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BLOOM-GILMAN DUALITY IN RENORMALIZABLE FIELD THEORY*

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ABSTRACT

We study the validity of Bloom-Gilman duality in the renormalizable field theory. It is found that, in the pseudoscalar theory and neutral vector-gluon theory, Bloom-Gilman duality is in general violated. We also discuss whether the leading logarithmic approximation is the cause of the breakdown of Bloom-Gilman duality.

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I. INTRODUCTION

A great deal of attention has recently been devoted to the idea of the asymptotic freedom.[1,2] It has been found that the non-abelian gauge theories have free-field-theory asymptotic behavior,[1,2] up to calculable logarithmic corrections. Gross and Wilczek[1] suggest that the non-abelian gauge theories may provide the explanation for Bjorken scaling.

Although, in a strict sense, Bjorken scaling is violated in asymptotically free-field theories, it is violated in a well-defined and calculable manner.[1,2] One way of checking the relevance of non-abelian gauge theories to the strong interactions is to look for the deviations from the exact Bjorken scaling in the deep-inelastic lepton-hadron scattering and to see if they agree with the theoretical predictions.

The moments[3] of the deep-inelastic structure functions have played important role in the discussion of the possible breakdown of Bjorken scaling. By inverting the moment equations, one can obtain useful information on the structure functions. These include the behavior of the structure function for $1 \leq \omega \leq 4$,[4] the asymptotic behavior of the nucleon form factors,[5,6] and the possible non-Regge behavior of electroproduction structure function.[7] De Rújula,[5] Gross and Treiman[6] have used the local duality of Bloom and Gilman[8] together with the large n behavior of the anomalous dimensions in asymptotically free-field theories to study the form factors of nucleons. They found that the nucleon form factors have the asymptotic form $(Q^2)^{-2G\ell n\, \ell n Q^2}$, which is otherwise very difficult to obtain from the other approaches. The breakdown effect is more prominent in the form factors than in the structure functions; therefore, it is much easier to get at the breakdown of Bjorken scaling indirectly through the form factors.

Since both the asymptotic freedom and Bloom-Gilman duality are assumed in the derivation, we have to know how to interpret the possible disagreement between experimental results on the one hand and the theoretical predictions on the other. If discrepancy does occur, we have to decide whether it should be interpreted as the irrelevance of asymptotically free field theories in strong interactions, or as the failure of Bloom-Gilman duality.

The local duality of Bloom and Gilman has been used to relate the threshold behavior of the structure functions to the elastic form factors of nucleons (Drell-Yan relation).[9] The Drell-Yan relation was first derived in the parton model which also predicts exact Bjorken scaling. Both Bjorken scaling and Drell-Yan relation are satisfied in the SLAC-MIT experiments.[10]

In view of the possible breakdown of Bjorken scaling at higher energy, it is very natural to ask whether Bloom-Gilman duality is satisfied in the context of renormalizable field theories.

In this paper we want to examine how well the local duality of Bloom and Gilman is satisfied in renormalizable field theories. Our approach is to find models in which both the structure function F_2 around $\omega = 1$ and the asymptotic behavior of the form factor can be calculated explicitly. We then use the results obtained to check the validity of Bloom-Gilman duality as well as the positivity constraint,[11] which is described in the next section. We consider as our examples the pseudoscalar field theory and the neutral vector-gluon theory. It is found that, in general, the local duality of Bloom and Gilman is not satisfied. We shall not attempt to study Bloom-Gilman duality in the context of asymptotically free field theories in this paper. Nevertheless, our conclusions in the pseudoscalar and the neutral vector-gluon theories may serve as a warning sign that the use of Bloom-Gilman duality may not be justified.

In Section II, we first review very briefly the moments of the structure functions in electroproduction, the local duality of Bloom and Gilman and the positivity constraint on the form factor. We then study the local duality in the pseudoscalar field theory. This problem is examined under several approximations. The first one is the approximation of Chang and Fishbane.[12] The second one is the leading logarithmic approximation of Gribov and Lipatov.[13] We also discuss the case that the pseudoscalar theory has an ultraviolet (UV) stable fixed point. We find that only in the approximation of Chang and Fishbane is Bloom-Gilman duality satisfied. Section III contains the discussion of Bloom-Gilman duality in the neutral vector-gluon theory. We use the leading logarithmic calculation of F_2 by Gribov and Lipatov together with the leading logarithmic calculation of form factor by Jackiw.[14] It is found that both Bloom-Gilman duality and the positivity constraint on form factor are violated. In the last section we summarize our conclusions and make some pertinent remarks about the breakdown of Bloom-Gilman duality.

II. PSEUDOSCALAR THEORY

In this section, we discuss Bloom-Gilman duality in the pseudoscalar field theory which contains the proton field and the neutral pion field. It is instructive to review very briefly here the moment equations of the structure function F_2, Bloom-Gilman duality and the positivity constraint on the form factor.

We recall that the moments of the structure function F_2 can be written as[3]

$$\int_1^\infty d\omega \; F_2(\omega, Q^2)\, \omega^{-n-2} = \sum_i c_n^{(i)} f_n^{(i)}(Q^2), \tag{2.1}$$

where $Q^2 \equiv -q^2$, $c_n^{(i)}$'s depend on the target and the functions $f_n^{(i)}(Q^2)$ depend only on the short-distance properties of the theory. In particular, if the theory has a UV-stable fixed point, the functions $f_n^{(i)}(Q^2)$ are given by

$$f_n^{(i)}(Q^2) = (Q^2)^{-A_n^{(i)}}, \text{ when } Q^2 \text{ is large,} \tag{2.2}$$

where $A_n^{(i)}$'s are the anomalous dimensions[15] of the spin n+2 operators in the Wilson's operator expansion[15] of the product of currents. For the asymptotically free field theories, the corresponding expressions are

$$f_n^{(i)}(Q^2) = (\ell n Q^2)^{-A_n^{(i)}} \tag{2.2'}$$

Conversely, the structure function F_2 in the asymptotic region can be expressed as a contour integral in the complex n-plane:

$$\begin{aligned} F_2(\omega, Q^2) &= \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} dn\, \omega^{n+1} \sum_i c_n^{(i)} f_n^{(i)}(Q^2) \\ &= \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} dn\, e^{(n+1)\ell n\, \omega} \sum_i c_n^{(i)} f_n^{(i)}(Q^2) \end{aligned} \tag{2.3}$$

It is easy to see that the behavior of the structure function F_2 around $\omega = 1$ is determined by the large n behaviors of $c_n^{(i)}$ and $f_n^{(i)}(Q^2)$, or equivalently the large n behaviors of $c_n^{(i)}$ and the anomalous dimensions $A_n^{(i)}$'s.

The local duality of Bloom and Gilman is the statement that

$$\int_1^{1 + \frac{S_o}{Q^2}} d\omega\, F_2(\omega, Q^2) = G^2(Q^2), \tag{2.4}$$

where

$$G^2(Q^2) \equiv \frac{[G_E(Q^2)]^2 + \frac{Q^2}{4m^2}[G_M(Q^2)]^2}{1 + \frac{Q^2}{4m^2}} \tag{2.5}$$

Since the structure function F_2 is positive definite, one expects the integral

$$\int_1^{1+\frac{S_o}{Q^2}} d\omega\, F_2(\omega, Q^2)$$

to be greater than the contribution coming from the nucleon, which is precisely $G^2(Q^2)$. Quantitatively, the positivity constraint on $G^2(Q^2)$ is expressed as

$$\int_1^{1+\frac{S_o}{Q^2}} d\omega\, F_2(\omega, Q^2) > G^2(Q^2) \tag{2.6}$$

It is the aim of this and the next sections to examine whether Eqs. (2.4) and (2.6) are satisfied.

We shall now concentrate on the pseudoscalar field theory in the remaining part of this section.

A. Chang-Fishbane Approximation

First we consider the approximation scheme of Chang and Fishbane.[12] Their result of summing leading logarithmic contributions in the outer rainbow graphs can be expressed in terms of moments:

$$\int_1^{\infty} d\omega\, \omega^{-n-2}\, F_2(\omega, Q^2) = (Q^2)^{\frac{g^2}{16\pi^2 (n+2)(n+3)}} \tag{2.7}$$

The result of Chang and Fishbane can also be reformulated in terms of the Callan-Symanzik equation. Because there are no self-energy or vertex corrections included in the outer rainbow graphs, the functions β, β', γ_1, and γ_2 in the Callan-Symanzik equation are all zero:

$$\beta = \beta' = \gamma_1 = \gamma_2 = 0 \tag{2.8}$$

where

$$\beta = (m \frac{d}{dm} + \mu \frac{d}{d\mu}) g$$

$$\beta' = (m \frac{d}{dm} + \mu \frac{d}{d\mu}) \lambda$$

$$2\gamma_1 = Z_2^{-1} (m \frac{d}{dm} + \mu \frac{d}{d\mu}) Z_2$$

$$2\gamma_2 = Z_3^{-1} (m \frac{d}{dm} + \mu \frac{d}{d\mu}) Z_3 \qquad (2.9)$$

It is understood that, in taking derivatives in Eq. (2.9), the unrenormalized coupling constants g_o, λ_o and cutoff Λ are held fixed.

According to our discussion before, the behavior of the structure function F_2 around $\omega = 1$, for the asymptotic Q^2, is

$$F_2(\omega, Q^2) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} dn\ \omega^{n+1}\ (Q^2)^{\frac{g^2}{16\pi^2 (n+2)(n+3)}}$$

$$= \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} dn\ \omega^{n+1} \left\{ 1 + \frac{g^2}{16\pi^2 (n+2)(n+3)} \ln Q^2 + \ldots \right\}$$

$$= \delta(\omega - 1) + O(\omega - 1) \ln Q^2 + \ldots \qquad , \qquad (2.10)$$

where dots denote less important terms.

Shei[17] has recently shown that the asymptotic behavior of the on-mass-shell form factor in the pseudoscalar theory is governed by the homogeneous Callan-Symanzik equation

$$(m \frac{\partial}{\partial m} + \mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} + \beta' \frac{\partial}{\partial \lambda} - 2\gamma_1) G(\text{asymptotic}) = 0 \qquad (2.11)$$

Under the same approximation as in Chang-Fishbane calculation:

$$\beta = \beta' = \gamma_1 = \gamma_2 = 0 \qquad ,$$

we obtain immediately the asymptotic expression for $G(Q^2)$:

$$G(Q^2) = 1, \quad \text{for asymptotic } Q^2. \tag{2.12}$$

It is now a simple matter to see that, indeed, Bloom-Gilman duality is satisfied.

B. Gribov-Lipatov Approximation

Next, we consider the Gribov-Lipatov calculation[13] of the structure function F_2. They sum up all leading logarithmic terms in the pseudoscalar field theory. In terms of the moments, their result is

$$\int_1^\infty d\omega\, \omega^{-n-2}\, F_2(\omega, Q^2) = c_n\, e^{\nu_n \xi} + c'_n\, e^{\nu'_n \xi} \quad , \tag{2.13}$$

where

$$\xi \equiv -\frac{1}{5}\ln\left[1 - \frac{5g^2}{16\pi^2}\ln Q^2\right]$$

$$\nu_n = -\frac{5}{4} + \frac{1}{2(n+2)(n+3)} + \left[\left(\frac{3}{4} + \frac{1}{2(n+2)(n+3)}\right)^2 + \frac{4}{(n+2)(n+3)}\right]^{\frac{1}{2}}$$

$$\nu'_n = -\frac{5}{4} + \frac{1}{2(n+2)(n+3)} - \left[\left(\frac{3}{4} + \frac{1}{2(n+2)(n+3)}\right)^2 + \frac{4}{(n+2)(n+3)}\right]^{\frac{1}{2}}$$

$$c_n = \frac{1}{\nu_n - \nu'_n}\left[\frac{1}{(n+2)(n+3)} - \frac{1}{2} - \nu'_n\right]$$

$$c'_n = \frac{1}{\nu'_n - \nu_n}\left[\frac{1}{(n+2)(n+3)} - \frac{1}{2} - \nu_n\right] \quad . \tag{2.14}$$

One finds that, in the limit of $n \to \infty$,

$$\nu_n \longrightarrow -\frac{1}{2} + O\left(\frac{1}{n^2}\right)$$

$$\nu'_n \longrightarrow -2 + O\left(\frac{1}{n^2}\right)$$

$$c_n \longrightarrow 1 + O\left(\frac{1}{n^2}\right)$$

$$c'_n \longrightarrow O\left(\frac{1}{n^2}\right) \quad .$$

The structure function F_2 around $\omega = 1$ is, therefore, given by

$$F_2(\omega, Q^2) = \frac{1}{2\pi i}\int dn\ \omega^{n+1} \exp\left[\frac{1}{10}\ln\left(1 - \frac{5g^2}{16\pi^2}\ln Q^2\right)\right]$$

$$+ \text{ less important terms}$$

$$= \left(1 - \frac{5g^2}{16\pi^2}\ln Q^2\right)^{\frac{1}{10}} \delta(\omega - 1) \quad + \text{ less important terms} \tag{2.15}$$

The on-mass-shell form factor $G(Q^2)$ can also be calculated in the leading logarithmic approximation. Appelquist and Primack[18] obtain

$$G(Q^2) = \left(1 - \frac{5g^2}{16\pi^2}\ln Q^2\right)^{\frac{1}{10}} \tag{2.16}$$

It is obvious that Bloom-Gilman duality is violated, i.e.,

$$\int_1^{1+\frac{S_o}{Q^2}} d\omega\ F_2(\omega, Q^2) \neq G^2(Q^2)$$

Nevertheless, the positivity constraint (Eq. (2.6)) is still valid.

C. Pseudoscalar Field Theory with a UV Fixed Point

As our last case, we consider the possibility that the pseudoscalar field theory has a UV-stable fixed point. In this case, two leading spin n+2 operators appear in the Wilson's operator expansion of the product of currents. They are

$$O^{(1)}_{\mu_1 \cdots \mu_{n+2}} = \frac{i^{n+2}}{(n+2)2^{n+2}}\left\{\bar{\psi}\gamma_{\mu_1}\overleftrightarrow{\partial}_{\mu_2}\cdots\overleftrightarrow{\partial}_{\mu_{n+2}}\psi\right.$$

$$\left. + \text{ permutations } - \text{ trace terms}\right\}$$

$$O^{(2)}_{\mu_1 \cdots \mu_{n+2}} = \frac{i^{n+2}}{2^{n+2}} \left| \varphi \overleftrightarrow{\partial}_{\mu_1} \cdots \overleftrightarrow{\partial}_{\mu_{n+2}} \varphi \; - \; \text{trace terms} \right| \tag{2.17}$$

Callan and Gross[19] have shown that the anomalous dimensions, corresponding to the appropriate linear combinations of $O^{(1)}_{\mu_1 \cdots \mu_{n+2}}$ and $O^{(2)}_{\mu_1 \cdots \mu_{n+2}}$, approach γ_ψ and γ_φ in the large n limit.[20] The moments of the structure function, in the asymptotic Q^2 region, are

$$\int_1^\infty d\omega\, \omega^{-n-2} F_2(\omega, Q^2) = c_n (Q^2)^{-\gamma_n} + c'_n (Q^2)^{-\gamma'_n} \tag{2.18}$$

As usual, the above equation can be inverted to yield

$$F_2(\omega, Q^2) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} dn\, \omega^{n+1} \left[c_n (Q^2)^{-\gamma_n} + c'_n (Q^2)^{-\gamma'_n} \right] \tag{2.19}$$

Following the analysis of Callan and Gross,[19] we find

$$c_n = 1 + O\left(\frac{1}{n}\right)$$

$$c'_n = O\left(\frac{1}{n}\right)$$

$$\gamma_n = \gamma_\psi + O\left(\frac{1}{n}\right)$$

$$\gamma'_n = \gamma_\varphi + O\left(\frac{1}{n}\right), \text{ in the limit of } n \to \infty. \tag{2.20}$$

The structure function around $\omega = 1$ is, therefore,

$$F_2(\omega, Q^2) = c(Q^2)^{-\gamma_\psi} \delta(\omega - 1) + c'(Q^2)^{-\gamma_\varphi} O(1) \tag{2.21}$$

Thus,

$$\int_1^{1 + \frac{S_o}{Q^2}} d\omega\, F_2(\omega, Q^2) = c(Q^2)^{-\gamma_\psi} + c'(Q^2)^{-\gamma_\varphi} O\left(\frac{S_o}{Q^2}\right) \tag{2.22}$$

Shei has shown that the asymptotic behavior of the on-mass-shell form factor $G(Q^2)$ is determined by the homogeneous Callan-Symanzik equation. His result is[17]

$$G(Q^2) = c''(Q^2)^{-\gamma_\psi} \tag{2.23}$$

Suppose the following inequality is true

$$c'(Q^2)^{-\gamma_\varphi} O\left(\frac{S_o}{Q^2}\right) < c(Q^2)^{-\gamma_\psi} \tag{2.24}$$

in the asymptotic region, then

$$c'(Q^2)^{-\gamma_\varphi} O\left(\frac{S_o}{Q^2}\right)$$

can be neglected in Eq. (2.22). One finds that

$$\int_1^{1+\frac{S_o}{Q^2}} d\omega\ F_2(\omega, Q^2) = c(Q^2)^{-\gamma_\psi} \tag{2.25}$$

and

$$G^2(Q^2) = c''(Q^2)^{-2\gamma_\psi} \tag{2.26}$$

Since the anomalous dimension of fermion $\gamma_\psi > 0$ by Kӓllen-Lehmann representation, one concludes that

$$\int_1^{1+\frac{S_o}{Q^2}} d\omega\ F_2(\omega; Q^2) > G^2(Q^2) \tag{2.27}$$

If, on the other hand, the following relation is true

$$c'(Q^2)^{-\gamma_\varphi} O\left(\frac{S_o}{Q^2}\right) > c(Q^2)^{-\gamma_\psi} \tag{2.28}$$

this implies that

$$\int_1^{1+\frac{S_o}{Q^2}} d\omega\ F_2(\omega, Q^2) > c(Q^2)^{-\gamma_\psi} > c''(Q^2)^{-2\gamma_\psi} \tag{2.29}$$

Combining these two possibilities, we obtain

$$\int_1^{1+\frac{S_o}{Q^2}} d\omega\ F_2(\omega, Q^2) > G^2(Q^2) \tag{2.30}$$

We therefore conclude that in this case too the Bloom-Gilman duality is violated. The positivity constraint is still all right.

III. NEUTRAL VECTOR-GLUON THEORY

In this section Bloom-Gilman duality will be examined in the neutral vector-gluon theory. We use here the leading logarithmic calculation of the structure function F_2 by Gribov and Lipatov.[13] Their result is

$$F_2(\omega, Q^2) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} dn\ \omega^{n+1} \left\{ \frac{e^{\nu_n \xi}}{1 + \frac{\psi_2(n+2)}{(\nu_n - \frac{4}{3})^2}} + \frac{e^{\nu'_n \xi}}{1 + \frac{\psi_2(n+2)}{(\nu'_n - \frac{4}{3})^2}} \right\} \tag{3.1}$$

where

$$\xi \equiv -\frac{3}{4} \ln \left[1 - \frac{g^2}{12\pi^2} \ln Q^2 \right]$$

$$2\nu_n = 3 + \psi_1(n+2) + \psi_3(n+2) - \left[(\psi_1(n+2) + \psi_3(n+2) + \tfrac{1}{3})^2 + 4\psi_2 \right]^{\frac{1}{2}}$$

$$2\nu'_n = 3 + \psi_1(n+2) + \psi_3(n+2) + \left[(\psi_1(n+2) + \psi_3(n+2) + \tfrac{1}{3})^2 + 4\psi_2 \right]^{\frac{1}{2}}$$

and

$$\psi_1(j) = \frac{2}{j(j+1)}$$

$$\psi_2(j) = \frac{8(j^2+j+2)^2}{(j-1)j^2(j+1)^2(j+2)}$$

$$\psi_3(j) = -4 \sum_{\ell=2}^{j} \frac{1}{\ell} \tag{3.2}$$

In the limit of large n, one can approximate the expressions inside the curly brackets in Eq. (3.1) by

$$e^{\nu_n \xi} + \frac{2}{(n+2)^2 \ell n^2(n+2)} e^{\nu'_n \xi}$$

and

$$\nu_n \text{ by } -4 \ell n(n+2) + 4(1-c_E) + \frac{5}{3}$$

$$\nu'_n \text{ by } \frac{4}{3}$$

where c_E is the Euler constant.

The behavior of F_2 around $\omega = 1$ is, therefore,

$$F_2(\omega, Q^2) = \frac{1}{2\pi i}\int dn\, \omega^{n+1} \left\{ e^{\xi\left[\frac{5}{3} + 4(1-c_E) - 4\ell n(n+2)\right]} + \frac{2\, e^{\frac{4}{3}\xi}}{(n+2)^2 \ell n^2(n+2)} + \ldots \right\}$$

$$= e^{\left[\frac{5}{4} + 4(1-c_E)\right]\xi} \frac{(\ell n\, \omega)^{4\xi-1}}{\omega\Gamma(4\xi)} + e^{\frac{4}{3}\xi} O(\omega-1) + \ldots \tag{3.3}$$

where dots denote less important terms.

Consider the following region of Q^2:

$$\ell n\, Q^2 >> 1 \quad , \quad g^2 \ell n\, Q^2 << 1 \quad ,$$

so that

$$4\xi = -3\,\ln\left[1 - \frac{g^2}{12\pi^2}\,\ln Q^2\right] << 1$$

In this region, one finds that

$$\int_1^{1+\frac{S_o}{Q^2}} d\omega\, F_2(\omega, Q^2) \approx \exp\left\{-3\left(\frac{-g^2}{12\pi^2}\,\ln Q^2\right)(-\ln Q^2)\right\}$$

$$= \exp\left\{-\frac{g^2}{4\pi^2}\,(\ln Q^2)^2\right\} \qquad (3.4)$$

The on-mass-shell form factor has also been calculated in the leading logarithmic approximation.[14] The asymptotic behavior is

$$G(Q^2) = \exp\left\{-\frac{g^2}{16\pi^2}\,(\ln Q^2)^2\right\} \qquad (3.5)$$

when

$$g^2(\ln Q^2) \sim 1, \quad g^2 \ln Q^2 << 1.$$

Comparing Eqs. (3.4) and (3.5), one obtains immediately

$$\int_1^{1+\frac{S_o}{Q^2}} d\omega\, F_2(\omega, Q^2) \neq G^2(Q^2)$$

Namely, the local duality of Bloom and Gilman is violated. Furthermore, the positivity constraint (Eq. (2.6)) is violated in the leading logarithmic approximation of the neutral vector-gluon theory.

We have seen, in the last section, several examples where Bloom-Gilman duality is violated. In the neutral vector-gluon theory we have the most serious trouble. The positivity constraint, which depends only on general physical principles, is violated. In the next section, we will return to this problem and discuss ways to resolve the puzzle.

IV. SUMMARY AND DISCUSSION

We have discussed, in the previous sections, the validity of Bloom-Gilman duality by comparing the threshold behavior of F_2 and the form factor. We found that in the pseudoscalar field theory and the neutral vector-gluon theory Bloom-Gilman duality is violated (except in the simplest example - A in Sec. II).

In the examples A and B in Sec. II, and in Sec. III, we have employed the leading logarithmic approximation. One might wonder whether the breakdown of Bloom-Gilman duality is due completely to the use of this approximation. It is interesting to see how the conclusions are changed if the higher order expression of the anomalous dimensions γ_n is used. We also would like to understand a more serious puzzle: why the positivity constraint is violated in the neutral vector-gluon theory.

First, let us concentrate on the pseudoscalar field theory. According to the analysis of Callan and Gross,[19] the difference of γ_n and γ_ψ is

$$\gamma_n - \gamma_\psi = O(\frac{1}{n}), \text{ when } n \to \infty,$$

independent of the orders in the perturbation theory. Therefore, even if the g^4 or higher order terms are kept in γ_n, the conclusions that we reached in Sec. II remain the same. Namely, there is no way to rescue Bloom-Gilman duality in the pseudoscalar theory by blaming the trouble on the non-uniformity of the limits $g \to 0$ and $n \to \infty$.

Next, we turn to the neutral vector-gluon theory. The expressions ν_n, ν'_n

in Eqs. (3.1) and (3.2) are related to the anomalous dimensions of the following spin n+2 operators:

$$O^{(1)}_{\mu_1\cdots\mu_{n+2}} = \sum_{r=2}^{n} \bar{\psi}\gamma_{\mu_1}\left(\overleftarrow{\partial}_{\mu_2} - igA_{\mu_2}\right)\cdots\left(\overleftarrow{\partial}_{\mu_r} - igA_{\mu_r}\right)$$

$$\left(\overrightarrow{\partial}_{\mu_{r+1}} - igA_{\mu_{r+1}}\right)\cdots\left(\overrightarrow{\partial}_{\mu_{n+2}} - igA_{\mu_{n+2}}\right)\psi ,$$

$$O^{(2)}_{\mu_1\cdots\mu_{n+2}} = F_{\mu_1\alpha}\overleftrightarrow{\partial}_{\mu_2}\cdots\overleftrightarrow{\partial}_{\mu_{n+1}}F^{\alpha}_{\mu_{n+2}}$$

The anomalous dimensions of the two linear combinations of $O^{(1)}$ and $O^{(2)}$ are:

$$\gamma_n \equiv \frac{g^2}{16\pi^2}\nu_n$$

and

$$\gamma'_n \equiv \frac{g^2}{16\pi^2}\nu'_n$$

The lowest order expression for ν_n and ν'_n is given in Eq. (3.2).

If the higher order terms in perturbation theory are kept, it is no longer true that

$$\nu_n \longrightarrow -4\,\ln(n+2) + 4\,(1-c_E) + \frac{5}{3}$$

and

$$\nu'_n \longrightarrow \frac{4}{3} .$$

There is no guarantee that the g^4 term in γ_n will not dominate over g^2 term in the large n limit. Indeed, there are indications[21] that

$$\gamma_n \longrightarrow -\frac{g^2}{4\pi^2}\ln(n+2) + dg^4\,\ln^2(n+2) + \ldots$$

Thus, in the case of neutral vector-gluon theory, we may blame the trouble on the non-uniformity of the limits $g \to 0$ and $n \to \infty$. This is the reason why the positivity constraint is violated in the neutral vector-gluon theory.

It is certainly true that in the neutral vector-gluon theory the uniformity assumption is not justified. However, in the pseudoscalar theory, where the uniformity condition is justified by Callan-Gross analysis, we find that Bloom-Gilman duality is still generally violated.

Therefore, we conclude that Bloom-Gilman duality is not valid in the field theory in general. Furthermore, the deviation from

$$\int_1^{1+\frac{S_o}{Q^2}} d\omega \, F_2(\omega, Q^2) = G^2(Q^2)$$

is not due solely to the non-uniformity of the limits $g \to 0$ and $n \to \infty$.

We have not touched upon Bloom-Gilman duality directly in the asymptotic free field theories in this paper. The possibility that Bloom-Gilman duality may be valid in the non-abelian gauge theories is not yet ruled out. Nevertheless, our results in the pseudoscalar and neutral vector-gluon theory strongly suggest that the validity of Bloom-Gilman duality can not be taken for granted.

Whether the asymptotic behavior of nucleon form factors is an important test of the non-abelian gauge theory of the strong interactions is still an open question.

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