# NEUTRINO MASS IMPLICATIONS OF DOUBLE BETA DECAY

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## ABSTRACT

It is shown that, under quite general circumstances, the detection of neutrinoless double beta decay would imply a significant lower bound on the mass of at least one neutrino. This bound would be $\sim$ 1 eV if the decay should be seen at a rate close to the present limit. An explanation, including the main numerical details, of the origin of this bound is given.

It has long been known that observation of the nuclear reaction $(A, Z) \rightarrow (A, Z+2) + 2e^-$, known as neutrinoless double beta decay or $\beta\beta_{o\nu}$, would imply *either* neutrino mass *or* right-handed currents. However, under quite general circumstances, the detection of $\beta\beta_{o\nu}$ would in fact imply a significant lower bound on neutrino mass, whether or not right-handed currents exist.[1] This bound may be expressed in terms of the lifetime $\tau_{Ge}$ for $\beta\beta_{o\nu}$ decay of $^{76}Ge$. At present, we have only a limit, $\tau_{Ge} > 5.3 \times 10^{23}$ yr.[2] However, should $\beta\beta_{o\nu}$ decay of $^{76}Ge$ actually be observed, then at least one neutrino must have a mass $M$ satisfying

$$M \gtrsim 1 \text{ eV} \left[10^{24} \text{ yr}/\tau_{Ge}\right]^{\frac{1}{2}} . \tag{1}$$

Interestingly, a mass of order 1 eV is large enough to be sought in neutrino oscillation experiments, and possibly also in the next generation of tritium beta decay experiments.

We shall derive the bound (1) by showing first that the observation of $\beta\beta_{o\nu}$ would imply *non-zero* neutrino mass, and then that this mass would satisfy Eq. (1). Our assumptions will be made explicit as we proceed.[3]

We assume as usual that $\beta\beta_{o\nu}$ is dominated by the neutrino exchange mechanism of Fig. 1(a), in which two quarks in the parent nucleus emit a pair of $W$ bosons, $W_a$ and $W_b$, each of which may or may not be the $W$ (82 GeV). Then, $W_a$ and $W_b$ exchange a neutrino mass eigenstate $\nu_m$. Of course, we must sum over all $\nu_m$ that may exist, and over all $W_a$ and $W_b$.

When a specific $W_a$ and $W_b$ both couple to left-handed currents, the amplitude for the upper, particle-physics part of the diagram in Fig. 1(a) is of a type we shall call "$A_{LL}$". When $W_a$ couples to left-handed currents but $W_b$ to right-handed ones, this amplitude is of a type we shall call "$A_{LR}$". The diagram and mathematical form for the contribution of a given $\nu_m$ exchange to $A_{LL}$ ($A_{LR}$) are shown on the left (right) side of Fig. 1(b).

If no right-handed currents exist, $A_{LL}$ is the only type of amplitude possible. From Fig. 1(b), we see that the contribution of each $\nu_m$ exchange to $A_{LL}$ vanishes as $M_m \rightarrow 0$. By contrast, if right-handed currents do exist, we can have amplitudes of $A_{LR}$ type. From Fig. 1(b), we see that the contribution of a given $\nu_m$ exchange to $A_{LR}$ is proportional to the $\not{q}$ part of the neutrino propagator rather than to the neutrino mass part, and so does not vanish when $M_m \rightarrow 0$. These characteristics of the $\nu_m$ contributions to $A_{LL}$ and $A_{LR}$ are the origin of our opening statement that

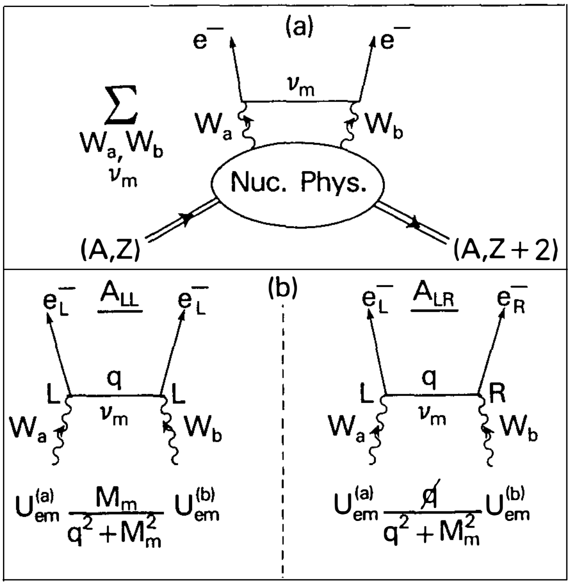


Fig. 1. (a) The neutrino exchange mechanism for $\beta\beta_{o\nu}$. The blob labelled "Nuc. Phys." is a nuclear process leading to emission of $W_a$ and $W_b$.

(b) The contribution of a given $\nu_m$ exchange to the amplitudes $A_{LL}$ and $A_{LR}$. The symbols $L, R$ denote the handedness of the couplings and the helicity of the produced electrons, neglecting their mass. In the formulae, $M_m$ is the mass of $\nu_m$, $q$ is the momentum carried by it, $U^{(a)}_{em}$ is the $e-\nu_m$ element of a mixing matrix describing the coupling of $W_a$ to leptons, and similarly for $U^{(b)}_{em}$.

$\beta\beta_{o\nu}$ requires either neutrino mass or else right-handed currents.

Suppose, now, that $W_a$ couples *only* to left-handed currents, and $W_b$ *only* to right-handed ones, and consider the process $W_aW_b \to e^-_L e^-_R$. An individual $\nu_m$ exchange contributes to this process with the same diagram and general mathematical form, given in Fig. 1(b), as to the $\beta\beta_{o\nu}$ amplitude $A_{LR}$. Furthermore, one can show that the individual $\nu_m$ exchange contribution to $W_aW_b \to e^-_L e^-_R$ violates unitarity as the energy goes to infinity. However, it is a very central feature of any gauge theory that the *complete* lowest-order amplitude for any process such as $W_aW_b \to e^-_L e^-_R$ has no such disease. That is, if the weak interactions are described by a gauge theory, the bad high-energy behavior of an individual $\nu_m$ exchange in $W_aW_b \to$

$e_L^- e_R^-$ must somehow be cancelled. This cancellation could, in principle, result from a diagram in which $W_a W_b \rightarrow e_L^- e_R^-$ via an intermediate virtual doubly-charged $Z$ boson, $Z^{--}$. We shall assume that no such exotic boson exists, especially since it could not couple to quarks of the known charges. Then the various $\nu_m$ exchanges must obviously cancel *each other* at high energy. From Fig. 1(b), we see that when the energy and $q^2$ are large, a cancellation among the $\nu_m$ contributions requires that

$$\sum_m U_{em}^{(a)} U_{em}^{(b)} = 0 \,. \tag{2}$$

Consider, then, the sum of the $\nu_m$ exchange contributions to the $\beta\beta_{o\nu}$ amplitude $A_{LR}$. The form of a single $\nu_m$ contribution is given in Fig. 1(b). Due to the constraint (2), the sum of these contributions obviously vanishes at all energies unless at least one $\nu_m$ has a mass $M_m \neq 0$. In summary, while an individual $\nu_m$ contribution to the $\beta\beta_{o\nu}$ amplitude $A_{LR}$ does not vanish when $M_m \rightarrow 0$, the $\nu_m$ contributions to this amplitude add up to zero in any gauge theory (with no $Z^{--}$) unless at least one $M_m \neq 0$. Since the remaining amplitude, $A_{LL}$, vanishes explicitly with the neutrino masses, this means that in any gauge theory, $\beta\beta_{o\nu}$ requires non-zero neutrino mass.

The previous argument that $A_{LR}$ (All $M_m = 0$) $= 0$ made the simplifying assumption that $W_a$ couples only to left-handed currents, and $W_b$ only to right-handed ones. When each boson couples to both kinds of currents, the $\nu_m$ contribution to $W_a W_b \rightarrow e_L^- e_R^-$ pictured in Fig. 1(b) is accompanied by a second diagram, in which the $e_R^-$ couples to the $W_a$, and the $e_L^-$ to the $W_b$. This second diagram could perhaps cancel the one in Fig. 1(b) at high energy. Thus, we can no longer argue that the various $\nu_m$ exchanges must cancel each other, and obtain the constraint (2). Nevertheless, this constraint still holds. To prove this, it has been shown[1)] by diagonalizing the most general possible mass matrix that in any gauge model with distinct neutrinos coupling to left- and right-handed currents,

$$\langle \overline{\nu_R'} | \nu_L \rangle = 0 \,. \tag{3}$$

Here $\nu_L$ is any weak eigenstate neutrino in the model that interacts via a left-handed current, and $\nu_R'$ is any one that interacts via a right-handed current. Now if, for example, $\nu_L$ couples to $W_a$ and an electron with a mixing matrix $U^{(a)}$, while $\nu_R'$ couples to $W_b$ with a matrix $U^{(b)}$, then

$$\langle \overline{\nu_{R'}} | \nu_L \rangle = \langle \sum_{m'} U_{em'}^{(b)*} \nu_{m'L} | \sum_m U_{em}^{(a)} \nu_{mL} \rangle = \sum_m U_{em}^{(b)} U_{em}^{(a)} \,.$$

Thus, Eq. (3) implies Eq. (2). Hence, $A_{LR}$ (All $M_m = 0$) still vanishes. (This argument fails in any model containing only one neutrino, since there is then no neutrino to which it can be orthogonal, and Eq. (3) is irrelevant. However, the bad high-energy behavior coming from the exchange of this neutrino must somehow be cancelled, so the model must also contain a $Z^{--}$. Thus, we simply learn again that $Z^{--}$-containing models are the exception to our rules.)

To see why detection of $\beta\beta_{o\nu}$ would imply the quantitative lower bound (1) on neutrino mass, let us assume that all $M_m << 10$ MeV, the typical momentum transfer in $\beta\beta$ decay. (Otherwise, Eq. (1) holds trivially.) Then, including the nuclear matrix element and the so-far suppressed strength factor $G^2_{Fermi}$, the $\beta\beta_{o\nu}$ amplitude $A_{LL}$ is (cf. Fig. 1(b))

$$A_{LL} \sim G^2_{Fermi} \sum_m U^{(a)}_{em} \frac{M_m}{q^2} U^{(b)}_{em} \times Nucl. = \left( \sum_m U^{(a)}_{em} U^{(b)}_{em} M_m \right) N . \tag{4}$$

Here $Nucl.$ is the nuclear matrix element, and $N$ is a calculable, neutrino-independent factor including this matrix element. If $\beta\beta_{o\nu}$ should be observed, we define the quantity $|M_{eff}|_{Exp}$, the experimentally measured effective neutrino mass, by

$$|M_{eff}|_{Exp} \equiv |\text{Observed } \beta\beta_{o\nu} \text{ Decay Amplitude}| \,/N . \tag{5}$$

Suppose there are no right-handed currents. Then we may neglect the presumably small contribution of any $W$ heavier than $W$(82 GeV). Hence, the $|M_{eff}|_{Exp}$ extracted from an observed $\beta\beta_{o\nu}$ decay would be given by Eqs. (5) and (4) with $W_a = W_b = W(82 \text{ GeV})$: $|M_{eff}|_{Exp} = \sum_m \left[U^{(W(82\text{ GeV}))}_{em}\right]^2 M_m$. Since the mixing matrix $U$ is unitary, we see that $|M_{eff}|_{Exp} \leq M^{max}_m$, where $M^{max}_m$ is the largest of the neutrino masses. *That is, at least one neutrino must have a mass no smaller than the measured effective neutrino mass.* If, using the definition (5), we express $|M_{eff}|_{Exp}$ in terms of parameters relating to $^{76}Ge$, this result implies the neutrino mass lower bound of Eq. (1).

Now suppose that right-handed currents do exist, and that an amplitude of type $A_{LR}$, arising when $W_a = W(82 \text{ GeV})$ but $W_b$ is some boson with right-handed couplings, dominates $\beta\beta_{o\nu}$. Including strength factors and the nuclear matrix element, this $A_{LR}$ is (cf. Fig. 1(b))

$$A_{LR} \sim G_{Fermi} G_R \sum_m U^{(W(82\text{ GeV}))}_{em} \frac{\not{q}}{q^2\left(1+\frac{M_m^2}{q^2}\right)} U^{(b)}_{em} \times \; Nucl. \tag{6}$$

Here $G_R$ is the analogue for $W_b$ of $G_{Fermi}$ for $W(82\text{ GeV})$, and "*Nucl.*" is, to within an order of magnitude, the same nuclear matrix element as in Eq. (4). To estimate the size of $A_{LR}$, we expand the $\nu_m$ propagators, use the constraint (2), and replace $q$ by its typical value of 10 MeV. This yields

$$A_{LR} \approx \left( \frac{G_R}{G_{Fermi}} \sum_m U_{em}^{(W(82\text{ GeV}))} (10\text{ MeV}) \frac{M_m^2}{(10\text{ MeV})^2} U_{em}^{(b)} \right) N \,, \tag{7}$$

with $N$ the same factor as in Eq. (4). Now, we assume that $G_R/G_{Fermi} \leq 1$ (which is *very* likely, since no right-handed interaction has yet been seen). Then, using the unitarity properties of the $U$ matrices, we see from Eqs. (5) and (7) that the $|M_{eff}|_{Exp}$ extracted from an observed $\beta\beta_{o\nu}$ decay would satisfy

$$|M_{eff}|_{Exp} \leq (M_m^{max})^2 / 10\text{ MeV} \,. \tag{8}$$

Since it is already known experimentally that $|M_{eff}|_{Exp} \lesssim 2$ eV,[2] Eq. (8) implies the much weaker constraint $|M_{eff}|_{Exp} \leq (M_m^{max})^2 / |M_{eff}|_{Exp}$, from which we learn that $|M_{eff}|_{Exp} \leq M_m^{max}$, *just as when* $A_{LL}$ *dominates.* (We have just weakened Eq. (8) by so much that the steps leading up to it can be rather approximate without affecting our final conclusion.)

We see that, regardless of whether $A_{LL}$ or $A_{LR}$ dominates, so long as $\beta\beta_{o\nu}$ results from neutrino exchange, the observation of this process would imply quite generally that some neutrino has a mass $M$ no smaller than the measured $|M_{eff}|_{Exp}$. For $^{76}Ge$ decay, this means that $M \gtrsim 1\text{ eV}\,[10^{24}\text{ yr}/\tau_{Ge}]^{\frac{1}{2}}$.

## REFERENCES

1. B. Kayser, S. Petcov, and S.P. Rosen, in preparation.

2. D. Caldwell, in these Proceedings.

3. Past work concerning the relation between $\beta\beta_{o\nu}$ and neutrino mass has been carried out by M. Doi, K. Enqvist, A. Goldhaber, B.K., C. Kim, T. Kotani, J. Maalampi, R. Mohapatra, K. Mursula, H. Nishiura, K. Okuda, J. Schechter, E. Takasugi, J. Valle, and L. Wolfenstein.