

Double Parton Interactions in pp and pA Collisions

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Abstract. As a consequence of the increasingly large flux of partons at small x , Double Parton Interactions (DPI) play an increasingly important role at high energies. A detail understanding of DPI dynamics is therefore mandatory, for a reliable subtraction of the background in the search of new physics. On the other hand, DPI are an interesting topic of research by themselves, as DPI probe the hadron structure in a rather different way, as compared with the large p_t processes usually considered. In this note we will make a short illustration of some of the main features characterizing DPI in pp and in pA collisions.

1 Introduction

The rapid increase of the parton flux at small x causes a dramatic rise of all cross sections with large momentum transfer exchange in high energy hadronic collisions [1,2]. The values of x , which contribute to a hard process, with a fixed lower cutoff in the exchanged momentum, are in fact increasingly smaller at large energies. One thus faces a unitarity problem in large p_t processes at high energy. A good example is the inclusive cross section to produce mini-jets which, as shown in fig.1, may become larger than the total inelastic cross section at high energy.

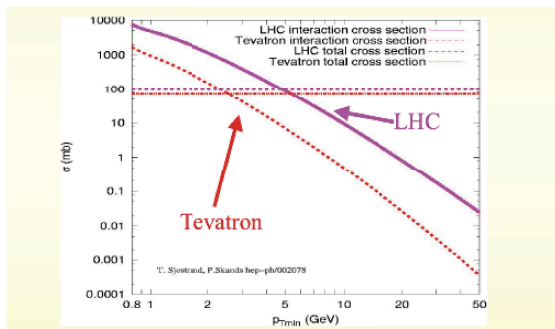


Figure 1. Minijets cross section as a function of the cutoff in p_t at Tevatron and at the LHC.

The problem with unitarity is solved when Multiple Parton Interactions (MPIs) are introduced in the interaction dynamics. MPIs introduce the possibility of different hard partonic interactions in a

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given inelastic event and, while each inelastic event contributes with multiplicity 'one' to the inelastic cross section, each inelastic event contributes with the multiplicity of hard partonic interactions to the inclusive hard cross section. In this way, when the average multiplicity of hard partonic interactions in a inelastic event is large, the inclusive cross section is no more bounded by the value of the total inelastic cross section [3].

The simplest case of MPIs is the Double Parton Interaction (DPI). A possible case to consider is the production of four large p_t jets, where transverse momenta are compensated pairwise. The process can thus be originated by two different pairs of initial state partons, which interact independently with large transverse momentum exchange. The corresponding contribution to the cross section maximizes the incoming parton flux and thus it gives an increasingly important contribution at large energies.

Given the large momentum transfer, hard interactions are localized in a space region much smaller as compared to the hadron size. In a DPI two hard partonic interactions are thus localized in two different points in transverse space, in the overlap region of the matter distribution of the two interacting hadrons. The hard component of the interaction is thus disconnected and the process can be described by the geometrical picture in fig.2.

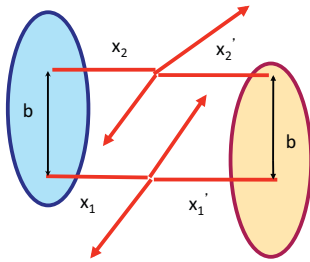


Figure 2. Geometrical picture of a DPI.

As apparent in fig.2, the non-perturbative components of a DPI are in this way factorized into functions which depend on two fractional momenta and on the relative transverse distance b between the two interaction points. The non-perturbative input to DPI, namely the double parton distribution functions, have thus dimensions of an inverse area and contain information on the hadron structure not accessible in a single scattering processes.

When neglecting spin and color, the inclusive double parton-scattering cross-section, for two parton processes A and B in a pp collision, is thus given by [4]

$$\sigma_D^{(A,B)} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_{i,j}(x_1, x_2; b) \hat{\sigma}_{i,k}^A(x_1, x'_1) \hat{\sigma}_{j,l}^B(x_2, x'_2) \Gamma_{k,l}(x'_1, x'_2; b) dx_1 dx'_1 dx_2 dx'_2 d^2b \quad (1)$$

where m is a symmetry factor ($m = 1$ when $A = B$ and $m = 2$ when A is different from B), the functions $\Gamma(x, x'; b)$ are the double parton distributions, which depend on two fractional momenta and of b , $\hat{\sigma}^{A,B}$ are the partonic scattering cross sections and the sum is over the different parton species contributing to the process.

Notice that the dependence of σ_D on the transverse momenta of final partons is very well characterised and it is very strong. Actually it is as strong as the square of a single hard scattering cross section. One should stress that this feature represents a rather non trivial experimental test of the DPI interaction dynamics.

By introducing the "effective cross section", σ_{eff} , one may express σ_D by the so called "pocket formula", utilised in the experimental analysis:

$$\sigma_D^{pp(A,B)} = \frac{m}{2} \frac{\sigma_A \sigma_B}{\sigma_{eff}} \quad (2)$$

where σ_A and σ_B are the single scattering inclusive cross sections. Of course the "pocket formula" makes sense only, if comparing with experiments, σ_{eff} turns out to be weakly dependent on the reaction channel and on kinematics. Which is indeed the case, as it may be seen by looking at fig.3, where the results of different measurements of σ_{eff} are shown [5-9].

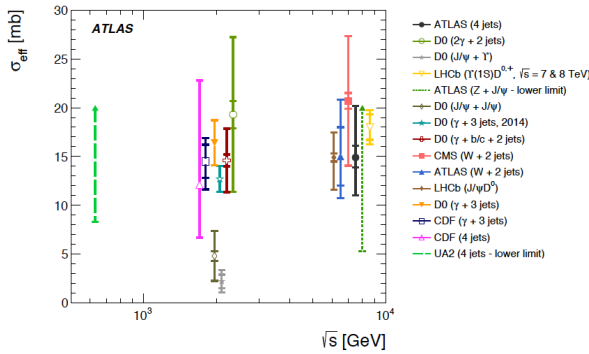


Figure 3. σ_{eff} as measured in different experiments.

When σ_A is small, the ratio σ_A/σ_{inel} represents the probability of having the process A in an inelastic collision. In the pocket formula $\sigma_D^{(AB)} = \sigma_A \sigma_B / \sigma_{eff}$ (here one assumes $A \neq B$) and in the biased case, where the process B takes place in presence of the process A , the effective cross section thus plays the role of the inelastic cross section.

By looking at fig.3, although within large experimental errors, the observed values of the effective cross section are approximately constant (~ 15 mb) and do not seem to depend on

the C.M. energy (Fermilab, LHC)

the reaction channel ($4j$; $\gamma 3j$; $\gamma \gamma 2j$; $\gamma b(c) 2j$; $W jj$; $J/\Psi J/\Psi$; $Z J/\Psi$; ΥD^0 ; $J/\Psi D^0$)

the values of x and Q^2

There is on the contrary a clear indication that the effective cross section is sizably smaller (≈ 2 -4 mb) in the case ($J/\Psi J/\Psi$) and ($J/\Psi \Upsilon$) production

Notice that σ_{eff} is sizably smaller as compared with σ_{inel} , which is an indication of strong partonic correlations in the hadron structure.

2 σ_{eff} and Partonic Correlations

One may write the double parton distribution functions as

$$\Gamma(x_1, x_2; b) = G(x_1, x_2) f_{x_1 x_2}(b), \quad G(x_1, x_2) = K_{x_1 x_2} G(x_1) G(x_2) \quad (3)$$

where $G(x)$ is a usual "one-body" parton distribution. The function f is normalised to one and the transverse scale, characterising f , may depend on fractional momenta. After integration on the transverse coordinate b , one thus obtains the average multiplicity of pairs with fractional momenta x_1 and x_2 :

$$\int f_{x_1 x_2}(b) d^2 b = 1; \quad G(x_1, x_2) = K_{x_1 x_2} G(x_1) G(x_2) = \langle n(n-1) \rangle_{x_1 x_2}; \quad G(x) = \langle n \rangle_x \quad (4)$$

Parton distributions are in this way understood as average number of partons with a given momentum fraction. In the simplest case one would have $K_{x_1 x_2} = 1$ which, after integrating on the transverse coordinate b , would be consistent with a Poissonian multiparton distribution in multiplicity [10]. By using relations in Eqs. (3) and (4), the double parton scattering cross section is given by

$$\begin{aligned} \sigma_D^{pp(A,B)}(x_1, x'_1, x_2, x'_2) &= \frac{m}{2} K_{x_1 x_2} K_{x'_1 x'_2} G(x_1) \hat{\sigma}_A(x_1, x'_1) G(x'_1) \\ &\quad \times G(x_2) \hat{\sigma}_B(x_2, x'_2) G(x'_2) \int f_{x_1 x_2}(b) f_{x'_1 x'_2}(b) db \\ &= \frac{m}{2} \frac{K_{x_1 x_2} K_{x'_1 x'_2}}{\pi \Lambda^2(x_1, x'_1, x_2, x'_2)} \sigma_A(x_1, x'_1) \sigma_B(x_2, x'_2) \end{aligned} \quad (5)$$

where

$$\int f_{x_1 x_2}(b) f_{x'_1 x'_2}(b) db = \frac{1}{\pi \Lambda^2(x_1, x'_1, x_2, x'_2)} \quad (6)$$

and the effective cross section is thus expressed in terms of the typical area of the interaction region Λ , which may depend on the fractional momenta of the interacting partons, and on the multiplicities of the interacting parton pairs:

$$\sigma_{eff}(x_1, x'_1, x_2, x'_2) = \frac{\pi \Lambda^2(x_1, x'_1, x_2, x'_2)}{K_{x_1 x_2} K_{x'_1 x'_2}} \quad (7)$$

Limiting cases are

a) partons are not correlated in multiplicity. In such a case $K_{x_1 x_2} = 1$

b) partons are not correlated in the transverse coordinates. In such a case one may write the generalised "one-body" parton distribution $\Gamma(x; b)$ as

$$\Gamma(x; b) = G(x) f_x(b), \quad \int f_x(b) d^2 b = 1 \quad \text{and one has : } f_{x_1 x_2} = \int f_{x_1}(b') f_{x_2}(b-b') d^2 b' \quad (8)$$

where $f_x(b)$ is the two gluon form factor of the nucleon. If one assumes that the effects of correlations are negligible, there are no unknowns and σ_{eff} can be evaluated. One obtains

$$\sigma_{eff} = \pi \Lambda^2 = 32 \text{ mb} \quad (9)$$

which is roughly a factor 2 larger as compared with the experimental indications. One may thus conclude that either K is not equal to 1 or partons are correlated in the transverse coordinates or, most likely, that partons are correlated both in multiplicity and in the transverse coordinates.

An additional experimental indication is that, in the kinematical region where DPI are important, the effective cross section depends only weakly on fractional momenta [5]. It makes therefore sense to assume that there is a weak dependence of both Λ and K on fractional momenta.

On the other hand it is clear that, since all new information on the hadron structure is summarized by a single quantity, namely the effective cross section, by studying DPI in pp collisions, one does not obtain enough information to discriminate between Λ and K . In other words one cannot disentangle parton correlations in the transverse coordinates and in multiplicity by studying DPI only in pp .

To obtain additional information on multi-parton correlations one needs to study DPI in pA collisions. In the case of a double parton interaction, in a collision of a proton with a nucleus, the effects of longitudinal and transverse correlations are in fact different when a single nucleon or two different target nucleons participate in the hard process.

3 DPI in pA Collisions

In the case of DPI in proton - nucleus collisions, in the regime where non additive corrections to the nuclear parton distributions are small, one may have a double parton interaction against a single or against two different target nucleons [11,12]. DPI have been measured in the WJJ production channel at the LHC, in pp collisions. It's interesting to compare the results, obtained in pp collisions, with the expectations of WJJ production in $p - Pb$ collisions in the same kinematical regime.

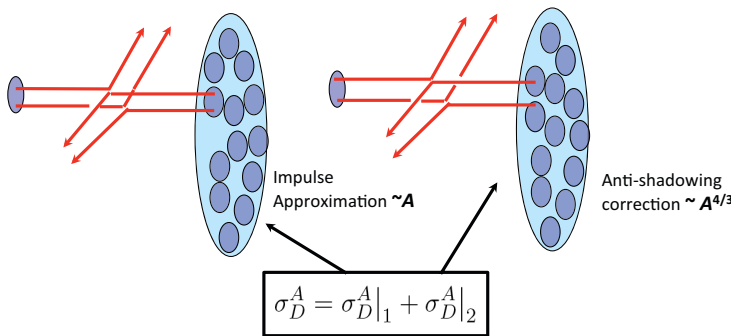


Figure 4. Different contributions to the DPI cross section in pA collision.

When neglecting the effects of the interference terms, which are estimated to produce a correction of the order of 10%, one obtains a simple expression for the DPI cross section in pA collisions:

$$\begin{aligned}\sigma^{pA}(WJJ) &= \sigma_S^{pA}(WJJ) + \sigma_D^{pA}(WJJ) \\ \sigma_D^{pA}(WJJ) &= \sigma_D^{pA}(WJJ)|_1 + \sigma_D^{pA}(WJJ)|_2\end{aligned}\tag{10}$$

The labels S and D here above correspond to the single and double parton scattering contributions, while the labels 1 and 2 distinguish the terms where one or two different target nucleons take

active part in the process. By neglecting the effects of interference terms, the explicit expressions of $\sigma_D^{pA}(WJJ)|_1$ and of $\sigma_D^{pA}(WJJ)|_2$ are [13]

$$\begin{aligned}\sigma_D^{pA}(WJJ)|_1 &= \frac{1}{\sigma_{eff}} [Z\sigma_S^{pp}(W) + (A-Z)\sigma_S^{pn}(W)]\sigma_S^{pp}(JJ) \\ \sigma_D^{pA}(WJJ)|_2 &= K \left[\frac{Z}{A}\sigma_S^{pp}(W) + \frac{A-Z}{A}\sigma_S^{pn}(W) \right] \sigma_S^{pp}(JJ) \int T(b)^2 d^2b\end{aligned}\quad (11)$$

where Z is the nuclear charge and $T(b)$ the nuclear thickness, as a function of the impact parameter b . In the kinematical regime of interest the production of jets is dominated by gluons, so in Eq.(11) we have put $\sigma_S^{pp}(JJ) = \sigma_S^{pn}(JJ)$. Notice that, while $\sigma_D^{pA}(WJJ)|_1$ grows linearly with A , $\sigma_D^{pA}(WJJ)|_2$ represents an additional positive contribution to the cross section, which grows as $A^{4/3}$. Therefore, in the case of DPI, the correction to the "impulse approximation term" is not the typical negative shadowing correction term. On the contrary, in the case of DPI, the correction term is positive and one has a anti-shadowing correction which, with heavy nuclei, may represent the dominant contribution to the cross section.

The first term in the equations above, $\sigma_D^{pA}(WJJ)|_1$, is proportional to the effective cross section and does not add much to the information on DPI obtained from pp collisions. The second term, $\sigma_D^{pA}(WJJ)|_2$, is on the contrary proportional to K , which measures the multiplicity of pairs of partons in the projectile hadron (one should remind that when $K = 1$ the multiplicity distribution is a Poissonian).

By measuring the amount of anti-shadowing, actually $\sigma_D^{pA}(WJJ)|_2$, one can thus obtain information on how the pairs of projectile partons are correlated in multiplicity.

Different contributions to the cross section are therefore possible, depending on the actual value of K . Two extreme cases can thus be considered in proton - lead collisions

a) There are no correlations in multiplicity:

$$\begin{aligned}K^2 &= 1 \text{ and } \pi\Lambda^2 = \sigma_{eff} \\ \frac{\sigma_D^{pA}(WJJ)|_2}{\sigma_D^{pA}(WJJ)|_1} &\approx 2\end{aligned}\quad (12)$$

b) There are no correlations in the transverse coordinates:

$$\begin{aligned}K^2 &= 2 \text{ and } \pi\Lambda^2 = K^2\sigma_{eff} \\ \frac{\sigma_D^{pA}(WJJ)|_2}{\sigma_D^{pA}(WJJ)|_1} &\approx 3\end{aligned}\quad (13)$$

Notice the huge values of the anti-shadowing contributions to the cross section: 200% and 300% in the two limiting cases.

A more detailed information is obviously obtained when looking at the differential distributions. In fig.5 the transverse distributions in $p-p$ and $p-Pb$ are compared. The leading Standard Model contribution (Single Parton Interaction) is represented by the pinkish histograms. The green histograms are the contributions due to Double Parton Interactions. The black histograms are the sum of Single and Double Parton Interactions.

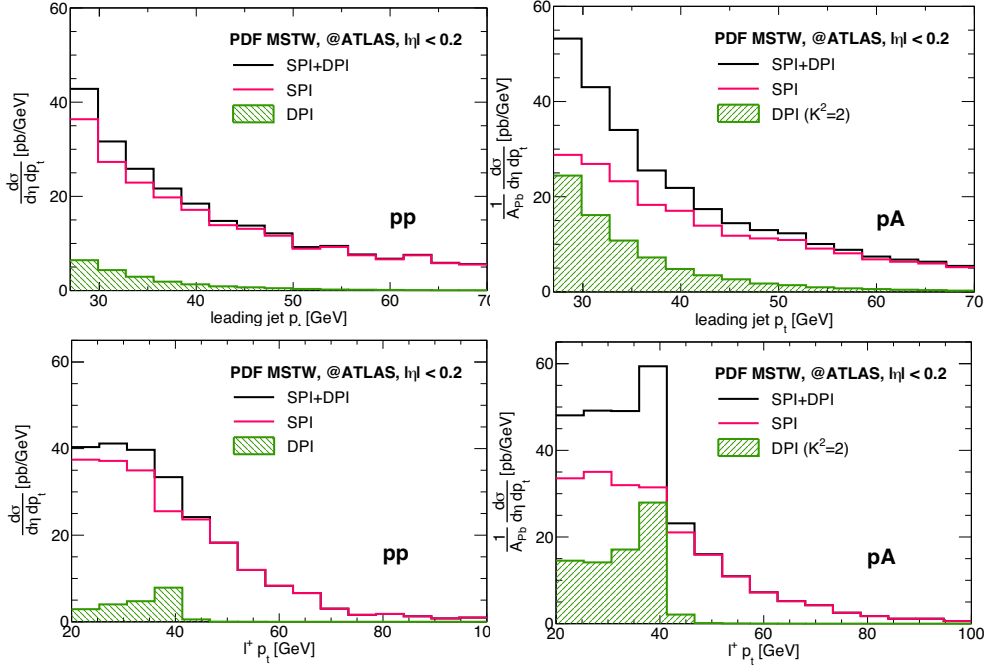


Figure 5. Upper figures: transverse spectra of the leading jet in $p-p$ (left panel) and $p-Pb$ collisions (right panel). Lower figures: transverse spectra of the W decay-lepton in $p-p$ (left panel) and $p-Pb$ collisions (right panel).

In the upper panels in fig.5, one compares the transverse spectra of the leading jet in $p-p$ (left panel) and $p-Pb$ collisions (right panel). One may notice that, consistently with the results of ATLAS and CMS, in $p-p$ DPI can only provide a minor modification to the SPI spectrum. On the contrary, the contribution of DPI is much more important in $p-Pb$, where the slope in p_t of the leading jet is substantially modified at $p_t \lesssim 40$ GeV.

In the lower panels in fig.5, one compares the transverse spectra of the W decay-lepton in $p-p$ and $p-Pb$ collisions. Again DPI have a small effect in $p-p$ while, as shown in lower panel on the right side, DPI produce a huge increase of the spectrum at $p_t \lesssim 40$ GeV in $p-Pb$. In a DPI the W boson is in fact produced with a rather small transverse momentum, since the observed jets do not recoil against the W . The transverse spectrum of the W decay-lepton produced by DPI (green histogram in the figure) is thus limited at $p_t \lesssim 40$ GeV and the huge increase of the spectrum of the decay lepton at 40 GeV can in this way give a direct indication on the contribution of DPI, and thus on the anti-shadowing term, to the cross section.

One may therefore conclude that one expects rather sizable effects from DPI in $p-Pb$ as compared to $p-p$ at the LHC, because of the different interaction mechanism in the two cases. The additional contribution to the cross section, namely $\sigma_D^{pA}(WJJ)|_2$, is dominant at large A and it is directly proportional to the multiplicity of interacting pairs of partons in the projectile (the factor K in the present note). In the case of WJJ production in $p-Pb$ one should therefore be able to obtain a

rather direct indication on the size of K by simply looking at the inclusive transverse spectrum of the W decay-lepton in $p - Pb$ collisions.

The sizable increase of the fraction of events due to DPI in $p - Pb$ collisions and the significant difference of the resulting value of the cross section, as a function of the correlation parameter K , are rather encouraging indications of the potential of DPI in $p - Pb$ collisions to obtain information on the parton correlation parameter K and, as a consequence, also on Λ . With a joint study of DPI in $p - p$ and $p - Pb$, one might thus be able to obtain information, to a large extent model independent, on the typical values of multiplicity of parton pairs and on the typical transverse distances between partons in the hadron structure; possibly also at different values of the parton's fractional momenta. The option of an experimental study of DPI in $p - Pb$ at the LHC could therefore be highly rewarding, offering a viable possibility of obtaining a remarkable insight into the three dimensional structure of the hadron.

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