Particle creation in a resonant microwave cavity?

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Abstract

Parametric photon creation via the so called dynamical Casimir effect is calculated numerically. We consider a model where a three-dimensional resonant cavity is bisected by a semiconductor diaphragm, which is irradiated by a pulse laser with frequency of the GHz order. Our preliminary results show that the photon number density depends on where the diaphragm is placed with the midpoint giving the largest contribution.

1 Introduction

In the pursuit of an experimental verification of the dynamical Casimir effect (DCE), the problem arises of how to oscillate a cavity wall with an extremely high frequency of the GHz or THz order? A particularly nice idea was by Yablonovitch [1], also see references in [2], who proposed an optical excitation of valence electrons of a semiconductor into the conduction band by a pulse laser, which makes the semiconductor metallic. The metallized semiconductor wall reflects electromagnetic waves and thus the semiconductor diaphragm (SCD) acts like an oscillating cavity wall. Quite recently, experimental schemes to detect DCE photons have been proposed using a semiconductor wall irradiated by a pulse laser [3].

From a theoretical standpoint there have been been some works on the *SCD idea* [2, 4, 5]. However, in [5] the prerequisite guaranteeing a perturbative treatment is not satisfied when the SCD is placed far from the cavity wall and a numerical approach should be used, e.g. [6]. Also, recently the work of Dodonov & Dodonov [7] discussed some possible problems with the *SCD idea*, relating to that fact that the dielectric constant of the semiconductor has a large positive imaginary part in the conducting (irradiated) state, which therefore leads to dissipative effects. A possible resolution to this problem was advocated in [8] by applying a single mode phenomenological dissipation model. The purpose of this work is to discuss how the location of the SCD affects the number of created photons assuming the SCD is a perfect conductor when irradiated (unitary evolution). Furthermore, we find that when the SCD is not attached to one of the cavity walls, such as at the midpoint, then the single mode approach used in [8] should somehow be generalized to multimode coupling.

2 Model for TE Modes

We evaluate numerically the number density for TE photons for an SCD placed in an aluminum cavity with dimensions $L_x \times L_y \times L_z (L_x = L_y \equiv L = 5$ cm, $L_z = 2L)$ which is bisected by an n-type semiconductor diaphragm (SCD) placed at a position d from the left wall along the z-axis (the exact details of the experimental design & detection will be presented elsewhere [9]). Electromagnetic waves in a vacuum can be conveniently decoupled into two scalar functions (or scalar Hertz potentials as they are commonly known) ψ_E and ψ_M instead of the usual scalar & vector potential $(\phi, \mathbf{A})$, e.g. see [10]. This allows us to find solutions for each respective scalar Klein-Gordon equation:

$$[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}]\psi_E(x,y,z,t) = V(t)\delta(z-d)\psi_E(x,y,z,t) \tag{1}$$

where the subscript E will be used to denote the TE mode. Similarly to the work of [4, 5], we model the SCD by a Dirac delta function, $\delta(z-d)$ with potential $V(t) = 4\pi\rho_e(t)\Delta z e^2/m^* c^2$; where ρ_e is the density of conduction electrons, Δz is the effective thickness of the SCD for laser absorption, e is the electronic charge and m^* the effective mass of the conduction electrons in the SCD with $m^* = 0.07m_0$ (m_0 being

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the electron rest mass and c, the velocity of light). Estimating a pulse laser power of around 100 J/pulse then we find $\rho_e \Delta z \cong 1 \times 10^{13}$ cm^{-2}, where we have assumed a donor density of 10^{18} atoms cm^{-3} and an energy interval of 10 meV between the conduction band minimum and donor level (at a temperature of 1 K). Thus one obtains the following maximum and minimum values for $V(t)$ of $V_{max} = 500$ cm^{-1} and $V_0 \approx 0$ cm^{-1}.

In the following we shall assume that the period of the laser pulse is set to $T = 149.07$ ps, which corresponds to a frequency of about three GHz for the TE fundamental modes: $(1,0,1)$ & $(0,1,1)$ respectively. The overall shape of $V(t)$ is assumed to asymmetric because the SCD excitation and recombination times are expected to differ [7]. We use a profile for $V(t)$ of the form of one Gaussian of half-width $\sigma_1 = 4$ ps going from V_0 to $V_{\max}$ where saturation at the maximum lasts for $t_{sat} = 7$ ps with the second Gaussian with $\sigma_2 = 11$ ps going back down to V_0 . We assume the pulse is offset by 30 ps. In practice these times can be measured experimentally and for example lifetimes of the order of 10 ps may be achievable. In order to avoid strong dissipation effects in the SCD we have also set the saturation time to a short time $t_{sat} = 7$ ps.

The scalar function ψ_E represents the longitudinal component of the magnetic field $\mathbf{B}_z$ and satisfies Dirichlet boundary conditions (BCs) on the longitudinal boundary and Neumann BCs on the transverse boundaries. Thus, the solution for the TE mode takes the form

$$\psi_E(\mathbf{x},t) = \begin{cases} \sqrt{\frac{2}{L_x}} \cos\left(\frac{\pi m_x x}{L_x}\right) \sqrt{\frac{2}{L_z}} \cos\left(\frac{\pi m_y y}{L_y}\right) \times A_m^E \sqrt{\frac{1}{d}} \sin\left(k_m z\right) & 0 < z < d \\ \sqrt{\frac{2}{L_x}} \cos\left(\frac{\pi m_x x}{L_x}\right) \sqrt{\frac{2}{L_z}} \cos\left(\frac{\pi m_y y}{L_y}\right) \times B_m^E \sqrt{\frac{1}{L_z - d}} \sin\left(k_m (L_z - z)\right) & d < z < L_z \end{cases}, \tag{2}$$

where m_x and m_y are integers $(0,1,2,3,\ldots)$ with $m_x = m_y \neq 0$ and m (dropping subscript z) denotes the eigenvalues of the function $k_m(t)$ in the z-direction A_m^E is a normalization constant satisfying

$$(\psi_n, \psi_n)_E = \left(1 - \frac{\sin(2dk_n)}{2dk_n}\right)(A_n^E)^2 + \left(\frac{\sin(2k_n(L_z - d))}{2k_n(d - L_z)} - 1\right)(B_n^E)^2 = 1\,. \tag{3}$$

The SCD δ-function in the wave equation leads to a discontinuity in the spatial derivative at $z = d$, while the field itself is continuous:

$$\psi_I(z = d, t) = \psi_{II}(z = L_z - d, t) \tag{4}$$

$$\frac{\partial}{\partial z}\psi_I(z = d, t) - \frac{\partial}{\partial z}\psi_{II}(z = L_z - d, t) = -V(t)\psi(z = d, t) \tag{5}$$

From the above relations, we obtain the following continuity and eigenvalue relations for the TE mode:

$$\frac{A_m^E}{B_m^E} = \sqrt{\frac{d}{L_z - d}\frac{\sin(k_m(L_z - z))}{\sin(k_m d)}} \qquad \frac{\sin(k_m L_z)}{\sin(k_m[L_z - d])\sin(k_m d)} = -V(t)L_z\,. \tag{6}$$

In this work we solve for the eigenvalues $k_m(t)$ exactly.

3 Photon Number Density

The second quantization of the equations of motion using the instanteneous basis approach leads to a set of infinitely coupled equations [11]. The TE field is quantized as $\psi_E(\mathbf{x},t) = \sum_m C_m \left[a_m u_m(\mathbf{x},t) + a_m^\dagger u_m^*(\mathbf{x},t)\right]$ with the standard harmonic oscillator solution $u_m(\mathbf{x},t) = e^{-i\omega_m^0 t}/(\sqrt{2\omega_m^0})\,\psi_m(\mathbf{x},0)$ for $t < 0$, before irradiation and the instantaneous basis $u_s(\mathbf{x}, t \geq 0) = \sum_m P_m^{(s)}(t)\psi_m(\mathbf{x},t)$ for $t \geq 0$ while irradiated. On substituting this expression into the wave equation (1) we obtain, after using orthonormality,

$$\ddot{P}_n^{(s)} + \omega_n^2(t)P_n^{(s)} = -\sum_m^\infty \left[\left(2\dot{P}_m^{(s)}\dot{k}_m + P_m^{(s)}\ddot{k}_m\right)g_{mn}^{\mathrm{A}} + P_m^{(s)}\dot{k}_m^2 g_{mn}^{\mathrm{B}}\right], \tag{7}$$

where

$$g_{mn}^{\mathrm{A}} = \frac{\delta_{m_x n_x}\delta_{m_y n_y}}{(\psi_n,\psi_n)}\left(\frac{\partial\psi_m}{\partial k_m}, \psi_n\right) \qquad g_{mn}^{\mathrm{B}} = \frac{\delta_{m_x n_x}\delta_{m_y n_y}}{(\psi_n,\psi_n)}\left(\frac{\partial^2\psi_m}{\partial^2 k_m}, \psi_n\right). \tag{8}$$

The g^A_{mn} and g^B_{mn} are very complicated functions and would require numerical integration in general (we have verified numerically that $g^A_{nn} = 0$ in all cases). However, for special cases (such as at the midpoint and for $d = L_z/3$) they can be integrated exactly to give a complicated function of $k_n(t)$. The wave number at a given instant of time is

$$\omega_n^2(t) = k_n^2(t) + \left(\frac{n_x\pi}{L_x}\right)^2 + \left(\frac{n_y\pi}{L_y}\right)^2 \qquad\qquad \omega_m(0) = \omega_m^0 \,. \tag{9}$$

and imposing continuity of u_n and $\dot{u}_n$ at $t = 0$ leads to the following initial conditions: $P^{(s)}_m(0) = 1/\sqrt{2\omega_m^0}$ and $\dot{P}^{(s)}_m(0) = -i\sqrt{\omega_m^0/2}$.

As has been well discussed in the literature [12] the number density, N_m, in a particular mode m is[2]

$$N_m = \frac{1}{C_m^2}\sum_n C_n^2|\beta_{mn}|^2 \qquad\qquad C_m^2 = 8\pi/\left[\left(\frac{\pi m_x}{L_x}\right)^2 + \left(\frac{\pi m_y}{L_y}\right)^2\right] \tag{10}$$

where C_m is a TE normalization [10] and β_{mn} is a Bogolubov coefficient [12]. These can be calculated by choosing the solution in $u_m(t)$ for time $t \geq 0$ as the *out* basis states and use the continuity conditions valid for $t < 0$ for the *in* basis states. A straightforward calculation leads to

$$\beta_{mn}(t) = \sqrt{\frac{\omega_m(t)}{2}}\left(P^{(n)}_m(t) - \frac{i}{\omega_m(t)}\left[\dot{P}^{(n)}_m(t) + \sum_\ell g^A_{\ell m}(t)P^{(n)}_\ell(t)\right]\right) , \tag{11}$$

with α_{mn} given by the complex conjugate. The choice of normalization in equation (11) is defined to satisfy the continuity conditions, which implies $\alpha_{mn}(0) = \delta_{mn}$ and $\beta_{mn}(0) = 0$. By solving equation (7) numerically we can find $\beta_{mn}(t)$ and hence the photon number density via equation (10).

4 Results & Discussion

There are various approaches to solve the set of equations (7), e.g. [6], and what we try here is to just solve the equations directly in MATHEMATICA. It may be worth mentioning that the larger the power of the pulse laser the more pulses which can fit into a given carrier wave pulse. In our case we expect the carrier wave pulse to be about 5000 ps long and thus the fundamental TE mode would contain about 33 pulses. However, due to limitations with our code we can only integrate the equations up to 1000 ps, about 7 pulses. In the numerics we went up to a given cutoff m_{max} in equation (7) such that the results converged, which we checked by verifying that the unitarity constraint, $\sum_n(|\alpha_{mn}|^2 - |\beta_{mn}|^2) = 1$, [12] is satisfied to a given accuracy, see Figure 1. For the midpoint this was at $m_{max} = 17$ while that for $d/L_z = 1/3$ was at $m_{max} = 10$.

A further point is that due to the $\delta_{m_x n_x}$, $\delta_{m_y n_y}$ terms in g^A_{mn} and g^B_{mn} we only consider the coupling of the modes in the z-direction to the $(1, 0, 1)$ mode: with $(1, 0, n_z)$. The equations effectively become equivalent to those of a one-dimensional *massive* scalar field in a cavity with Dirichlet BCs [6], where the effective mass acts as a damping term. Thus, although there are some limitations with the code (larger the cutoff m_{max} the slower the code), these results at the very least give an upper bound on the number of photons produced for 1000 ps.

The results are presented in Figure 1 and show that the largest amount of photon production occurs for the SCD placed at the midpoint (at least as compared to the case $d/L_z = 1/3$ for 1000 ps). Also our numerics show that assuming single mode coupling leads to an over-prediction in the number of photons produced, which is simply because we must include the damping terms coming from the effective scalar field mass (though there are cases where the effective damping is negligible, see [6]).

We are now currently writing code in FORTRAN to deal with the limitations of the integration of equation (7) over time, the cutoff m_{max} (which must be increased as we go to larger times) and the fact that the values for the g^A_{mn} and g^B_{mn} also require numerical integration in general. However, although the results presented here have their limitations, if the results are converging then we should be able to partially extraploate themto larger times.

[2] The method of detection relies on a Rydberg atom beam which can detect individual photons of single frequency [9].

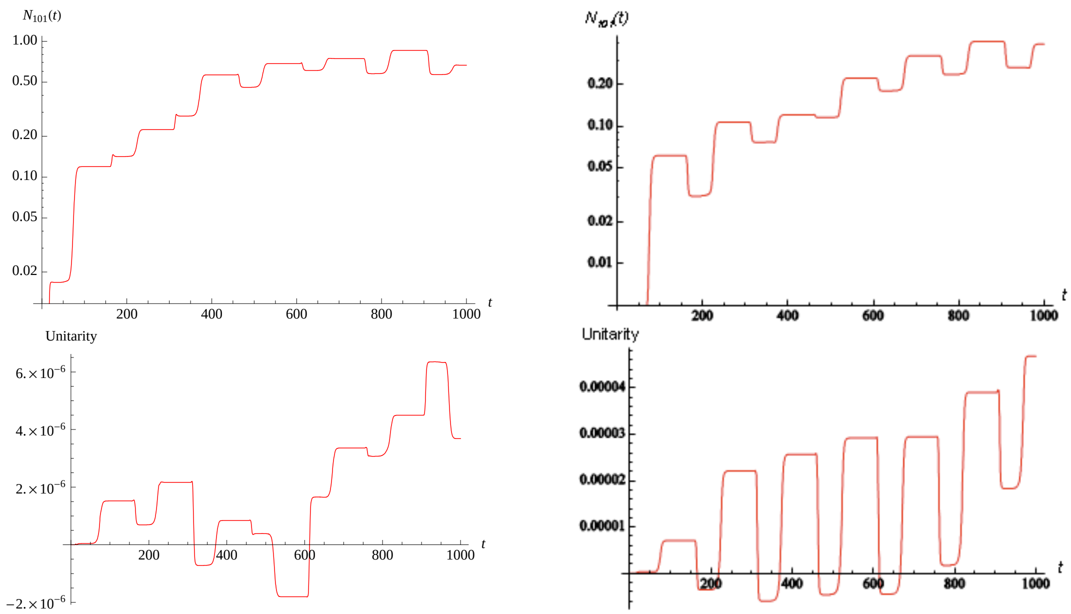

Figure 1: Top: the number of photons produced in the $(1, 0, 1)$ fundamental mode against time. Bottom: the unitarity constraint $\sum_n (|\alpha_{mn}|^2 - |\beta_{mn}|^2) = 1$. Left & right panels are for the SCD at the midpoint and $d/L_z = 1/3$ respectively.

References

[1] E. Yablonovitch, Phys. Rev. Lett. **62**, 1742 (1989).

[2] A. V. Dodonov and V. V. Dodonov, J. Opt. B: Quantum Semiclass. Opt. **7**, S47 (2005).

[3] C. Braggio, G. Bressi, G. Carugno, C.D. Noce, G. Galeazzi, A. Lombardi, A. Palmieri, G. Ruoso, and D. Zanello, Europhys. Lett. **70**, 754 (2005).

[4] G. Barton and A. Calogeracos, Anals of Phys. **238**, 227 (1995).

[5] M. Crocce, D.A.R. Dalvit, F.C. Lombardo, and F.D. Mazzitelli, Phys. Rev. A 70, 033811 (2004).

[6] M. Ruser, Phys. Rev. A **73**, 043811 (2006) [arXiv:quant-ph/0509030].

[7] V. V. Dodonov and A. V. Dodonov, J. Phys. A: Math. Gen. **39**, 6271 (2006); *ibid.* S749.

[8] V. V. Dodonov, J. Opt. B: Quantum Semiclass. Opt. **7**, S445 (2005) .

[9] S. Matsuki, H. Hatano, Y. Kido, T. Nishimura, W. Naylor, and K. Yamamoto, submitted to Europhys. Lett.

[10] M. Crocce, D.A.R. Dalvit, F.C. Lombardo, and F.D. Mazzitelli, J. Opt. B: Quantum Semiclass. Opt. **7**, S32 (2006).

[11] C. K. Law, Phys. Rev. A **51**, 2537 (1995).

[12] N. D. Birrell and P. C. W. Davies, Quantum Fields In Curved Space (Cambridge University Press, UK, 1982).