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Modified Gravity and Cosmic Acceleration: Now and in the Early Universe

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Title: Modified Gravity and Cosmic Acceleration: Now and in the Early Universe

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Abstract: We review our previous works which explored the problems of dark energy and cosmological constant problem in the context of modified gravity.

In the first part, we present extensions of mimetic dark matter. The latter is a Weyl-invariant scalar-tensor theory able to describe dark matter on cosmological scales. In our works we have extended the mimetic construction to vector fields and Yang-Mills gauge fields. The resulting theories provided a novel Weyl-invariant and higher derivative formulations of unimodular gravity. We also introduced a mixture of this mimetic dark energy with mimetic dark matter and showed that it results in k-essence like scalar theory.

In the second part we reviewed a minimal modification of Einstein equations, in which their trace part is made trivial. This results in the Newton constant becoming a global degree of freedom. Consequently, the Newton constant is subjected to quantum fluctuations and uncertainty relations. We find that the same applies to the effective Planck constant in a certain classically equivalent formulation of this gravity modification.

Finally, we present an analysis of tensor perturbations in the recently proposed models of minimally varying cosmological constant. In these theories, extending Einstein-Cartan gravity, the cosmological constant is allowed to vary by means of a balancing torsion. These models in general allow for a parity-odd torsion. We have found that the speed of left and right handed gravitational waves differ in the presence of this parity-odd torsion. We use recent observations to put severe constraints on the parameters of this model.

Keywords: dark energy, mimetic gravity, unimodular gravity, Einstein-Cartan gravity, gravity waves

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Introduction

General Relativity (GR) is one of the most successful theories in history, and, together with quantum field theory (QFT), it is considered to be a cornerstone of modern physics. GR is remarkably elegant, and, with minimal amount of free parameters, it has been able to explain such phenomena as the precession of the Mercury perihelion and to provide numerous unexpected predictions like the expansion of the Universe, gravitational lensing, existence of black holes and only recently observed gravitational waves. These unprecedented accomplishments are significantly underscored by the fact that GR has remained completely unchanged in the past 100 years withstanding the enormous observational scrutiny.

During the past several decades, we have witnessed substantial advances in cosmological and astrophysical observations, which revealed that the Universe is dominated by non-luminous forms of energy. These are the dark matter (DM) and dark energy (DE). DM has been introduced in order to explain dynamics of galaxy clusters and the rotation curves of spiral galaxies [1–3], and it is a crucial ingredient for large scale structure formation. DE drives the late time acceleration of the Universe [4, 5]. Collectively these are dubbed as the dark sector (DS). According to the recent precision observations of the anisotropies in the cosmic microwave background radiation (CMB) by the Planck collaboration [6], dark matter makes up 26%, while dark energy constitutes 69% of the total energy density. In contrast, the luminous matter contributes only the remaining 5%.

One can accommodate this dark Universe within the framework of General Relativity and obtain a very good fit of the cosmological evolution by considering a very simple effective description of the dark sector. Namely, we can model dark energy as the cosmological constant (CC), denoted as Λ, and dark matter as a cold, nearly pressureless gas (cold dark matter - CDM). This simple model of DS, together with General Relativity and homogeneity and isotropy of the Universe on large scales, constitutes the ΛCDM model, the standard model of cosmology, for review see [7]. This theory has been very successful in describing cosmological evolution, and it provided us with substantial understanding of the formation of large scale structures, as well as the structure of anisotropies of CMB, along with the relative abundance of elements [8]. Unfortunately, ΛCDM has been unreasonably efficient in its predictions, as it achieved the above feats without any need for a detailed knowledge of the nature of the dark sector. As a result, despite these spectacular successes, we did not have an opportunity to learn about the microscopic nature of the dark sector in our Universe yet.

Fortunately, ΛCDM has not been without flaws. Some of them have already given rise to promising new physics. One of the most prominent examples is the inflationary paradigm [9–15]. Inflation postulates that the Universe underwent a period of rapid accelerated expansion in the early epoch of the Universe as a way to address the problem of (non)existence of magnetic monopoles, as well as the horizon, the flatness and the homogeneity problems of the hot Big Bang cosmology, see [7]. It has been shown that inflation provides an origin for the large scale structures in the Universe [16–22]. The simplest way to incorporate inflation into ΛCDM is to introduce a novel scalar field, the inflaton, whose potential then drives the early accelerated expansion of the Universe.

Maybe the most prominent shortcoming of the ΛCDM is the famous cosmological constant problem [23–30] and the related problem of dark energy. The dark energy is made out of several contributions, one of which is the bare cosmological constant that is introduced as a coupling constant in the Einstein-Hilbert action. Furthermore, there are multiple other sources of vacuum energy that exist within the Standard Model of particle physics (SM), which provide a cosmological constant-like contribution to the total DE. For example, these are the zero point energies of all quantum fields, the Higgs potential in the electroweak (EW) sector of SM [7, 31] or the quark and gluon condensates in quantum chromodynamics (QCD) [32, 33]. These constituents of the total dark energy are many orders of magnitude larger than the observed value of DE today. We discuss some of these contributions in Chapter 1. Cancellation of such an unprecedented precision among seemingly unrelated sources seems to be exceedingly unlikely unless there is some underlying symmetry at play. However, no such symmetry has been observed so far. A full resolution of the cosmological constant problem has to explain how to properly calculate these contributions, why they almost cancel or why they can be neglected, as well as to provide a mechanism to generate the needed tiny value of the cosmological constant. This value has to be tiny to be consistent with current observations of dark energy. This problem has attracted major attention and presents a very active field of research.

The picture of ΛCDM has been recently challenged further. The measurements of the Hubble parameter coming from CMB anisotropies [6] or baryonic acoustic oscillation [34, 35] substantially differ from the local measurements [36–42]. The tension between Hubble parameter values measured in these ways has been recently reported to have grown to 5σ [42]. This suggests that the simple cosmological constant may not be sufficient to describe the effects of dark energy or that the value of the cosmological constant has changed during the cosmological evolution of the Universe.

The prospects of resolving the dark problems of ΛCDM (for recent summary see [8]) within the framework of the SM seem to be bleak at best. Indeed, we are very likely dealing with new physics, possibly of gravitational origin, for review see [25]. The idea of modifying gravity is certainly not new. People have sought out alternatives and extensions to GR ever since its formulation [43–47]. While this may have been driven by curiosity at first, we now have reasons to believe that going beyond Einstein's theory may be inevitable. Indeed, when considering high energies beyond the Planck scale, one definitely cannot ignore the quantum nature of the Universe: we must subject gravity to quantization. However, the quantization procedure does not guarantee a consistent result for any theory subjected to it. For gravity this leads to a breakdown, as GR fails to be renormalizable [48–51]. Here we should stress that on scales smaller then the Planck scale, GR, taken as an effective field theory, is still valid and extremely successful, see for example [52, 53]. While observational/experimental confirmation of breakdown of GR in high energies is far beyond our reach, it provides further motivation to consider deviations from General Relativity.

Modified gravity has become an active field of research in addressing many of the problems of cosmology and the dark sector in particular. Indeed, modifications of gravity often introduce novel degrees of freedom, which may provide us with components of the DS or to drive inflation in the early Universe. A common approach to modifying gravity is to supplement General Relativity with additional fields, which interact non-trivially with the gravitational degrees of freedom or couple universally to matter. The oldest example of this type is the Brans-Dicke theory [47]. Other examples include the Horndeski scalar-tensor theories [54–57], a vector-tensor theory (for instance Einstein aether [58]) or a tensor-vector-scalar theory (TeVeS) [59–61]. A different class of gravity modifications focuses on changing the self interactions of the metric directly without explicitly introducing novel variables into the action. A prime examples of such modifications are the $f(R)$ models [62], see also [10, 63–67], which are the non-linear generalizations of the Einstein-Hilbert action. It has been shown that these models can be rewritten as a scalar-tensor theory [68–70]. Another example of this are the so-called minimally modified gravity theories, which surprisingly do not introduce any new local propagating degrees of freedom [71–73].

Of particular interest for this thesis is the recently proposed gravity modification called the mimetic dark matter [74], which we discuss in more detail in Chapter 2. This theory builds upon the idea that the "physical" metric of GR, whose geodesics correspond to free fall of test bodies, may be a composite object rather then an independent field. The physical geometry is described through an auxiliary metric $h_{\mu\nu}$ and a novel scalar field ϕ as:

$$g_{\mu\nu} = h_{\mu\nu}\ h^{\sigma\rho}\partial_\sigma\phi\partial_\rho\phi\ . \tag{1}$$

The crucial property here is that the physical metric is invariant under the Weyl transformations of the auxiliary metric. This degeneracy gives rise to an effective fluid behavior, which can provide us with a simple model of DM, the so-called mimetic dark matter. This proposal has attracted a considerable attention in the recent years, e.g. [75–83], and has been central to parts of our research [84–86]. We discuss first of these works in Chapter 3. In this work [84] we have proposed a generalization of the mimetic DM scenario, where the physical metric is built from an auxiliary metric $h_{\mu\nu}$ and a vector field V^μ as

$$g_{\mu\nu} = h_{\mu\nu}\sqrt{\nabla_\sigma V^\sigma}\ . \tag{2}$$

Here the covariant derivative is compatible with the auxiliary metric. The key idea in our paper was that the above ansatz can be made Weyl invariant by providing a non-trivial conformal weight to the vector field V^μ. This results in a degeneracy in the physical metric, which leads to an additional vacuum energy component, arising as a global degree of freedom. In this sense, we obtain "mimetic dark energy". We show that this proposal provides the same classical dynamics as unimodular gravity [87–90], which we review in Section 1.3. In fact, by going to Weyl invariant variables, it can be directly related to the Henneaux and Teitelboim unimodular gravity [88]. Due to the underlying Weyl invariance, our formulation produces equations that are manifestly invariant under any constant change of the vacuum energy. This may be of particular use for addressing the cosmological constant problem. We show that our proposal can be viewed as a scalar-vector-tensor theory with a nontrivial universal coupling to matter.

We further expanded upon the above idea in our work [85], which we review in Chapter 4. There, we considered the possibility that the vector field V^μ is itself a composite field. Namely, we traded V^μ for the Chern-Simons current of an auxiliary Yang-Mills gauge field A_μ. This produces the following ansatz

$$g_{\mu\nu} = h_{\mu\nu}\,\sqrt{F_{\sigma\rho}\tilde{F}^{\sigma\rho}}\ , \tag{3}$$

where $\tilde{F}^{\sigma\rho}$ is the Hodge dual of the field strength $F_{\sigma\rho}$. Thus, we utilize the Pontryagin term to define the physical metric. Our construction is in a sense complementary to the proposal [82, 83], where the kinetic term of a gauge field is used for this purpose. Surprisingly, we found that the composite nature of the vector field does not spoil the classical dynamics of the theory, which are still equivalent to unimodular gravity. This novel formulation is particularly useful as a starting point for additional extensions of unimodular gravity, as it involves fields that are natural to the SM. We show that, by rewriting the theory in (Weyl) gauge invariant variables, the novel global degree of freedom, which corresponds to the vacuum energy, obtains an axion-like coupling to the gauge field A_μ. In this way, it behaves both like a cosmological constant and as the θ parameter of the gauge theory. This line of thought has been previously suggested but not explored in [91]. We further discuss the similarity with axion and speculate that unimodular gravity can be recovered as a particular dynamical limit of an axion coupled to a Yang-Mills theory.

Following the ideas of mimetic DM [74] and of our mimetic DE [85], we ask ourselves a natural question: Can the mimetic construction provide us with both components of the dark sector at once? In Chapter 5 we review a possible approach to this question, which has been proposed in our paper [92]. There we consider a more general mimetic substitution, which combines the original kinetic term of a scalar field (1) and the Pontryagin term (3). For example:

$$g_{\mu\nu} = h_{\mu\nu}\left[A\,h^{\sigma\rho}\partial_\sigma\phi\partial_\rho\phi + B\,\sqrt{\tilde{F}^{\sigma\rho}F_{\sigma\rho}}\right]\ , \tag{4}$$

where A and B can in general depend on ϕ. The conformal factor multiplying $h_{\mu\nu}$ can in fact be arbitrarily complicated. We only require that the physical metric $g_{\mu\nu}$ is invariant under Weyl transformations of $h_{\mu\nu}$. We have shown that a general substitution of the type (4) introduces additional dynamical sector that is in equivalent to a k-essence theory [93–95] accompanied by an additional global degree of freedom. This global degree of freedom provides an overall energy scale for the k-essence[1]. This resembles what happens in the so-called generalized unimodular gravity [96, 97]. We show how one can map nearly any k-essence to an equivalent mimetic description. Interestingly, the mimetic counterpart exists only for theories that never cross an ultra-relativistic equation of state. We also discuss the relevance of the Weyl transformation. We show that, in case of Weyl violating substitutions, the theory may dynamically restore this symmetry and provide us with k-essence anyway. Finally, we found a class of Weyl violating substitutions, which produce ordinary mimetic DM with a potential [76].

In Chapter 6 we discuss a modification of Einstein equations that we introduced in [86]. In this work we have proposed a minimal modification of Einstein equations in which we trivialized their trace in the following way:

$$\frac{G_{\mu\nu}}{G} = \frac{T_{\mu\nu}}{T}\ . \tag{5}$$

[1]For a better readability we briefly discuss k-essence in Section 2.2.

The trace part of this equation results in a useless identity $1 = 1$. Interestingly, the above equation is scale-free and eliminates the need for the Newton constant. We show that this theory is in fact classically equivalent to GR, where the Newton constant appears as an integration constant or a global degree of freedom. This occurs in the same way in which the cosmological constant is recovered in unimodular gravity. We discuss various formulations of this theory and show that in certain cases the Planck constant also becomes an integration constant. The promotion of fundamental constants to degrees of freedom has an interesting consequence. Upon quantization these "constants" become operators [86, 98] subjected to quantum fluctuations with corresponding uncertainty relations. These may be significant close to cosmological singularities [86, 99].

In Chapter 7 we switch our focus to Einstein-Cartan gravity (ECG) [100, 101], which we discuss in Section 7.1. For a more detailed review see [102]. Unlike in GR, the form of the Einstein equation in ECG does not force the cosmological term to be a constant. Indeed, due to the possibly non-vanishing torsion, the Bianchi identity for the Einstein tensor may be violated. This opens a possibility of a varying cosmological "constant" term in the Einstein equation. This possibility has been explored in [103–105]. In these works the cosmological constant is promoted to a dynamical field, which is coupled to the topological terms of gravity providing its dynamics. Interestingly, the cosmologies arising in these models may admit an unusual parity-odd part of torsion, which is consistent with the homogeneity and isotropy of the Universe. We discuss these ideas in Section 7.2. After that, we review our findings from the paper [106] in Section 7.3. There, we further investigated the above proposal, in particular [105]. We have found that, in order to recover a valid cosmological evolution in these models, the parity-odd torsion must be non-vanishing. We further provide an analysis of the propagation of gravitational waves on cosmological backgrounds. We find that in these models the speed of propagation of gravity waves is affected by the parity-odd contributions and differs for the two helicities of the graviton. Based on this finding, in [106] we put strong constraints on the parameters of the model from the results of LIGO/Virgo and Fermi/INTEGRAL measurements of the gravitational waves and electromagnetic signal coming from a binary neutron star merger GW170817 and GRB 170817A [107, 108]. The theory is still viable under the new found constraints.

For the most part of this thesis (Chapters 1 to 6), we are using the mostly negative signature $(+, -, -, -)$, and we work in the natural units $\hbar = c = 8\pi G_N = 1$. We make an exception in Chapter 6, where the constants $\hbar$ and G_N play a major role, and thus their presence is restored when needed. Throughout this thesis we mostly use the reduced Planck mass $M_{Pl}^2 = \hbar c/8\pi G_N$ except for cases when its said otherwise.

In Chapter 7 we switch to the signature $(-, +, +, +)$ in order to remain consistent with the choices made in the published version of the work on which this chapter is based on. The dependence on the Newton constant is being kept explicit as $8\pi G_N = \kappa$.

1. Cosmological constant problem

The cosmological constant problem envelops multiple questions about the vacuum energies that appear in the Standard Model of particle physics and their interaction with General Relativity. With the advent of QFT, we have learned that the vacuum states of field theories carry non-vanishing energy density. While these zero-point energies are often blissfully ignored in the standard treatment of QFT in Minkowski spacetime, in the presence of gravity, they cannot be neglected, and they should in principle contribute to the dark energy [23, 109, 110].

The most popularized part of the problem is the apparent disagreement between the observed density of the dark energy and the theoretical estimates of the magnitude of zero point energies in the SM. It is often stated that the natural order of these contributions is of the order of Planck scale (see [26, 111]):

$$\rho_{\rm vac} = \frac{M_{Pl}^4}{16\pi^2} \ . \tag{1.1}$$

Here M_{Pl} is the reduced Planck mass defined as $M_{Pl}^2 = \hbar c/8\pi G$. This vacuum energy is then added to the bare cosmological constant[1] $\rho_{\rm bare}$ to provide the total dark energy density that we observe (in absence of additional sources of DE)

$$\rho_{\rm obs} = \rho_{\rm bare} + \rho_{\rm vac} \ . \tag{1.2}$$

This is in stark contrast with the recent measurements of the cosmological constant [6]:

$$\Lambda_{\rm obs} = 4.24 \times 10^{-84} \ {\rm GeV}^2 \ , \tag{1.3}$$

or in terms of energy density

$$\rho_{\rm obs} = 5.839 \ \times \ 10^{-33} \ {\rm g} \cdot {\rm cm}^{-3} \ . \tag{1.4}$$

This value is 120 orders of magnitude smaller then the estimate (1.1). In order to resolve this disparity, the bare CC would have to cancel out the vacuum energy contribution with an incredible precision. This requires an unprecedented level of fine tuning, which is physically unacceptable.

However, we would like to stress that the particular discrepancy above is only a smaller part of the problem. Indeed, the estimate (1.1) is actually often regarded as incorrect [26, 29, 112]. Various other attempts to evaluate these zero-point contributions have been proposed (see for example [23, 26, 29, 109, 112]), producing a range of answers that differ in orders and orders of magnitude. However, none have been able to provide the observed value of CC. The large part of the problem and the lesson here is that we currently have no reliable way of estimating the zero point contributions to the dark energy.

In the following section, we are going to review two approaches to estimating the zero point energies. First, we are going to discuss the frequently stated

[1] We will often refer to the vacuum energy density and the cosmological constant interchangeably. The two can always be related as $\Lambda = 8\pi G_N \rho_\Lambda$, where G_N is the Newton constant. This can be extended to other sources of vacuum energy as well.

proposal, which produces the result (1.1) and relies on a cut-off regularization to evaluate the zero-point energy. Second, we are going to discuss the dimensional regularization and renormalization of vacuum energy. We argue that naively applying these simplified results to the SM particles is generally incorrect.

Furthermore, the zero-point energies are not the only corrections to the dark energy from the SM sector. There are also vacuum energies that arise from the non-vanishing potentials of the Higgs field and from the quark and gluon condensates [32, 33]. We are going to briefly review how phase transitions between two minima of a scalar field potential induces a change in the effective CC. However, we will not go to further details as these contributions are not the focus of this thesis.

Finally, we are going to discuss the unimodular gravity as a possible candidate to avoid the problem of zero point energies altogether.

1.1 Quantum cosmological constant problem

The simplest toy model for calculation of the zero point energies is the free massive scalar field in Minkowski spacetime given by the action:

$$S = \frac{1}{2}\int d^4x \Big[\partial_\mu\phi\partial^\mu\phi - m^2\phi^2\Big] , \tag{1.5}$$

where m is the mass of the field. Its associated energy momentum tensor is

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\eta_{\mu\nu}\Big(\partial_\sigma\phi\partial^\sigma\phi - m^2\phi^2\Big) . \tag{1.6}$$

Here $\eta_{\mu\nu}$ is the Minkowski metric. Upon quantization the field operators can be expanded in terms of the associated annihilation operators $a_{\mathbf{k}}$ and creation operators $a^\dagger_{\mathbf{k}}$ as

$$\phi(t,\mathbf{x}) = \frac{1}{(2\pi)^{3/2}}\int \frac{d^3\mathbf{k}}{\sqrt{2\omega_k}}\Big[a_{\mathbf{k}}e^{-i\omega_k t+i\mathbf{k}\cdot\mathbf{x}} + a^\dagger_{\mathbf{k}}e^{+i\omega_k t-i\mathbf{k}\cdot\mathbf{x}}\Big] , \tag{1.7}$$

where

$$\omega_k = \sqrt{k^2+m^2} . \tag{1.8}$$

The bold-face $\mathbf{x}$ and $\mathbf{k}$ refer to the spatial components of $x^\mu = (t,\mathbf{x})$ and the spatial components of the associated four momenta $k^\mu = (\omega_k,\mathbf{k})$ respectively. The Lorentz invariant vacuum state $|0\rangle$ is defined as the unique state annihilated by all the operators $a_{\mathbf{k}}$:

$$a_{\mathbf{k}}\,|0\rangle = 0 . \tag{1.9}$$

The associated vacuum expectation value of the above energy momentum tensor is

$$\langle T_{\mu\nu}\rangle = \langle 0|\,T_{\mu\nu}\,|0\rangle . \tag{1.10}$$

From the above expression, we can infer the vacuum energy density

$$\rho = \langle T_{00}\rangle = \frac{1}{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\omega_k . \tag{1.11}$$

Due to the vacuum averaging, all the off-diagonal terms of $\langle T_{\mu\nu}\rangle$ vanish identically. Furthermore, due to the isotropy of the vacuum, the diagonal spatial terms are equal and can be characterised by a single quantity, the pressure:

$$p = \frac{1}{3}\langle T_{ij}\delta^{ij}\rangle = \frac{1}{6}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\mathbf{k}^2}{\omega_k} \ . \tag{1.12}$$

Both the vacuum energy density and the vacuum pressure are represented as divergent integrals. This can be naively alleviated by introducing a hard cut-off scale. Eliminating arbitrarily high frequencies from the integrals renders them finite. For the above example, we get

$$\rho_M = \frac{1}{(2\pi)^2}\int_0^M dk\, k^2\omega_k \simeq \frac{M^4}{(4\pi)^2} \ , \tag{1.13}$$

where M is the cut-off mass scale. Taking this as an estimate for the zero point energy by taking the Planck mass as the cut-off parameter produces the result:

$$\rho_M \approx 10^{120}\rho_{\rm obs} \ , \tag{1.14}$$

which is famously 120 orders of magnitude larger then the observed value of the cosmological constant (1.3). We can, however, easily argue that the above procedure is in fact incorrect. Indeed, introducing the same cut-off regularization to the pressure (1.12), we find

$$p_M = \frac{1}{(2\pi)^2}\frac{1}{3}\int_0^M dk\frac{k^4}{\omega_k} \simeq \frac{1}{3}\frac{M^4}{16\pi^2} \ . \tag{1.15}$$

At the leading order, this produces the equation of state of radiation

$$p_M = \frac{1}{3}\rho_M \ . \tag{1.16}$$

However, the Lorentz invariance of the vacuum state implies that the energy momentum tensor of the vacuum is of the form

$$T_{\mu\nu}^{\rm vac} = const\ \eta_{\mu\nu} \ . \tag{1.17}$$

Hence the vacuum energy density and pressure should have the following equation of state

$$p_{\rm vac} = -\rho_{\rm vac} \ . \tag{1.18}$$

This is clearly not satisfied by the above results (1.13) and (1.15); therefore, the Lorentz symmetry is broken. In particular this has been pointed out in [29].

This violation of symmetry can be traced to the hard cut-off regularization, which explicitly differentiates between the spatial components and the zeroth component of the four momentum. By performing the hard cut-off in the Euclidean space, the violation of the symmetry can be avoided. The results in the Minkowski spacetime can be then obtained through analytical continuation. This naively yields the same expression for the energy density (1.13); however, the result for the pressure is given by (1.18). Other Euclidean methods point toward this result as well [113]. So, while the introduction of a hard cut-off in Minkowski spacetime is arguably incorrect, the result (1.13) cannot be dismissed as easily.

The breaking of Lorentz symmetry can be avoided in the Minkowski spacetime by using a different scheme to regularize the divergent integrals (1.11) and (1.12). This can be done by using the dimensional regularization on which we focus from now on. The expression for the energy density extended to d spacetime dimensions reads:

$$\rho_{\text{dim}}(\mu) = \frac{\mu^{4-d}}{(2\pi)^{d-1}} \frac{1}{2} \int d^{d-1}\mathbf{k}\, \omega_k \ , \tag{1.19}$$

where μ is parameter with dimension of mass introduced in order to balance the dimensions of the extra momenta in the integral. Evaluating the integral and expanding in $4-d$, we obtain

$$\rho_{\text{dim}}(\mu) \simeq \frac{m^4}{64\pi^2}\left[-\frac{2}{4-d} - \frac{3}{2} + \gamma + \ln\left(\frac{m^2}{4\pi\mu^2}\right)\right] \ . \tag{1.20}$$

Here γ is the Euler-Mascheroni constant. Performing the same procedure for the pressure reveals that the resulting energy density indeed has the correct equation of state:

$$\rho_{\text{dim}}(\mu) = -p_{\text{dim}}(\mu) \ . \tag{1.21}$$

The above expression (1.20) is clearly divergent in $d=4$. We can eliminate this divergence along with the $\gamma - 3/2$ term by using the modified minimal subtractions renormalization scheme [26, 31]. This yields the following estimate for the vacuum energy density:

$$\rho_{\text{vac}} = \frac{m^4}{64\pi^2} \ln\left(\frac{m^2}{\mu^2}\right) \ . \tag{1.22}$$

This result is very different from the expression obtained using the hard cut-off (1.13), as it is fourth order in the mass of the scalar field, not the cut-off parameter. In this approach, massless fields do not contribute any vacuum energy. We should note that in comparison with (1.13), the result (1.22) is renormalized as we have subtracted the divergent terms. We would like to stress that the meaning of the parameter μ is not entirely clear at this point, which diminishes the usefulness of the result (1.22).

A similar calculation can be performed for fermion fields as well as vector fields, giving the same answer up to an opposite overall sign for fermions. The total contribution of these zero point energies for multiple particles yields the following sum:

$$\rho_{\text{vac}} = \sum_i n_i \frac{m_i^4}{64\pi^2} \ln\left(\frac{m^2}{\mu^2}\right) \ . \tag{1.23}$$

The index i denotes the respective particles, n_i is the number of degrees of freedom each particle species carries, as well as the sign of the contribution. This formula has been naively applied to the particles of the SM [112] to produce the following estimate:

$$\rho_{\text{vac}} \approx -10^{56} \rho_{\text{obs}} \ . \tag{1.24}$$

We would like to stress that simply evaluating the above expression for the masses of the SM particles is not really well justified. Indeed, going beyond this basic approximation, we find that, in general, particle masses run with the energy scale. Thus, it is a priori unclear, which mass should enter the above formula. This is even less clear in the SM, where most of the particle masses are generated through the Higgs mechanism and vanish in ultraviolate (UV) limit.

Finally, any interactions in the theory can produce further contributions to the vacuum energy density [26, 30]. These appear from the vacuum Feynman diagrams in the theory. In fact, the expression (1.22) appears as the first loop correction in this picture. With each successive loop, we receive additional contributions to the vacuum energy density, which do not diminish for higher loop orders. Furthermore, while gravitons are massless, they will also produce higher order loop contributions and should be included in the calculation. These considerations prevent us from obtaining any reliable estimate for the zero-point energy. Indeed, any result we obtain for a fixed number of loops gets spoiled by including an additional loop order. Clearly, the zero-point corrections are very sensitive to the UV regimes of the theory.

1.2 Phase transitions

The zero-point energies of quantum fields are not the only possible contribution to dark energy. In this section, we will discuss how non-vanishing effective potentials of scalar fields can induce CC when they undergo a phase transition. We will demonstrate this on the example of a canonical scalar field in the presence of gravity with a cosmological constant Λ and a potential $V(\phi)$. The total action is

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R - \Lambda + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right] . \tag{1.25}$$

Let us consider that the potential has a minimum at $\phi = 0$, and that the field ϕ resides in the associated vacuum. The potential can be in general expanded around this minimum as

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 + \dots \ , \tag{1.26}$$

where m is the mass of the scalar. By plugging this expansion into the action, we can clearly see that the constant term of the potential V_0 can be equally well grouped together with the gravitational sector. Thus, it effectively behaves as a contribution to the cosmological constant

$$\Lambda_{\text{tot}} = \Lambda + V_0 \ . \tag{1.27}$$

These two constituents of the dark energy are indistinguishable, as only the total vacuum energy is observable. The problem arises when there exists an additional minimum of the potential with a lower energy. The expansion (1.26) around this minimum will in general have different parameters:

$$V(\phi) = V_- + \frac{1}{2}\tilde{m}^2(\phi - \phi_-)^2 + \dots \ . \tag{1.28}$$

Here ϕ_- is the location of the minimum, and $\tilde{m}$ is the mass of the scalar associated with the second vacuum. The energy difference between the the vacua is $\Delta V = V_0 - V_- > 0$. Clearly, using this expansion in the action (1.25) results in the total vacuum energy

$$\Lambda_{\text{tot}} = \Lambda + V_- \ . \tag{1.29}$$

Since the second vacuum is energetically preferable, the theory may undergo a (first order) phase transition and tunnel into the second vacuum. This induces a change in the the dark energy by ΔV.

A similar effect can result due to the interaction of the scalar with other fields, for example with a scalar ψ. This interaction may cause an effective change in the total potential, which can lead to a phase transition. To demonstrate this, let us assume the following form of the entire potential

$$V(\phi, \psi) = V(\phi) + \frac{\lambda}{4}\phi^2\psi^2 \ , \tag{1.30}$$

where λ is a coupling constant, which we assume to be positive for the purpose of this discussion. Assuming that the field ψ is in a thermal equilibrium in a relativistic regime with temperature $T \gg m_\psi$, where m_ψ is the mass of ψ, one may replace ψ^2 with an average $\langle\psi^2\rangle_T = T^2/12$ taken in a thermal state with temperature T [114]. The effective potential then reads

$$V(\phi, T) = V(\phi) + \frac{\tilde{\lambda}}{2}\phi^2 T^2 \ . \tag{1.31}$$

Here we have redefined $\tilde{\lambda} = \lambda/24$. Clearly, the effective potential now depends on the temperature. In an expanding universe, this temperature decreases, and the shape of the potential changes accordingly. This may result in some minima of the effective potential to become maxima instead. Consequently, a stable vacuum may turn into a false one. Let us demonstrate this by assuming the potential $V(\phi)$ has a maximum at $\phi = 0$. The second derivative of the effective potential $V(\phi, T)$ at $\phi = 0$ is thus

$$V''(0, T) = V''(0) + \tilde{\lambda}T^2 \ . \tag{1.32}$$

The first term is by assumption negative. For large enough temperatures, the effective potential will have a minimum at $\phi = 0$, as the thermal correction overcomes the negative contribution of the $V''(0)$. However, once the temperature drops bellow the threshold $T^2 = -V''(0)/\tilde{\lambda}$, the minimum changes character and becomes a maximum instead[2]. The associated vacuum turns to a false one as a result. The scalar field then rolls away from this false vacuum until it stabilizes in a new one. The energy difference between the old vacuum and the new one then induces a contribution to the total dark energy as we have seen in the previous example.

A similar thing occurs for the Higgs field during the electroweak crossover [7, 31]. The relevant parameters of the Higgs field potential can be determined completely from the parameters of the SM, which allows us to obtain the energy difference between the two vacua ΔV [26], see also [116]:

$$\Delta V \simeq 1.2 \times 10^8 \ \text{GeV}^4 \simeq 10^{55}\rho_{\text{obs}} \ . \tag{1.33}$$

This energy difference then induces a change in the vacuum energy. Since we measure a tiny cosmological constant today, this suggests that the vacuum energy should have been huge prior to electroweak crossover. The electroweak transition is not unique in the SM. A similar argument can be applied to the QCD transition as well, which produces the following contribution [26]:

$$\Delta V \simeq 10^{-2} \ \text{GeV}^4 \simeq 10^{45}\rho_{\text{obs}} \ . \tag{1.34}$$

[2] This situation may in fact happen in reverse as well, turning a maximum into a minimum when the coupling constant $\tilde{\lambda}$ is negative [115].

1.3 Unimodular gravity

A possible solution to the above problems is unimodular gravity (UG), which has been originally proposed by Einstein [87]. This modification can be introduced on the level of equations of motion as a traceless version of Einstein equations; however, many other classically equivalent formulations exist [88–90, 117, 118]. The Einstein traceless equations read:

$$G_{\mu\nu} - \frac{1}{4} G g_{\mu\nu} = T_{\mu\nu} - \frac{1}{4} T g_{\mu\nu} \ , \tag{1.35}$$

where G and T are the traces of the Einstein tensor and the energy momentum tensor respectively. Since the cosmological constant term is a pure trace term it is nowhere to be found in the above equation. Due to the modified form of Einstein equations the covariant divergence does not vanish identically and instead produces a novel differential constraint in the theory. This way the cosmological constant makes a comeback; however, this time, it enters as an integration constant rather then a fundamental constant. Indeed, taking the divergence of the traceless Einstein equations we obtain

$$\partial_\mu (G - T) = 0 \ . \tag{1.36}$$

Therefore any solution of equation (1.35) must satisfy

$$G - T = 4\Lambda = const. \tag{1.37}$$

Substituting this back into (1.35) we obtain

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu} \ . \tag{1.38}$$

This is equivalent to the Einstein equations with the cosmological constant. It should be stressed that any choice of Λ here is valid, therefore the correct conclusion is not that UG is equivalent to GR, but rather that UG contains all GR theories with various cosmological constants. UG only reduces to GR for a particular choice of this integration constant.

1.4 Actions for unimodular gravity

Unimodular gravity can be derived from an action principle in multiple ways. Historically the most prevalent way was to vary the action with an additional condition of keeping the volume element fixed [89, 90]. That is

$$\delta \sqrt{-g} = 0 \ . \tag{1.39}$$

This condition is usually implemented in a non-covariant way by introducing a Lagrange multiplier that fixes the volume element to a constant, usually a unity, hence the name unimodular gravity:

$$S_{\mathrm{UG}} = \int d^4x \left[-\frac{\sqrt{-g}}{2} R + \lambda\left(\sqrt{-g} - 1\right) \right] \ . \tag{1.40}$$

This formulation has an intriguing advantage: the constraint allows us to eliminate all instances of $\sqrt{-g}$ in the action and thus render the Einstein-Hilbert action and all its couplings to the matter sector polynomial in the metric components [117]. This is potentially advantageous for quantization[3]. This comes at a cost of breaking the diffeomorphism invariance, seemingly requiring us to work with coordinate systems that satisfy the unimodular condition

$$\sqrt{-g} = 1 \ . \tag{1.41}$$

On the classical level any solution in unimodular gravity in arbitrary coordinates can be brought to satisfy the unimodular constraint locally via an appropriate change of coordinates. For this reason the above constraint can be ignored at least for the purpose of classical solutions. The variation of the action (1.40) gives us Einstein equations with a cosmological term that is given by the Lagrange multiplier λ

$$G_{\mu\nu} + \lambda g_{\mu\nu} = T_{\mu\nu} \ . \tag{1.42}$$

By utilizing the Bianchi identities and by assuming the covariant conservation of the energy momentum tensor it follows that λ must be a constant.

The most important form of unimodular gravity for this thesis is due to Henneaux and Teitelboim [88]. In their work, they have performed a canonical analysis of the above scenario and in the process they have found a way to restore the diffeomorphism invariance by introducing a novel vector field. This is the first fully diffeomorphism invariant version of unimodular gravity. The theory also has a form of an additional Lagrange constraint, which, this time, forces a covariant divergence of the novel vector field to be 1:

$$S_{HT}[\lambda, V^\mu, g_{\mu\nu}] = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}R + \lambda\Big(\nabla_\mu V^\mu - 1\Big)\Big] \ . \tag{1.43}$$

Variation of V^μ enforces the constancy of the Lagrange multiplier λ. This way the spacetime dependence of λ is eliminated by a separate equation of motion rather then the Bianchi identity. The constraint produces a non-conservation law

$$\nabla_\mu V^\mu = 1 \ , \tag{1.44}$$

for an associated global charge - cosmic time

$$\mathcal{T} = \int d^3x\sqrt{-g}V^0 \ , \tag{1.45}$$

that is constantly being produced. Equation (1.44) allows us to determine that the cosmic time measures the four volume between two Cauchy hypersurfaces

$$\mathcal{T}_2 - \mathcal{T}_1 = \int d^4x\sqrt{-g} \ . \tag{1.46}$$

The cosmic time is an additional global degree of freedom[4] in the theory. Due to the first order nature of the action in both λ and V^μ it is easy to check [120, 121]

[3] It has been reported that the corresponding quantum theory is equivalent to GR in case of localized perturbative quantities, but inequivalent for non-perturbative quantities on backgrounds of finite volume [119].

[4] A global degree of freedom corresponds to a single extra pair of dimensions in the phase space as opposed to ordinary degrees of freedom which usually contribute a pair per point in space.

that its associated momentum is the Lagrange multiplier λ, which plays the role of cosmological constant. It has been pointed out that this cosmic time does not have all the appropriate properties to assume the role of actual time [118].

It is noteworthy to mention that neither of the above formulations reproduce the traceless equations (1.35), as they introduce a Lagrange multiplier that acts as the cosmological constant instead.

1.5 Eliminating the zero point energies

The crucial property of equation (1.35) is that it is invariant with respect to the shifts of the trace of both the Einstein tensor and the energy momentum tensor respectively. Indeed, both transformations

$$G_{\mu\nu} \to G_{\mu\nu} + c_1 g_{\mu\nu} \ , \qquad \text{or} \qquad T_{\mu\nu} \to T_{\mu\nu} + c_2 g_{\mu\nu} \ , \tag{1.47}$$

leave the equation (1.35) intact. At this point, $c_{1,2}$ are seemingly allowed to have a spacetime dependence; however, from equation (1.36) we can infer that $c_{1,2}$ need to be constants. It is exactly this shift that is produced via the zero-point energies from the previous section (1.17). This property is still manifest in the equation (1.36) as the partial derivative annihilates the constants $c_{1,2}$. Seemingly, these equations are insensitive to any shifts in the vacuum energy and thus we may hope that this would decouple the vacuum energies from gravity at the semiclassical level. At the same time, we must note that once we specify Λ, as in (1.37), the theory reduces to GR and the problem seemingly arises again. From this point of view, it seems that we have merely hidden the issue away. Whether the unimodular gravity indeed solves the problem of quantum corrections to vacuum energy is a matter of ongoing debate. It has been argued that UG does not do anything to alleviate the issue [27, 30, 122]. On the other hand, it has been reported that the loop corrections in unimodular gravity indeed do not contribute to the effective cosmological constant, via a direct calculation in certain models [123–127]. In this discussion we tend to lean toward the opinion that UG does decouple the quantum corrections to the cosmological constant.

In order to understand the issue better, let us briefly discuss the nature of the traceless equations (1.35). Imagine, we are provided with an initial data set on a given Cauchy surface as we would be in GR. We cannot evolve this data as we normally would, because one piece of information is missing, namely the cosmological constant. Indeed, as we have shown, any solution of the traceless equations corresponds to a solution of the standard Einstein equations with some cosmological constant. Thus, for each cosmological constant we get a solution of the Einstein traceless equations. In order to get a unique evolution[5] one needs to pick one of these solutions by hand. This observation is further supported by the canonical analysis of unimodular gravity, which reveals that the corresponding phase space contains one additional global degree of freedom [128]. The subtlety here is in how we specify this solution in our initial data.

In [27, 30] an argument against unimodular gravity has been presented based on the fact that this freedom is fixed by providing the value for Λ in equation (1.37). Indeed, if we fix this value, all the vacuum contributions directly affect

[5] Up to the standard gauge freedoms of GR.

the trace of the Einstein tensor. So, in order to keep the effective (gravitational) cosmological constant small, we need to counter these contributions by retuning our choice of Λ. Alternatively, one can take our choice of Λ in (1.37) and plug it back into (1.35) to obtain

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu} \ . \tag{1.48}$$

This is clearly just the standard GR with a fixed cosmological constant, thus we have gained nothing.

This issue may be circumvented by choosing our constant of integration with reference to only a part of the energy momentum tensor. Consider isolating the quantum vacuum contributions (1.17) to the energy momentum tensor in our semi-classical limit:

$$T_{\mu\nu} = T^{\text{main}}_{\mu\nu} + T^{\text{vac}}_{\mu\nu} \ , \tag{1.49}$$

where $T^{\text{main}}_{\mu\nu}$ describes the entire energy momentum tensor up to the vacuum corrections to the trace. The corrections will be attributed to $T^{\text{vac}}_{\mu\nu}$. Using this split in (1.36) we obtain

$$\partial_\mu\Big(G - T^{\text{main}} - T^{\text{vac}}\Big) = \partial_\mu\Big(G - T^{\text{main}}\Big) = 0 \ , \tag{1.50}$$

where we have used that the quantum corrections contribute only a constant term as in (1.17). Integrating this equation we get

$$G - T^{\text{main}} = 4\Lambda^{\text{main}} \ . \tag{1.51}$$

By definition T^{main} does not receive the quantum corrections and thus the cosmological constant Λ^{main} is stable. Plugging this result back into the traceless equations, we see that the vacuum energies do not play any role in gravitational dynamics

$$G_{\mu\nu} + \Lambda^{\text{main}} g_{\mu\nu} = T^{\text{main}}_{\mu\nu} \ . \tag{1.52}$$

We would like to note that the situation looks to be different when we deal with the theory given from the Lagrangian (1.40). There the Lagrange multiplier λ enters the normal Einstein equations exactly like the cosmological constant would. Thus, we cannot invoke the above argument. From this point of view it might seem that the Lagrange multiplier λ should obtain corrections from the zero-point energies. However, despite this, it has been reported that a direct calculation confirmed that quantum contributions do not affect the observable cosmological constant in this formulation of unimodular gravity [123, 124, 126].

We would like to stress that our argument does not meaningfully depend on the above splitting of the energy momentum tensor (1.49). The split is but a convenient way to present the argument. To make our point stronger, we derive the same conclusion in another way, while avoiding this arbitrary distinction. We will instead work with the entire right hand side as it is given in (1.35). This way we can explicitly see that we never have to worry about any corrections of the form (1.47) in any of the steps. Let us denote the traceless energy momentum tensor as

$$\tilde{T}_{\mu\nu} = T_{\mu\nu} - \frac{1}{4} T g_{\mu\nu} \ . \tag{1.53}$$

At first glance one may think that the above tensor has no information about the trace of the original energy momentum tensor. However, this is in fact not the case! We can infer almost the entire trace from $\tilde{T}_{\mu\nu}$, up to its zero mode which remains obscured. Indeed, consider the following equation for a scalar quantity τ:

$$\partial_\nu \tau = -4\nabla^\mu \tilde{T}_{\mu\nu} \; . \tag{1.54}$$

This equation is always integrable since, by assumption, the traceless tensor $\tilde{T}_{\mu\nu}$ originates from a conserved energy momentum tensor. Any function τ that satisfies the above equation can be added to the traceless tensor $\tilde{T}_{\mu\nu}$ to obtain a conserved energy momentum tensor

$$\tau_{\mu\nu} = \tilde{T}_{\mu\nu} + \frac{1}{4}\tau g_{\mu\nu} \; . \tag{1.55}$$

The quantity τ clearly carries the information about the spacetime dependence of the classical trace of the original energy momentum tensor. The crucial point here is that τ is calculated from the tracless tensor $\tilde{T}_{\mu\nu}$ and as such it does not receive any quantum corrections. Consequently, the new energy momentum tensor $\tau_{\mu\nu}$ is free from any correction of the type (1.47) since it is a sum of two stable tensors. It is this tensor that sources gravitational field in UG. We can show this by taking the divergence of traceless Einstein equations. Hence we find:

$$\partial_\nu G = -4\nabla^\mu \tilde{T}_{\mu\nu} = \partial_\nu \tau \; , \tag{1.56}$$

which is trivially solved as

$$G = \tau - 4\Lambda^{\text{obs}} \; . \tag{1.57}$$

Λ^{obs} is an integration constant and as its name suggests it has the role of the observable cosmological constant. Indeed, plugging the above equation into the traceless equations (1.35) we obtain

$$G_{\mu\nu} + \Lambda^{\text{obs}} g_{\mu\nu} = \tilde{T}_{\mu\nu} + \frac{1}{4}\tau g_{\mu\nu} = \tau_{\mu\nu} \; . \tag{1.58}$$

These are again Einstein equations with a cosmological constant Λ^{obs}. The effective energy momentum tensor on the right hand side is given by the novel $\tau_{\mu\nu}$. Every term in this equation is invariant under the constant shifts of the vacuum energy (1.47) and therefore we again see that the value Λ^{obs} is stable against any vacuum corrections.

While, the unimodular gravity is seemingly able to alleviate the problem of quantum corrections of the cosmological constant it does very little to actually explain its measured value. Indeed, by decoupling CC from the quantum effects of the matter sector the only relevant scale left for comparison is the Planck scale. It seems that UG cannot solve the entire problem, however, it does seem to be a step in the right direction.

2. Recap of Mimetic dark matter and k-essence

In this chapter we introduce two modifications of gravity, which play a key role in the upcoming chapters. These theories are mimetic dark matter [74] and k-essence [93–95].

2.1 Mimetic dark matter

The physical geometry of our spacetime in General Relativity is described solely through the metric tensor $g_{\mu\nu}$ which fully describes the gravitational field. As a way to extend GR it has been proposed that the physical geometry that governs the motion of free falling bodies may not be a fundamental field itself, but rather a composite object. In such a scenario one may require additional equations of motion in order to fully describe the resulting dynamics. A particularly interesting example of such principle is the disformal transformation originally introduced in [129], in which an additional scalar field participates on the final form of the gravitational field. The proposed form of the physical metric is

$$g_{\mu\nu} = C(\phi, Y)h_{\mu\nu} + D(\phi, Y)\partial_\mu\phi\partial_\nu\phi \ . \tag{2.1}$$

Here Y is the kinetic term of ϕ with respect to the auxiliary metric $h_{\mu\nu}$

$$Y = h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \ . \tag{2.2}$$

The upper indices metric $h^{\mu\nu}$ is meant as the inverse of $h_{\mu\nu}$. The functions C and D are subject to few constraints, which ensure that the resulting metric have the expected properties (invertibility, proper signature, etc.). These assumptions are carefully described in the original paper [129].

Considering the above ansatz (2.1) one may generate novel theories of gravity by simply inserting this expression into an existing theory of gravity (a *seed theory*) that includes a metric $g_{\mu\nu}$ as an independent field.

$$S_{dis}[h_{\mu\nu}, \phi, \Psi] = S_{seed}[g_{\mu\nu}(h_{\sigma\rho}, \phi), \Psi] \ . \tag{2.3}$$

The resulting theory clearly falls in the class of scalar-tensor theories. Indeed, the action now depends on $h_{\mu\nu}$ and ϕ together with any other matter fields Ψ that have been present in the theory in the first place. The matter sector does not couple minimally to the metric $h_{\mu\nu}$. Interestingly, for a large class of functions C and D the novel theory describes the exact same dynamics as the original action. By applying (2.1) to GR, we find that as long as

$$C(\phi, Y) - C_Y(\phi, Y)Y + 2D_Y(\phi, Y)Y \neq 0 \ , \tag{2.4}$$

the resulting theory retains the GR dynamics [130, 131]. Note that this relation has to be satisfied for all possible configurations of ϕ as it has been pointed out in [132].

The proposal of mimetic dark matter represents a particular case of disformal transformation of GR. In the original work [74] the authors have aimed to isolate the conformal mode of the metric in a covariant manner. This has been achieved by considering the following composite structure of the physical metric

$$g_{\mu\nu} = h_{\mu\nu} h^{\sigma\rho} \partial_\sigma \phi \partial_\rho \phi \ . \tag{2.5}$$

The key idea here is that the right hand side is manifestly invariant under the Weyl transformations of the auxiliary metric $h_{\mu\nu}$.

$$h_{\mu\nu} \to e^{2\omega} h_{\mu\nu} \ , \tag{2.6}$$

where ω is an arbitrary function parametrizing the transformation. This way the conformal mode of the metric $h_{\mu\nu}$ becomes a pure gauge and is in a way replaced by the dynamics of the scalar field ϕ. By comparing the coefficients in (2.5) with (2.1) we see that

$$C(\phi, Y) = Y \ , \qquad \text{and} \qquad D(\phi, Y) = 0 \ . \tag{2.7}$$

This clearly does not satisfy the condition (2.4) so we can expect deviations from standard GR dynamics.

The action for mimetic dark matter is obtained by taking the standard Einstein-Hilbert action for $g_{\mu\nu}$ and substituting the above expression for every instance of $g_{\mu\nu}$.

$$S_{mim}[h_\mu, \phi, \Psi] = S_{EH}[g_{\mu\nu}(h_{\sigma\rho}, \phi), \Psi] \ . \tag{2.8}$$

To obtain the equations of motion we vary with respect to these new fields. As a consequence of the underlying Weyl symmetry the resulting equations of motion for $h_{\mu\nu}$ have a special form, namely they are traceless. This can be seen by performing an infinitesimal Weyl transformation of the Lagrangian. Let δ_{Weyl} be the generator of the Weyl transformation

$$\delta_{Weyl} h_{\mu\nu} = 2\omega h_{\mu\nu} \ . \tag{2.9}$$

Then its action on the Lagrangian is

$$0 = \delta_{Weyl} \mathcal{L} = \frac{\partial \mathcal{L}}{\partial h_{\mu\nu}} \delta_{Weyl} h_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial h_{\mu\nu}} h_{\mu\nu} 2\omega \ . \tag{2.10}$$

Since this is true for all ω, we get

$$\frac{\partial \mathcal{L}}{\partial h_{\mu\nu}} h_{\mu\nu} = 0 \ . \tag{2.11}$$

The variation of the Lagrangian in this case gives us the equations of motion for the auxiliary metric $h_{\mu\nu}$ up to total derivatives. This implies that the resulting equation is traceless. This holds for both the metric $h_{\mu\nu}$ as well as $g_{\mu\nu}$ since they are conformally related.

Before we derive this explicitly let us show that there is an inherent constraint in the theory that is ultimately responsible for the vanishing of the trace. Following from the ansatz (2.5) the inverse physical metric is

$$g^{\mu\nu} = \frac{1}{h^{\mu\nu} \partial_\mu \phi \partial_\nu} \ . \tag{2.12}$$

Using this we find that the kinetic term for ϕ with respect to the physical metric is fixed to be unity. Indeed, by direct substitution we get

$$g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = \frac{h^{\mu\nu}}{h^{\sigma\rho}\partial_\sigma\phi\partial_\rho\phi}\partial_\mu\phi\partial_\nu\phi = 1 \ . \tag{2.13}$$

Going back to the equations of motion, by varying with respect to the auxiliary metric $h_{\mu\nu}$ we obtain the following equation

$$G_{\mu\nu} - (G-T)\partial_\mu\phi\partial_\nu\phi = T_{\mu\nu} \ . \tag{2.14}$$

The above equation turns out to be Weyl invariant, which allows us to reduce all instances of $h_{\mu\nu}$ to $g_{\mu\nu}$. This is not surprising since the conformal factor differentiating the physical metric from the auxiliary metric is pure gauge. Therefore, we cannot ever determine its evolution from a gauge invariant action without gauge fixing. The gauge invariant information in the theory is fully captured by the composite $g_{\mu\nu}$ and ϕ. For that reason the above equation may be treated as equation for $g_{\mu\nu}$ rather then $h_{\mu\nu}$. Taking the trace of this equation with respect to $g^{\mu\nu}$ reveals that

$$(G-T)\Big(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 1\Big) = 0 \ , \tag{2.15}$$

which is satisfied identically due to the mimetic constraint (2.13). The information about the traces of the Einstein tensor and the energy momentum tensor are clearly lost due to this property. The missing information can be recovered from the equation of motion for ϕ

$$\nabla^\mu\Big((G-T)\partial_\mu\phi\Big) = 0 \ . \tag{2.16}$$

One can equivalently obtain this from taking the divergence of (2.14) and using the *mimetic constraint* (2.13).

Modified Einstein equations (2.14) describe a very simple system, namely, GR with an additional irrotational dust. In this way, it provides a simple candidate for dark matter. The velocity potential of the dust is given by ϕ, while the constraint (2.13) plays the role of normalization of the 4-velocity

$$g^{\mu\nu}u_\mu u_\nu = 1 \ . \tag{2.17}$$

It's energy density is given by

$$\rho_{\text{mim}} = G - T \ . \tag{2.18}$$

The equation of motion for ϕ (2.16) then has the interpretation of the conservation of this density as it reads

$$\nabla_\mu(\rho_{\text{mim}}u^\mu) = 0 \ . \tag{2.19}$$

To evolve these equations we need to provide an additional initial information about the energy density ρ_{mim} together with the initial data for ϕ and the metric $g_{\mu\nu}$.

Unlike, in most gauge theories the Weyl symmetry in mimetic gravity is an empty symmetry [133–135]. This is due to the fact that the conformal mode of the auxiliary field actually vanishes identically from the action. As a consequence there is no associated current or constraint resulting from the symmetry. This allows us to completely factor out the conformal mode by redefining our degrees of freedom in terms of pure Weyl invariants [78]. Doing so, brings the theory into a constraint form where the mimetic constraint (2.13) is enforced via a Lagrange multiplier

$$S[g_{\mu\nu}, \lambda] = \int d^4x\sqrt{-g}\Big(-\frac{1}{2}R(g) + \lambda(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 1)\Big) \ . \tag{2.20}$$

This form of the action was originally discovered in [77] and it is this action that is mostly used in the literature. It is noteworthy that this action can arise in a number of other models, for example in low energy limit of Hořava-Lifshitz gravity [136] or in the "pre-geometric" model [137].

The equation (2.14) has a similar property to the unimodular traceless equations in that both sides are invariant under certain shifts. This time the shifts are

$$G_{\mu\nu} \to G_{\mu\nu} + c_1\partial_\mu\phi\partial_\nu\phi \ , \qquad \text{and} \qquad T_{\mu\nu} \to T_{\mu\nu} + c_2\partial_\mu\phi\partial_\nu\phi \ . \tag{2.21}$$

and can be carried out independently for both Einstein tensor and the energy momentum tensor. The parameters $c_{1,2}$ at this stage are seemingly free to have an arbitrary spacetime dependence. However, upon inspecting the equation of motion for ϕ (2.16) we find that $c_{1,2}$ must satisfy

$$\nabla^\mu\Big(c_{1,2}\partial_\mu\phi\Big) = 0 \ . \tag{2.22}$$

This is similar to what happened with shifts of the vacuum energies in (1.47), where $c_{1,2}$ are forced to be constant. These solutions for c_2 may be interpreted as the only shifts of the energy momentum tensor of the form (2.21) that respect its conservation.

2.2 K-essence

While the cosmological constant has been a very good fit for the model of cosmic expansion, the observational constraints for the dark energy equation of state do not rule out other candidates. This opens avenues for a slowly changing vacuum energy that can dynamically relax to a small value. Such theories are often realized using a scalar field, which, under certain dynamical constraints, is able to provide a very good candidate for the dark energy component. This behavior is familiar from the inflationary theories where the same mechanism is employed to drive the early rapid accelerated expansion of the Universe. This is very intriguing as these models may leave additional imprint in the Universe in the form of clustering perturbations, which could be potentially observable [138]. In order to resolve the dark energy problem, these theories must provide us with a small vacuum like energy density without requiring equally small parameters defining the theory in the first place. For the same reason any need for extreme fine tuning of initial conditions falls flat in resolving the issue. We have to stress that any such resolution of the DE problem assumes that the cosmological constant problem is solved through some other mechanism.

One of the first models that have been proposed along these lines has been the Quintessence [139–143], which utilizes the standard canonical scalar field with a potential. The action for such scalar is:

$$S = \int d^4x\sqrt{-g}\Big(\frac{1}{2}X - V(\phi)\Big) \ , \tag{2.23}$$

with

$$X = \partial_\mu\phi\partial^\mu\phi \ , \tag{2.24}$$

being the kinetic term for ϕ. The associated energy momentum tensor is given as

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\Big(\frac{1}{2}\partial_\sigma\phi\partial^\sigma\phi - V(\phi)\Big) \ . \tag{2.25}$$

Upon requiring that the derivatives are time-like this energy momentum tensor is of a perfect fluid form with the energy density and pressure

$$\rho = \frac{1}{2}X + V \ , \qquad \text{and} \qquad p = \frac{1}{2}X - V \ . \tag{2.26}$$

Hence, we can immediately see that the corresponding equation of state parameter w is

$$w = \frac{p}{\rho} = \frac{X - 2V}{X + 2V} \ . \tag{2.27}$$

Consequently w falls in the range $w > -1$ (given that $\rho > 0$). The lower bound corresponds to a solution where the field sits in a potential well, which yields the entire energy density and pressure. In order to provide the appropriate dark energy behavior the scalar field must evolve slowly so that the energy density and pressure are dominated by the potential energy. This regime is referred to as the slow roll. The potential for the inflaton field is chosen in such a way that it dominates early universe and then dissipates. The potential for quintessence must do the exact opposite, it must dominate in late times yet stay hidden in the early epochs. This presents an additional challenge for the model as its dynamics are significantly affected by the expansion of the Universe during the radiation/matter domination era. This has turned out to be a powerful feature of quintessence as for many potentials [140, 144–149] there exists an attractor solution, during which the quintessence tracks the dominating form of matter. This makes the evolution insensitive to the initial conditions and naturally relaxes the energy density to small values. In order for the quintessence to become the dominating component in the universe the scalar must exit the tracking solution eventually. This mechanism provides an elegant solution to explain the current density of dark energy. However, the mechanism of escaping the tracking solution is largely controlled by choice of the parameters of the potential [95]. In this sense one does introduce a form of fine tuning.

The quintessence theory has been further expanded by considering deviation from the canonical scalar theories. It has been demonstrated that many of the behaviors that are achieved by considering various potentials in quintessence can be alternatively driven by a non-canonical kinetic terms. This is further justified from the point of view of string theory where higher order powers of the kinetic term appear generically in the effective description of massless scalar degrees of freedom. These models have been introduced as k-inflation [150] for early accelerated expansion and later to model dark energy as kinetically driven quintessence or k-essence [93–95]. The most general action for k-essence has the form

$$S = \int d^4x\sqrt{-g}\ K(X,\phi)\ . \tag{2.28}$$

where X is again the kinetic term of ϕ. The associated energy momentum tensor is:

$$T_{\mu\nu} = 2K_X\partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}K\ , \tag{2.29}$$

where K_X denotes a partial derivative of the Lagrangian density K with respect to X. Akin to quintessence, as long as the derivatives of k-essence are time-like, the above energy momentum tensor is of a perfect fluid form with the following energy density

$$\rho = 2K_XX - K\ , \tag{2.30}$$

and the pressure given as the Lagrangian density $p = K(X,\phi)$. The w parameter is

$$w = \frac{K}{2K_XX - K}\ . \tag{2.31}$$

Furthermore, the background configurations of the k-essence field provide an interesting implications for evolution of linear perturbations [151]. Indeed, these perturbation evolve as a free massive field on an emergent geometry that is formed by the background configuration of the k-essence and the underlying spacetime geometry. This geometry first appears on the level of the background equations of motion:

$$\tilde{G}^{\mu\nu}\nabla_\mu\nabla_\nu\phi + 2XK_{X\phi} - K_\phi = 0\ . \tag{2.32}$$

We can see that the characteristic of the equation and thus its causal structure is determined by an emergent metric $\tilde{G}^{\mu\nu}$ [151–153] which is given as

$$\tilde{G}^{\mu\nu} \equiv 2K_Xg^{\mu\nu} + 4K_{XX}\partial^\mu\phi\partial^\nu\phi\ . \tag{2.33}$$

Clearly this metric only supports time evolution in case it retains a Lorentzian signature, that is

$$1 + 2X\frac{K_{XX}}{K_X} > 0\ . \tag{2.34}$$

The above expression arises also as the speed of sound for the associated k-essence fluid [154]

$$c_s^2 \equiv \frac{K_X}{2XK_{XX} + K_X}\ . \tag{2.35}$$

As a consequence the above condition (2.34) is equivalent to hydrodynamical stability

$$c_s^2 > 0\ . \tag{2.36}$$

The evolution of the linear perturbations π of the k-essence field in the absence of perturbations of the metric itself are determined by a conformally related metric $G^{\mu\nu}$. Hence the causal structure that is seen by these perturbations is already encoded within $\tilde{G}^{\mu\nu}$. The perturbed equations of motion read

$$\frac{1}{\sqrt{-G}}\partial_\mu\Big(\sqrt{-G}G^{\mu\nu}\partial_\nu\pi\Big) + M^2\pi = 0\ . \tag{2.37}$$

The metric $G^{\mu\nu}$ is related to $\tilde{G}^{\mu\nu}$ as

$$G^{\mu\nu} \equiv \frac{c_s}{4K_X^2}\tilde{G}^{\mu\nu} \ , \qquad (2.38)$$

and $G = 1/\det G^{\mu\nu}$. The effective mass M^2 can be found in [151]. The metric $G^{\mu\nu}$ can be inverted as

$$G^{-1}_{\mu\nu} = \frac{2K_X}{c_s}\left(g_{\mu\nu} - 2c_s^2\left(\frac{K_{XX}}{K_X}\right)\partial_\mu\phi\partial_\nu\phi\right) \ . \qquad (2.39)$$

Interestingly this metric has the form of a disformal transformation [129] of the physical metric of the underlying spacetime. Since the relation between these two metrics is in general not conformal, the causal structure is expected to differ [151].

The key property that has made k-essence a powerful tool for studying the accelerated expansion of the Universe and dark energy is the presence of tracker and attractor solutions. In contrast to the tracking potentials in quintessence, k-essence can be set up in such a way that it tracks the dominant form of matter only in the radiation-dominated era. On the onset of matter domination the k-essence leaves this tracker and moves toward the de-Sitter attractor where it mimics the cosmological constant. K-essence energy density drops several orders of magnitude during this transition. Despite this drop it eventually becomes the dominant matter component as the Universe expands. This behavior is generic for a wide range of possible models and the tracking and attractor behavior set k-essence on the desired ”track” for a wide range of initial conditions. In this sense k-essence can provide a natural mechanism to escape the tracking of the dominant form of matter [94, 95].

This behavior has been found generically in a simpler class of k-essence models in which the ϕ and X dependence factorizes in the Lagrangian. The dependence on ϕ has to be ϕ^{-2} to ensure the existence of the tracking solutions

$$K(X,\phi) = \frac{1}{\phi^2}\tilde{p}(X) \ . \qquad (2.40)$$

Thanks to the particular ϕ dependence, the equation of motion (2.32) allows for dynamical regimes in which $X = const$. This is necessary for the equation of state (2.31) to become fixed since in this class of models the w parameter depends on X alone.

The crucial advantage of k-essence over the quintessence is the non-vanishing speed of scalar perturbations (2.35). Providing a small but non-vanishing c_s makes k-essence useful as a model of dark matter [153, 155, 156]. K-essence has been also used as a model of superfluids in the limit of zero temperatures and low energies [157–161]. Unfortunately, the higher powers of the kinetic terms give rise to various sorts of instabilities and singularities as well as caustic formation [162–167]. This has motivated search for further extensions of k-essence that could alleviate these problems [168, 169]. K-essence is also a starting point for such a gravity modification as the ghost condensate [170].

3. Mimetic unimodular gravity

This chapter is based on our work ”New Weyl-invariant vector-tensor theory for the cosmological constant” [84] which has been published in the Journal of Cosmology and Astroparticle Physics.

As we have suggested in the previous chapter the original mimetic dark matter scenario [74] shares multiple similarities with unimodular gravity. Indeed, both theories can be described by modified Einstein equations, that are characteristically traceless. This tracelessness implies an invariance of these equations under certain types of shifts of the energy momentum tensor. These shifts are given as (1.47) for unimodular gravity and (2.21) for mimetic dark matter. Furthermore, both theories can be described through an additional Lagrange constraint in the Einstein-Hilbert action. For unimodular gravity this description corresponds to the Henneaux and Teitelboim unimodular gravity (1.44). For the mimetic dark matter, this is just the usual description (2.20). In our paper we have aimed to make this connection concrete by recovering unimodular gravity by modifying mimetic dark matter.

Prior to our work [84], most efforts at extending the mimetic dark matter scenario have focused on modifications of the seed action. These extensions usually introduced additional ϕ dependent terms, like a potential for the mimetic scalar or higher order derivative terms [76, 79–81, 171–174]. Through these extensions the mimetic dark matter can mimic nearly any gravitational properties of normal matter. This way mimetic dark matter could be used as a candidate for dark energy or to drive inflation. The higher derivative terms can further provide nontrivial speed of sound for the mimetic scalar perturbations or even change the propagation of gravity waves and thus makes for a more realistic model of dark matter. However, despite these desirable properties, this route always leads to the same mimetic constraint appearing in the action.

In our effort we have used a different strategy at extending mimetic dark matter. Namely, we have changed the mimetic substitution itself. A similar route has been explored in [82]. Where the authors have considered replacing the kinetic term of a scalar field by a kinetic term for Yang-Mills gauge field. Aiming to obtain the Henneaux and Teitelboim constraint (1.44) we have proposed the following mimetic substitution:

$$g_{\mu\nu} = h_{\mu\nu}\sqrt{\nabla_\sigma V^\sigma} \ . \tag{3.1}$$

Here the covariant derivative is compatible with the auxiliary metric $h_{\mu\nu}$. Our key observation for this construction is that the above ansatz can be made Weyl invariant by extending the action of the Weyl group onto the vector field V^μ:

$$V^\mu \to \omega^{-4} V^\mu \ , \qquad \text{as} \qquad h_{\mu\nu} \to \omega^2 h_{\mu\nu} \ . \tag{3.2}$$

In our paper [84] we have explored this ansatz in detail, and have confirmed that it indeed classically reproduces unimodular gravity. We have analyzed this equivalence both on the level of equations of motion as well as on the level of the action. We have explicitly shown that by reparametrizing this theory in terms of (Weyl) gauge invariant variables, the theory reduces to the Henneaux and Teitelboim unimodular gravity. We have shown that in the original variables our model is a higher derivative vector-tensor theory. Higher order derivatives often signify the presence of the Ostrogradsky instability [175]. In our model this instability is avoided as a result of the underlying Weyl symmetry. Interestingly, the Hamiltonian of the theory [128] is in fact unbounded from bellow, but the unbounded piece is constrained to be a constant.

For the full text of the paper [84], please see Attachment 1 of this thesis.

4. Axionic cosmological constant

This chapter is based on our work "Axionic cosmological constant" [85] that is available on the server arxiv.org, which will be submitted to a peer-reviewed journal.

Following our paper [84], we have further investigated the possible modifications of the mimetic substitutions. We have noticed that our previous proposal can be alternatively realized by demoting the vector field V^μ from an independent variable to a composite one. In particular, we have considered V^μ as the Chern-Simons current for a Yang-Mills gauge field A_μ:

$$V^\mu = 2E^{\mu\nu\sigma\rho} A_\nu D_\sigma A_\rho \ , \tag{4.1}$$

where $E^{\mu\nu\sigma\rho}$ is the Levi-Civita tensor defined with respect to the auxiliary metric $h_{\mu\nu}$ and D_σ is the covariant derivative associated to A_σ. The mimetic ansatz thus becomes

$$g_{\mu\nu} = h_{\mu\nu} \sqrt{F_{\sigma\rho}\tilde{F}^{\sigma\rho}} \ , \tag{4.2}$$

where $\tilde{F}^{\sigma\rho}$ is the Hodge dual of the field strength $F_{\sigma\rho}$. In our paper [85] we have explored this proposal in detail. We have found that the solutions of the theory are, in fact, unaffected by the novel composite structure of V^μ. In other words, the above mimetic substitution produces a new formulation of unimodular gravity. This novel formulation has several notable advantages over our original proposal [84]. First of all, the Chern-Simons current is naturally a vector field of conformal weight 4 due to its dependence on the Levi-Civita tensor. This eliminates the need for the introduction of non-trivial Weyl transformations for the gauge fields themselves. Secondly, the resulting divergence of the Chern-Simons current is the Pontryagin term, which, unlike its predecessor $\nabla_\mu V^\mu$, does not depend on the derivatives of the metric. This inherently simplifies the structure of the equations of motion, which, in this case, results in the Einstein traceless equations (1.35). Additionally, the appearance of the Pontryagin term in the mimetic substitution highlights a closer connection of our proposal with the model from [82, 83], where the kinetic term of a gauge field has been used to provide the mimetic substitution. Finally, the gauge fields A_μ are very natural objects in the SM. This makes our proposal [85] an advantageous starting point for further extensions of the unimodular gravity.

We have shown that our theory can be re-expressed in terms of (Weyl) gauge invariant variables. There the cosmological constant takes the form of a Lagrange multiplier, which is linearly coupled to the Pontryagin term. This way the multiplier obtains an axion-like coupling to the gauge fields A_μ. Consequently, the cosmological constant is equal to the θ parameter of the corresponding gauge theory. In our work [85] we have suggested that the similarity with axion can be in fact taken much further. In particular, we have suggested that one can promote the Lagrange multiplier to a scalar field, and provide it with a potential and a kinetic term. Surprisingly, doing so has very little consequences for the theory, which still describes unimodular gravity.

Substituting the ansatz (4.2) in the square root of the physical metric determinant effectively results in a replacement of the metric volume by the Pontryagin density. In this sense, the notion of volume is given by the Yang-Mills fields rather then by the metric. Furthermore, the Pontryagin density is a total derivative. Any form of vacuum energy induced from quantum corrections enters the action as a cosmological constant. In other words it only couples to $\sqrt{-g}$. Upon the substitution (4.2) this term turns to a total derivative and thus the induced vacuum energy naively drops from the action. In this sense we can see why our proposal decouples the zero-point energies from gravity. This mechanism has been discovered from a different perspective in [176] prior to our work.

The specific structure of the gauge group in our proposal is unimportant for most of the discussion. For this reason we work mostly with the $U(1)$ group for simplicity. We only generalize to non-Abelian groups later on. However, the gauge group becomes important when we address the existence of solutions of our theory. The equations of motion in our model correspond to those of unimodular gravity and as such they are guaranteed to provide solutions. However, on top of these equations, there is additionally the mimetic constraint. This constraint is completely decoupled from the physical sector of the theory, yet, in order for our model to be consistent, we have to guarantee that it is solvable in general. We have found that this is indeed the case as long as the gauge group contains $SU(2)$ as a subgroup, and the spacetime is globally hyperbolic. We include a proof of this existence in the next section.

For the full text of the paper [85], please see Attachment 2 of this thesis.

4.1 Existence of solutions for the mimetic constraint

In the above work we have neglected to prove that the mimetic constraint

$$1 = F^a_{\mu\nu} F^{\star\mu\nu}_a \ . \tag{4.3}$$

has a solution for the gauge fields A_μ for arbitrary volume element $\sqrt{-g}$. It turns out that these solutions exist in general as long as the gauge group contains $SU(2)$ as a subgroup and the spacetime manifold is globally hyperbolic. The key observation is that any globally hyperbolic spacetime is diffeomorphic to $M = \mathbb{R} \otimes N$ [177], where the $\mathbb{R}$ may be associated with a time direction in the spacetime. N is a 3-dimensional manifold, which, we further assume, is orientable. This ensures that N supports a global frame [178] and by extension the entire spacetime does as well [179]. Let us denote the global basis on N as

$$e^a_i \ . \tag{4.4}$$

The latin index takes on values $a = 1, 2, 3$. Note, that the global basis is in general not orthonormal. This basis can be extended to the full manifold M as e^a_μ by prescribing

$$e^a_0 = 0 \ , \tag{4.5}$$

$$\partial_0 e^a_\mu = 0 \ , \tag{4.6}$$

where the 0 direction is associated with the $\mathbb{R}$ part of the manifold M. We consider the following ansatz for the $SO(2)$ gauge fields[1]

$$g' A^a_\mu = \alpha e^a_\mu \ . \tag{4.7}$$

Here g' is the coupling constant of the $SU(2)$ theory. This clearly does not respect the gauge symmetry as on one hand the gauge fields transform under $SO(2)$, while on the other side the frame fields do not. The above equation should be understood as equality of components in a certain gauge. The corresponding field strength is

$$g' F^a_{\mu\nu} = \partial_\mu(\alpha e^a_\nu) - \partial_\nu(\alpha e^a_\mu) + \alpha^2 \epsilon^a{}_{bc} e^b_\mu e^c_\nu \ , \tag{4.8}$$

$$= 2\partial_{[\mu}\alpha e^a_{\nu]} + 2\alpha\partial_{[\mu} e^a_{\nu]} + \alpha^2 \epsilon^a{}_{bc} e^b_\mu e^c_\nu \ . \tag{4.9}$$

Calculating the Pontryagin density yields

$$\frac{g'^2}{2}\epsilon^{\mu\nu\sigma\rho} F^a_{\mu\nu} F^b_{\sigma\rho} = 2\alpha^2 \partial_\mu \alpha e^a_\nu e^b_\sigma e^c_\rho \epsilon_{abc}\epsilon^{\mu\nu\sigma\rho} + 4\alpha\partial_\mu\alpha e^a_\nu \partial_\sigma e^b_\rho \epsilon^{\mu\nu\sigma\rho}\delta_{ab} \ . \tag{4.10}$$

The only terms that are left after the anti-symmetrization have a single derivative of α. Terms with two derivatives get annihilated due to the anti-symmetry of $\epsilon^{\mu\nu\sigma\rho}$, while terms with no derivative have no time components and thus vanish as well. In total we get

$$\frac{g'^2}{2}\epsilon^{\mu\nu\sigma\rho} F^a_{\mu\nu} F^b_{\sigma\rho} = 2\alpha^2 \dot\alpha e^a_i e^b_j e^c_k \epsilon_{abc}\epsilon^{ijk} + 4\alpha\dot\alpha e^a_i \partial_j e^b_k \epsilon^{ijk}\delta_{ab} \ . \tag{4.11}$$

We may further simplify this expression by introducing the factor β as[2]

$$2 e^a_i \partial_j e^b_k \epsilon^{ijk}\delta_{ab} = -\beta e^a_i e^b_j e^c_k \epsilon_{abc}\epsilon^{ijk} \ . \tag{4.14}$$

Note that β is not an unknown in the problem as it is given completely by the frame fields that we are using for the construction. Using this definition we may write

$$\frac{g'^2}{2}\epsilon^{\mu\nu\sigma\rho} F^a_{\mu\nu} F^b_{\sigma\rho} = 2\alpha(\alpha-\beta)\dot\alpha e^a_i e^b_j e^c_k \epsilon_{abc}\epsilon^{ijk} \ , \tag{4.15}$$

$$= 12\alpha(\alpha-\beta)\dot\alpha e \ . \tag{4.16}$$

[1] Note that the gauge fields and the volume element actually live on the original spacetime and not on M. These two manifolds are, however, by assumption diffeomorphic. Therefore, there exists a diffeomorphism which can be used to map these objects from one manifold to the other. For the sake of simplicity we do not write this explicitly in the equations.

[2] The function β can be derived from the coefficients of anholonomy $c^a{}_{bc}$ (see for example [180]) that are defined for any frame as

$$\partial_{[\mu} e^a_{\nu]} + \frac{1}{2} c^a{}_{bc} e^b_\mu e^c_\nu = 0 \ . \tag{4.12}$$

Plugging this into (4.12) we can see that

$$\beta = \frac{1}{6}\epsilon^{abc} c_{abc} \ . \tag{4.13}$$

Note that we raised latin indices using δ_{ab}.

The e stands for the determinant of the frame fields $\det(e_i^a)$ and it is not equal to unity since the frame does not need to be orthonormal. Similarly to β, e is a known quantity fixed by our choice of e_i^a. Plugging this result into the mimetic constraint we obtain

$$12\alpha(\alpha - \beta)\dot{\alpha}e = g'^2\sqrt{-g} \ . \tag{4.17}$$

This is a first order ordinary differential equation that can be easily solved. Indeed, integrating both sides along the time direction we obtain

$$\alpha^3 - \frac{3}{2}\alpha^2\beta = \frac{1}{4}\frac{g'^2}{e}\int dt\sqrt{-g} \ . \tag{4.18}$$

Since this is a cubic equation we are guaranteed to have at least one real solution for α. Plugging it back into (4.7) yields the sought after solution for the gauge fields that satisfies the mimetic constraint.

5. Mimetic k-essence

This work is based on a yet unpublished work ”Mimetic k-essence”, that will be made available soon on arxiv.org

As we have seen, the mimetic gravity is able to provide us with both a simple model of dark matter, in the form of the original proposal [74], or a simple model of dark energy, in the form of our models [84, 85]. This leads to a natural question: Can mimetic gravity provide us with both components of the dark sector? In our work [92] we investigate a possible approach to answering this question. The key idea in this paper is that the mimetic conformal factor does not need to be made out of a single term. By allowing multiple fields to participate in the definition of the physical metric we can propose more complicated mimetic substitutions. We only require that the physical metric is Weyl invariant. This clearly opens doors to a vast amount of possible extensions. Indeed, allowing for any additional term or field in the mimetic substitution increases the amount of combinations we may consider significantly.

In [92] we have attempted to combine the dark matter [74] with the dark energy [85] by allowing both the kinetic term of a scalar field and the Pontryagin term to enter the mimetic conformal factor. This results generically in a k-essence theory with an extra global degree of freedom, which corresponds to the overall energy scale of the k-essence. This alternative mimetic description exists for nearly arbitrary k-essence. A notable exception to this stems from the fact that the energy component coming from a mimetic theory can never have the ultra-relativistic equation of state. Trying to find a mimetic description for an ultra-relativistic k-essence results in the associated conformal factor being ill-defined. We have found a general method at finding a mimetic description for a given k-essence.

The effective k-essence Lagrangian arises from the mimetic theory from an implicit equation. This equation may, in principle, have multiple solutions and thus our model may provide multiple k-essence theories simultaneously. These may then appear in superposition once we turn to quantum theory. Such cases may be engineered through our above method as well.

Finally we investigate the importance of Weyl symmetry of the setup. Interestingly, the conformal mode that typically disappears from the mimetic theory, becomes an auxiliary field or a Lagrange multiplier. We show that in the former case this mode can be integrated out of the action leaving us with a Weyl invariant theory of the above mimetic type. Surprisingly, if the conformal mode behaves as a Lagrange multiplier, the theory becomes equivalent to GR with two constraints. If these constraints are compatible with each other the theory describes mimetic dark matter with a potential, whose overall scale is given by the additional global degree of freedom, that is still present in the theory. If the associated potential is flat, then our model provides both mimetic dark matter and unimodular gravity simultaneously.

For the full text of the paper [92], please see Attachment 3 of this thesis.

6. Losing the trace of the Einstein equations

This chapter is based on the work ”Losing the trace to find dynamical Newton or Planck constants” [86] that has been published in Journal of Cosmology and Astroparticle Physics.

The origins of unimodular gravity can be traced to the proposal of traceless Einstein equations [87]

$$G_{\mu\nu} - \frac{1}{4} G g_{\mu\nu} = T_{\mu\nu} - \frac{1}{4} T g_{\mu\nu} \ . \tag{6.1}$$

A notable feature of these equations is that the trace is eliminated in a way that hides any information about the cosmological constant. As we have seen in mimetic theories one can loose the trace in other ways using additional fields in the theory. In this sense the original traceless Einstein equations are somewhat minimal in that they do not require any new field content to postulate.

The work presented in this chapter is based on a single key observation: the trace part of Einstein equations can be trivialized in another minimal way without the need to introduce additional fields. Instead of eliminating the trace of each side of the equation, we propose that the trace is instead made to be equal to 1. This can be achieved by dividing each side of Einstein equations by its respective trace as:

$$\frac{G_{\mu\nu}}{G} = \frac{T_{\mu\nu}}{T} \ . \tag{6.2}$$

This makes the trace part of the equations a trivial identity $1 = 1$. Interestingly, the proposed modification eliminates the Newton constant from the equation. In this sense, the above equations (6.2) are scale free. We have shown that this model is in fact equivalent to ordinary General Relativity with an arbitrary Newton constant, which enters as a constant of integration in our setup. The comeback of the Newton constant is very similar to the appearance of the cosmological constant in Einstein traceless equations (1.35). They both result from the Bianchi identity and the covariant conservation of the energy momentum tensor. In this sense, our theory is complementary to unimodular gravity as we have two free parameters in Einstein equations (G_N and Λ) and two ways to eliminate the trace. Each of them results in one parameter vanishing only to return as an integration constant.

We further explore various other implementations of our proposal on the level of the equations of motion and on the level of the action. Drawing on the similarities with unimodular gravity we have found that our model can be realized by a Lagrange multiplier that is constrained to a constant by an additional vector field, like in Henneaux and Teitelboim unimodular gravity [88]. The constancy of the Lagrange multiplier can be alternatively achieved by coupling it to a Pontryagin term of a gauge field, like in [85]. The Lagrange multiplier in this approach either couples to the Einstein-Hilbert term of the action, where its plays the role of an inverse of the Newton constant, or it couples linearly to the entire matter Lagrangian. In the latter option the multiplier rescales the Newton constant, which classically provides the same effect. Upon quantization, we have found that the latter option additionally leads to a rescaling of the Planck constant.

By promoting the Newton constant to a global degree of freedom, the "constant" becomes an operator upon quantization. Consequently it is subjected to quantum fluctuations and their corresponding uncertainty relations. These fluctuations may play an important role near cosmological and black hole singularities. It is interesting to note that the conjugate quantity of the promoted constant may vary for different formulations. When the Lagrange multiplier couples to the Einstein-Hilbert term the conjugate quantity to the inverse Newton constant is the spacetime integral of the Ricci curvature or in other words the Einstein Hilbert action. In the second case, where the multiplier couples to the matter Lagrangian, the conjugate quantity is the matter sector action. Note that these quantities are also subjected to the quantum fluctuations.

For the full text of the paper [86], please see Attachment 4 of this thesis.

7. Varying cosmological ”constant” in parity violating Friedman universe

In this chapter we focus on a class of models that allow for a variable cosmological ”constant” without changing the form of Einstein equations. This idea has been proposed in [103, 104] and developed in [105]. This approach is very different from the dark energy scenarios like quintessence since the promoted cosmological constant has no kinetic term. Rather its dynamics are driven by its interaction with the topological terms of the gravitational sector. In this chapter we first introduce the framework of Einstein-Cartan gravity, on which the above papers are based on, and briefly review the above extensions. Then we present our findings that are based on the work ”Gravity waves in parity-violating Copernican Universes” that has been published in Physical Review D [106]. In this work we analyse the propagation of tensor perturbation in the above extensions and through it we provide constraints on the parameters of the model.

7.1 Einstein-Cartan gravity

At first sight the description of General Relativity seem to be much different from the description of the other fundamental forces of the Standard Model [181, 182]. The latter are different realization of the Yang-Mills theory [183] for various groups and the forces themselves are being mediated by the connection forms associated to the said groups [184–187]. The Yang-Mills gauge fields A_μ couple to matter only through the covariant derivative

$$D_\mu = \partial_\mu + igA^c_\mu \mathrm{f}^{abc} \ . \tag{7.1}$$

Here g is the coupling constant, f^{abc} are the structure constants of the associated gauge group and the latin indices are indices of the Lie algebra of the group. The presence of the gauge fields in the covariant derivative ensures that the differentiated objects transform covariantly under the action of the underlying group. This is very similar to the covariant derivative of general relativity, where such compensation is needed in order to preserve the covariant transformation of derivatives of tensors under diffeomorphisms:

$$\nabla_\mu = \partial_\mu + \Gamma^\rho_{\nu\mu} \ . \tag{7.2}$$

The stark difference between (7.1) and (7.2) is that A_μ is an independent field while $\Gamma^\sigma_{\nu\mu}$ is not. Instead, it is the Levi-Civita connection given through the standard expression from the metric:

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}\Big(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}\Big) \ . \tag{7.3}$$

The metric itself plays a very different role in the theory. It couples to the matter sector through the associated volume form and by contracting indices in kinetic terms of the matter fields. This clearly provokes the question: Why is the affine connection in GR not the independent field and how does metric fit into the picture of gauge theory?

Einstein-Cartan Gravity is one of several theories that tries to bridge the gap between GR and Yang-Mills by capitalizing on their similarities [188, 189]. It does this by promoting the affine connection in GR to an independent field, while the metric stays in the picture. This promotion can be done in several ways as there is a lot of freedom in our choice of the affine connection. These ways are easily understood by recalling which considerations lead us to the Levi-Civita connection in the first place. These are the metric compatibility condition

$$\nabla_\mu g_{\nu\sigma} = 0 \ , \tag{7.4}$$

and the requirement of vanishing torsion

$$\Gamma^\sigma_{\mu\nu} - \Gamma^\sigma_{\nu\mu} = 0 \ . \tag{7.5}$$

Upon these two constraints the affine connection is uniquely fixed as (7.3). To make the connection independent we must abandon at least one of these requirements. In Einstein-Cartan gravity we relax the vanishing torsion condition while keeping the metric compatibility. Note, that a complementary approach is the Palatini formalism [190, 191] in which the metric compatibility is relaxed, while torsion is fixed to vanish [25]. There is also the most general approach called metric affine gravity in which both conditions are relaxed [192].

The curvature tensor for an independent connection has the same form as in GR and as the field strength for a Yang-Mills field. That is

$$R^\sigma_{\rho\mu\nu} = \partial_\mu \Gamma^\sigma_{\rho\nu} - \partial_\nu \Gamma^\sigma_{\rho\mu} + \Gamma^\sigma_{\iota\mu}\Gamma^\iota_{\rho\nu} - \Gamma^\sigma_{\iota\nu}\Gamma^\iota_{\rho\mu} \ . \tag{7.6}$$

Since we have abandoned the symmetry in the lower indices of $\Gamma^\sigma_{\nu\mu}$ one has to be wary of the index order in this expression. Note that the curvature depends purely on the connection $\Gamma^\sigma_{\nu\mu}$. The Ricci tensor and Ricci scalar can be formed in the standard way, however, the Ricci tensor $R_{\mu\nu}$ no longer needs to be symmetric. The Einstein-Cartan action is a minimal deviation from the Einstein-Hilbert action:

$$S[g_{\mu\nu}, \Gamma^\rho_{\iota\sigma}] = \int d^4x \sqrt{-g} g^{\mu\nu} R_{\mu\nu}(\Gamma) \ . \tag{7.7}$$

In contrast to GR this action is first-order in derivatives. As long as no other fields in the theory couple to the connection $\Gamma^\rho_{\nu\mu}$ this action replicates GR as the equation of motion for $\Gamma^\rho_{\nu\mu}$ provides the condition of vanishing torsion. Once additional couplings are introduced, this is no longer the case. Einstein-Cartan gravity is usually formulated in a different set of variables than $g_{\mu\nu}$ and $\Gamma^\rho_{\nu\mu}$. This set is the spin connection $\omega^A{}_{B\mu}$ and the tetrad fields e^A_μ which appear very naturally from the need to accommodate spinors on a curved spacetime. In order to do so, one has to forgo the metric in favor of the tetrad fields. These two are related by the following relation

$$g_{\mu\nu} = e^A_\mu e^B_\nu \eta_{AB} \ . \tag{7.8}$$

The Latin index is acted upon by an additional Lorentz group[1] that is disassociated from the diffeomorphisms of the spacetime manifold. This Lorentz group

[1] We use capital Latin indeces from the beginning of the alphabet ($A,\ B,\ C\ldots$) for the Lorentz group indeces and capital Latin indeces from the middle of the alphabet ($I,\ J,\ K\ldots$) for the $SO(3)$ subgroup indeces.

is essential for the introduction of fermionic fields. Indeed, there are no spinorial representations of general diffeomorphisms and thus we need a new Lorentz group to define spinors. Using the tetrad fields the affine connection induces a connection fields for this Lorentz group - the spin connection:

$$\omega^A_{\ B\mu} = e^A_\nu \Gamma^\nu_{\ \sigma\mu} e^\sigma_B + e^A_\nu \partial_\mu e^\nu_B \ . \tag{7.9}$$

Note that the spin connection can be introduced in GR as well.

In Einstein-Cartan gravity the tetrad e^A_μ and $\omega^A_{\ B\mu}$ are usually taken to be the fundamental variables, making the Lorentz group the fundamental group of this theory[2]. Furthermore, since these fields only carry a single lower spacetime index, they can be naturally thought of as differential 1-forms as it is the case in other gauge theories. The entire theory can be very elegantly described within the exterior algebra. For a detailed introduction to differential forms, exterior algebra and exterior calculus see for example [180, 193]. This allows us to drop all the spacetime indices, keeping in mind that the objects are forms and any multiplication carried among them is meant to be the exterior product $\wedge$:

$$e^A_\mu \to e^A \ , \tag{7.10}$$

$$\omega^A_{\ B\mu} \to \omega^A_{\ B} \ . \tag{7.11}$$

To provide a quick reference for the further sections we list several standard identities in the language of forms. The exterior covariant derivative is schematically given as

$$D = d + \omega^A_{\ B} \ , \tag{7.12}$$

here d is the exterior derivative. This allows us to express the torsion 2-form as

$$De^A = T^A \ , \qquad \text{where} \qquad T^A = \frac{1}{2} e^A_\rho T^\rho_{\mu\nu} dx^\mu \wedge dx^\nu \ , \tag{7.13}$$

The curvature 2-forms are given as a field strength of the spin connection

$$R^A_{\ B} = d\omega^A_{\ B} + \omega^A_{\ C}\omega^C_{\ B} \ , \qquad \text{where} \qquad R^A_{\ B} = \frac{1}{2} e^A_\rho e^\sigma_B R^\rho_{\sigma\mu\nu} dx^\mu \wedge dx^\nu \ . \tag{7.14}$$

The curvature 2-forms are annihilated by the covariant derivative

$$DR^A_{\ B} = 0 \ . \tag{7.15}$$

In general the repeated use of the exterior covariant derivative reduces to a multiplication by the curvature 2-forms. For example

$$DT^A = D^2 e^A = R^A_{\ B} e^B \ . \tag{7.16}$$

The Einstein-Cartan action (7.7) with cosmological constant expressed in terms of forms can be written as

$$S[e^A, \omega^A_{\ B}] = \int \epsilon_{ABCD} \Big(e^A e^B R^{CD} - \frac{\Lambda}{6} e^A e^B e^C e^D \Big) \ . \tag{7.17}$$

[2]One can further consider the tetrads to be a connection for translations of the representation space of the Lorentz group. This makes the theory a Poincare gauge theory.

Here the integral sign denotes the integration of 4-forms on a manifold. The resulting equations of motion for e^A are

$$\epsilon_{ABCD}\Big(e^B R^{CD} - \frac{\Lambda}{3} e^B e^C e^D\Big) = -2\kappa\tau_A \ , \tag{7.18}$$

where $\tau_A = \frac{1}{2}\frac{\delta S}{\delta e^A}$ and $\kappa = 8\pi G_N$. The equations for the spin connection $\omega^A{}_B$ are

$$\epsilon_{ABCD} e^C T^D = 0. \tag{7.19}$$

The right hand side may be considered to be non-zero in case we consider matter couplings to the spin connection. These couplings naturally appear in the kinetic terms of spinors and they cause deviations from GR.

7.2 The varying cosmological constant

In this chapter we introduce some of the ideas that were proposed in the papers [103–105].

As we have seen in the previous chapters a possible strategy in addressing the cosmological constant problem is to promote the CC to a dynamical field. This can be beneficial in multitude of ways. In unimodular gravity the cosmological constant becomes a Lagrange multiplier, which allows us to decouple the zero-point energies from the gravitational dynamics. In other proposals the spacetime dependence may provide a mechanism to relax the value of the cosmological constant to the unnaturally small value that we observe today. This is exactly what has been explored in quintessence scenarios and other various dark energy candidates [143].

Our ability to provide a spacetime variability to the cosmological constant is severely restricted by the Bianchi identities and the covariant conservation of the energy momentum tensor. Assuming the standard form of Einstein equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \ , \tag{7.20}$$

and taking the four divergence we quickly find that the only consistent option is $\Lambda = const.$ Indeed, any spacetime dependence of Λ must be compensated somehow. Since the form of the Einstein tensor is fixed in GR, the price must be paid on the right hand side of the equation by breaking the conservation of the energy momentum tensor. Consequently, any vacuum solution is still forced to a constant Λ as the right hand side is conserved identically.

This can be circumvented in Einstein-Cartan gravity. There the Einstein tensor is made out of torsionful curvature and as a consequence the usual Bianchi identities do not apply. This allows us to balance the spacetime dependence of Λ against the torsion contributions in the Einstein tensor, while keeping the conservation of the energy momentum tensor intact. This makes the varying cosmological ”constant” possible even in the absence of matter.

These considerations result in a severely constrained space of options we can look at. Indeed, any kinetic terms for Λ are forbidden as they produce contributions to the energy momentum tensor. In fact, Λ cannot be coupled to the tetrad anywhere else but the cosmological constant term. By similar reasoning one can find that Λ cannot couple to the matter fields either as that would spoil the conservation of energy and momentum. Indeed, any coupling to matter fields results in Λ appearing in the matter equations of motion, which are necessary for the vanishing of the divergence of the stress energy tensor. Λ cannot remain uncoupled either as that would make it a Lagrange multiplier constraining an unphysical constraint $\sqrt{-g} = 0$. This leaves us only with terms that are purely formed from the spin connection $\omega^A{}_B$ and since the spin connection does not transform homogeneously under the Lorentz group we are in fact left with terms that depend only on Lorentz invariants of the curvature $R^A{}_B(\omega)$.[3] This way we find that Λ may only couple to the Euler term and the Pontryagin invariant[4]

$$I_{\text{Euler.}} = \epsilon_{ABCD} R^{AB} R^{CD}, \tag{7.21}$$

$$I_{\text{Pont.}} = R^{AB} R_{AB} \ . \tag{7.22}$$

The remaining freedom in the choice of this coupling has been fully fixed in [103, 104] by requiring that the action is parity even and is invariant under the following swap operation

$$R^{AB}(\omega) \leftrightarrow \frac{\Lambda}{3} e^A e^B \ . \tag{7.23}$$

This has lead to a simple extension of the Einstein-Cartan gravity where the Λ is coupled to the Euler term in such a way to achieve the above invariance [103, 104]:

$$S[e^A, \omega^A{}_B, \Lambda] = \int \epsilon_{ABCD} \Big(e^A e^B R^{CD} - \frac{\Lambda}{6} e^A e^B e^C e^D - \frac{3}{2\Lambda} R^{AB} R^{CD} \Big) \ . \tag{7.24}$$

Since this action does not include any new terms dependent on the tetrad field e^A the form of Einstein equations remains unchanged as intended:

$$\epsilon_{ABCD} \Big(e^B R^{CD} - \frac{\Lambda}{3} e^B e^C e^D \Big) = -2\kappa \tau_A \ . \tag{7.25}$$

On the other hand, the equation for the spin connection obtains a non-trivial right hand side in comparison to (7.19)

$$T^{[A} e^{B]} = -\frac{3}{2\Lambda^2} d\Lambda R^{AB} \ . \tag{7.26}$$

This produces a non-trivial torsion. Since we have promoted Λ to an independent field we also obtain an associated equation

$$\epsilon_{ABCD} \Big[\frac{\Lambda^2}{9} e^A e^B e^C e^D - R^{AB} R^{CD} \Big] = 0 \ . \tag{7.27}$$

Remarkably by following the self duality condition this model has fewer free parameters then General Relativity. Due to this severely constrained nature the above model unsurprisingly fails to provide a viable cosmology.

[3] This is allowed since we take $\omega^A{}_B$ as part of the gravitational sector, it naturally enters the Einstein tensor in ECG and it does not couple to matter as we will not consider fermions at this point.

[4] One may in general consider functions of these terms but since they are 4-forms this space becomes limited to only a function of a single variable.

The above model (7.24) has been further extended in [105], where the requirement on parity is lifted. This frees up the coupling of the cosmological "constant" Λ to the Pontryagin invariant as well. This can be achieved while retaining the duality symmetry (7.23), indeed the proposed action has the form

$$S = -\int \frac{3}{2\Lambda}\Big(\epsilon_{ABCD} + \frac{2}{\gamma}\eta_{AC}\eta_{BD}\Big)\Big(R^{AB} - \frac{\Lambda}{3}e^A e^B\Big)\Big(R^{CD} - \frac{\Lambda}{3}e^C e^D\Big) - \frac{2}{\gamma}\int T^A T_A \,. \tag{7.28}$$

Note that in order to preserve the duality (7.23) the Holst term [194] appears in the action, however, its contribution is balanced out by the torsion squared term. Together forming the last topological invariant in the theory, the Nieh-Yan term

$$I_{\mathrm{NY}} = e^A e^B R_{AB} - T^A T_A. \tag{7.29}$$

The Nieh-Yan invariant is a novelty in comparison to GR as it vanishes identically in case of $T^A = 0$ as can be seen from (7.16). Note that this invariant appears uncoupled and thus is truly a surface term in the action and has zero contribution to the equations of motion. The coupling of Λ to the Pontryagin term allows us to introduce a single parameter to the theory, the "Immirzi" parameter γ. The resulting equations of motion generalize the above equations of motion (7.25),(7.26) and (7.27) in the following way

$$\epsilon_{ABCD}\Big(e^B R^{CD} - \frac{1}{3}\Lambda e^B e^C e^D\Big) = -2\kappa\tau_A \,, \tag{7.30}$$

$$T^{[A}e^{B]} + \frac{3}{2\Lambda^2}d\Lambda R^{AB} - \frac{3}{4\gamma\Lambda^2}\epsilon^{ABCD}d\Lambda R_{CD} = 0 \,, \tag{7.31}$$

$$\epsilon_{ABCD}\Big(R^{AB}R^{CD} - \frac{1}{9}\Lambda^2 e^A e^B e^C e^D\Big) + \frac{2}{\gamma}R^{AB}R_{AB} = 0 \,. \tag{7.32}$$

On flat ($k = 0$) cosmological solutions the above system can be characterized by 4 scalars. The standard scale factor which enters through the tetrad fields

$$e^0 = dt, \tag{7.33}$$

$$e^I = a dx^I. \tag{7.34}$$

The next scalar is the cosmological constant $\Lambda(t)$ and the final pair characterises the torsion degrees of freedom

$$T^0 = 0, \tag{7.35}$$

$$T^I = -T(t)e^0 e^I + P(t)\epsilon^I{}_{JK}e^J e^K. \tag{7.36}$$

The crucial consequence of allowing of parity breaking is the appearance of the scalar P which has been disregarded in [103, 104]. This results in a new branch of solutions in the theory with novel degrees of freedom as has been shown through Hamiltonian analysis [105] of these models. Curiously this branch of solutions exists even without the parity breaking Pontryagin term (7.24) or in other words in the limit $\gamma \to \infty$. Both of the torsion scalars P and T contribute to the full curvature tensor. Interestingly, P is able to source Weyl curvature even on homogeneous and isotropic background.

7.3 Gravity waves in parity-violating Copernican Universes

This section is based on our paper "Gravity waves in parity-violating Copernican Universes" that has been published in Physical Review D [106]. Unfortunately, we have recently found an error in the original manuscript, which we have corrected. Here we include a revised version of our work.

In our paper [106] we have further investigated the role of the parity-odd scalar P in the model (7.28). We study the background cosmology in this setting for finite values of the Immrizi parameter γ. We have found that the scalar P in general tends to dominate over the other energy components during the radiation and matter domination era. A large contribution of P is clearly incompatible with observations. We have found that there exists a tracker solution for P. On this solution P may remain small in comparison to other forms of energy producing a viable cosmological history. P is allowed to leave the tracking solution on the onset of DE domination, which is driven by the varying cosmological constant Λ. In this dynamical regime it no longer grows. Unfortunately, we have found that the tracking solution is unstable, and thus we have to fine tune the initial conditions for P to a high degree so that it stays on the tracking solution for as long as we need.

We further perform an analysis of linear tensor perturbation in this setting. We find parity dependent deviations from GR that affect the speed of propagation of gravity waves. These deviations differ for the left and right helicities of the wave. This effect depends on the magnitude of both the parity violating background torsion P and the value of the Immirzi parameter γ. Comparing these with the observations the gravitational waves and their electromagnetic counterpart from the binary neutron star merger event GW170817 and GRB 170817A by the LIGO/Virgo and Fermi/INTEGRAL [107, 108], allowed us to constrain the value of the Immirzi parameter to $\gamma^2 < 10^{-15}$. Under these constraint the model is still able to provide a viable cosmology.

For the full text of the paper [106], please see Attachment 5 of this thesis.

Conclusion

In this thesis we have discussed various modifications of gravity mostly related to the cosmological constant problem and to the problem of dark energy.

In Chapter 1 we have provided a brief exposition of the cosmological constant problem(s). Mainly we have focused on the problem of zero-point energies of quantum fields. We have argued that these vacuum energies are far from understood. Indeed, various past attempts at estimating their size result in vastly different values by orders and orders of magnitude. We have reviewed two approaches to estimating the zero-point energies for a toy model scalar field: the Minkowski hard cut-off and the dimensional regularization. The first method produces an estimate that is quartic in the cut-off mass parameter, while the second provides a contribution, which is quartic in the mass of the field itself. We have discussed several flaws of these methods and explained why they cannot be trusted to provide a prediction for the value of the cosmological constant. Indeed, the hard cut-off method produces a contribution, which violates the appropriate equation of state of vacuum energy and as a consequence breaks the Lorentz symmetry of the vacuum. On the other hand, the dimensional regularization introduces an arbitrary parameter that, unlike in scattering experiments, has no entirely clear interpretation. The application of this result to the real world is rather limited. Indeed, going beyond the free-field approximation, the masses of quantum fields in general run with energy scale. Thus, it is unclear which mass should provide the vacuum energy in this approach. This shortcoming becomes even more pronounced when we consider the SM. There almost all particle masses are generated by the Higgs mechanism and vanish in the UV limit of the theory. We have also briefly discussed how vacuum energy may be induced by a transition of a scalar field from one vacuum to another in either the first or the second order phase transition or crossover.

We followed this discussion by an introduction to unimodular gravity in Section 1.3. We went over several possible ways this theory can be realized. In particular, we have reviewed how it arises from the Einstein traceless equations, from a diffeomorphism violating constraint in the action (1.40) or from a diffeomorphism invariant constraint in the action (1.44). We have argued that unimodular gravity is subtly different from GR in that it contains solutions of all Einstein equations with all possible cosmological constants. Note that any particular classical solution is, however, indistinguishable from GR with a corresponding cosmological constant. We have discussed that in unimodular gravity the quantum fluctuations of maximally symmetric vacuum do not gravitate. This is best seen from the Einstein traceless equations which are insensitive to any vacuum shifts of the energy momentum tensor. We have presented a simple argument to support this conclusion.

In Chapter 2 we have reviewed the theory of mimetic dark matter [74], which has been a key idea in Chapters 3 to 5. In mimetic dark matter the physical geometry, $g_{\mu\nu}$, is constructed from an auxiliary metric $h_{\mu\nu}$ and a scalar field ϕ in a Weyl invariant way (2.5). The gauge degeneracy resulting from the Weyl symmetry leads to a novel dynamical sector in the theory, which, in this case, corresponds to a simple irrotational dust. We have discussed that this proposal can be viewed as a special case of disformal transformation [129]. We have highlighted that the underlying Weyl symmetry leads to a traceless Einstein equation (2.14), which is similar to but also distinct from that of unimodular gravity (1.35). These equations are insensitive to shifts of the energy momentum tensor of the form (2.21). This is analogous to the shifts of the vacuum energy in unimodular gravity. In this sense, mimetic dark matter is related to unimodular gravity. We made this connection more concrete in Chapter 3 and Chapter 4. In Chapter 2 we have also briefly discussed the theory of k-essence [93–95] in order to introduce it in preparation for Chapter 5. We have made a short exposition of its origins and of its basic features. We have highlighted that the background configurations can provide a small sound speed for the propagation of the k-essence scalar perturbations. This makes this system useful not only as a candidate for dark energy, but also an interesting model for dark matter [153, 155, 156].

In Chapter 3 we have reviewed our findings from [84]. In this work we have explored the relation between the mimetic DM and unimodular gravity in detail. We have found that the mimetic scenario can be modified by introducing a different form of the physical metric, $g_{\mu\nu}$, which, in this case, is constructed from an auxiliary metric $h_{\mu\nu}$ and vector field V^μ. In order to preserve the Weyl invariance, which was a key ingredient of the original mimetic proposal [74], we have extended the action of the Weyl group onto the vector field. In particular, V^μ has a conformal weight 4 in our setting. We have demonstrated that this different realization (in comparison to (2.5)) of the underlying Weyl symmetry of the physical metric results in the simple dynamics of unimodular gravity discussed in Section 1.3. We have shown that this connection can be made explicit on the level of the action by going to Weyl invariant variables, in which this theory reproduces the Henneaux-Teitelboim formulation of unimodular gravity (1.44). Due to the underlying Weyl invariance, the resulting equations of motion for the original variables are manifestly invariant under constant shifts of the vacuum energy. Our model is a novel vector-tensor representation of unimodular gravity which contains higher order derivatives of the vector field. Our formulation may also be very interesting in providing a new origin for additional extensions of unimodular gravity and may facilitate further links with other modifications of gravity. In particular, a non-trivial Weyl transformation of the vector field allows us to impose a gauge condition $h_{\mu\nu}V^\mu V^\nu = 1$. In this sense our vector construction becomes similar to Einstein aether [58]. Finally, the equivalence of classical dynamics may not imply full quantum equivalence. Therefore, our proposal may represent a distinct theory in the quantum regime.

We pursued these ideas further in our paper [85], which we have reviewed in Chapter 4. In this work, we have explored an extension of the above idea in which the vector field V^μ is considered as a composite object, namely the Chern-Simons current of a gauge field A_μ. This change has several advantages. First of all, the Chern-Simons current is naturally a vector field of conformal weight 4 due to the determinant of the metric appearing in its definition. Thus, we may achieve the Weyl invariance of the physical metric without prescribing any non-trivial Weyl transformation to the gauge fields themselves. Secondly, the mimetic conformal factor appearing in this proposal is free from derivatives of the metric, which results in a simpler structure of the resulting equations of motion. Finally, the gauge fields are a very natural object in field theory as opposed to the unusual vector V^μ with a non-trivial Weyl transformation. This provides a better starting point for proposing additional extensions of this theory. The mimetic conformal factor itself is the square root of the Pontryagin term of the gauge field A_μ. Our construction can thus be considered as a complementary one to the proposal [82, 83], which features the kinetic term of a gauge field in this role. Despite the introduced composite structure of V^μ, this theory is still classically equivalent to unimodular gravity. Therefore, it is not surprising that our model only contains a single additional global degree of freedom, which corresponds to the cosmological constant. We have shown, that in (Weyl) gauge invariant variables this degree of freedom is represented as a Lagrange multiplier constraining the Pontryagin term to a constant. In this sense the Lagrange multiplier has an axion like coupling to the gauge fields and therefore plays the role of the θ parameter of the corresponding Yang-Mills theory as well as the cosmological constant. We have discussed this similarity further, and we have shown that the model can be extended to resemble the axion even more. Indeed, one can promote the Lagrange multiplier to an ordinary scalar field and equip it with a kinetic term, as well as a potential term without significantly impacting the dynamical content of the theory. Only after the introduction of a kinetic term for the gauge fields themselves, the theory becomes significantly different from unimodular gravity. In this sense unimodular gravity may be naively recovered as a dynamical regime of an axion, in which the kinetic term of the Yang-Mills fields is suppressed. It would be very interesting, to see if any of these departures from unimodular gravity could provide us a selection mechanism for the value of the cosmological constant.

In Chapter 5 we have reviewed a possible approach [92] to combining the original mimetic dark matter [74] with our above proposal [85] in an attempt to produce a "mimetic dark sector". The key observation in our work [92] has been that the conformal factor in a mimetic substitution does not need to consist of a single term (for example the kinetic term $(\partial\phi)^2$ or the Pontryagin term $\tilde{F}^{\sigma\rho}F_{\sigma\rho}$). We can consider various combinations of them instead, only requiring that the overall conformal weight is 2. Interestingly, we have found that this mixture results generically in a k-essence theory with an additional global degree of freedom, which provides an overall energy scale for the k-essence. We have examined the extent of this correspondence between our mimetic description and k-essence theories to find that our proposal can support almost arbitrary k-essence. The only exception is when the equation of state of the k-essence becomes ultra-relativistic, that is when its energy momentum tensor becomes traceless. At these points the mimetic description breaks down, and the associated mimetic conformal factor becomes ill defined. We have provided a general method for deriving a corresponding mimetic picture for a given k-essence. We have further discussed the importance of Weyl symmetry in this picture. Interestingly, we have found that in models that violate this symmetry the conformal mode of the auxiliary metric behaves either as an auxiliary field or a Lagrange multiplier. In the former case, integrating this auxiliary mode out of the action restores the Weyl symmetry dynamically. In the latter case, the conformal mode enforces additional constraint on the theory. In this regime the dynamics of the theory are equivalent to those of mimetic DM with a potential [76]. The overall scale of the potential is given by the global degree of freedom of the theory. If the potential is flat (constant), this theory provides us with both unimodular gravity and mimetic dark matter using a single mimetic substitution.

In Chapter 6 we have reviewed our paper [86]. In this work we have proposed a minimal scale-free modification of the Einstein equations. This modification is obtained by dividing each side of the original Einstein equations by their respective traces. Doing so removes any dependence on the Newton constant from the equations, and thus, the scale is lost. Similarly to unimodular gravity, this change does very little to the classical solutions of the theory. In fact the model yields the same dynamics as GR; however, the Newton constant appears as a constant of integration or a global degree of freedom, rather then a coupling constant. We have discussed various alternative formulations, of our proposal both on the level of the action and on the level of the equations of motion. We have highlighted the additional similarities with unimodular gravity. Interestingly, in some of these formulations the effective Planck constant $\hbar$ also becomes a global degree of freedom. We have discussed that promoting the fundamental constants to global degrees of freedom has substantial consequences upon quantization. There, all degrees of freedom are subjected to quantum fluctuations and their corresponding Heisenberg uncertainty relations. Thus, we get quantum fluctuations of the Newton constant itself. This may be of significance near classical cosmological and black hole singularities. It would be very interesting to further investigate the implications of these fluctuations. Furthermore, it has been suggested that unimodular gravity appears naturally from thermodynamic or emergent gravity settings [195–198]. It would be thus very interesting to see if the scale-free Einstein equations could arise in a similar manner.

In Chapter 7 we have discussed an extension of Einstein-Cartan gravity. We have reviewed the Einstein-Cartan gravity in Section 7.1 and the extension itself [103–105] in Section 7.2. In these models the cosmological constant is promoted to an auxiliary field, that is coupled to the topological terms of gravity. In particular, the Euler term (7.21) and the Pontryagin term (7.22). In the setting of ECG, such couplings give rise to a non-trivial torsion tensor and consequently to a varying cosmological "constant". Interestingly, it has been found that this setting naturally supports an unusual parity-odd scalar P, which constitutes a part of the torsion tensor and is consistent with both homogeneity and isotropy of cosmological solutions. This parity-odd piece further gives rise to a non-trivial Weyl curvature. In Section 7.3 we have reviewed our paper [106], in which we further investigate the role of this parity violating piece in the above models. We have found that the presence of a non-zero P is necessary for obtaining cosmological solutions that are compatible with observations. In these potentially viable solutions, the scalar P tracks the dominating form of matter for most of cosmological history and leaves the tracking solution on the onset of DE domination. We found that the tracking solution itself is unstable, and any deviations from it quickly grow. Thus, the phenomenological viability of this model rests on the fine tuning of the initial conditions of P so that it finds itself close enough to the tracking solution. We have performed a technically challenging analysis of the tensor perturbations in this model, which has been carried out in the first order formalism. We have found that the speed of propagation of gravitational waves is in general affected by the parity-odd piece, and that this effect differs for the left and right helicities of the graviton. These results have allowed us to put constraint on the single dimensionless parameter γ in the model. By comparing our results with the LIGO/Virgo and Fermi/INTEGRAL measurements of the gravitational waves and electromagnetic signal coming from a binary neutron star merger GW170817 and GRB 170817A [107, 108] we have been able to determine that $\gamma^2 < 10^{-15}$. Under this constraint the theory can still provide a viable cosmological evolution. Our findings are potentially very interesting for phenomenological approaches to quantum gravity, where modified dispersion relations of tensor modes play a major role [199–202]. The chiral modification that is realized in the above model adds a novel layer to these dispersion relation modifications. Finally, as we have seen in Section 1.3 and in Chapter 6, the cosmological constant and the Newton constant in GR are restricted to a constant in a very similar fashion. This immediately raises a question whether a similar style of torsion balancing, that has been employed here to make CC vary, could be used to support a spacetime varying Newton constant, and if this would imply similar parity violations. It would be interesting to further explore these mechanisms in ECG or in the broader context of metric-affine gravities [192].

We hope that we have been able to demonstrate that modified gravity is a very interesting subject that allows us to provide novel insights to the problem of dark energy and cosmological constant. As we have discussed, a reliable evaluation of the zero point energies will likely require knowledge of the UV physics of the SM, which is likely not going to be understood sufficiently in the near future. Thus, we feel that modifications of gravity are currently the best tool we have to address the cosmological constant problem. Additionally, any novel degrees of freedom that appear due to the modification may be significant for the problems of DE or DM. Studying modified gravity is particularly exciting at the present day due to the recent enormous improvements in cosmological observations as well as gravitational wave measurements. These allow us to directly confront many of the proposed models with observations and to consequently gain deeper understanding of gravity.

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List of Abbreviations

GR - General Relativity
QFT - Quantum Field Theory
DM - Dark Matter
DE - Dark Energy
DS - dark sector
CMB - Cosmic Microwave Background
CC - Cosmological Constant
CDM - cold dark matter
SM - Standard Model of particle physics
EW - Electroweak
QCD - Quantum Chromodynamics
TeVeS - Tensor-Vector-Scalar
ECG - Einstein-Cartan gravity
UV - Ultraviolet
UG - Unimodular Gravity

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