnetic fields do not satisfy the continuity conditions of the corresponding components of fields and inductions. For these conditions to be satisfied the solutions of the homogenous Maxwell equations with the arbitrary Fourier coefficients should be added to the charge fields in each of the media and then they should be determined from the continuity conditions at the media interface. These will be the transition radiation fields. Thus, everything depends on the proper choice of solutions for the homogenous Maxwell equations. For example, in the case of the second medium we take [57]

$$\vec{E}_2'(\vec{r},t) = \int \vec{E}_2^1(\vec{k}) e^{i(\vec{k}\vec{\rho} + \lambda_2 z - \omega t)} d\vec{k} \quad (2)$$

where $\vec{\rho}$ and $\vec{K}$ are the components of vectors $\vec{r}$ and $\vec{k}$ in the (x, y) plane. It is evident that the integral in Eq.(2) will readily be a solution for the homogenous Maxwell equations in the second medium, with any arbitrary

$$\vec{E}_2'(\vec{K}), \text{ if } \lambda_2^2 = \frac{\omega^2}{c^2}\varepsilon_2 - K^2.$$

On the other hand, it is seen from comparison with Eq. (1), that by choosing the radiation fields to be in the form of Eq.(2) we can discard all the integrals because of the relationship which stems from the continuity equation. These conditions are reduced to a system of algebraic equations for the Fourier coefficients of the transition radiation fields in both media. We will neither write out this system nor solve it here because all this is simple enough and already done [57] (see also ref. 58). The final result will only be given below.

In the case where the particle moves from vacuum into the medium with $\varepsilon_2 = \varepsilon$ and $\mu_2 = 1$, we obtain for the Poynting vector flux of the transition radiation emitted in the backward direction into the vacuum and in a solid angle $d\Omega = \sin\theta d\theta d\rho$ throughout the particle' s entire flight time the expression (3)

$$\frac{dS}{d\Omega} = \frac{e^2\beta^2\sin^2\theta\cos^2\theta}{\pi^2 c(1-\beta^2\cos^2\theta)^2}\int_0^\infty \left|\frac{(\varepsilon-1)(1-\beta^2+\beta\sqrt{\varepsilon-\sin^2\theta})}{(\varepsilon\cos\theta+\sqrt{\varepsilon-\sin^2\theta})(1+\beta\sqrt{\varepsilon-\sin^2\theta})}\right|^2 d\omega$$

Here θ is the angle between the radius-vector toward the point of observation and the negative z direction (see Fig. 1) and $\beta = \frac{v}{c}$. This formula was first derived by Ginzburg and Frank[1] using a different method of calculation, (see also refs. 59-61). In the extreme relativistic case it is seen from Formula (3) that the radiation has a sharp maximum in the direction $\theta = 1-\beta^2$ and the Poynting vector flux becomes:

$$S = \frac{e^2}{\pi c}\int\left(\frac{\sqrt{\varepsilon}-1}{\sqrt{\varepsilon}+1}\right)^2\left(\ln\frac{2}{1-\beta^2} - 1\right)d\omega. \quad (4)$$

From the two latter formulae, one can also see that the spectral distribution of intensity of transition radiation in the backward direction covers the frequency range where $\epsilon(\omega)$ is markedly different from unity, that is the optical frequency region.

Let us now consider the transition radiation produced by an extremely relativistic particle along the direction of its motion. It is evident that the appropriate formula can be derived from Eq. (3), if we replace $\beta \rightarrow -\beta$. In doing so, only the second parenthesis in the denominator of the expression under the integral sign will undergo a significant change, namely, it will become

$$(1 - \beta\sqrt{\epsilon - \sin^2\theta}).$$

It means that this term may become small as $\theta \rightarrow 0$ if $\epsilon \rightarrow 1$. That is, the range of frequencies emitted in the extremely relativistic case will extend toward the x-rays at the expense of frequencies largely in the optical region [62, 63], because, in the x-ray frequency range $\epsilon = 1 - \sigma/\omega^2$, where $\sigma = 4\pi ne^2/m$ is the square of plasma frequency, n is the number of electrons per unit volume and m is the electron mass. (The case where $\epsilon \rightarrow 1$ in the optical range is treated elsewhere [64]).

We write Formula (3) with $\beta \rightarrow -\beta$ for the case of the extremely relativistic particles and x-ray transition radiation, in the form of

$$\frac{dS}{d\Omega} = \frac{e^2}{\pi^2 c}\int\left(\frac{1}{1-\beta^2+\theta^2} - \frac{1}{1-\beta^2+\frac{\sigma}{\omega^2}+\theta^2}\right)^2\theta^2 d\omega \quad (5)$$

This will be used in Section III. Finally, we turn to the approximation of small angles since the maximum in the angular distribution falls at the angles of $\theta_{max} \sim \sqrt{1-\beta^2}$. After the integration of this expression over angles, we obtain the relation [65]

$$\frac{dS}{d\omega} = \frac{2e^2}{\pi c}\left[\left(\frac{1}{2} + \frac{\omega^2(1-\beta^2)}{\sigma}\right)\ln\left(1 + \frac{\sigma}{(1-\beta^2)\omega^2}\right) - 1\right]. (6)$$

It follows from the latter formula that there exists a boundary frequency

$$\omega_b = \frac{\sqrt{\sigma}}{\sqrt{1-\beta^2}} \quad (7)$$

such that

$$\frac{dS}{d\omega} = \frac{2e^2}{\pi c}\ln\frac{\omega_b}{\omega}, \quad \text{when } \omega \ll \omega_b \quad (8)$$

$$\text{and, } \frac{dS}{d\omega} = \frac{e^2}{6\pi c}\left(\frac{\omega_b}{\omega}\right)^4, \quad \text{for } \omega \gg \omega_b$$

Now, if we integrate relation (6) over the entire frequency spectrum up to ∞, we obtain the full radiated energy [63]:

$$S = \frac{1}{3}\frac{e^2\sqrt{\sigma}}{c\sqrt{1-\beta^2}}, \quad (9)$$

i.e., the intensity, emitted into the x-ray transition radiation, depends only linearly on the particle energy (see also ref. 66).

III. X-RAY TRANSITION RADIATION FROM A SINGLE PLATE.

In this Section, we derive Formula (5) by another method [65-67, 68] which apart from its simplicity, is very convenient because it is readily generalized for the cases of a single plate or stack of plates*). To do this, we shall consider the field generated by a system of moving charges observed at distances greater than the system's dimensions. Although in our case of interest a charge moves from $-\infty$ to $+\infty$, we can still use the field of such a system since it is clear that the radiation will take place at the end portion of the trajectory in the vicinity of the interface between the media. Accordingly [69], the energy radiated into the solid angle element $d\Omega$ and in the frequency range $d\omega$ is given by:

* A more rigorous derivation of this method is found in ref. 121

$$dE_{\vec{n},\omega} = \frac{c}{2\pi} \, |\vec{H}_\omega|^2 R_o^2 d\Omega \, \frac{d\omega}{2\pi} \tag{10}$$

where,

$$\vec{H}_\omega = e \, \frac{i\omega e^{ikR_o}}{c^2 R_o} \int e^{i(\omega t - \vec{k}\vec{r}_o(t))} [\vec{n} d\vec{r}_o] \tag{11}$$

and R_o is the distance between the point where the radiation took place (here, it is the interception of the charge trajectory with the boundary) and the point of observation $\vec{r}_o$ (t) is the charge radius-vector, n is the unit vector along R_o and $K = (\omega/c)\,\vec{n}$ is the wave vector of the radiation field. Formulas (10) and (11) are written for the case of vacuum. These should be generalized not merely for the case of a single medium but moreover for the case of two different media. To do this, one has to recall that if these media have a flat interface (z = const.) then the wave vector of the radiation field K should have the values of $\vec{\kappa}$, λ_1 and $\vec{\kappa}$, λ_2 as its components in the first and the second media respectively. See Formula (2), where

$$\lambda_{1,2}^2 = \frac{\omega^2}{c^2} \varepsilon_{1,2} - \kappa^2. \tag{12}$$

When generalizing Formula (11), the second consideration lies in the fact that the electromagnetic field of radiation generated in one of the media on its passage into the other one undergoes reflections and refractions which should be taken into account by means of appropriate coefficients. However, in the problem under study, apart from free fields there is also a particle field. From this it follows that, if the familiar coefficients of reflection and refraction are used for free fields, then the generalization of Formulas (10) and (11) will provide correct results for the transition radiation only in the case of ultrarelativistic particles because it is only here that the charge field is close to the field of free radiation.

Thus, let us consider a particle moving from the medium into the vacuum and calculate the field of radiation emitted forward with respect to the particle's direction of motion. According to the generalized Formula (11), the integral corresponding to the charge's motion in matter becomes:

$$v \sin\theta' \int_{-\infty}^{o} e^{i(1-\beta\sqrt{\varepsilon - \sin^2\theta})\frac{\omega}{v} t} dz \tag{13}$$

and in vacuum it is

$$v \sin\theta \int_{o}^{\infty} e^{i(1-\beta \cos\theta)\frac{\omega}{v} t} dz. \tag{14}$$

In these integrals we put $\kappa = \frac{\omega}{c} \sin \theta$, where θ is the angle between the wave vector in vacuum and the Z axis, θ' determines the angle of radiation in matter. It is evident from the latter integrals that the main contribution comes when the argument of the exponents are equal or less than unity and from this it follows that the regions of the charge trajectory in medium, Z_{med}, and in vacuum, Z_{vac}, which are essential for the production of transition radiation, are given by:

$$Z_{med} = \frac{c/\omega}{1-\beta\sqrt{\varepsilon - \sin^2\theta'}}, \quad Z_{vac} = \frac{c/\omega}{1-\beta \cos\theta}. \tag{15}$$

These are referred to as the zones of formation of transition radiation or the lengths of coherence/4, 57/.

If we take into account refractions and reflections of waves and make use of the appropriate laws, we can derive formulas which are the ultrarelativistic limit both for Formula (3) with $\beta \rightarrow -\beta$ for the forward direction, and for the case of Formula (3) itself, to calculate the backward radiation.

However, we do not perform this calculation here and restrict ourselves to the case of x-ray transition radiation where reflected waves may be neglected, i.e. the passage coefficients are assumed to be equal to unity and $\theta = \theta'$. Then from Formulas (10) and (11) one immediately obtains Formula (5) with all the consequences following from it. Therefore, let us consider the case of the ultrarelativistic particle flight through a plate of thickness α placed in vacuum. If we make use of the coefficients which describe the passage [70] of radiation in the present case they have absolute values equal to unity and provide only a phase shift. Thus the integral in Formula (11) will be of the form:

$$v \sin\theta \Big\{ \int_{-\infty}^{o} e^{i(1-\beta\cos\theta)\omega t} e^{i(\lambda - \lambda_o)a} dt + \int_{o}^{\frac{a}{v}} e^{i(1-\beta\sqrt{\varepsilon - \sin^2\theta})\omega t} e^{i(\lambda-\lambda_o)a} dt + \int_{\frac{a}{v}}^{\infty} e^{i(1-\beta\cos\theta)\omega t} dt \Big\} \tag{16}$$

where $\lambda_o^2 = \frac{\omega^2}{c^2} - \kappa^2$. After certain simple transformations we obtain [71]

$$\frac{dS}{d\Omega} = \frac{e^2}{\pi^2 c} \int \left(\frac{1}{1-\beta^2+\theta^2} - \frac{1}{1-\beta^2+\frac{\sigma}{\omega^2}+\theta^2} \right)^2 \theta^2 \left[4 \sin^2 \frac{a\omega}{c4v} (1-\beta^2+\frac{\sigma}{\omega^2}+\theta^2) \right] d\sigma \tag{17}$$

It is readily seen that, if the thickness of plates is much greater than the formation zone of transition radiation in the medium, then the function in brackets under the integral has the value of 2 (on averaging over a short range of frequencies), i.e. the plate radiation is equal to double the radiation on one boundary. If this condition is not satisfied, oscillations take place in the spectral distribution of transition radiation. Its maxima fall on the frequencies [72] of:

$$\omega_S = \frac{\omega_a''}{S+\frac{1}{2}}, \tag{17a}$$

where $\omega_a = a\sigma/4\sigma v$ with S as integers. This relation holds for the region of $\omega_S \ll \omega_a'$, where $\omega_a' = 4\sigma v/a(1-\beta^2)$. Formula (17) describes only the x-ray portion of the spectrum of the ultrarelativistic particle transition radiation. In the general case the formula was derived by Pafomov [73] (see also references 70, 74-77).

IV. X-RAY TRANSITION RADIATION FROM A STACK OF PLATES.

We now turn to the case of a stack of plates and x-ray transition radiation. To this end, first we shall consider a stack of two plates (see Fig. 2). Using derived passage coefficients [70] the integral contained in the generalized Formula (11) is written as follows:

$$v \sin\theta \Big\{ \int_{-\infty}^{0} e^{i(1-\beta\cos\theta)\omega t} \, e^{2i(\lambda-\lambda_o)a} dt \, +$$

$$+ \int_{0}^{\frac{a}{v}} e^{i(1-\beta\sqrt{\varepsilon-\sin^2\theta})\omega t} \, e^{2i(\lambda-\lambda_o)a} dt \, +$$

$$+ \int_{\frac{a}{v}}^{\frac{a+b}{v}} e^{i(1-\beta\cos\theta)\omega t} \, e^{i(\lambda-\lambda_o)a} dt \, +$$

$$+ \int_{\frac{a+b}{v}}^{\frac{2a+b}{v}} e^{i(1-\beta\sqrt{\varepsilon-\sin^2\theta^2})\omega t} \, e^{i(\lambda-\lambda_o)(2a+b)} dt \, +$$

$$+ \int_{\frac{2a+b}{v}}^{\infty} e^{i(1-\beta\cos\theta)\omega t} dt \Big\} \qquad (18)$$

where b is the distance between the plates. After appropriate transformations we obtain Formula (17) which now contains under the integral the additional factor of:

$$\frac{\sin^2[(\frac{\omega}{v}-\lambda)\frac{a}{2}+(\frac{\omega}{v}-\lambda_o)\frac{b}{2}]\cdot 2}{\sin^2[(\frac{\omega}{v}-\lambda)\frac{a}{2}+(\frac{\omega}{v}-\lambda_o)\frac{b}{2}]} \quad . \qquad (19)$$

It is evident that in the general case of a stack of N plates for the x-ray transition radiation intensity, using Equation (3) we obtain

$$\frac{dS}{d\Omega}=\frac{e^2}{\pi^2 c}\int\Big(\frac{1}{1-\beta^2+\theta^2}-\frac{1}{1-\beta^2+\frac{\sigma}{\omega^2}+\theta^2}\Big)^2\theta^2[4\sin^2\frac{a\omega}{4v}(1-\beta^2+\frac{\sigma}{\omega^2}+\theta^2)].$$

$$\left\{ \frac{\sin^2[\frac{a\omega}{4v}(1-\beta^2+\frac{\sigma}{\omega^2}+\theta^2)+\frac{b\omega}{4v}(1-\beta^2+\theta^2)]N}{\sin^2[\frac{a\omega}{4v}(1-\beta^2+\frac{\sigma}{\omega^2}+\theta^2)+\frac{b\omega}{4v}(1-\beta^2+\theta^2)]} \right\} d\omega \quad . \qquad (20)$$

As mentioned earlier, while deriving Formula (17) we neglected the reflected field, considering the reflection coefficient in this case to be small. But in the case of a stack of plates, reflection takes place on each plate. Therefore in the region of applicability of Formula (20) the smallness of the total reflected field as compared with the passing field should be considered. This results in the condition of:

$$\frac{(\sqrt{\varepsilon}-1)}{2} \quad N \ll 1 \qquad (21)$$

Formula (20) for transition radiation without condition (21) was derived by Ter-Mikaelian and Gazazian [78,79] and by Garibian and Goldman [80]. In references 78, 79, 81 this radiation was referred to as "resonance" radiation.

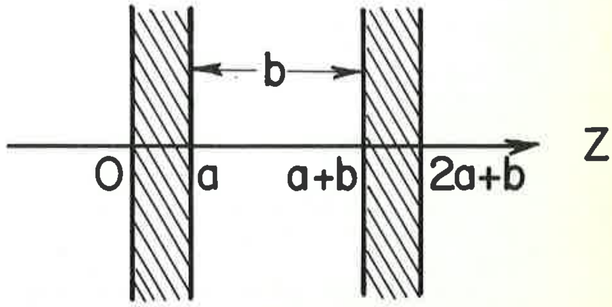

Figure 2

When analyzing Formula (20), it is natural to compare the average radiation emitted per unit of plate in a stack with the radiation generated only on a single separate plate, i.e. Formula (20), divided by N, should be compared with Formula (17). The analysis under examination has been performed [72] and only its results will be presented below. Let us introduce the frequency $\omega'_p = \frac{4\pi v}{p(1-\beta^2)}$, where p=a+b. Then with conditions of $\omega \gg \omega'_p$ and $p \gg a$, an independent addition of intensities of radiation from all the plates takes place, i.e., an average radiation on one plate is equal to the radiation obtained from Eq. (17). Physically, these conditions imply that the distances between the plates are much larger than the zone of formation of transition radiation in vacuum. In order to consider other frequencies, it should be noted that with the conditions of $\omega \ll \omega'_p$ and $\omega \sim \omega'_p$, maxima and minima occur in the spectral distribution of radiation emitted from a single plate, since $\omega'_p < \omega'_a$. It has been shown [72] that the radiation maxima both in a stack of plates and on a single plate fall nearly at the same frequencies. In the region of frequencies of $\omega \ll \omega'_p$, the average radiation intensity emitted from one plate in a stack of plates has a smaller amplitude at the maxima of radiated frequencies $\bar{\omega}_S$, as compared with the amplitude at the maxima of the same radiated frequencies generated from one single plate. At $\omega \sim \omega'_p$ and $p \gg a$ the amplitude emitted from one plate in a stack of plates at these frequencies becomes somewhat larger than the intensities from a single plate.

Formula (20) was derived for x-ray transition radiation of extremely relativistic particles and a stack of N plates. In the general case of radiation of any wavelength, solutions exist [70,82-84] (see also ref. 85, 86) for a particle of arbitrary velocity and for a stack made up of a finite number of plates. Accurate solutions are also derived [87] for an infinite layered medium and for a periodical continuously changing inhomogenous medium [88]. Similarly, the problem is solved [89] for an infinite medium and a smoothly changing $\epsilon(\omega)$, in the approximation where $\epsilon(\omega)$ is close to unity. Also the problem is solved [90] for the case where the medium's density changes periodically and smoothly. Particularly, for the case of x-ray frequencies the first two terms in the expansion series of the radiation field were obtained. These were derived elsewhere by another method [89]. The case of a thin stack of very thin plates [91] and a stack of optically blue non-reflecting plates [92] was also investigated. A formal accurate solution of the problem for an infinite periodic medium is given in ref. 93. The case of a dielectric medium, interlayed with thin metal foils is also studied [94,95].

V. PROBLEMS RELATED TO TRANSITION RADIATION.

It is evident from Formula (15) that the formation zone of transition radiation in matter for ultrarelativistic particles may become very large. If it becomes so large that the particle begins to experience multiple scattering in this length, then the theory of transition radiation should be modified properly [99]. The effect of multiple scattering on transition radiation in the presence of one interface between media has been treated by a number of authors [100-108].

The other question is connected with the thermal motion of electrons in the medium resulting in a spatial dispersion of dielectric permeability. Since in this case longitudinal waves develop, the boundary conditions appear to be fewer in number than the unknown components of the field. Some additional conditions may be introduced in such a manner that the problem can be solved [40, 109]. On the other hand, Silin and Fetisov [110] demonstrated that a missing boundary condition for the electric field components can be substituted by a boundary condition on the distribution function of the medium electrons, if the problem is solved in a kinetic gas approximation. In such a formulation the problem of transition radiation was solved while taking into account the spatial dispersion of the dielectric constant [12, 111-113]. The same problem was also solved in a hydrodynamic approximation [144].

The influence of diffused boundaries on the formation of transition radiation is studied by Amatuny and Korkhmazian [115] and Galeev [116]. The transition radiation of dipole moments [117] and of bunches of charged particles periodically following one another is examined by Amatuny [118]. The formation of transition radiation by a charged particle with an arbitrarily changing velocity and for the case of flat interfaces between the media is investigated [97, 98, 119-121]. Similarly the case of spherical interfaces between media is given [122]. The fields produced by a charge passing through joint two-layered and multilayered plates are derived [123-125].

Transition radiation resulting from oblique incidence of a charge has been studied for the cases of one interface between media [31, 36, 126-132], a plate (that is, two interfaces) [41, 133, 134] and a stack of plates [135].

There are also investigations which treat the generation of the transition radiation fields under the conditions of a moving interface between media [136], when one of the media is isotropic and optically active [137] and again, when one of the media is of gyrotropic ferromagnetic nature [138]. The quantum theory of transition radiation is developed in some detail [103, 139].

Finally, we remark that an investigation exists which treats the radiation generated by a charge moving in a randomly inhomogenous medium [140], in a medium where parameters are coordinate and time dependent according to the law of travelling waves [141]. Also, Barsukov [62, 76] has treated the problem of transition radiation when a charge travels in a waveguide.

REFERENCES

1. V. L. Ginzburg and I. M. Frank. JETP 16, 15 (1946).
2. I. M. Frank. Nobel Lecture, Uspekhi Fiz. Nauk 68, 397 (1961).
3. J. E. Lilienfeld. Physik. Zeit. 20, 280 (1919).
4. H. Boersch, et al. Phys. Rev. Lett. 7, 52 (1961), Zeit. Phys. 165, 464 (1961).
5. J. V. Jelley. Cerenkov radiation and its applications. Pergamon Press, 1958.
6. P. Goldsmith and J. V. Jelley. Phil. Mag. 4, 836 (1959).
7. S. Mikhaliak. Dissertation, Moscow State University, 1961.
8. W. Steinmann. Phys. Rev. Lett. 5, 470 (1960).
9. R. W. Brown et al. Phys. Rev. Lett. 5, 472(1960).
10. R. A. Ferrell. Phys. Rev. 111, 1214 (1958).
11. V. P. Silin and E. P. Fetisov. Phys. Rev. Lett. 7, 374 (1961).
12. V. P. Silin and E. P. Fetisov. JETP 45, 1572 (1963).
13. R.H. Ritchies and H. B. Eldridge. Phys. Rev. 126, 1935 (1962).
14. E. A. Stern. Phys. Rev. Lett. 8, 7 (1962).
15. R. A. Ferrell and E. A. Stern Am. J. Phys. 30, 810 (1962).
15a. M. Hattori and K. Yamada Journ. Phys. Soc. Japan, 18, 200 (1963).
16. A. L. Frank, et al. Phys. Rev. 126, 1947 (1962).
17. E. T. Arakawa, et al. Phys. Rev. 131, 719 (1963).
18. E. T. Arakawa, et al. Phys. Rev. Lett. 12, 319 (1964).
19. S. Tanaka and Y. Katayama. J. Phys. Soc. Jap. 19, 40 (1964).
20. E. T. Arakawa, et al. Jour. de Physique 25, 129(1964).
21. E. T. Arakawa, et al. Phys. Rev. 135, A224 (1964).
22. R. J. Herickoff, et al. Phys. Rev. 137, A1433 (1965).
23. R. J. Herickoff, et al. Phys. Rev. 139, A1455 (1965).
24. R. F. Arutunian, et al. JETP. Lett. 3, 193 (1966).
25. F. R. Arutunian, et al. JETP 51, 760 (1966).
26. F. R. Arutunian, et al. Opt. Spect. 21, 399, (1966).
27. L. A. Ananova, et al. Dokl. Akad. Nauk Arm. SSR 43, 87 (1966).
28. S. Mikhaliak. Yadernaya Fiz. 3, 89 (1966).
29. S. Michaliak and J. Rutkowski. Acta Phys. Polon. 29, 65 (1966).
30. P. von Blanckenhagen, et al. Phys. Lett. 11, 296(1964).
31. H. Boersch, et al. Zeit. Phys. 187, 97 (1965).
32. G. E. Jones, et al. Phys. Rev. 147, 515 (1966).
33. L. S. Cram and E. T. Arakawa. Phys. Rev. 153, 455 (1967).
34. J. C. Ashley, et al. Phys. Rev. 160, 313 (1967).
35. I. M. Frank. Usp. Fiz. Nauk 87, 189 (1965).
36. I. M. Frank. Preprint Joint Inst. Nucl. Res., P4-4980 (1910).
37. E. A. Stern. Optical Properties and Electronic Structure of Metals and Alloys, Amsterdam, 1966, p. 396.
38. V. E. Pafomov, E. P. Fetisov. JETP 53, 965(1967).
39. E. P. Fetisov. Izv. Vuzov. Radiofiz. 12, 265 (1969).
40. F. Forstmann. Zeit. Phys. 217, 416 (1968).
40a. P. Dobberstein. Phys. Atat. Sol. 38, 649 (1970).
41. J. C. Ashley. Phys. Rev. 155, 208 (1967).
42. J. Oostens, et al. Phys. Rev. Lett. 19, 541 (1967).
43. S. Prunster, et al. Phys. Lett. 28B, 47 (1968).
44. D. K. Aitken, et al. Proc. Phys. Soc. 82, 710 (1962).
44a. F. W. Inman and J. J. Muray. Phys. Rev. 142, 272(1966).
45. K. A. Ispirian, A. G. Oganesian. Izv. Akad. Nauk Arm. SSR, Fizika 3, 290 (1968).
46. A. I. Alikhanian, et al. JETP 56, 1796 (1969).
47. A. I. Alikhanian, et al. Proc. Int. Simp. Nucl. Electr. V. 3. High energy Instr, Versailles, 1968.
48. A. I. Alikhanian, et al. Izv. Akad. Nauk Arm. SSR, Fizika 4, 233 (1969).
49. A. I. Alikhanian, et al. Yadernaya Fiz. 10, 549(1969).

50. E. Janikova, et al. Nuc. Instr. Methods 74, 61(1969).
51. A. I. Alikhanian, et al. JETP 41, 2002 (1961).
52. F. R. Arutunian, et al. Yadernaya Fizika 1, 842(1965).
53. F. R. Arutunian, et al. JETP 52, 1121 (1967).
54. L. C. L. Yuan, et al. Phys. Rev. Lett. 23, 496(1969).
54a. L. C. L. Yuan, et al. Phys. Lett. 31B, 603(1970).
54b. L. C. L. Yuan, 11 Int. Conf. on Cosmic Rays, Hungary, August, 1969.
54c. L. C. L. Yuan, Scintillation and Semicon. Count. Simp., Washington, D. C. 1970.
54d. L. C. L. Yuan, et al. Phys. Rev. Lett. (to be published).
55. A. L. Alikhanian, et al. Lett. JETP 11, 347 (1970).
55a. A. I. Alikhanian, et al. Nucl. Inst. Meth. (to be published).
56. A. I. Alikhanian, et al. Izv. Akad. Nauk Arm. SSR, Fizika 5, 4, (1970); Phys. Rev. Lett. 25, 635 (1970).
57. G. M. Garibian. JETP 33, 1403 (1957).
58. F. G. Bass and V. M. Yakovenko. Uspekhi Fiz. Nauk 86, 189 (1965).
59. G. Beck. Phys. Rev. 74, 795 (1948).
60. N. P. Klepikov. Vestnik Moscow State University 8, 61 (1951).
61. N. A. Korkhmazian. Izv. Akad. Nauk Arm. SSR, Fiz-Mat. 10, 4, 29 (1957).
62. K. A. Barsukov. JETP 37, 1106 (1959).
63. G. M. Garibian JETP 37, 527 (1959).
64. Ts. Amatuny, G. M. Garibian, Izv. Akad. NaukArm. SSR, Fizika 1, 342 (1966).
65. G. M. Garibian, JETP 39, 332 (1960).
66. V. P. Zrelov. "Vavilov-Cerenkov radiation and its application in high energy physics" in Russian Vol. 1. Atomizdat, Moskva, 1968.
67. A. I. Alikhanian. Physics of elementary particles. Vol. 5, Yerevan, 1966. pp. 666.
68. A. I. Alikhanian. Loeb Lectures. Harvard University February, 1965.
69. L. D. Landaw and E. M. Lifshitz. Field Theory Moskva. 1967 66.
70. V. A. Arakelian and G. M. Garibian. Izv. Akad. Nauk Arm. SSR, Fizika 4, 339 (1969).
71. G. M. Garibian. Dok. Akad. Nauk Arm. SSR 33, 105(1961); Izv. Akad. Nauk SSR, Ser. Fiz. 36, 754 (1962).
72. G. M. Garibian. preprint Yerevan Physics Institute. TF-4(70); JETP (to be published).
73. V. E. Pafomov. JETP 33, 1074 (1957).
74. G. M. Garibian and G. A. Chalikian. JETP 35, 1282 (1958).
75. G. M. Garibian and G. M. Chalikian. Izv. Akad. Nauk Arm. SSR Fiz-mat. 12, 13, 49 (1959).
76. K. A. Barsukov. JETP 30, 1337 (1960).
77. V. A. Arakelian and G. M. Garibian. Izv. Akad. Nauk Arm. SSR, Fizika 5, No. 4 (1970).
78. M. L. Ter-Mikaelian and A. D. Gazazian. JETP 39, 1993 (1960).
79. M. L. Ter-Mikaelian. Izv. Akad. Nauk Arm. SSR, Fiz-mat. 14, No. 2, 103(1961); Nucl. Phys. 24, 43 (1961).
80. G. M. Garibian and I. I. Goldman. Dokl. Akad. Nauk Arm. SSR 31, 219, (1960).
81. M. L. Ter-Mikaelian. Effect of Medium on Electromagnetic Processes on High energies. Izd. Akad. Nauk Arm. SSR. Yerevan, 1969.
82. G. M. Garibian. JETP 35, 1435 (1958).
83. V. E. Pafomov. JTP 33, 557 (1963).
84. V. A. Arakelian, et al. Izv. Akad. Nauk Arm. SSR. Fizika 4, 287 (1969).
85. G. M. Garibian. Dokl. Akad. Nauk Arm SSR 33, 193 (1961).
86. V. A. Djrbashian. Izv. Akad. Nauk Arm. SSR Fiz-mat 16, No 6, 113 (1963).
87. J. B. Fainberg and N. A. Khijniak. JETP 32, 883(1957).
88. P. V. Bliokh. Izv. Vuzov, Radiofiz. 2, 63 (1959).
89. M. L. Ter-Mikaelian. Dok. Akad. Nauk SSSR 134, 318 (1960).
90. A. Ts. Amatuny, N. A. Korkhmazian. Izv. Akad. Nauk Arm. SSR. Fiz-mat. 13, No. 5, 55 (1960).
91. V. E. Pafomov. JETP 39 134 (1960).
92. V. E. Pafomov, I. M. Frank. Yadernaya Fiz. 5, 631 (1967).
93. K. F. Casey, et al. Phys. Rev. 140, B768 (1965).
94. R. Pratesi, et al. N. Cim. 25, 754 (1962).
95. L. Ronchi and G. Toraldo di Francia. "Alta frequenza" 32, 25, 8, (1963).
96. I. M. Frank. Izv. Akad. Nauk SSSR, ser. Fiz. 6, 3, (1942).
97. V. E. Pafomov. Uzv. vuzov, Radiofiz. 10, 240 (1967).
98. V. E. Pafomov. Trudy EIAN, vol. 44, 28 (1969).
99. G. M. Garibian, I. I. Pomeranchuk. JETP 37, 1828(1959).
100. V. E. Pafomov. Dokl. Akad. Nauk SSSR 133, 135 (1960).
101. I. I. Gol' dman. JETP 38, 1866 (1960).
102. G. M. Garibian. JETP 39, 332 (1960).
103. F. F. Ternovsky. JETP 39, 171, 491 (1960).
104. V. E. Pafomov. JETP 47, 530 (1964).
105. R. H. Ritchie, et al. Phys. Rev. 135, A759 (1964).
106. V. E. Pafomov. JETP 52, 208 (1967).
107. M. L. Ter-Mikaelian. Lett. JETP 8, 100 (1968).
108. F. R. Arutunian. Dokl. Akad. Nauk Arm. SSR, 48, 32 (1969).
109. B. L. Zhelnov. JETP 40, 170 (1961).
110. V. P. Silin and E. P. Fetisov. JETP 41, 159 (1961).
111. E. A. Kaner and V. M. Yakovenko. JETP 42, 471(1962).
112. A. Ts. Amatuny. JTP 34, 1354 (1964).
113. S. S. Elbakian. Izv. Akad. Nauk Arm. SSR, Fizika 5, 29 (1969).
114. V. M. Yakovenko. JETP 41, 385 (1961).
115. A. Ts. Amatuny and N. A. Korkhmazian. JETP 39, 1011 (1960).
116. A. A. Galeev. JETP 46, 1335 (1964).
117. A. Ts. Amatuny Izv. Adad. Nauk Arm. SSR Fiz-mat. 13, No 1, 111 (1960).
118. A. Ts. Amatuny. Izv. Akad Nauk Arm. SSR Fiz. -mat. 15, No 1, 109 (1962).
119. G. M. Garibian and M. R. Magomedov. Akad. Nauk Arm. SSR 36, 77 (1963).
120. M. R. Magomedov. Dokl. Akad. Nauk Arm. SSR, 36, 157 (1963).
121. M. R. Magomedov. Izv. Akad. Nauk Arm. SSR, Fizika 2, 170 (1967).
122. M. R. Magomedov. Izv. Akad. Nauk Arm SSR, Fizika 4, 271 (1969).
123. G. M. Garibian and M. M. Muradian. Izv. Akad. Nauk. Arm. SSR, Fizika 1, 310 (1966).
124. M. M. Muradian. Izv. Akad. Nauk Arm. SSR, Fizika 2, 343 (1967).
125. G. M. Garibian and M. M. Muradian. Izv. Akad. Nauk Arm. SSR, Fizika 3, 103 (1968).
126. G. M. Garibian Izv. Akad. Nauk Arm. SSR, Fiz-mat. 11, No 4, 7, (1958).
127. N. A. Korkhmazian. Izv. Akad. Nauk Arm. SSR, Fiz-mat 11, No 6, 87 (1958).
128. N. A. Korkhmazian. Izv. Akad. Nauk Arm. SSR Fiz-mat. 13, No 2, 139 (1960).
129. G. M. Garibian. JETP 38, 1814 (1960).
130. N. A. Korkhmazian. Izv. Akad. Nauk Arm. SSR Fiz-mat. 15, No 1, 115 (1962).
131. V. E. Pafomov. Izv. vuzov. Radiofiz. 5, 484 (1962).
132. N. A. Korkhmazian, S. S. Elbakian. Izv. Akad. Nauk Arm. SSR, Fizika 4, 3 (1969).
133. V. A. Enghibarian and B. V. Khachatrian. Izv. Akad. Nauk Arm. SSR, Fizika 1, 11 (1966).
134. G. M. Garibian and S. S. Elbakian. Izv. Akad. Nauk Arm. SSR, Fizika 3, 224 (1968).
135. V. A. Arakelian and G. M. Garibian. Izv. Akad. Nauk Arm. SSR, Fizika 5, No 5 (1970).
136. K. A. Barsukov and B. M. Balotovsky. Izv. vuzov, Radiofiz. 3, 336 (1960).
137. B. M. Bolotovsky and O. S. Mergelian. Opt. Spektr. 18, 3 (1965).
138. O. S. Mergelian. Izv. Akad. Nauk Arm. SSR, Fizika 3, 285 (1968).
139. G. M. Garibian JETP 39, 1630 (1960).
140. S. P. Kapitsa. JETP 39, 1367 (1960).
141. K. A. Barsukov and B. M. Bolotovsky. Izv. vuzov, Radiofiz. 7, 291 (1964).