HIGHER TWIST EFFECTS IN THE UNPOLARIZED PARTON DISTRIBUTIONS AT SMALL x

Alexei Yu. Illarionov[1,2†], Anatoly V. Kotikov[2,3] and Gonzalo Parente[4]

(1) *Dipartimento di Fisica "Enrico Fermi", Università di Pisa, and INFN, Sezione di Pisa, I-56100 Pisa, Italy*
(2) *Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia*
(3) *Institut für Theoretische Teilchenphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany*
(4) *Departamento de Física de Partículas, Universidade de Santiago de Compostela, E-15706 Santiago de Compostela, Spain*
† *E-mail: Alexei.Illarionov@pi.infn.it*

Abstract

Higher twist corrections to F_2 at small x are studied for the case of a flat initial condition for the twist-two QCD evolution at next-to-leading order approximation. We present an analytical parameterization of the contributions from the twist-two and higher twist operators of the Wilson operator product expansion. Higher twist terms are evaluated using two different approaches, one motivated by BFKL and the other motivated by the renormalon formalism. The results of the latter approach are in very good agreement with deep inelastic scattering data from HERA.

For more than a decade the various models of the behavior of quarks and gluons at small x has been confronted by a large amount of experimental data from HERA on the deep-inelastic scattering (DIS) structure function (SF) F_2 [8, 2]. In the small x regime, non-perturbative effects are expected to give a substantial contribution to F_2. However, what is observed up to very low $Q^2 \sim 1$ GeV^2 values, traditionally explained by soft processes, is described reasonably by perturbative QCD evolution (see for example [3]). Thus, it is important to find the kinematical region where the well-established perturbative QCD formalism can be safely applied.

At small x the Q^2 dependence of quarks and gluons is usually obtained from the numerical solution of the DGLAP equations [4]. The x profile of partons at some initial Q_0^2 and the QCD energy scale Λ are determined from a fit to experimental data.

On the other hand, when analyzing exclusively the small x region, a much simpler analysis can be done by using some of the existing analytical approaches of DGLAP equations in the small x limit. In Refs. [5, 6, 7] it was pointed out that HERA small x data can be interpreted in terms of the so called doubled asymptotic scaling (DAS) phenomenon related to the asymptotic behavior of the DGLAP evolution discovered many years ago.

In the present talk we report the new results of [8] (reffered to as I hereafter) about the incorporation of the contribution from higher twist (HT) operators of the Wilson operator product expansion to our previous analysis [7] (reffered to as II hereafter). The semianalytical solution of DGLAP equations obtained in Ref. II using a flat initial condition, is the next-to-leading order (NLO) extension of previous studies performed at the leading order (LO) in perturbative QCD [5, 6]. The flat initial conditions correspond to the case when parton distributions tend to some constant value at $x \to 0$ and at some initial value Q_0^2.

In Ref. II, both the gluon and quark singlet densities are presented in terms of the diagonal '+' and '−' components obtained from the DGLAP equations in the Mellin moment space. The '−' components are constants at small x for any values of Q^2, whereas the '+' components grow for $Q^2 \geq Q_0^2$ as

$$\sim \exp\left(2\sqrt{\left[a_+ \ln\left(\frac{a_s(Q_0^2)}{a_s(Q^2)}\right) - \left(b_+ + a_+\frac{\beta_1}{\beta_0}\right)\left(a_s(Q_0^2) - a_s(Q^2)\right)\right]\ln\left(\frac{1}{x}\right)}\right), \quad (1)$$

where $a_+ = 4C_A/\beta_0$ and $b_+ = 8[23C_A - 26C_F]T_Rf/(9\beta_0)$. Hereafter we use the notation $a_s = \alpha_s/(4\pi)$.

The first two coefficients of the QCD β-function in the $\overline{\text{MS}}$-scheme are $\beta_0 = (11/3)C_A - (4/3)T_Rf$ and $\beta_1 = (2/3)[17C_A^2 - 10C_AT_Rf - 6C_FT_Rf]$ where f is the number of active flavors. This new presentation as a function of the $SU(N)$ group casimirs, with f active flavors, $C_A = N$, $T_R = 1/2$, $T_F = T_Rf$ and $C_F = (N^2-1)/(2N)$ permits one to apply our results to, for example, the popular $N = 1$ supersymmetric model. Of course, for $N = 3$ one obtains the QCD result II.

The analysis performed in our previous work (see II) has shown very good agreement with HERA 1994 data [8, 2] at $Q^2 \geq 1.5$ GeV^2. In Ref. I we added the contribution from higher twist operators with the hope to describe also more modern 1996/97 data at lower Q^2.

1. The basic results are the twist-four and twist-six corrections for the SF F_2

$$F_2(x,Q^2) = F_2^{\tau 2}(x,Q^2) + \frac{1}{Q^2}F_2^{\tau 4}(x,Q^2) + \frac{1}{Q^4}F_2^{\tau 6}(x,Q^2), \quad (2)$$

where for the higher twist parts $F_2^{\tau 4,6}$ BFKL-motivated evaluations [9] (in the case only the twist-four correction has been estimated) and the calculations [10] in the framework of the renormalon model (hereafter the results are marked like $F_2^{R\tau 4,6}$) have been used.

The latter case is essentially more complete and the predicted HT corrections can be expressed through the twist-two ones as follows

$$F_2^{R\tau 4}(x,Q^2) = e \sum_{a=q,G} a_a^{\tau 4}\, \widetilde{\mu}_a^{\tau 4}(x,Q^2) \otimes f_a^{\tau 2}(x,Q^2) = \sum_{a=q,G} F_{2,a}^{R\tau 4}(x,Q^2), \quad (3)$$

where the symbol $\otimes$ marks the Mellin convolution, the functions $\widetilde{\mu}_a^{\tau 4}(x,Q^2)$ are given in [10] and $e = (\sum_1^f e_i^2)/f$ is the average charge square for f active quarks. We mark the parts of HT corrections proportional to the twist-two quark and gluon densities as $F_{2,q}^{R\tau 4}$ and $F_{2,G}^{R\tau 4}$, respectively.

Note that the parton distributions $f_a^{\tau 2}(x,Q^2)$ are multiplied on x, i.e., $f_q^{\tau 2}(x,Q^2) = x\,q(x,Q^2)$ and $f_G^{\tau 2}(x,Q^2) = x\,G(x,Q^2)$. Note also that we neglect the nonsinglet quark density $f_\Delta(x,Q^2)$ and the valent part $f_V(x,Q^2)$ of the singlet quark distributions.

At the leading twist part we have (see II) at the LO and LO&NLO approximations, respectively,

$$F_{2,\mathrm{LO}}^{\tau2}(x,Q^2) \;=\; e\, f_{q,\mathrm{LO}}^{\tau2}(x,Q^2)\;, \tag{4a}$$

$$F_2^{\tau2}(x,Q^2) \;=\; e\,\left(f_q^{\tau2}(x,Q^2) \;+\; \frac{4T_Rf}{3}\, a_s(Q^2)\, f_G^{\tau2}(x,Q^2)\right)\;. \tag{4b}$$

Let us to keep the LO&NLO relation (4b) beyond the leading twist approximation. Then for the total SF F_2 (2) we obtain (see I)

$$F_2(x,Q^2) \;=\; e\,\left(f_q(x,Q^2) \;+\; \frac{4T_Rf}{3}\, a_s(Q^2)\, f_G(x,Q^2)\right)\;, \tag{5}$$

where $f_a(x,Q^2)$ are the total parton distributions containing both the twist-two part, presented in II, and the twist-four and twist-six contributions,

$$f_a(x,Q^2) \;=\; f_a^{\tau2}(x,Q^2) \;+\; \frac{1}{Q^2}\, f_a^{R\tau4}(x,Q^2) \;+\; \frac{1}{Q^4}\, f_a^{R\tau6}(x,Q^2)\;. \tag{6}$$

For the HT part $f_a^{R\tau4,6}(x,Q^2)$ calculations in the framework of the renormalon model have been used [1].

We would like to note that the each HT term $f_a^{R\tau4,6}(x,Q^2)$ can be chosen in quite arbitrary form and only the combination

$$f_q^{R\tau4,6}(x,Q^2) \;+\; \frac{4T_Rf}{3}\, a_s(Q^2)\, f_G^{R\tau4,6}(x,Q^2) \tag{7}$$

is uniqueness, because we keeped the originally twist-two relation (4b) to be same in the case when HT corrections are incorporated (see Eq. (5)).

In the Ref. I we study the x and Q^2 dependences of the structures F_2, $\partial F_2/\partial \ln Q^2$ and $\partial \ln F_2/\partial \ln(1/x)$, that needs to define the parton densities in a proper way. So, we take quite *natural* choice

$$f_q^{R\tau4,6}(x,Q^2) \;=\; a_q^{\tau4,6}\, \widetilde{\mu}_q^{\tau4,6}(x,Q^2) \otimes f_q^{\tau2}(x,Q^2) \;\equiv\; \frac{1}{e}\, F_{2,q}^{R\tau4,6}(x,Q^2)\;, \tag{8a}$$

$$f_G^{R\tau4,6}(x,Q^2) \;=\; \frac{3/4T_Rf}{a_s(Q^2)}\, a_G^{\tau4,6}\, \widetilde{\mu}_G^{\tau4,6}(x,Q^2) \otimes f_G^{\tau2}(x,Q^2) \;\equiv\; \frac{3/4T_Rf}{e a_s(Q^2)}\, F_{2,G}^{R\tau4,6}(x,Q^2)\;, \tag{8b}$$

i.e., the HT quark (gluon) part of F_2 relates only to the corresponding quark (gluon) twist-two density. This choice corresponds exactly to the Eq. (5), i.e. to generalization of the standard twist-two relation (4b) between F_2 and parton densities at the LO&NLO approximation with the purpose to include the HT contributions.

Note also that at any choices of parton densities the DGLAP equation will be violated by the HT corrections.

2. As it has been already noted above it is useful to split the parton distributions in two parts

$$f_a(x,Q^2) \;=\; f_a^+(x,Q^2) \;+\; f_a^-(x,Q^2)\;, \tag{9}$$

[1]Note that twist-four corrections are studied in Ref. I in two approaches based on BFKL and DGLAP. However, we give here the results only for the DGLAP approach based on the infrared renormalon model because it contains a more complete calculation and the agreement with experimental data is much better.

where the both '+' and '−' components contain twist-two and HT parts. The two component representation follows directly form the exact solution of DGLAP equation in the Mellin moment space at the leading twist approximation.

The twist-two contribution is presented in the Ref. II and the twist-four and twist-six parts can be expressed through the twist-two one as follows (here for simplicity we restrict our consideration by LO approximation):
for the (singlet) quark distribution

$$\frac{f_q^{R\tau 4,+}(x,Q^2)}{f_{q,\mathrm{LO}}^{\tau 2,+}(x,Q^2)} = \frac{64C_F T_R f}{15\beta_0^2}\, \mathrm{a}_q^{\tau 4} \left\{ \frac{2}{\rho_{\mathrm{LO}}^2} + \ln\left(\frac{Q^2}{|\mathrm{a}_q^{\tau 4}|}\right) \frac{\widetilde{I}_0(\sigma_{\mathrm{LO}})}{\rho_{\mathrm{LO}}\, \widetilde{I}_1(\sigma_{\mathrm{LO}})} \right\} + \mathcal{O}(\rho_{\mathrm{LO}}) , \quad (10a)$$

$$\frac{f_q^{R\tau 4,-}(x,Q^2)}{f_{q,\mathrm{LO}}^{\tau 2,-}(x,Q^2)} = \frac{64C_F T_R f}{15\beta_0^2}\, \mathrm{a}_q^{\tau 4} \left\{ \ln\left(\frac{1}{x_q}\right) \ln\left(\frac{Q^2}{x_q\, |\mathrm{a}_q^{\tau 4}|}\right) - p'(\nu_q) \right\} + \mathcal{O}(z) . \quad (10b)$$

for the gluon distribution

$$\frac{f_G^{R\tau 4,+}(x,Q^2)}{f_{G,\mathrm{LO}}^{\tau 2,+}(x,Q^2)} = \frac{8}{5\beta_0^2}\, \frac{\mathrm{a}_G^{\tau 4}}{a_s(Q^2)} \left\{ \frac{2}{\rho_{\mathrm{LO}}} \frac{\widetilde{I}_1(\sigma_{\mathrm{LO}})}{\widetilde{I}_0(\sigma_{\mathrm{LO}})} + \ln\left(\frac{Q^2}{|\mathrm{a}_G^{\tau 4}|}\right) \right\} + \mathcal{O}(\rho_{\mathrm{LO}}) , \quad (10c)$$

$$\frac{f_G^{R\tau 4,-}(x,Q^2)}{f_{G,\mathrm{LO}}^{\tau 2,-}(x,Q^2)} = \frac{8}{5\beta_0^2}\, \frac{\mathrm{a}_G^{\tau 4}}{a_s(Q^2)} \ln\left(\frac{Q^2}{x_G^2\, |\mathrm{a}_G^{\tau 4}|}\right) + \mathcal{O}(z) , \quad (10d)$$

where $\mathrm{a}_a^{\tau 4}$ are the magnitudes which should be extracted from the fits of the experimental data. The variables $x_a = x\exp[p(\nu_a)]$, where $p(\nu_a) = [\Psi(1+\nu_a) - \Psi(\nu_a)]$ and ν_a are the powers of the $x \to 1$ asymptotics of the parton distributions, i. e. $f_a \sim (1-x)^{\nu_a}$ at $x \to 1$. From the quark counting rules we know that $\nu_q \approx 3$ and $\nu_G \approx 4$. Then, we get $p(\nu_q) \approx 11/6$ and $p(\nu_G) \approx 25/12$, and there derivatives $p'(\nu_q) \approx -49/36$ and $p'(\nu_G) \approx -205/144$ (see Ref. I for further details).

The functions $\widetilde{I}_\nu$ in Eqs. (10a, 10c) are related to the modified Bessel function I_ν and to the Bessel function J_ν by:

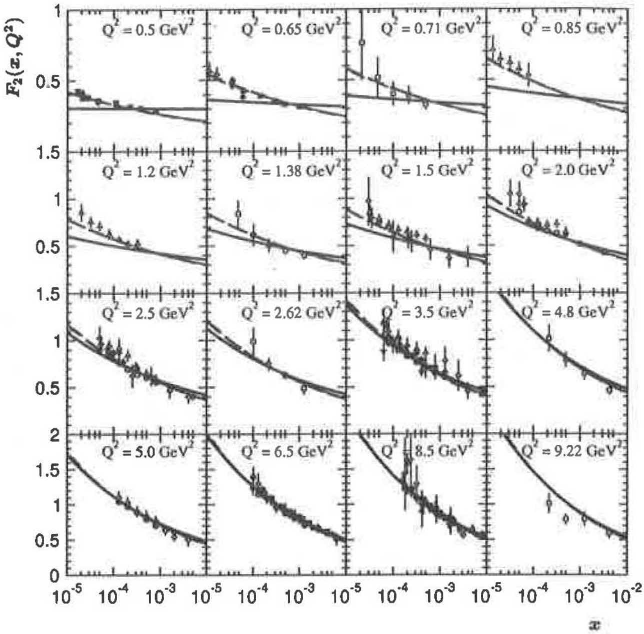
$$\widetilde{I}_\nu(\sigma) = \begin{cases} I_\nu(\bar{\sigma}), & \text{if } \sigma^2 = \bar{\sigma}^2 \geq 0 , \\ i^\nu J_\nu(\bar{\sigma}), & \text{if } \sigma^2 = -\bar{\sigma}^2 < 0 , \end{cases} \quad (11)$$

σ and ρ are the generalized Ball-Forte variables (see I). Note that the upper (down) line in the r.h.s. of Eq. (11) corresponds to the solution of the DGLAP equation for the "direct" ("backward") evolution in the DAS approximation.

The twist-six part can be easy obtained from the corresponding twist-four one as

$$f_a^{R\tau 6}(x,Q^2) = -\frac{8}{7} \times \left[f_a^{R\tau 4}(x,Q^2) \text{ with } \mathrm{a}_a^{\tau 4} \to \mathrm{a}_a^{\tau 6},\ \ln\left(\frac{Q^2}{|\mathrm{a}_a^{\tau 4}|}\right) \to \ln\left(\frac{Q^2}{\sqrt{|\mathrm{a}_a^{\tau 6}|}}\right) \right] . \quad (12)$$

3. The typical fits for the SF $F_2(x,Q^2)$ as a function of x for different Q^2 bins are presented on the figure below. The experimental points are from H1 [8] (open points) and ZEUS [2] (solid points). The solid line represents the NLO fit alone with χ^2/n.d.f. = 1.31. The dashed curve are obtained from the fit at the NLO, when the renormalon contributions of higher-twist terms have been incorporated. The corresponding χ^2/n.d.f. = 0.86. The dash-dotted curve (hardly distinguished from the dashed one) represents the fit at the



LO together with the renormalon contributions of higher-twist terms. The corresponding χ^2/n.d.f. = 0.84. The results demonstrate excellent agreement between theoretical predictions and experimental data for the region $Q^2 \geq 0.5$ GeV^2 as for SF $F_2(x, Q^2)$ as for the effective slope (see I and discussions therein).

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Discussion

Q. (J.Nassalski, SINS, Warsaw): The quality of LO and NLO fits is comparable (NLO slightly worse). Can you comment it?

A: It looks so, that the perturbation theory works well in the small x regime, which is in agreement with many other analysis, where it was shown, that the argument of the strong coupling constant is effectively much larger than Q^2 in the small x domain.