

ENERGY OF W DISTRIBUTION IN TOP QUARK DECAYS*

M. JEŻABEK

Institute of Nuclear Physics, Kawory 26a, PL-30055 Cracow, Poland
and

Institut f. Theoretische Teilchenphysik, Universität Karlsruhe
D-76128 Karlsruhe, Germany

CH. JÜNGER

Institut f. Theoretische Teilchenphysik, Universität Karlsruhe
D-76128 Karlsruhe, Germany

(Received August 4, 1993)

A relatively simple analytical formula is derived for the energy spectrum of W boson in top quark decays $t \rightarrow Wb$ including $\mathcal{O}(\alpha_s)$ radiative corrections. We discuss the accuracy of this formula and compare it to a more general albeit more complicated one derived in (A. Czarnecki, M. Jeżabek, J.H. Kühn, *Acta Phys. Pol.* **B20**, 961 (1989); (E) **B23**, 173 (1992)). A Monte Carlo algorithm for generation of W energy spectrum is briefly described.

PACS numbers: 12.15. Ji

1. Introduction and summary

Radiative QCD corrections to the energy distribution of $\bar{f}f'$ in $t \rightarrow b\bar{f}f'$ and $t \rightarrow bg\bar{f}f'$ decays have been calculated some time ago [1]. In the meantime the lower limit for top quark mass m_t has been pushed up by CDF and D0 collaborations well above the threshold for $t \rightarrow bW$ channel. Although the results of [1] are applicable also in this regime it seems reasonable and useful to derive a new formula assuming dominance of decays into real W .

* Work partly supported by the Polish Committee for Scientific Research (KBN) Grants 203809101 and 223729102, and by EEC Contract ERB-CIPA-CT-92-2077.

Such a formula, albeit less general than that given in [1], can be very useful in studies of top quark physics at future e^+e^- colliders [2].

Our present approach is based on an appropriately modified narrow width ($\Gamma_W = 0$) approximation, where Γ_W is the width of W boson. In contrast to [1], where the rates are manifestly infrared finite, we introduce an explicit infrared cutoff λ on the effective mass squared of the system b quark + gluon. Thus the formula for $\mathcal{O}(\alpha_s)$ contribution from virtual gluons also depends on λ . This apparent failure turns out to be an advantage in Monte Carlo simulations, which are indispensable for a precise determination of the strong coupling constant α_s and m_t from the total $\sigma(e^+e^- \rightarrow t\bar{t})$ and differential cross sections [3–6]. We wrote a Monte Carlo program based on the results of the present article and found a very good agreement with the formulae given in [1].

We have checked also that massless b approximation which is known to be a satisfactory one for the total decay rate and m_t above 120 GeV [7, 8], cannot be used in calculations of W energy distribution. One reason is purely kinematical: Born distribution in the narrow width approximation has a Dirac delta shape, i.e. the energy of W is fixed by two body kinematics. A shift from realistic $m_b = 4.7$ GeV to $m_b = 0$ results in a non-negligible shift in this energy. We attempted to correct the massless formula (see Appendix A) for this trivial effect but the result was still a rather poor approximation. Thus, we conclude that for realistic top and bottom masses one has to use the complete $\mathcal{O}(\alpha_s)$ result rather than its massless approximation.

Our article is organized as follows. In Section 2 we derive our formula for the energy of W distribution in the narrow width approximation. In Section 3 we include a non-zero W width and describe our Monte Carlo program based on our calculations. Then, we compare the results of this program with those of [1]. In Appendix A our formulae for $m_b \rightarrow 0$ are given.

2. Energy of W distribution in narrow width approximation

We use throughout the same notation as in [9]. We stay in the top quark rest frame and some variables are expressed in units of m_t . In particular y — effective mass squared of $\bar{f}f'$ system, z — the mass squared of bg system and $\varepsilon = m_b/m_t$. We define also the energy of W :

$$x_W = \frac{E_W}{m_t} = \frac{1}{2}(1 + y - z), \quad (1)$$

which for twobody Born decay mode $t \rightarrow bW$ is replaced by

$$\bar{x}_W = \frac{E_W}{m_t} = \frac{1}{2}(1 + y - \varepsilon^2), \quad (2)$$

and $x_W \leq \bar{x}_W$. In general "barred" quantities are evaluated at $z = \varepsilon^2$. Other useful variables are:

$$w_3 = \sqrt{x_W^2 - y} = \frac{1}{2} \sqrt{\lambda(1, y, z)}, \quad (3)$$

$$\lambda(u, v, w) = u^2 + v^2 + w^2 - 2(uv + uw + vw) \quad (4)$$

the length of W momentum¹, which is equal to

$$p_3 = \sqrt{x_h^2 - z} \quad (5)$$

the momentum of bg system of energy $x_h = 1 - x_W$, the light cone variables

$$\begin{aligned} w_{\pm} &= x_W \pm w_3, \\ p_{\pm} &= x_h \pm p_3, \end{aligned} \quad (6)$$

and rapidities

$$\begin{aligned} Y_W &= \frac{1}{2} \ln \frac{w_+}{w_-}, \\ Y_p &= \frac{1}{2} \ln \frac{p_+}{p_-}. \end{aligned} \quad (7)$$

In the narrow width approximation $\gamma = \Gamma_W/m_W \rightarrow 0$ one can replace a factor resulting from W propagator by Dirac delta function

$$\frac{1}{\pi} \frac{\gamma y_0}{(y - y_0)^2 + \gamma^2 y_0^2} \longleftrightarrow \delta(y - y_0), \quad (8)$$

where $y_0 = (m_W/m_t)^2$. Thus, when the decay through real W dominates the effective mass is close to $\sqrt{y_0}$. In this section we consider y as fixed and give the formula for the differential rate $\left. \frac{d\Gamma}{dx_W} \right|_y$. Then, in Section 3 we relax this assumption using (8). For fixed y the energy distribution of W including $\mathcal{O}(\alpha_s)$ corrections can be written as follows:

$$\begin{aligned} \left. \frac{d\Gamma}{dx_W} \right|_y &= \frac{G_F m_t^3}{8\sqrt{2}\pi} \left[\delta(z - \varepsilon^2) \left(F_0 - \frac{2\alpha_s}{3\pi} \tilde{g}_1(y, \lambda) \right) \right. \\ &\quad \left. + \frac{2\alpha_s}{3\pi} \Theta(z - \varepsilon^2 - \lambda) \Theta(x_W - \sqrt{y}) \tilde{g}_1(y, z) \right], \end{aligned} \quad (9)$$

¹ In unpolarized case discussed here we choose z -axis in the direction of W .

where z is fixed through (1),

$$F_0(y, \varepsilon) = \frac{1}{2} \sqrt{\lambda(1, y, \varepsilon^2)} C_0(y, \varepsilon), \quad (10)$$

$$C_0(y, \varepsilon) = 4[(1 - \varepsilon^2)^2 + y(1 + \varepsilon^2) - 2y^2]. \quad (11)$$

$$\begin{aligned} \bar{G}_1 &= g_1 C_0(y, \varepsilon) \bar{x}_h + g_2 C_0(y, \varepsilon) \bar{p}_3 + g_3 \bar{x}_h \bar{p}_3 + g_4 \bar{p}_3 + g_5 \bar{Y}_p + g_6 \\ g_1 &= 4\bar{Y}_p^2 - 2\text{Li}_2(\bar{w}_0) + 4\text{Li}_2(\bar{w}_-/\bar{w}_+) \\ &\quad - 4\text{Li}_2\left(\frac{\bar{p}_- \bar{w}_-}{\bar{p}_+ \bar{w}_+}\right) + 2\text{Li}_2(\bar{w}_+) - 8\bar{Y}_p \bar{Y}_w \\ &\quad + 4\bar{Y}_p (2 \ln \varepsilon - \ln \lambda + 2 \ln(1 - \bar{p}_-/\bar{p}_+) - \ln y) \\ g_2 &= 4 - 6 \ln \varepsilon + 4 \ln \lambda \\ g_3 &= 24(1 - \varepsilon^2) \ln \varepsilon \\ g_4 &= 8(-1 + 2\varepsilon^2 - \varepsilon^4 - y - \varepsilon^2 y + 2y^2) \\ &\quad - \left[18\varepsilon^2(2y^2 - y - 1) + 4\varepsilon^2 C_0(y, \varepsilon)\right] \frac{\lambda}{\varepsilon^2(\varepsilon^2 - \lambda)} \\ g_5 &= 4(-1 + \varepsilon^2 + \varepsilon^4 - \varepsilon^6 - 3y + 2\varepsilon^2 y - 3\varepsilon^4 y + 9y^2 + 9\varepsilon^2 y^2 - 5y^3) \\ g_6 &= \left[9\varepsilon^2(2y^2 - y - 1) + 2\varepsilon^2 C_0(y, \varepsilon)\right] \frac{1+y}{1-y} \ln(1 + \lambda/\varepsilon^2) - \\ &\quad \left[7(2y^2 - y - 1) + 2C_0(y, \varepsilon)\right] (1-y) \ln(1 + \lambda/\varepsilon^2), \end{aligned} \quad (12)$$

and

$$\begin{aligned} \bar{g}_1 &= 2a_1 p_3(z) + 4a_2 Y_p(z) - \frac{4\varepsilon^4 C_0(y, \varepsilon)}{z^2(z - \varepsilon^2)} p_3(z) + \frac{4\bar{x}_h C_0(y, \varepsilon)}{z - \varepsilon^2} Y_p(z) \\ a_1 &= \frac{\varepsilon^2}{z^2} [-9 + 15\varepsilon^2 - 8\varepsilon^4 - y(9 + 7\varepsilon^2 - 18y)] \\ &\quad + \frac{1}{z} [-7 + \varepsilon^2(20 - 5\varepsilon^2 - 11y) + 7y(2y - 1)] \\ &\quad + 2y - 3(1 + \varepsilon^2) \\ a_2 &= 2 + \varepsilon^2(\varepsilon^2 - 5) - 2y(2y - 1) + (1 + \varepsilon^2 + 2y)z. \end{aligned} \quad (13)$$

In the above formulae λ denotes an infrared cutoff on the effective mass $z \geq \varepsilon^2 + \lambda$. Let us sketch now the derivation of Eq. (9): The contribution resulting from real gluon radiation (Θ -piece) is obtained by direct integration of the fully differential decay rate, whereas the (Born + soft) contribution (δ -piece) is derived from the requirement that integrating (9) over x_w one obtains the narrow width limit of the expression for the total

rate given in [7]; see also (16) in the following section. The formula (9) simplifies considerably in the limit $m_b \rightarrow 0$, cf. Appendix A. However, for realistic b quark masses this is a rather poor approximation.

3. Finite W width and results

We generalize now the results of the previous section and include a nonzero W width. Let us note that y is not fixed for $\gamma \neq 0$. The double differential distribution

$$\frac{d\Gamma}{dx_w dy} = \frac{\gamma}{\pi} \frac{y_0}{(y - y_0)^2 + \gamma^2 y_0^2} \frac{d\Gamma}{dx_w} \Big|_y \quad (14)$$

is, however, closely related to the narrow width result (9), cf. (8). We use the standard model result for Γ_w :

$$\Gamma_w = \frac{G_F m_W^3}{6\sqrt{2}\pi} \left(9 + 6 \frac{\alpha_s}{\pi} \right). \quad (15)$$

Integrating (14) over x_w we obtain of course the standard model result for $d\Gamma/dy$ [7,8]²

$$\frac{d\Gamma}{dy} = \frac{G_F^2 m_t^5}{192\pi^3} \left(9 + 6 \frac{\alpha_s}{\pi} \right) \frac{1}{(1 - y/y_0)^2 + \gamma^2} \left[F_0(y, \varepsilon) - \frac{2\alpha_s}{3\pi} F_1(y, \varepsilon) \right], \quad (16)$$

$0 \leq y \leq (1 - \varepsilon)^2$, where F_0 is defined in (10) and

$$\begin{aligned} F_1(y, \varepsilon) = & \frac{1}{2} C_0(y, \varepsilon) (1 + \varepsilon^2 - y) \left[2\pi^2/3 + 4\text{Li}_2(u_w) - 4\text{Li}_2(u_q) \right. \\ & - 4\text{Li}_2(u_q u_w) - 4 \ln u_q \ln(1 - u_q) \\ & \left. - 2 \ln u_w \ln u_q + \ln y \ln u_q + 2 \ln \varepsilon \ln u_w \right] \\ & - 2F_0(y, \varepsilon) [\ln y + 3 \ln \varepsilon - 2 \ln \lambda(1, y, \varepsilon^2)] \\ & + 4(1 - \varepsilon^2) [(1 - \varepsilon^2)^2 + y(1 + \varepsilon^2) - 4y^2] \ln u_w \\ & + [3 - \varepsilon^2 + 11\varepsilon^4 - \varepsilon^6 + y(6 - 12\varepsilon^2 + 2\varepsilon^4) \\ & - y^2(21 + 5\varepsilon^2) + 12y^3] \ln u_q \\ & + 6\sqrt{\lambda(1, y, \varepsilon^2)}(1 - \varepsilon^2)(1 + \varepsilon^2 - y) \ln \varepsilon \\ & + \sqrt{\lambda(1, y, \varepsilon^2)} [-5 + 22\varepsilon^2 - 5\varepsilon^4 - 9y(1 + \varepsilon^2) + 6y^2], \quad (17) \end{aligned}$$

² we derived (9) using this condition.

where

$$u_q = \frac{\bar{p}_-}{\bar{p}_+} \quad u_w = \frac{\bar{w}_-}{\bar{w}_+}. \quad (18)$$

We can also integrate (14) over y . In this way we obtain a new formula for

$$\frac{d\Gamma}{dx_w} = \int_0^{(1-\varepsilon)^2} dy \frac{d\Gamma}{dx_w dy}. \quad (19)$$

We compared (19) with the result of [1] and found a perfect numerical agreement for small λ . For $\lambda = 10^{-8}$ the relative error is 10^{-6} .

The formulae (9) and (16) can be also used as a starting point for Monte Carlo simulations. A key observation is that (16) gives the distribution of y for $0 \leq y \leq (1 - \varepsilon)^2$ whereas (9) gives relative probabilities for x_w at fixed y . The distribution (14) can be generated as follows: y is generated first according to (16). Then, for given y , x_w is generated according to (9). Both steps can be performed by a standard combination of importance sampling and von Neumann rejection. The only difficulty is to find the value of the infrared cutoff λ such that λ is small enough and the δ -piece in (9) remains positive. This well known difficulty limits relative accuracy of our Monte Carlo to about 1%.

In Fig. 1 we show the normalized distribution $\Gamma^{-1} d\Gamma/dx_w$ for $m_t = 120$ GeV obtained from (19) for $\lambda = 10^{-8}$. The curve obtained using the result from [1] is identical up to the resolution of the plot.

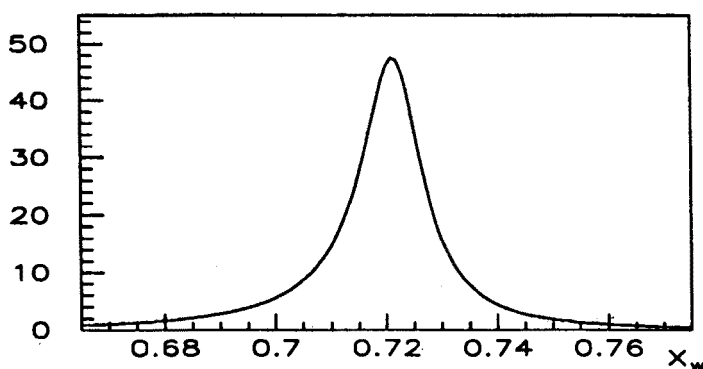


Fig. 1. Normalized energy distribution of W : $\Gamma^{-1} d\Gamma/dx_w$ for $m_t = 120$ GeV evaluated from (19).

In Fig. 2 we compare the analytical result (19) ($\lambda = 10^{-8}$) with our

Monte Carlo program for $\lambda = 3 \cdot 10^{-4}$. We plot the ratio (in percent)

$$1 - \frac{d\Gamma/dx_w|_n}{d\Gamma/dx_w|_a}, \quad (20)$$

where the subscript "n" refers to the Monte Carlo and "a" to the result obtained from (19).

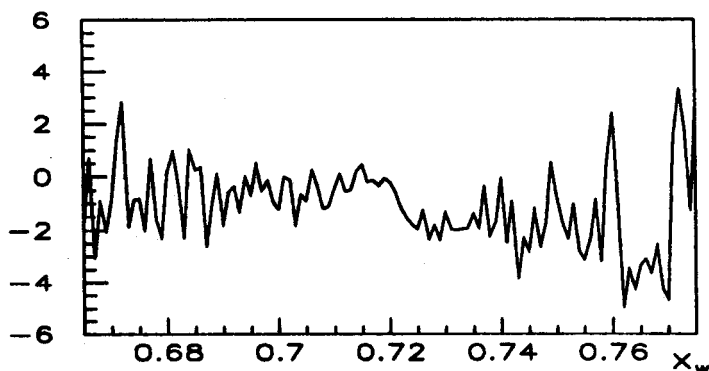


Fig. 2. Comparison of Monte Carlo for infrared cutoff $\lambda = 3 \cdot 10^{-4}$ and analytic result (19) for $\lambda = 10^{-8}$. The ratio (20) is shown as function of x_w .

APPENDIX A

In this Appendix we give the limit of (9) for $m_b \rightarrow 0$. These formulae can be used for $t \rightarrow s$ or $t \rightarrow u$ transitions. It would be, however, a rather poor approximation to use these formulae for the dominant $t \rightarrow b$ transition. For $\varepsilon \rightarrow 0$ (9) is replaced by

$$\begin{aligned} \left. \frac{d\Gamma}{dx_w} \right|_y &= \frac{G_F m_t^3}{8\sqrt{2}\pi} \left\{ \delta(z) F_0 \left[1 - \frac{2\alpha_s}{3\pi} G_1(y, \lambda) \right] \right. \\ &\quad \left. + \frac{2\alpha_s}{3\pi} \Theta(z - \lambda) \Theta(x_w - \sqrt{y}) g(z, y) \right\}. \end{aligned} \quad (21)$$

$$F_0 = 2(1 - y)^2 (2y + 1). \quad (22)$$

$$\begin{aligned} G_1(y, \lambda) &= \frac{2}{3} \pi^2 + \frac{5}{2} + 2 \text{Li}_2(y) + 4 \ln^2(1 - y) - 7 \ln(1 - y) + \ln^2(\lambda) \\ &\quad + \frac{1}{2} \ln(\lambda) [7 - 8 \ln(1 - y)] + \frac{5 + 4y}{1 + 2y} \ln(1 - y). \end{aligned} \quad (23)$$

$$g(z, y) = p_3(z) \left[2(2y - 3) - 14(1 - y)(2y + 1) \frac{1}{z} \right] + 4Y_p(z) \left[(2y + 1)(2 - 2y + z) + \frac{F_0}{z} \right]. \quad (24)$$

Then, integrating the double differential distribution (14) over y one obtains $d\Gamma/dx_w$ in the massless limit $m_b = 0$.

REFERENCES

- [1] A. Czarnecki, M. Jeżabek, J.H. Kühn, *Acta Phys. Pol.* **B20**, 961 (1989). (E) **B23** (1992) 173
- [2] P.M. Zerwas (ed.), e^+e^- Collisions at 500 GeV: The Physics Potential, DESY Orange Report DESY 92-123A, Hamburg 1992.
- [3] J.M. Strassler, M.E. Peskin, *Phys. Rev.* **D43**, 1500 (1991).
- [4] Y. Sumino, K. Fujii, K. Hagiwara, H. Murayama, C.-K. Ng, *Phys. Rev.* **D47**, 56 (1992).
- [5] M. Jeżabek, J.H. Kühn, T. Teubner, *Z. Phys.* **C56**, 653 (1992); M. Jeżabek, T. Teubner, *Z. Phys.* **C** (1993) in print; Karlsruhe Univ. preprint TTP 93-11.
- [6] M. Martinez *et al.*, in: Proceedings of the Workshop on Physics and Experiments at Linear e^+e^- Colliders, Waikoloa, Hawaii, April 1993.
- [7] M. Jeżabek, J.H. Kühn, *Nucl. Phys.* **B314**, 1 (1989).
- [8] M. Jeżabek, J.H. Kühn, *Phys. Rev. D*, (Rapid Commun.) (1993) in print; Karlsruhe Univ. preprint TTP 93-4.
- [9] M. Jeżabek, J.H. Kühn, *Nucl. Phys.* **B320**, 20 (1989).