

Revisiting the oscillations in the cosmic microwave background angular power spectra at $\ell \sim 120$ in the Planck 2015 data

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While the observed nearly scale-invariant initial power spectrum is regarded as favorable evidence of the standard inflationary cosmology, precision observations of the cosmic microwave background anisotropies also suggest possible existence of nontrivial features such as those observed around multipoles $\ell \sim 120$ by WMAP. Here, we examine the Planck data and investigate the effects of these features on cosmological parameter estimation by performing Markov-chain Monte Carlo analysis. We find that the features exist in the Planck data at the same position as in the WMAP data, but they do not affect the cosmological parameter estimation significantly.
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Subject Index E60, E63

1. Introduction

In modern cosmology, the origin of the large-scale structure of the universe is attributed to tiny initial quantum fluctuations [1–4] produced during the inflation epoch [5–7] (for a review of inflation, see, e.g., Ref. [8]). The simplest models of inflation predict a nearly scale-invariant initial power spectrum of curvature perturbations that can be characterized by the amplitude and the power-law spectral index [2–4]. Motivated by this theoretical background, as well as for the sake of simplicity, the power-law initial spectrum has been tested by precision observations of cosmic microwave background (CMB) radiation by WMAP [9] and Planck [10], and shown to provide a good fit. Furthermore, values of the cosmological parameters have been determined by CMB data mostly under the assumption of the power-law initial spectrum. From a purely observational point of view, however, the shape of the initial spectrum of our Universe should be determined from observational data alone, without any theoretical prejudice.

In fact, much work has been done to reconstruct the primordial spectrum from observed CMB data using a number of methods such as Markov-chain Monte Carlo (MCMC) analysis of the parameterized spectrum [11–14], cosmic inversion [15–19], and maximum likelihood reconstruction methods [19–21], to name a few. As a result, a number of possible features imprinted on an otherwise power-law spectrum have been reported in the literature.

Although it is difficult to interpret them in the context of inflationary cosmology, namely, to judge if models predicting featured spectra are really necessary, the presence of spectral features can affect the estimation of the cosmological parameters of the homogeneous and isotropic background universe. Thus, unless we fully incorporate such features in the parameter analysis we may not obtain correct values of cosmological parameters.

The purpose of this paper is to examine whether the feature found around the multipole $\ell \sim 120$ [11] based on TT and TE data of the five-year WMAP observation (WMAP5) [9] is persistent in the latest Planck data [10], which now includes more polarization data as well, and how its presence affects the estimation of other cosmological parameters. Here, T refers to temperature anisotropy and E to the E-mode polarization.

Note that this feature has been observed in all sky regions in the WMAP data [22]. If it is a real cosmological feature, it should be found in the Planck data as well, and we should take these features into account in cosmological analysis.

In addition, previous works [12,23,24] have reconstructed some local features from recent observational data on other scales. These features probably originate from fine distortions of the initial power spectrum that were induced in the inflation epoch. There are a number of inflation mechanisms that could induce such distortions, for instance the dynamics of a heavy field in multi-field inflation [25–29], brane wrapped inflation [30], features from the modified slow-roll inflation potential called local features [14,31], change in the sound velocity in the inflation epoch [32–34], and the non-local inflationary feature from wiggly whipped inflation [35]. Analyses of these features could be a good index for inflation models.

A previous study [11] estimated the cosmological parameters with and without the features in the MCMC analysis, and they found that these features particularly affect the estimation of the parameters for the initial power spectrum, the amplitude A , and the spectral index n_s when using the five-year WMAP data. In their analysis, the resultant amplitude A turned out to be smaller and the spectral index n_s larger than those based on the power-law initial power spectrum without features. We need to examine whether these trends could be seen or not using the Planck data. We pay particular attention to the impact of the newly released E-mode autocorrelation data on analyzing the features.

The rest of the paper is organized as follows. In Sect. 2, we will explain the parameterization of the features and the setup of the analysis. To understand the effects of the features, we will perform an MCMC analysis using the Planck unbinned angular power spectrum data under the standard Λ CDM cosmology. In Sect. 3, we present the results of the analysis and discuss the existence of the features and their impacts on the parameter estimation. Then we check the effects of the Planck polarization data on the analysis of the features. Section 4 is devoted to the conclusion.

2. Method

We examine whether the features around multipole $\ell \sim 120$ found in the WMAP data [11,19,22,36,37] are persistent in the Planck data in terms of MCMC analysis using the COSMOMC [38] code. In this section, we give our parameterized model of the features and explain the setup of the analysis.

2.1. Model of the features

In the standard cosmology, we usually adopt the power-law initial power spectrum,

$$P(k) = \frac{k^3 P_\zeta(k)}{2\pi^2} = A \left(\frac{k}{k_0} \right)^{n_s-1}, \quad (1)$$

Table 1. Best-fit χ^2 for several models in Ref. [11].

Model	χ^2	$\Delta\chi^2$
Standard power law	24190.9	—
Λ -type	24183.1	−7.8
V^Λ -type	24182.1	−8.8
W -type	24181.7	−9.2
S-type [Eq. (2)]	24174.8	−16.1

where $P_\zeta(k)$ is the power spectrum of primordial curvature fluctuation in the comoving gauge, A is the amplitude of the fluctuation, n_s is the spectral index, and $k_0 = 0.05 \text{ Mpc}^{-1}$ is the pivot scale. Here we assume that the features in the CMB power spectra originate in the initial power spectrum and explain the features modifying the initial power spectrum Eq. (1). Besides this overall power-law component, we incorporate a feature around a comoving wavenumber k_* following [11], where several models of features in the primordial spectrum were tested. From these we adopt the following functional form:

$$P(k) = A \left(\frac{k}{k_0} \right)^{n_s-1} + B \left(\frac{k}{k_0} \right)^{n_s-1} \exp \left(-\frac{(k-k_*)^2}{\kappa^2} \right) \cos \left(\frac{\pi(k-k_*)}{\kappa} \right), \quad (2)$$

which reproduced the WMAP5 data the best. Here B , κ , and k_* correspond to the amplitude, the width, and the position of the oscillations, respectively. The product of k_* and the angular diameter distance to the last scattering surface d_{ang} is equivalent to the position of the features in the multipole space ($k_* d_{\text{ang}} \sim \ell$). We call the parameters B , κ , and $k_* d_{\text{ang}}$ the feature parameters hereafter.

In Ref. [11], the authors analyzed other models of the initial power spectrum, namely Λ -, V^Λ -, and W -type. We employed the same models in fitting the Planck 2015 TT, TE, EE combined data, and found that these three models do not improve the χ^2 values compared with the model given by Eq. (2); see Table 1. In this paper, therefore, we will restrict our attention to the two initial power spectrum models given by Eq. (1) and Eq. (2), to investigate the statistical significance of the features below.

2.2. Analysis setup

We perform the MCMC analysis under the standard Λ CDM cosmology. Here we run the feature parameters in addition to the cosmological parameters with flat priors. We set the range of the feature parameters as $0 \leq 10^{10}B \leq 150$, $1 \leq 10^4 \kappa d_{\text{ang}} \leq 30$, and $100 \leq k_* d_{\text{ang}} \leq 140$ for the amplitude, the width, and the position, respectively. To understand the effects of the polarization data on the estimation of the cosmological parameters and the feature parameters, we performed the MCMC analysis for two different data sets. One consists only of the TT autocorrelation data, and the other consists of the combined data, which contains the TT, EE autocorrelation, and the TE cross-correlation data of the unbinned Planck 2015 angular power spectra.

3. Results and discussion

In this section, we show the result of the analysis. We performed several MCMC analyses to compare the standard initial power spectrum Eq. (1) and the initial power spectrum with oscillations given by Eq. (2). Here we show the best-fit values in Table 2 and the mean values in Table 3 for the cosmological and feature parameters, and χ^2 values in Table 4.

Table 2. Best fit cosmological and feature parameters for the Planck 2015 TT data and for the TT, TE, EE combined data. Here, τ is the optical depth, $H_0 = 100h$ [km s⁻¹ Mpc⁻¹] is the Hubble parameter, and $\Omega_b h^2$ and $\Omega_c h^2$ represent the density parameters of baryon and cold dark matter, respectively. We investigated models with the standard initial power spectrum given by Eq. (1), the oscillating initial power spectrum of Eq. (2), and the fixed oscillation models in which we used the oscillating initial power spectrum of Eq. (2) with the feature parameters fixed to the best-fit values.

	Standard TT	With oscillations TT	Standard TTTEEE	With oscillations TTTEEE	Fixed oscillations TTTEEE
$10^{10}A$	21.6	21.6	21.2	21.3	21.4
n_s	0.969	0.970	0.965	0.967	0.967
τ	0.0719	0.0707	0.0613	0.0644	0.0649
H_0	68.0	68.0	67.4	67.6	67.6
$\Omega_b h^2$	0.0223	0.0223	0.0222	0.0223	0.0223
$\Omega_c h^2$	0.118	0.118	0.119	0.119	0.119
$10^{10}B$	—	46.3	—	37.4	—
$10^4\kappa$	—	3.09	—	3.14	—
k_*d_{ang}	—	124.0	—	123.5	—

Table 3. As Table 2, but here we show the mean cosmological and feature parameters for the Planck 2015 TT data and for the TT, TE, EE combined data with 1σ errors.

	Standard TT	With oscillations TT	Standard TTTEEE	With oscillations TTTEEE	Fixed oscillations TTTEEE
$10^{10}A$	$21.5^{+0.9}_{-1.0}$	$21.6^{+0.9}_{-1.1}$	$21.2^{+0.7}_{-0.6}$	$21.3^{+0.6}_{-0.8}$	$21.3^{+0.6}_{-0.8}$
n_s	$0.968^{+0.007}_{-0.008}$	$0.970^{+0.008}_{-0.008}$	$0.965^{+0.005}_{-0.005}$	$0.966^{+0.005}_{-0.005}$	$0.966^{+0.005}_{-0.005}$
τ	$0.0694^{+0.0229}_{-0.0261}$	$0.0721^{+0.0246}_{-0.0267}$	$0.0609^{+0.00171}_{-0.00173}$	$0.0634^{+0.0170}_{-0.0184}$	$0.0631^{+0.0171}_{-0.0192}$
H_0	$68.0^{+1.1}_{-1.3}$	$68.1^{+1.2}_{-1.2}$	$67.4^{+0.7}_{-0.7}$	$67.5^{+0.7}_{-0.7}$	$67.5^{+0.7}_{-0.7}$
$\Omega_b h^2$	$0.0223^{+0.0002}_{-0.0003}$	$0.0223^{+0.0003}_{-0.0003}$	$0.0222^{+0.0002}_{-0.0001}$	$0.0222^{+0.0002}_{-0.0002}$	$0.0222^{+0.0002}_{-0.0002}$
$\Omega_c h^2$	$0.118^{+0.003}_{-0.002}$	$0.118^{+0.003}_{-0.003}$	$0.119^{+0.002}_{-0.001}$	$0.119^{+0.002}_{-0.002}$	$0.119^{+0.002}_{-0.001}$
$10^{10}B$	—	$43.2^{+11.6}_{-43.2}$	—	$32.5^{+13.8}_{-11.5}$	—
$10^4\kappa$	—	$4.32^{+0.18}_{-2.57}$	—	$3.57^{+0.51}_{-1.04}$	—
k_*d_{ang}	—	$123.1^{+2.7}_{-0.9}$	—	$123.0^{+0.9}_{-0.1}$	—

Table 4. Best-fit χ^2 for the same models as Table 2.

Model data	Standard TT	With oscillations TT	Standard TTTEEE	With oscillations TTTEEE	Fixed oscillations TTTEEE
χ^2	8431.9	8418.2	24190.9	24174.8	24174.8
$\Delta\chi^2$	—	-13.7	—	-16.1	-16.1

3.1. Features in the Planck data

Let us focus on the feature parameters first. Reference [11] has shown that the best-fit values are $10^{10}B = 55.6$, $10^4\kappa = 3.58$, and $k_*d_{\text{ang}} = 124.5$ using the TT autocorrelation and TE cross-correlation spectra of the five-year WMAP data. Comparing the new results shown in Table 2 and Table 3 with these values, we may say that the new result is basically in good agreement with the previous one [11]. The main difference in the feature parameters between the WMAP5 data and the Planck data is the amplitude of the oscillatory feature. To understand this difference, let us compare

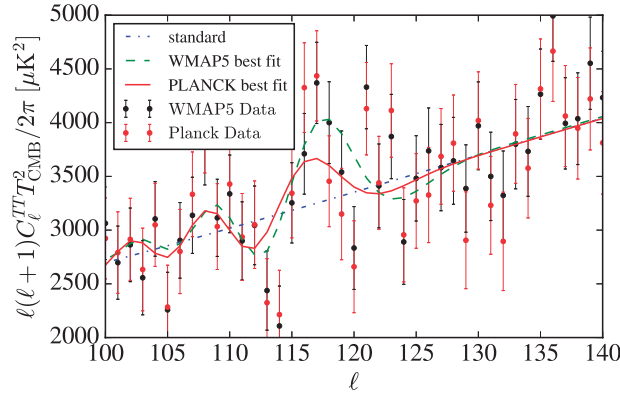


Fig. 1. The angular power spectrum of the TT autocorrelation. The black (red) dots and bars show the WMAP5 (Planck 2015) data points and their error bars. The green dashed line, the blue dot-dashed line, and the red solid line represent the best-fit model for the TT and TE combined data of WMAP5 using the oscillating initial power spectrum given by Eq. (2); for the TT, TE, and EE combined data of Planck 2015 using the standard initial power spectrum of Eq. (1); and the oscillating initial power spectrum of Eq. (2), respectively.

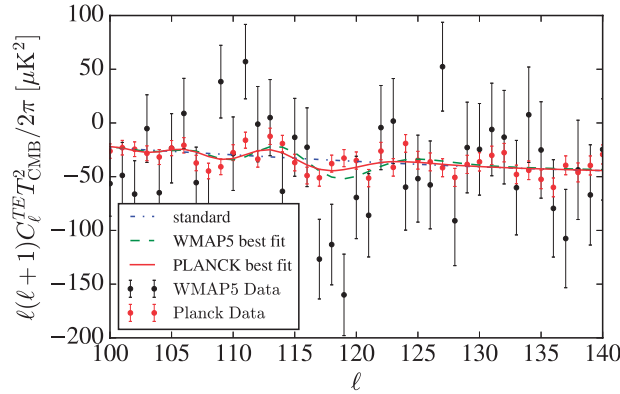


Fig. 2. As Fig. 1, but for the Planck TE cross-correlation data.

the data points of the angular power spectra of WMAP5 and Planck 2015. Figure 1 shows that at first glance the data points of the TT autocorrelation are almost the same between the WMAP5 and the Planck 2015 data, as both measurements are cosmic-variance limited in these multipoles. Therefore, these points are not responsible for the change of the feature parameters, although it is interesting to note that the Planck 2015 and WMAP5 data deviate from each other well beyond their measurement errors. In Fig. 2, where we show the TE cross-correlation data, we can see that both the amplitude of the oscillations and errors have become smaller in the Planck data. Therefore, we conclude that this improved TE cross-correlation data makes the value of the amplitude parameter B somewhat smaller for the Planck 2015 data compared with the WMAP5 data.

The E-mode polarization autocorrelation data have been newly released by the Planck collaboration in 2015. In Fig. 3, we can see that the prediction by the WMAP5 data fits pretty well to the Planck EE power spectrum. For more accurate understanding of features in the polarization data, we show the likelihoods of the feature parameters in Fig. 4. There we show a comparison between the likelihood of the feature parameters only from the TT autocorrelation data and that from the TT, TE, and EE combined data with smaller uncertainties. We can see that all of the likelihoods are sharpened, and the amplitude parameter becomes slightly smaller if we use the combined data. These indicate that

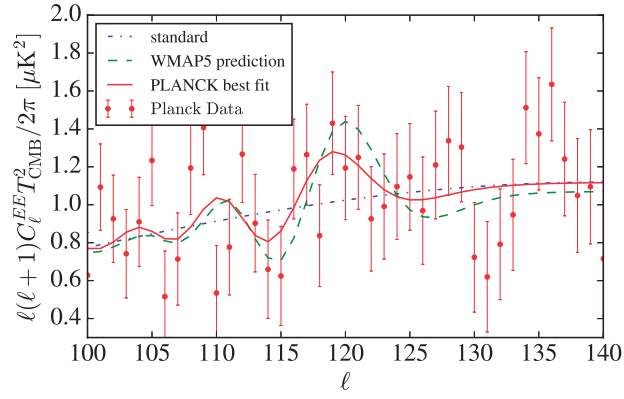


Fig. 3. As Fig. 1, but for the Planck EE autocorrelation data.

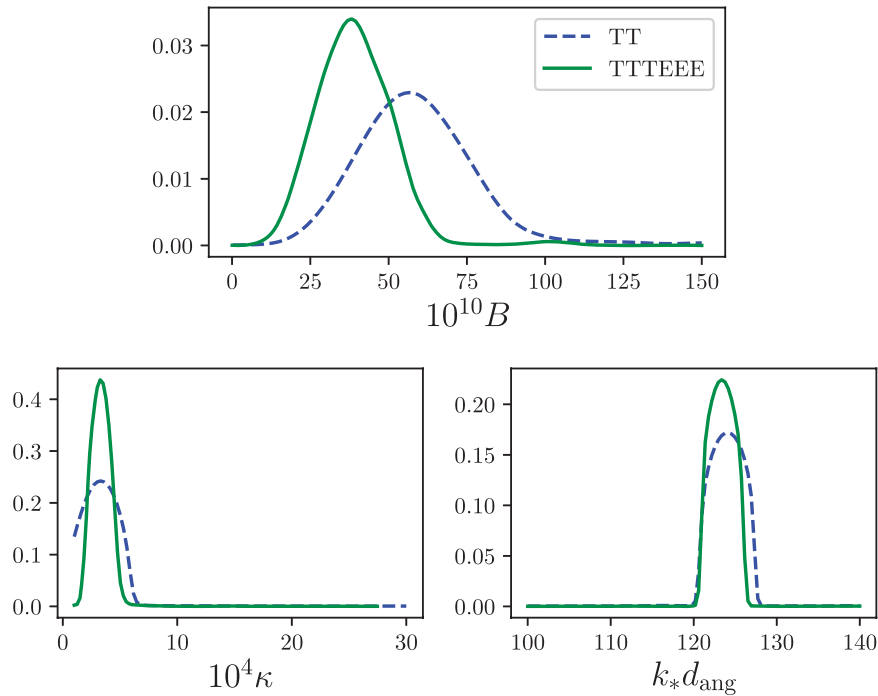


Fig. 4. The normalized likelihoods of the amplitude of the oscillations (top), the width (bottom left), and the position (bottom right) from the TT autocorrelation data of Planck 2015 (blue dashed line) alone and the TT, TE, and EE combined data of Planck 2015 (green solid line).

there are features with a smaller amplitude, but with the same width and at the same position in the polarization data as in the temperature data.

Table 4 shows that the χ^2 values are improved by using the oscillating initial power spectrum, and the improvement is more than twice the number of added parameters. In view of Akaike's information criterion [39], this implies that the oscillating initial power spectrum (2) is the better model to describe the primordial spectrum of curvature perturbation of our Universe. This does not necessarily mean that inflation model which produced our Universe as it is today must realize such a featured spectrum as a mean value, because the theoretical cost to realize such an ad hoc spectrum cannot be taken into account in the Akaike approach. What is important here is the fact that our Universe is imprinted with such a feature whether it was generated as a result of a non-standard

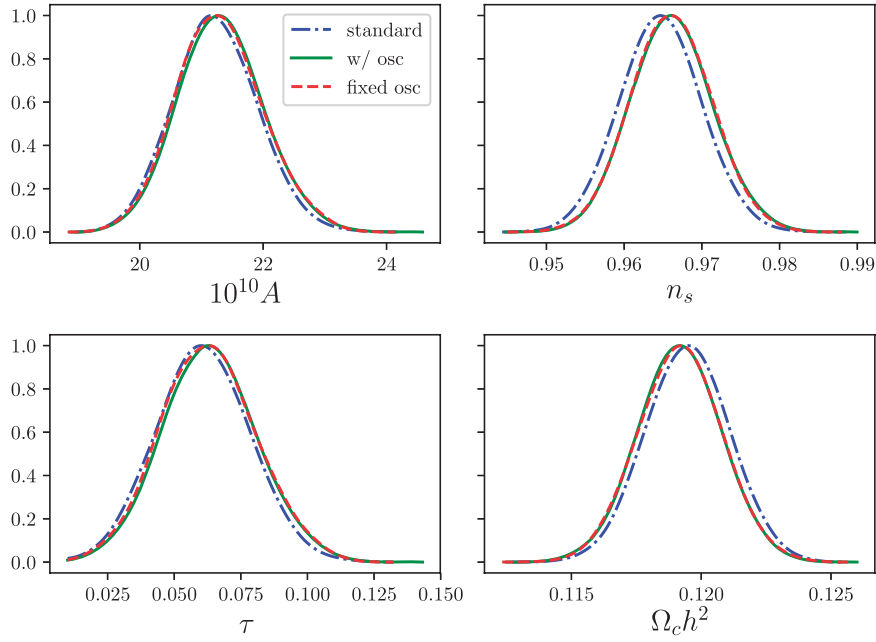


Fig. 5. The posterior distributions of the cosmological parameters $10^9 A$ (top left), n_s (top right), τ (bottom left), and $\Omega_c h^2$ (bottom right) from the standard power-law model (blue solid dot and dash line), oscillations (green solid line), and the fixed oscillation model (red dashed line) derived from the TT, TE, and EE combined data.

inflation model or as a very rare realization of the conventional inflation model predicting a simple power-law spectrum. In either case, in order to determine precise values of cosmological parameters of our Universe, we should incorporate these features in the MCMC analysis.

If they are real imprinted features, they can be seeds of the large-scale structure of the Universe, and we will be able to see them in the galaxy correlations [40].

3.2. The effects on the cosmological parameters

We have seen the existence of the features in the Planck data. We need to take into account these features when we estimate the other cosmological parameters. In fact, the inclusion of the features in the initial power spectrum has considerably affected the estimation of cosmological parameters if we use the WMAP5 data [11], especially the estimation of the amplitude of the initial power spectrum A and the spectral index n_s . From Table 2 and Table 3, we can check the effects of the features on the estimation of the best-fit values and the mean values of cosmological parameters for the Planck data. In these tables, the estimated values, including the amplitude A and the spectral index n_s , have barely changed among the models for each data set, namely the standard initial power spectrum, the oscillating initial power spectrum, and the fixed oscillation initial power spectrum. From the posterior distributions of some cosmological parameters (Fig. 5), we can confirm that the distributions do not vary between the different models of the initial power spectrum. These indicate that the local features in the Planck data do not affect the estimation of cosmological parameters. This result is consistent with the analysis for other scales using a similar feature model and Planck data [33,34]. The improved angular resolution of Planck enables us to observe a greater number of multipoles at small scales. These rich small-scale data have sufficient statistical power to determine the cosmological parameters. This is why the local features do not affect the estimation of the cosmological parameters.

4. Conclusion

In this paper, we have studied the features at $\ell \sim 120$ in the Planck 2015 CMB angular power spectra data which were obtained by the WMAP5 data analysis [11].

We have performed an MCMC analysis using the TT autocorrelation data and TT, TE, and EE combined data of Planck 2015, and confirmed the existence of the features in the Planck 2015 data. We checked the consistency between the temperature fluctuation data and the polarization data, confirming that the features exist in both cases with almost the same width and at the same position, albeit at a slightly smaller amplitude in the latter data. Then we have investigated the effects of these features at $\ell \sim 120$ in determining cosmological parameters, and found that they do not affect the estimation of cosmological parameters, unlike the case of WMAP5 data, because there are enough data in the higher multipoles to determine the cosmological parameters.

We expect they are the real features of the initial power spectrum and will be observed by future experiments without interrupting the estimation of the cosmological parameters.

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