Noise factor and reception bandwidth in optoacoustical GW antenna

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Abstract. An expression has extracted from the OGRAN project theory, which provides connection between numerical values of noise factor F and achieved displacement resolution and antenna's threshold signal in metric variations. Noise factor and "reception bandwidth" connects across displacement resolution. There is defined analytical expression and numerical value for design displacement resolution (sensitivity) on the base intention $F = 1$. It has appeared that the extracted analytical expression for readout resolution does not correspond to applied Pound-Drever-Hall technique and AURIGA circuitry. This requires an improvement in theoretical design. The achieved resolution value $2 \cdot 10^{-15}$ cm/Hz$^{1/2}$ is matched to the value for metric sensitivity in pulse $h_{min} \simeq 10^{-18}$, which is 15 dB higher than the thermal sensitivity limit.

1. Threshold signal of a bar detector

The effect of a gravitational wave is represented as an equivalent force F_G acting on the test body as the cylindrical acoustic resonator (bar) [1];

$$F_G = h_m M \omega_0^2 L/2. \quad (1)$$

Here h_m is amplitude of metric variation; L, ω_0 and M are length, resonant frequency and equivalent mass of the oscillator.

Force is registered through the mechanical displacement that it causes.

While relaxation time τ_{ref} is long, amplitude of forced oscillations of an oscillator increases linearly; at the end of exposure it reaches the value [1]

$$\Delta A = F_G \tau (2M\omega_0)^{-1},$$

where τ is duration of forecast supernova burst pulse.

Certain antenna "sensitivity bandwidth" appears $\Delta f_S \simeq (1/\tau)$.

Oscillator is in state of permanent thermal Brownian motion; corresponding energy is $k_B T/2$.

Motion has form of a narrow-band stochastic quasi-harmonic process $x(t) = A(t)\cos[\omega_0 t + \varphi(t)]$.

The amplitude and phase change stochastically and slowly. With phase coincidence, signal creates amplitude variation ΔA.

Under strong metric variation affect, thin dissipation effects that determine an oscillator quality factor Q are irrelevant. Consequently, no significant requirements are imposed on accuracy of coincidence of frequency of quasi-harmonic metric signal and resonance frequency of the oscillator ω_0.

During the time τ of signal exposition, stochastic variations of amplitude take place. Its average statistical value is [2]

$$\sigma_B = (2k_B T\tau / M\omega_0^2 \tau_{rel})^{1/2} = (k_B T\tau / QM\omega_0)^{1/2}.$$

Here there is dependence on quality factor.

From comparison of signal and fluctuation variations, threshold signal is determined. This is how a short pulse should detect by a resonant bar antenna.

For general case of arbitrary phase mismatch, the process is represented using quadrature components $x(t) = A_C(t)\cos\omega_0 t + A_S(t)\sin\omega_0 t$. The components A_C and A_S are highlighted by synchronous detection. The algorithm for primary data processing has implemented in the "ULITKA" detector [3,4].

In 2005, development of a full-scale facility OGRAN ($L \simeq 2$ m, $M \simeq 1000$ kg and $f_0 = \omega_0/2\pi \simeq 1300$ Hz) has begun [5]. Somewhat earlier, development of the pilot model had begun ($L \simeq 0.5$ m and $M \simeq 50$ kg). In 2004 [6], the first article appeared with the title: «Gravitational wave experiments and Baksan project «OGRAN». The article presents calculation formulas provided by theory (GWA sensitivity analysis), formed in 1996 [7]. In conjunction with predefined parameter values the "theoretical basis" of the OGRAN project had formed. This basis has confirmed in the final article on the pilot model [8].

In 2013, the Moscow adjustment period of full-scale facility OGRAN had completed; the facility has moved to Baksan Neutrino Observatory of the Institute for Nuclear Research of RAS into camera inside the mount. As result, two articles has published [9,10]; in modern view of OGRAN foundation has presented: "Under optimal filtration procedure, the minimum registered perturbation of the OGRAN optical length in the bandwidth $\Delta f_S = (1/\tau)$ is read as" [9]:

$$h_{min} \approx (4/L)\sqrt{(k_B T/M\omega_0^2)(1/Q\omega_0\tau)} \approx 10^{-20}\sqrt{F\Delta f_s}Hz^{-1/2}. \quad (2)$$

Here F is a noise factor; it represents an unique scheme for registration of mechanical displacements and its properties. Further, intended parameters have substituted into the formula: $F = 1$ and $\Delta f_S = 100$ Hz [9]; then forecast of antenna's sensitivity in variations of metrics is $h_{min} \approx 10^{-19}$. The astrophysical forecast $\tau = 10$ ms has become the base parameter. This is intention as of 2014; it is result of evolution of indevour and aspirations, because earlier in 2004, the more optimistic forecast had presented; it was assumed that antenna should register pulses of $\tau = 1$ ms [6]; well-selected set of parameter values in formula for noise factor had allowed to obtain the value $F = 1$. Then expressions (2) for $\Delta f_S = 1000$ Hz provide forecast estimation of $h_{min} = 3\cdot 10^{-19}$. In [6] the forecast some differs numerically because, in particular, parameters of full-scale acoustic oscillator had not known definitely.

Change in theoretical basis in 2014 has led to change in "design sensitivity". This term and numerical value has announced once in [6] for the OGRAN pilot model as 10^{-16} cm/Hz$^{1/2}$. This indicator for the full-scale facility has not announced yet, while this term mentions sometimes and somewhere. In article [10], the similar value of $3\cdot 10^{-16}$ cm/Hz$^{1/2}$ has presented without connection with experimental data. The pilot model had intended to achieve extra high sensitivity of the registration scheme. In it, the calculation of design sensitivity has not opened.

The task of the presented report is to define and calculate design sensitivity for the OGRAN GW antenna to compare it with test result achieved in 2013 [9,10,11].

The expressions (2) has presented as approximate. The analytical expression for spectral density has obtained when changing $\tau \simeq 1/\Delta f_S$. As alternative, it is possible to derive the exact expression of this spectral density and avoid non-exact intermediate analytical transfigurations. So, the expression (1) connects metric variation amplitude and force affecting the oscillator. The Nyquist force is also affected; it is expressed as spectral density: $G_T(f) = 4k_B T H_\mu = 4kTM\omega_0/Q$, where H_μ is oscillator viscosity. When comparing the forces, we find directly the spectral density of antenna's metric limit thermal threshold signal $S_{h0}(f) = (4/L)^2 k_B T(MQ\omega_0^3)^{-1}$. The threshold signal expression takes the view:

$$h_{min} = (S_{h0}\Delta f_S F)^{1/2}. \quad (3)$$

Substituting parameter values, we get the more exact estimation $(S_{h0})^{1/2} = 1.7\cdot 10^{-19}$ Hz$^{-1/2}$ for $Q = 10^5$ [9,10]. It is noticeably different from (2). The exact value is required to specify the resultant value of "reception bandwidth". In article [12] for bar without readout mirrors the first measured value $Q = 1.6\cdot 10^5$ has presented, and the limit threshold value of $(S_{h0})^{1/2} = 1.5\cdot 10^{-19}$ Hz$^{-1/2}$ had calculated.

2. Noise factor in the OGRAN theory

Antenna's registration scheme becomes meaningful when F > 1.

In the OGRAN project, the laser optoelectronic scheme uses Fabry-Perot interferometer; its mirrors has fixed at top ends of the cylinder. The scheme registers interferometer eigenfrequency deviations δν, and small displacements δx are defined by calculation through the ratio ($\nu = c/\lambda$)

$$\frac{\delta x}{L} = \frac{\delta \nu}{\nu}. \tag{4}$$

Fabry-Perot interferometer has used in AURIGA GW detector in the room-temperature version. The full scheme and its description has published and available at the AURIGA website in master's thesis of L.Conti of 1995 [13] and somewhere else in proceedings. This scheme had later presented in the article [14]. In the scheme, Pound-Drever-Hall (PDH) [15,16] technique is applied. Moreover, the article [16] contains formula for maximum achievable resolution of the displacement meter for GW experiment without taking into account laser noise.

In 1996 (soon), the SAI MSU had published the paper [7] with title: “Room-temperature gravitational bar-detector with cryogenic level of sensitivity”. Here the theory of article [17] had simplified and elaborated, and term “noise factor” has entered into consideration. Certain optimism in sensitivity presented in the title [7] is because the new PDH technique effectively eliminates noise of laser power technical phenomenological fluctuations, whereas this noise was significant in [17,18]. Afterwards, the presented sensitivity analyze has become the theory of the OGRAN project [6].

MSU has certain groundwork in theoretical investigations on Fabry-Perot application to displacement measurement. There is the article “A combined optical-acoustical gravitational antenna” [17]. The formula for resolution of displacement measurer with a F-P resonator has presented by Braginskii [18]. Moreover, MSU has significant hardware experience [19,20,21,22] in addition to review [5].

Institute of Laser Physics (ILP) RAS has relevant hardware experience too; there is the article with the title [23]: “Use of narrow optical resonances for measuring small displacements and for building gravity-wave detectors”. Here the steep slope of the spectral line of methane had used. Moreover, ILP possesses extensive groundwork in Pound-Drever-Hall technology. According to collaborated “Agreement in Intentions”, “ILP RAS works out, mounts and adjusts the optoelectronic part of the facility”. In fact, ILP has created the meter of small mechanical oscillations. Much earlier, such measurers had created by MSU; now this work became the task of ILP, and SAI MSU accomplishes scientific control.

At MSU, the displacement meters had investigated and developed separately from the probe body [24,25,26]; small vibration were created by a rigid plate with electrostatic force actuator in quasi-static mode. In the OGRAN project, this stage of development has skipped, and the facility should be considered as a whole during customization. As a consequence, SAI MSU and INR RAS has increased essentially their participation in adjustment.

INR represents interest of the Presidium of RAS in the project and from 2006 INR assists hardware adjusters of SAI MSU in study of readout circuitry functioning; there are a number of reports and articles [26,27,28,29]. In report [26], influence of intrinsic laser frequency fluctuations had considered and it had shown how to suppress them; the feedback depth should be increased at least to value $K_f = 1000$ (60 dB) [27]. This led to the need to expand the frequency response of the servo amplifier that controls retuning of laser radiation frequency [30] in order to prevent self-excitation in locked feedback loop. In turn, this has required introduction of electro-optical crystal (EOC) inside the laser cavity as a third, high-speed laser frequency retuning device. This fast control has presented in the functional scheme [31]. Accordingly, the value $K_f = 2000$ has realized [8].

In article [28] there has considered the residual amplitude modulation (RAM) of laser radiation and noise that it induces. Certain efforts have presented at the report [11], thesis [32] and articles [33,34] to suppress this modulation.

Under theoretical consideration of GW antenna in a complex, there have proposed two versions of excess noise origin [35,36].

The OGRAN theory has established the expression for the noise factor as a part of general design procedure [7,6,9]:

$$F = (2M/\tau)\sqrt{G_e/G_T}. \quad (5)$$

Here G_T is spectral density of bar noise

$$G_T(\omega) = 2kTM\omega_0/\pi Q. \quad (6)$$

Spectral density G_e of readout "optical noise" [10] is defined as [7, 37, 38]

$$G_e(\omega) = B\omega_0^2(2h\nu/\eta P)(\lambda/2\pi N)^2. \quad (7)$$

Here N is the "sharpness or the effective number of reflections" in the FP resonator, B is "the phenomenological factor that indicates by how many times laser noise exceeds the Poisson level" [10], η is a photodiode yield. In (7) P is "optical power"; there is not complete certainty in its definition. So, in previous articles [17,18] P is incident power at the interferometer. In [9] it is "effective power". In [39] outside the project, P is the "optical power available at the photodetector". This ambiguity is consequence of incompleteness of optoelectronic circuit functioning theory.

In (5) the noise factor is a function of astrophysical forecast τ. The formula for it has transformed to the "suitable" form [9,10]:

$$F \cong \left(\frac{\sqrt{B}}{N}\right)\left(\frac{1W}{P}\right)^{1/2}\left(\frac{1s}{\tau}\right). \quad (8)$$

This format simplifies selection of parameter values to calculate the desired value $F = 1$. Substituting parameter values P = 0.01 W, τ = 0.01 s [9,10] and as examples $N = 3\cdot 10^3$ and $B = 10$, we find $F = 1$. The formula (8) is convenient for a theorist-designer making general sensitivity forecast; it shows that "the task has a solution". Executor should implement this set of figures in hardware.

In the initial OGRAN article [6] ($\tau = 10^{-3}$ s) one can read: "For the designed OGRAN optical parameters: $P = (1 - 3)$ W, $B \simeq (1 - 10)$, λ = 1.064 μ, η = 0.8, $N = (10^3 - 10^4)$ one can find the estimation F ≃ 1." So, in the example presented above, making substitutions τ = 0.001 s and $P = 1$ W, we also get $F = 1$. An acceptable version of initial design set of parameters is formed, which is used below in the formula for calculated (design) resolution value of an optoelectronic meter.

According to reference in intermediate final article [8], "the presented values constitute the theoretical basis of the OGRAN project."

In 2014 intended astrophysical forecast duration τ has changed. Comparison of above selected parameters shows that increase in duration τ in design permits hardware worker reduce significantly power of laser radiation P according to (8). This is result of an attempt to register power of 1 W by creating two complex photodetectors with 16 photodiodes each using light splitting cubes [31,8,10]. This is a response to technical reality. The next parameter to be revised is the laser excess noise factor ***B***.

In addition, perspective is associated with increasing the parameter $N = 1/(1 - R)$ [17]; here R is energy reflectance of interferometer mirrors. For this, there had been acquired mirrors of high quality manufactured by the LAM factory Lyon (France) [40,9,10,33,34]. "A serious problem in the process of OGRAN assembling was a method of mirror installation so as the high quality mirrors require a clear atmosphere conditions" [9]. The clean room as a "laminar box" with dust protection has designed and erected in the BNO INR chamber around the facility in 2015. Results of mirror replacing have presented in articles [33,34].

In the article [8], in sensitivity forecast, calculation there used two base values: $\Delta f_S = 1000$ Hz (τ = 1 ms) and $F = 10$. Also, further there has proposed to decrease in bandwidth towards $\Delta f_S = 10$ Hz, because it should lead to increase in sensitivity. On this way, as the first step we put $\Delta f_S = 100$ Hz; it means τ = 10 ms. Replacing this time value in the expressions (5), (8) we obtain the value $F = 1$. Thus, we have obtained just the same pair of parameter values on which the sensitivity analysis in articles [9,10] has based. Thus, the transition to desired value $F = 10$ already in 2010 means change of intention preventively.

Further in this way, supposing $\Delta f_S = 10$ Hz [8] we obtain the value of $F = 0.1$. It is a significant departure from reality in forecast. Actually, value of $F \gg 1$ is realized in the facility so far.

The initial sensitivity forecast calculations assume values of $B = (1 \div 10)$; it means practical absence of technical noise in laser radiation. This initial optimism, as indicated above, had based on employment of new advanced and complicated PDH technique.

To take into account significant real excess noise in hardware tests, the conclusion has made that acceptable values of factor B may be expanded in the range: $B = 1 \div 1000$ [9,10]. So, according to the final article on the pilot model [8], excessive technical noise did not allow observing (revealing) thermal noise of the bar. Final estimate of displacement resolution has estimated as $4 \cdot 10^{-16}$ m/Hz$^{-1/2}$ [11]. This level of noise makes it impossible to identify thermal noise at the spectrogram [8]. Further, in the full-scale facility, joint efforts of two institutes succeed in definition and reduction of dominant noise source as of 2010-2012, and thermal noise has revealed quite confidently [41].

In articles [37,38], value range $B = (1 \div 10)$ in the noise factor formula provides a capacious source of perspective again.

3. Latent content of the OGRAN theory

The task is to find connection between two result sensitivity signicatives. The first is the initial threshold signal h_{min} jointed with the noise factor [7]. The second signicative is a novel - the "reception bandwidth". This signicative has introduced in [9,10]. It based on sensitivity result representation in the AURIGA project as a graph at Fig.7 [42] of metric variation spectral density; there the novelty is apportionment the abscissa distance in frequency 10 Hz (865-855) between two points with the ordinate of 10^{-19} Hz$^{-1/2}$.

The reception bandwidth unequivocally relates to the actual achieved displacement resolution.

To define mentioned connection, we should deep into the theory of the OGRAN project to extract the analytical expression for displacement resolution.

There is also a relationship between displacement resolution expression and noise factor expression. Thus, displacement resolution is the third, intermediate significance.

While the noise factor formula is convenient for a design theorist, it is desirable to add to it no less convenient formula for hardware developer measuring resolution directly, because the noise factor formula "with a clear physical sense" is not relevant in test. In particular, one should strive to ensure that traditional correspondence between theoretical introduction and result value inside an article; absence of this connection some signs of theory's inadequacy become not meaningful.

The example of desired correspondence is contained in the fragment of the report [31] on the pilot model. So, we read: "Experiments, had shown sensitivity of $\sim (1 \div 2) \cdot 10^{-14}$ cm/Hz$^{1/2}$ for this setup while theoretical estimation was $\sim 3 \cdot 10^{-15}$ cm/Hz$^{1/2}$. The reason for this mismatch was revealed as technical noise of detection system."

These affirmations allow us to conclude that the required design formula really exists, albeit in a latent form. Moreover, that the presented test result differs moderately from the calculated value.

In the next article [8] there has written: "The equivalent limiting (theoretically expected) spectral density of noise displacements is $0.5 \cdot 10^{-15}$ cm/Hz$^{1/2}$." In both sources, obviously, values of parameters in calculation formula differ. In the message [31], as earlier [6] and later [8], test resultant signicative has presented in the displacement noise format.

In the AURIGA detector, there is significant difference between measured values and calculated ones [42,8].

Meanwhile, the aim of scientific part of instrument development is to achieve coincidence of test result with previously calculated numerical value within limits of measurement errors and accuracy of parameter determination. It is advisable to solve this problem without choosing an appropriate value for parameter B.

Revealing of appropriate resolution formula in OGRAN theory may be considered as the inherent part of general design procedure; the sought formula should connect rigidly with antenna's design sensitivity.

Information about displacement measurement using FP interferometer exists outside the referred above articles. So, the formula for displacement resolution can revealed in the article [17]:

$$\Delta x = \frac{\lambda}{2\pi N}\sqrt{\frac{h\nu}{\eta P}B\Delta f}.$$

Parameter N is not measureable, whereas a generally accepted term "finesse" $\mathcal{F} = \pi (1 - R)^{-1}$ is used [16,14]. It determines relative transmission bandwidth of an interferometer at the level of -3 dB. This value has determined experimentally in the OGRAN facility by scanning interferometer transmission function using scanning retune of laser light frequency ν_L with the piezoceramic actuator (driver) [30]. The value $\mathcal{F}$ has announced as 3000 [9,10]. The brief derivation of above finesse expression has implemented in [36]. Accordingly, we get the relation $N = \mathcal{F}/\pi$, and then

$$\Delta x = \frac{\lambda}{2\mathcal{F}}\sqrt{\frac{h\nu}{\eta P}B\Delta f}. \quad (9)$$

Elsewhere, this formula has presented in the article [18]; details of its derivation has presented in monography [2]. Without technical noise ($B = 1$) displacement resolution formula has presented outside the project in the article [39] at page 262. At the beginning of the OGRAN project, the formula (9) without technical noise had presented to ILP RAS as the initial scientific foundation to hardware implementation of the optoelectronic displacement readout; certain optimism with laser noise was due to PDH technique virtue. Later, certain efforts to derive 'readout noise spectral density' in improved form has presented in [12,40].

The project OGRAN theory contains the required analytical expression for displacement noise in latent form. In order to extract it, we should refer to the base article [7]. One can find there the expression for spectral density G_N of the equivalent stochastic force F_N, formally applied to massive body of the equivalent oscillator

$$G_N = G_T + G_f + |Z_\mu|^2 G_e. \quad (10)$$

Here G_f is spectral density of inverse fluctuation force acting by readout circuit; this force represents stochastic component of photon flux pressure.

The third term $|Z_\mu|^2 G_e$ in (10) is output noise of registration circuit adducted to the measured value, in this case, to force. Here Z_μ is the oscillator mechanical impedance: $Z_\mu = M\,[j\omega + 2\delta + \omega_0^2/(j\omega)]$, $2\delta = \omega_0/2Q$. Spectral density G_e contains information about the optical registration scheme, which extremely high resolution is the aim of presenting scientific and engineering development.

It had shown that $G_f \ll G_B$ [7]; this is in contrast to MSU's capacitive displacement meters [19,20,25]. To find displacement noise, we write out the equation of motion

$$M\ddot{x} + H_\mu \dot{x} + k_\mu x = F_N.$$

Here $x(t)$ is instantaneous coordinate, k_μ is oscillator stiffness.

The solution of this equation "in spectra" allows us to find an analytical expression for spectral density of total displacement noise G_X:

$$G_X(\omega) = G_N/\omega^2|Z_\mu|^2 = G_T/\omega^2|Z_\mu|^2 + G_e/\omega^2 \equiv G_{XT} + G_{XS}. \quad (11)$$

The first term represents thermal displacement noise of the bar. The sense of the second term is spectral density of optoelectronic scheme noise. The frequency range near resonance ($\omega \approx \omega_0$) is of interest here. There is important consequence for determining spectral density of optoelectronic readout noise:

$$G_{XS}(\omega) = G_e(\omega)/\omega_0^2. \quad (12)$$

Thus, the general relationship between displacement resolution and noise factor has determined.

Substituting expressions (6) and (7) into (11), we have obtained the specific noise expression ($\Delta\omega = 2\pi f - \omega_0$):

$$G_X(f) = \frac{4k_B T Q}{M\omega_0^3}\frac{1}{1+(\Delta\omega/\delta)^2} + B\frac{\pi h\nu}{\eta P}\left(\frac{\lambda}{\pi N}\right)^2. \quad (13)$$

It is expression for spectral density of total displacement noise, observed by developer-adjuster in experiment (test) at output spectrogram at the figure 4 in [9] or at the figure 6 in [10]. The first term describes quantitatively thermal noise of the bar as the narrow resonant peak. The second term has no dependence on frequency; it represents spectrum "background". It is the formula for resolution of displacement measurer

$$\Delta x = \frac{\lambda}{\mathcal{F}}\sqrt{\frac{\pi h\nu}{\eta P}B\Delta f}. \qquad (14)$$

This expression contains parameters of the design set of parameters. Substituting the mentioned values $N = 3 \cdot 10^3$ ($\mathcal{F} = 9.5 \cdot 10^3$), $P = 1$ W, $\eta = 0.7$, $B = 10$, we obtain variant of design sensitivity estimation $\Delta x = 3 \cdot 10^{-17}$ cm/Hz$^{1/2}$.

The formula (14) has scientific view. However, its engineering significance is reduced by arbitrary defined parameter $B = 1 \div 1000$ [9,10]. In the AURIGA project "laser power noise spectral density" has measured quite accurately [14]; laser frequency noise has mentioned; but its relative contribution has estimated as negligible [14,42]. It is a consequence of smallness of the "transducer-cavity length".

The formula and the presented theory do not take into account significant energy losses in the interferometer mirrors due to multiple reflections. Phenomenologically this effect has observed and fixed in the facility. It defines sufficient decrease in signal transmission in the readout and accordingly decrease in sensitivity (resolution) of displacement meter. To describe this effect the term "interference contrast" has introduced in [8]. Its measured values are of 0.3 in [9] and of 0.2 in [10]. In addition, we read: "… due to losses in the light guide elements and the interference contrast ~10% the real value of effective power is $P = 0.01$W", whereas "the power of radiation that reached the photodiodes was P = 50 mW."

The attempt to derive analytical dependence of signal transmission and displacement resolution on these energy losses had devoted in the article [12]. The solution that is more adequate has presented in the article [28].

The formula (14) corresponds to the conceptual scheme of Braginskii [18,2]. The resulting formulas for displacement resolution (9), (14) describe Braginskii's conceptual laser scheme as of 1967 too. Slight differences in coefficients [18,39] is due to some uncertainty in interpretation of parameter P and the value $G_T(6)$ in [7,17]. The Pound-Drever-Hall scheme differs significantly. It uses not transmitted beam, but reflected one.

The functional optoelectronic scheme of OGRAN facility has attached to the article [6]. It repeats the AURIGA scheme [13,14]. This scheme has also presented clearly in [8,10,31,37]. In the AURIGA PDH circuit laser radiation is phase modulated by high frequency Ω (~ 10 MHz). The beam reflected from the interferometer falls on photodiode; variable power component has the form $\delta P_{\mathrm{ref}} = D_P(\nu_L - \nu)\sin\Omega t$. Here D_P is a decrement [16,28]. After synchronous detection, a sign alternating discriminator characteristic of the automatic control system of laser frequency is formed. When $\nu_L - \nu = 0$, zero voltage is presented at output of the synchronous detector and further to retune laser frequency. When the feedback loop is locked, the AURIGA circuit implements fast auto-tuning of laser frequency; it exactly follows fast signal variations $\delta\nu$ of interferometer mode frequency (4). In this way frequency manipulated laser radiation is formed; signal and noise encode in laser radiation frequency.

Under condition S/N = 1, the threshold displacement signal in radiation has determined [26]:

$$\Delta x = \frac{\lambda}{8\mathcal{F}}\sqrt{\frac{h\nu}{\eta P_C}\Delta f}. \qquad (15)$$

Here P_C is a carrier power spectral component of phase modulated laser radiation at a photodiode; $P_C \approx P/2$ [16]. When $\eta = 1$ expression (15) coincides with the resolution expression in the article [16]. There signal and noise are presented at input of the photodetector as variations of laser radiation power, whereas the formula (15) presents signal and noise containing in laser frequency deviations.

Formulas (14) and (15) present shot noise of photoelectrons and in this sense have single physical meaning. This consideration gives occasion to neglect difference in descriptions of two registration schemes in the OGRAN theory. When focusing on their difference, one can understand the origin of excess noise in the PDH optoelectronic registration scheme [35].

Frequency manipulated radiation is fed to the interferometer of discriminator, second PDH channel. It uses the slope of sign alternating discriminator characteristic. On this slope frequency deviations transforms into voltage variations in output of the facility. The slope is easily measured. Through the slope steepness, the output voltage noise is recalculated (adducted) to frequency noise at test spectrogram and then using (4) - to displacement noise of the facility. The channel discriminator introduces its intrinsic noise; it has not taken into account by the formula (14), and in the noise factor formula. Account of the discriminator's contribution has presented in the report [27].

Regardless analytical expressions, we can use the numerical value of displacement resolution. To connect it with factor F we use the expression (12). The numerous value of noise factor can determined. In addition, we can determine the design value of sensitivity.

4. Consequences and estimates

The readout system registers fast variations in eigenfrequency of the FP resonator, and achieved resultant frequency noise value is available at spectrograms. Noise in this format converts into resultant displacement noise by means of the ratio (4). These two noise dimensions correspond to left and right scales at spectrograms in articles [9,10,38]; where $\delta x/\delta\nu = 0.7\cdot 10^{-12}$ cm/Hz.

The best-achieved resolution can be revealed as background at test low-resolution spectrograms at fig. 6 of [10] and at fig. 3 of [9]. In addition, this important numerical value has pointed out in the text [10]: “The measured level of the spectral density of the total antenna noise (background above which a thermal peak dominates) in the operating antenna range is ~ 0.003 Hz/Hz$^{1/2}$.”

Recalculation (4) gives the value of ~ $2\cdot 10^{-15}$ cm/Hz$^{1/2}$. Just the same value has announced in the previous report [11] as a manifestation of ILP RAS competence.

To transform this achieved result into metric format, the absolute and relative height of the thermal peak on the first test low-resolution spectrogram at $f_0 \simeq 1300$ Hz should be determined. We can see on the spectrogram that the peak height is of 0.3 Hz/Hz$^{1/2}$; it corresponds to $2\cdot 10^{-13}$ cm/Hz$^{1/2}$. This absolute height coincides with the value calculated by means of the general formula (13) in the term of the heat peak. The relative height A is of 100 or 40 dB. This important achievement has fixed in the synopsis [42]: “The level of intrinsic fluctuations of the optical displacement sensor has been practically reached, which is two orders of magnitude lower than the level of the thermal acoustic resonance noise.”

At two points of resonance curve in (13) thermal noise spectral density is equal to background. Distance between them is Δf_{h0}. Using the expression for shape of the thermal noise curve (13), the bandwidth corresponding the level of - 40 dB is $\Delta f_{h0} = \Delta f_{40} = A\Delta f_0 = Af_0/Q$, where $\Delta f_0 = 0.013$ Hz. Within the specified frequency bandwidth $\Delta f_{h0} = 1.3$ Hz, thermal noise dominates, and limit achievable spectral density is $S_{h0}(f)$.

There is a graph of spectral density of metric variations at the fig. 5 of [9]. This graph is result of processing the experimental data, which has simultaneously represented by the previous spectrum at the fig. 4 as displacement noise. Spectrogram at the fig.5 is connected with spectrogram at the fig. 4 by means of the formula $(G_h)^{1/2} = (G_X)^{1/2}/K_{xh}$, where $K_{xh} \equiv x_m/h_m = Q(L/2)/[1+(\Delta\omega/\delta)^2]^{1/2}$. It is a product of analytical conversion algorithm. In the lowest small part of the spectrum at the fig. 5 (inside frequency bandwidth Δf_{h0}) the spectral density is constant: $S_{h0}(f)$.

Outside the band Δf_{h0}, metric spectral density increases sharply and there are two spectrum sections increasing with distancing from the resonant frequency f_0. This spectrum exactly determined analytically; it is analogous in sense to metric spectrum in the article [42]. The bar thermal peak and this conversion are scientific contribution of SAI MSU into the OGRAN collaboration, whereas ILP provides the unique displacement measurer and its actual noise.

There is decision to extend the bandwidth up to reception bandwidth Δf_{rec}. We are to calculate the distance in frequency between two points with ordinates of 10^{-19} Hz$^{-1/2}$ on the metric graph [9,10]. When using the shape of the resonant curve (13) the distance is determined by relation completing the transformation algorithm:

$$\Delta f_{rec} = \Delta f_{h0}\,[10^{-19}\ \text{Hz}^{-1/2}/(S_{h0})^{1/2}] \simeq 8\ \text{Hz}. \qquad (16)$$

However, articles [9,10] present the value of $\Delta f_{rec} = 4$ Hz. To implement the proper calculation and explain the divergence in figures, we should consider the other, high-resolution spectrogram at fig. 4 [9]. In it, the visual relative peak height is $A \simeq 30$, and then $\Delta f_{h0} = \Delta f_{30} \simeq 0.4$ Hz. In addition, since in articles [9, 10] the approximate value of $(S_{h0})^{1/2} \approx 10^{-20}$ Hz$^{-1/2}$ has established, we obtain using (16) $\Delta f_{rec} \approx 10\Delta f_{h0} = 4$ Hz.

To connect displacement resolution with noise factor we apply expression (12). Spectral density of achieved displacement noise has the numerical value of $G_X(\omega) = 6.3\cdot 10^{-35}$ m^2/Hz. For the value $\omega_0 = 8.2\cdot 10^3$ s^{-1}, we find $G_e(\omega) = 4.2\cdot 10^{-27}$ m^2/s. For thermal noise we have $G_T(\omega) = 2.2\cdot 10^{-19}$ N^2/Hz. Then, for $\tau = 10^{-2}$ s according (5) we find $F \simeq 30$. According (3) we find conjugate metric threshold signal $h_{min} \simeq 9\cdot 10^{-19}$. It corresponds to $\Delta f_{\rm rec} \simeq 8$ Hz.

Design displacement resolution (sensitivity) we can find when $F = 1$; the estimate is of $0.7\cdot 10^{-16}$ cm/Hz$^{1/2}$ as a response to "main technical challenge" [9]. It means A = 3000, $\Delta f_{\rm h0} = 40$ Hz and $\Delta f_{\rm rec} \simeq$ 250 Гц.

Hardware executors should realize this outstanding design resolution value; it considers in the sense that has presented in [6]. This is response to the challenge [9]. For this purpose, it is planned to increase interferometer finesse and reduce the contribution of excess noise.

The intermediate value of signal spectrum bandwidth of $\Delta f_S = 250$ Hz is of interest. It means intermediate value $\tau = 4$ ms [7]; then $F = 2.5$, $h_{min} = 4.3\cdot 10^{-19}$. This is moderate distinction of 8 dB from limit thermal threshold.

If the main part of signal pulse energy locates within the bandwidth $\Delta f_{\rm h0} = 40$ Hz ($\tau = 25$ ms), thermal noise dominates, and maximum achievable antenna's sensitivity S_{h0} is realized.

In this case, according to the concept (challenge), a pulse with bandwidth of 100 Hz has detected when the condition $F = 1$ is fulfilled. Some mismatch in band widths has revealed, which is subject to further comprehension.

The article [8] the question is raised for further understanding and elaboration; we read: "... the main question of the pilot experiment is the closeness of the results to the aforementioned theoretical bound." Also, in the article [9] "problems" have pointed out. Similarly, the more significant aspect has arisen in comprehending the OGRAN project theory. Namely, it is shown that executor must reduce the threshold signal of the displacement measurer by 30 times. In this case, value h_{min} changes by the factor of 5.5 (15 dB) from the calculated result of 10^{-18}.to thermal limit $1.7\cdot 10^{-19}$. This difference between the current metric threshold signal and the limit one can consider as moderate and acceptable. Similarly, when establishing the reception bandwidth, somewhat arbitrary 6-fold widening of band $\Delta f_{\rm h0}$ has led to moderate and permissible decrease in sensitivity in comparison with the thermal limit one. The question is to admit the achieved threshold signal in metric of 10^{-18} sufficient to stop making efforts to further improvement the readout scheme. This means recognition of the achieved resolution of readout circuit as satisfactory and the scientific part of the OGRAN project is completed. As an alternative version, the formula (2) is not valid for engineering application for the case $F \neq 1$.

In principle, the noise factor expression as of 1996 requires redefinition to avoid values F < 1. This has done in articles [38,39] and before in report [29].

5. Conclusions

The procedure for preliminary design of sensitivity of bar gravitational antenna is selection of such set of parameter values in the noise factor formula to realize the value $F = 1$ when registering pulse from supernova burst with duration $\tau = 1$ ms [6]. Based on results of tests of the pilot model, significant difficulties have appeared with realization in hardware required values of photodiode power P and excess laser noise factor B. The initial intention has corrected [8,9,10] by increasing pulse duration to value of $\tau = 10$ ms. Then the task of calculating the desired value $F = 1$ has obtained its solution.

Deepening into of the OGRAN project theory [7] the general analytical relationship (12) has revealed between expression for the noise factor and expression for displacement resolution of registration scheme. Regardless of extracted specific expression for resolution, the corresponding numerical relationship has defined; namely, the achieved value of resolution $2\cdot 10^{-15}$ cm/Hz$^{1/2}$ [11] has assigned the value of $F = 30$ and value of threshold signal in metric of $h_{min} \simeq 10^{-18}$.

There has determined analytical dependence of design displacement resolution on initial intention of F =1 and there has determined its numerical value of $0.7\cdot 10^{-16}$ cm/Hz$^{1/2}$. While returning to astrophysical forecast of $\tau = 1$ ms the required resolution of readout is $0.7\cdot 10^{-17}$ cm/Hz$^{1/2}$.

The expression for displacement resolution has restored coherence of the OGRAN theory - from initial intention towards engineering realization. The extracted formula does not provide adequate description of features of the registration scheme implemented in hardware. In this way, the analytical expression for value G_e in the formula for noise factor must be much more complex. The absence of connection between theory and experiment allows designer to present inadequate theoretical relations. This may be a reason for difficulties in comprehension of the Pound-Drever-Hall registration scheme operation and, in particular, in determining origin of excessive technical noise in the facility.

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