SLAC-PUB-1464
August 1974

RESIDUE SUM RULES FOR INELASTIC (ANTI)NEUTRINO NUCLEON SCATTERING*

B. F. L. Ward
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

We discuss the complete set of current algebra sum rules for the Regge residues and asymptotic constant S-limits of the structure functions of inelastic (anti) neutrino nucleon scattering.

(Submitted to Phys. Rev.)

*Work supported by the U. S. Atomic Energy Commission.

I. INTRODUCTION

This paper is essentially a continuation of two previous papers[1, 2] on the subject of sum rules for the structure functions of forward current-hadron scattering. (Hereafter, we shall refer to Refs. 1 and 2 as A and B, respectively.) The principal concern of the present discussion will be the complete set of current algebra constraints on the Regge and asymptotic fixed hadronic mass[3] residues of the structure functions of inelastic (anti) neutrino nucleon scattering. For the sake of completeness and continuity, we shall begin by briefly reviewing the connection of the discussion here with that of A and B.

In A, we introduced[4] a method which in principle permits the systematic discussion of all equal-time current algebraic sum rules. However, there, for convenience, we made a convergence assumption about the states near $x = -q^2/2M\nu = 0, -1$. This assumption amounted to suppressing the possible presence of leading[5] Regge poles and asymptotic fixed hadronic mass states. Of course, it is well known that such a suppression is of no effect insofar as convergent scaling and fixed $-q^2$ sum rules are concerned. For divergent sum rules, such a suppression is understood to mean that the resulting expressions are only formal and require some kind of truncation[6] in general. In B, we showed that continuity in dynamics allows one to treat the states at $x=0, -1$ systematically in the framework of A. By applying the resulting formalism to the spin independent Schwinger term sum rule, it was shown that the required truncations of formal expressions occur naturally. Moreover, it was shown that this formalism yields new surprising sum rules for the Regge and asymptotic fixed hadronic mass limit residues. Only the isospin symmetric, spin independent, 0-i aspect of current algebra was considered in this latter connection in B. Here, as we have remarked above, we shall use the methods of A and B to discuss the entire set of residue sum rules following from equal-time current algebra taken together with continuity in dynamics.

We should point out that theoretical prejudice would suggest that the fixed hadronic mass limit residues which will be under discussion are in fact trivial.[6] However, for the most part, this will not concern us here. For definiteness we shall presume a form for the behavior of the structure functions in the $q^2 \to \infty$, $\mu^2 = (p+q)^2$ fixed limit which is general enough that the extension of our results to an arbitrary behavior will be immediate. The analogue of this last statement can be made about the form of Regge behavior which we shall presume.

Let us also remark that in the course of deriving our principal results (the residue sum rules) we shall naturally obtain the truncated (when necessary) versions of the conventional fixed $-q^2$ and scaling sum rules (both spin independent and spin dependent). The spin independent (untruncated) results are of course well known whereas the (untruncated) spin dependent results are discussed in A,[7] for example. We shall always record these (truncated) sum rules here mainly in the interest of completeness.

It will be apparent in what follows that the residue sum rules presented here are very restrictive insofar as the form of the residue functions is concerned. This fact alone makes these results very interesting. They should prove very useful theoretically in constraining the parameters of detailed analytic models of forward current hadron scattering. However, the models of current hadron scattering which have appeared to date have not been constructed in sufficient detail to permit their serious consideration in what follows. A sufficiently detailed model is under investigation.

We shall not belabor the details of the method which we shall employ in deriving the residue sum rules, since this procedure is described in some detail in B. Of course, a few details will always appear—in the interest of completeness.

The paper is organized as follows. In Section II, we briefly review the formalism developed in A and B. In Section III, we apply this formalism to all aspects of equal-time current algebra and obtain the complete set of residue sum rules. For clarity, we only list in III those results associated with the more familiar fixed $-q^2$ and scaling sum rules. The remaining results are relegated to Appendix II. Appendix I contains the forms we employ for the various asymptotic limits of the structure functions. Section IV contains some concluding remarks and an explicit statement of our interpretation of current algebra.

II. PRELIMINARIES

In this section we shall briefly review the method which we shall employ to discuss residue sum rules. As stated in the Introduction, a detailed description of this approach to sum rules may be found in B. We begin by setting the notation.

We shall be concerned with the structure functions parametrizing forward current-hadron scattering

$$\underset{q}{\text{current a}} + \underset{p}{\text{hadron } \alpha} \rightarrow \underset{q}{\text{current b}} + \underset{p}{\text{hadron } \beta} \quad ,$$

specializing always to the case where the hadron is a nucleon and the current is the full V-A weak current $J^a_\mu = V^a_\mu - A^a_\mu$, with $a = 1+i2$. (We take Cabbibo angle, θ_c, to be zero without loss of generality.) The corresponding hadronic tensor $W^{ab}_{\mu\nu}$ will be written in the convention of paper A, ignoring the possibility of time reversal violations:

$$
\begin{aligned}
W^{ab}_{\mu\nu} &= \frac{1}{2\pi}\int d^4y\, e^{iq\cdot y} \langle p,s| \left[J^b_\nu(y), J^a_\mu(0)\right] |p,s\rangle \\
&= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1 + \frac{1}{M^2}\left(p_\mu - \frac{q\cdot p}{q^2} q_\mu\right)\left(p_\nu - \frac{q\cdot p}{q^2} q_\nu\right) W_2 \\
&\quad - \frac{i}{2M^2}\,\epsilon_{\mu\nu\alpha\beta}\, p^\alpha q^\beta W_3 + \frac{q_\mu q_\nu}{M^2} W_4 + \frac{(p_\mu q_\nu + p_\nu q_\mu)}{M^2} W_5 \\
&\quad - \frac{1}{M}\left[\left(p_\mu - \frac{q\cdot p}{q^2} q_\mu\right)\left(s_\nu - \frac{q\cdot s}{q^2} q_\nu\right) + \left(p_\nu - \frac{q\cdot p}{q^2} q_\nu\right)\left(s_\mu - \frac{q\cdot s}{q^2} q_\mu\right)\right] V_1 \\
&\quad - \frac{1}{M}(q_\mu s_\nu + q_\nu s_\mu) V_3 + \frac{s\cdot q}{M}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) V_5 \\
&\quad - \frac{s\cdot q}{M^2 \nu}\left(p_\mu - \frac{q\cdot p}{q^2} q_\mu\right)\left(p_\nu - \frac{q\cdot p}{q^2} q_\nu\right) V_6 - \frac{s\cdot q\, q_\mu q_\nu}{Mq^2} V_7 \\
&\quad - \frac{s\cdot q}{Mq^2}(p_\mu q_\nu + p_\nu q_\mu) V_8 - \frac{i}{\nu}\,\epsilon_{\mu\nu\sigma\rho}\, s^\sigma q^\rho G_1 \\
&\quad - \frac{i}{Mq^2}\left[q^2 \epsilon_{\mu\nu\sigma\rho}\, s^\sigma p^\rho + (q_\mu \epsilon_{\nu\sigma\rho\delta} - q_\nu \epsilon_{\mu\sigma\rho\delta})\, s^\sigma p^\rho q^\delta\right] G_2 \\
&\quad - \frac{i}{Mq^2}(q_\mu \epsilon_{\nu\sigma\rho\delta} - q_\nu \epsilon_{\mu\sigma\rho\delta})\, s^\sigma p^\rho q^\delta\ G_4
\end{aligned}
\tag{2.1}
$$

where s is the nucleon spin, M the nucleon rest mass, and W_i, V_k, and G_j are functions only of q^2 and ν (we are suppressing U(3) labels).

We shall take a standard form for the equal-time commutation relations of the octet currents V_μ^a, A_μ^a:

$$\left[V_0^k(x),\ V_0^\ell(y)\right]_{x_0=y_0} = \left[A_0^k(x),\ A_0^\ell(y)\right] = i\delta^3(\vec{x}-\vec{y})\ f_{k\ell m}\ V_0^m(y) \tag{2.2a}$$

$$\left[V_0^k(x),\ A_0^\ell(y)\right]_{x_0=y_0} + \left[A_0^k(x),\ V_0^\ell(y)\right]_{x_0=y_0} = 2i\ \delta^3(\vec{x}-\vec{y})\ f_{k\ell m}\ A_0^m(y) \tag{2.2b}$$

$$\left[V_0^k(x),\ V_r^\ell(y)\right]_{x_0=y_0} + \left[A_0^k(x),\ A_r^\ell(y)\right]_{x_0=y_0} = 2i\ \delta^3(\vec{x}-\vec{y})\ f_{k\ell m}\ V_r^m(y) + i(\partial_x)^t\ S_{tr}^{k\ell}(y) \tag{2.2c}$$

$$\left[V_0^k(x),\ A_r^\ell(y)\right]_{x_0=y_0} + \left[A_0^k(x),\ V_r^\ell(y)\right]_{x_0=y_0} = 2i\ \delta^3(\vec{x}-\vec{y})\ f_{k\ell m}\ A_r^m(y) + i(\partial_x)^t\ S_{tr}^{5\,k\ell}(y) \tag{2.2d}$$

$$\left[V_r^k(x),\ V_s^\ell(y)\right]_{x_0=y_0} + \left[A_r^k(x),\ A_s^\ell(y)\right]_{x_0=y_0} = -2i\ \delta^3(\vec{x}-\vec{y})\left[g_{rs}f_{k\ell m}\ V_0^m(y) + \epsilon_{rs}{}^t d_{k\ell m}\ A_t^m(y)\right] + \text{(gradient terms)} \tag{2.2e}$$

$$\left[V_r^k(x),\ A_s^\ell(y)\right]_{x_0=y_0} + \left[A_r^k(x),\ V_s^\ell(y)\right]_{x_0=y_0} = -2i\ \delta^3(\vec{x}-\vec{y})\left[g_{rs}f_{k\ell m}\ A_0^m(y) + \epsilon_{rs}{}^t d_{k\ell m}\ V_t^m(y)\right] + \text{(gradient terms)} \tag{2.2f}$$

Here k, ℓ, m are U(3) labels ranging from 0 to 8; r, s, and t are spatial indices varying from 1 to 3. The objects S and S^5 are Schwinger terms.

The residue sum rules, like the more familiar fixed $-q^2$ and scaling sum rules, will be seen to follow from the familiar observation that, for $\vec{q}\cdot\vec{p}=0$,

$$\int_{-\infty}^{\infty} d\nu\ \frac{M}{p_0}\ W_{\mu\nu}^{ab} = \int d^3y\ e^{-i\vec{q}\cdot\vec{y}} \langle p,s|\left[J_\nu^b(y),\ J_\mu^a(0)\right]\Big|_{y_0=0}|p,s\rangle$$

$$\equiv C_{\mu\nu}^{ab}(\vec{q},p,s) \tag{2.3}$$

where $C^{ab}_{\mu\nu}$ is clearly determined by (2.2). Specifically, isolating the kinematically independent parts of (2.3) yields in general identities of the form

$$\int_0^\infty d\nu \; I(q,p) = D \tag{2.4}$$

where D is given by (2.2) and I is a linear combination of structure functions with coefficients determined by (2.1) as functions of q, p. As in B, in what follows, we shall augment (2.4) with the following assumptions:

1. Bjorken scaling in the form of (AI.1).
2. Regge and fixed hadronic mass limit behavior in the form (AI.2) and (AI.3), respectively.
3. Continuity in dynamics.
4. Commutativity of x-integration and $\lim_{bj}$ in the region $x < -1$, where the scaling limits vanish.

With these assumptions, it follows (see B, Eq. (2.10)) from (2.4) that

$$D = \lim_{\eta\to\infty}\left[\int_0^\eta d\nu \lim_{p_0\to\infty} I - \int_{-1}^{-\vec{q}^{\,2}/2\eta M} dx \lim_{p_0\to\infty} \frac{\nu^- I(q_0^-, \vec{q}, p)}{x}\right.$$

$$+ \lim_{p_0\to\infty}\left\{\frac{1}{2M}\int_{\mu_0^2}^{2p_0^2\vec{q}^{\,2}/\eta M} d\mu^2 \frac{H}{\sqrt{1+\dfrac{\vec{q}^{\,2}+\mu^2-M^2}{p_0^2}}}\right.$$

$$\left.\left.+\int_{-\vec{q}^{\,2}\left(1-\eta^2M^2/\vec{q}^{\,2}p_0^2\right)}^{\vec{q}^{\,4}p_0^2/\eta^2M^2} dq'^2 \frac{R}{2M\sqrt{q'^2+\vec{q}^{\,2}}}\right\}\right] \tag{2.5}$$

where H is the asymptotic $\nu\to-\infty$, $\mu^2 = (p+q)^2$-fixed form of I

$$I \underset{\substack{\nu\to-\infty\\ \mu^2\text{-fixed}}}{\longrightarrow} H \tag{2.6}$$

and R is the Regge asymptotic form of $p_0 I$. Thus, for D constant, the leading residue sum rules occur as the coefficients of p_0^{γ}, $\gamma > 0$, in (2.5). The (truncated) fixed $-q^2$ and scaling sum rules occur as the constant term (in P_0) in equations like (2.5), with the contributions of J=0 Regge poles and of terms constant in the limit (2.6) naturally included.

Our forms (AI.1 and AI.2) for the Bjorken and Regge asymptotic behavior of the structure functions are standard. The form (AI.3) for the fixed hadronic mass limit is as suggested by the DGS spectral representation.[6] Let us again point out that theoretical prejudice would suggest that the residue functions $g_{i', \alpha}$ which characterize the fixed hadronic mass limit are in fact trivial.[6] To repeat, this will not concern us here.

Having made these preliminary remarks, we shall now turn to the complete set of residue constraints.

III. RESIDUE SUM RULES

As we remarked above, in general residue sum rules may be obtained from each kinematically independent statement of the form (2.4). We shall now systematically discuss these results. Since the results are quite numerous, many of the them have been relegated to Appendix II in the interest of clarity. In this section we shall only list explicitly those residue sum rules which derive from consideration of the familiar sum rules of Adler,[8] Gross-Llewellyn Smith,[9] and Bjorken[10] in the context of the formalism of Section II.

We begin by considering the 0-0 aspect of (2.2), which is a convenient starting point for the derivation of Adler's sum rule for W_2. We have from (2.1) and (2.3)

$$\int_0^{\infty} d\nu \, \frac{M}{p_0} \left[\left(-1 + \frac{q_0^2}{q^2} \right) W_1^{\bar{\nu}-\nu} + \frac{1}{M^2} \left(p_0 + \frac{q_0}{2x} \right)^2 W_2^{\bar{\nu}-\nu} + \frac{q_0^2}{M^2} \, W_4^{\bar{\nu}-\nu} + \frac{2q_0 p_0}{M^2} \, W_5^{\bar{\nu}-\nu} \right] = 4g_V^{I_3} \frac{p_0}{M} \tag{3.1}$$

and

$$\int_0^\infty d\nu \; \frac{Ms_0}{p_0}\left[\left\{\frac{2}{M}\left(p_0+\frac{q_0}{2x}\right)+\frac{q_0}{Mx}\left(1-\frac{q_0^2}{q^2}\right)\right\}V_1^{\bar{\nu}-\nu}+\frac{2q_0}{M}\,V_3^{\bar{\nu}-\nu}\right.$$

$$\left.-\frac{q_0}{M}\left(1-\frac{q_0^2}{q^2}\right)V_5^{\bar{\nu}-\nu}+\frac{q_0}{M^2\nu}\left(p_0+\frac{q_0}{2x}\right)^2 V_6^{\bar{\nu}-\nu}+\frac{q_0^3}{Mq^2}\,V_7^{\bar{\nu}-\nu}-\frac{q_0}{Mx}\,V_8^{\bar{\nu}-\nu}\right]=4g_A\,\frac{I_3}{s_0} \tag{3.2}$$

where g_V and g_A are the vector and axial vector couplings, respectively. (Remember, we have set $\theta_c = 0$.) From (2.5) we obtain, for $q^2 < 0$,

$$\int_0^\infty d\nu\; W_2^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}}+\frac{1}{2M}\int_{\mu_0^2}^\infty d\mu^2\left[-q^2M^2\hat{g}_{1,0}^{2,0,1}+q^4\hat{g}_{2,0}^{0,0,2}\right.$$

$$\left.+\hat{g}_{4,0}^{2,2,0}+2\hat{g}_{5,0}^{1,1,0}\right]^{\bar{\nu}-\nu}\Bigg|_{q^2\text{-fixed}}=4g_V\,I_3 \tag{3.3}$$

$$\frac{2}{(\alpha-1)M^\alpha}C_{2,\alpha}^{\bar{\nu}-\nu}(q^2)\,(-q^2)^{\left(\frac{\alpha-1}{2}\right)}+\int_{q^2}^\infty\frac{ds}{\sqrt{s-q^2}}\left[\left(\frac{-q^2}{s}\,C_{1,\alpha}^{\bar{\nu}-\nu}(s)+\frac{2}{M}\,C_{5,\alpha}^{\bar{\nu}-\nu}(s)\right)\left(\frac{\sqrt{s-q^2}}{M}\right)^\alpha+\frac{q^4}{M^2s^2}\,\tilde{C}_{2,\alpha}^{\bar{\nu}-\nu}(s)\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-2}\right.$$

$$\left.+\,C_{4,\alpha}^{\bar{\nu}-\nu}(s)\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha+2}\right]$$

$$+\frac{1}{M^2}\int_{\mu_0^2}^\infty d\mu^2\left[-q^2M^2\hat{g}_{1,\alpha}^{1,0,1}+q^4\hat{g}_{2,\alpha}^{-1,0,2}+\hat{g}_{4,\alpha}^{1,2,0}+2\hat{g}_{5,\alpha}^{0,1,0}\right]^{\bar{\nu}-\nu}\Bigg|_{q^2\text{-fixed}}=$$

$$\alpha\in\mathscr{R} \tag{3.4}$$

$$\int_{\mu_0^2}^\infty d\mu^2\left[-q^2M^2\hat{g}_{1,\gamma}^{1,0,1}+q^4\hat{g}_{2,\gamma}^{-1,0,2}+\hat{g}_{4,\gamma}^{1,2,0}+2\hat{g}_{5,\gamma}^{0,1,0}\right]^{\bar{\nu}-\nu}\Bigg|_{q^2\text{-fixed}}=0\quad,$$

$$0<\gamma\notin\mathscr{R} \tag{3.5}$$

$$\int_0^\infty d\nu\ (2V_1+V_6)^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} + \frac{1}{2M}\int_{\mu_0^2}^\infty d\mu^2 \Big[2q^4\hat{g}^{0,0,2}_{7,0} + 2\hat{g}^{1,1,0}_{9,0} - q^2\hat{g}^{1,1,1}_{11,0}$$

$$+ q^4\hat{g}^{0,0,2}_{12,0} + \hat{g}^{1,3,1}_{13,0} + 2\hat{g}^{0,2,1}_{14,0}\Big]^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} = 4g_A^{I_3} \qquad (3.6)$$

$$\frac{2}{(\alpha-1)M^{\alpha-1}}\left(2C^{\bar{\nu}-\nu}_{7,\alpha}(q^2)+C^{\bar{\nu}-\nu}_{12,\alpha}(q^2)\right)(-q^2)^{\frac{\alpha-1}{2}} + \int_{q^2}^\infty \frac{ds}{\sqrt{s-q^2}}\left[\left(2C^{\bar{\nu}-\nu}_{9,\alpha}(s) - \frac{q^2}{s}C^{\bar{\nu}-\nu}_{11,\alpha}(s) + \frac{2M}{s}C^{\bar{\nu}-\nu}_{14,\alpha}(s)\right)\left(\frac{\sqrt{s-q^2}}{M}\right)\right.$$

$$\left. + \frac{q^4}{Ms^2}\left(2\tilde{C}^{\bar{\nu}-\nu}_{7,\alpha}(s) + \tilde{C}^{\bar{\nu}-\nu}_{12,\alpha}(s)\right)\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-2} + \frac{M^2}{s}C^{\bar{\nu}-\nu}_{13,\alpha}\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha+2}\right]$$

$$+ \frac{1}{M}\int_{\mu_0^2}^\infty d\mu^2\left[2q^4\hat{g}^{-1,0,2}_{7,\alpha} + 2\hat{g}^{0,1,0}_{9,\alpha} - q^2\hat{g}^{0,1,1}_{11,\alpha} + q^4\hat{g}^{-1,0,2}_{12,\alpha} + \hat{g}^{0,3,1}_{13,\alpha} + 2\hat{g}^{-1,2,1}_{14,\alpha}\right]^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}}$$

$$= 0 \quad , \qquad \alpha \in \mathscr{R} \qquad (3.7)$$

and

$$\int_{\mu_0^2}^\infty d\mu^2\left[2q^4\hat{g}^{-1,0,2}_{7,\gamma} + 2\hat{g}^{0,1,0}_{9,\gamma} - q^2\hat{g}^{0,1,1}_{11,\gamma} + q^4\hat{g}^{-1,0,2}_{12,\gamma} + \hat{g}^{0,3,1}_{13,\gamma} + 2\hat{g}^{-1,2,1}_{14,\gamma}\right]^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}}$$

$$= 0 \quad , \qquad 0 < \gamma \notin \mathscr{R} \qquad (3.8)$$

where the $\hat{g}_{i,\lambda}$ are defined in Appendix I, $\mathscr{R}$ is the set of all leading contributing Regge trajectory intercepts, and we have introduced

$$\tilde{C}^{\bar{\nu}-\nu}_{i,\alpha}(s) = C^{\bar{\nu}-\nu}_{i,\alpha}(s) - \theta(-s)\frac{s^2}{q^4}C^{\bar{\nu}-\nu}_{i,\alpha}(q^2) \quad , \quad i = 2, 7, 12 \qquad (3.9)$$

In obtaining (3.3)-(3.8) we have used the standard inequality

$$\max \mathscr{R} < 1$$

as well as the result

$$\lim_{bj}\left[2x\nu V_3 - \nu V_8\right] = 0$$

which follows from positivity[1] and the scaling behavior (AI. 1).

The result (3.3), taken together with the apparent validity of Adler's famous result[8] for W_2, implies that

$$\int_{\mu_0^2}^{\infty} d\mu^2 \left[-q^2M^2\hat{g}^{2,0,1}_{1,0} + q^4\hat{g}^{0,0,2}_{2,0} + \hat{g}^{2,2,0}_{4,0} + 2\hat{g}^{1,1,0}_{5,0}\right]^{\bar{\nu}-\nu}\Bigg|_{q^2\text{-fixed}} \approx 0.$$

(3.10)

We turn next to the $i \neq j$ aspect of current algebra. Proceeding precisely as above we have, for $q^2 < 0$,

$$\int_0^1 dx\, F_3^{\bar{\nu}+\nu}(x) + \frac{1}{4M}\int_{\mu_0^2}^{\infty} d\mu^2 \left[\hat{g}^{1,1,0}_{3,0}\right]^{\bar{\nu}+\nu}\Bigg|_{q^2\text{-fixed}} = 2g_V^{2B+Y}$$

(3.11)

$$Mf^{\bar{\nu}+\nu}_{3,\alpha}B\left(1-\alpha,\frac{\alpha-1}{2}\right)(-q^2)^{\left(\frac{1-\alpha}{2}\right)} + \int_{q^2}^{\infty} ds\, \tilde{C}^{\bar{\nu}+\nu}_{3,\alpha}(s)\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-1} + \int_{\mu_0^2}^{\infty} d\mu^2\left[\hat{g}^{0,1,0}_{3,\alpha}\right]^{\bar{\nu}+\nu}\Bigg|_{q^2\text{-fixed}} = 0,$$

$\alpha \in \mathscr{R}$ (3.12)

$$\int_{\mu_0^2}^{\infty} d\mu^2\left[\hat{g}^{0,1,0}_{3,\gamma}\right]^{\bar{\nu}+\nu}\Bigg|_{q^2\text{-fixed}} = 0 \quad , \quad 0 < \gamma \notin \mathscr{R} \qquad (3.13)$$

$$\int_0^1 dx\, F_{16}^{\bar{\nu}+\nu}(x) + \frac{M}{4}\int_{\mu_0^2}^{\infty} d\mu^2\left(\hat{g}^{1,0,0}_{16,1} + \hat{g}^{1,0,0}_{17,1}\right)^{\bar{\nu}+\nu}\Bigg|_{q^2\text{-fixed}} = -g_A^{2B+Y}$$

(3.14)

$$Mf^{\bar{\nu}+\nu}_{16,\alpha}B\left(1-\alpha,\frac{\alpha-1}{2}\right)(-q^2)^{\left(\frac{1-\alpha}{2}\right)} + \int_{q^2}^{\infty} ds\, \tilde{C}^{\bar{\nu}+\nu}_{16,\alpha}(s)\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-1} + M\int_{\mu_0^2}^{\infty} d\mu^2\left(\hat{g}^{1,0,0}_{16,\alpha} + \hat{g}^{1,0,0}_{17,\alpha}\right)^{\bar{\nu}+\nu}\Bigg|_{q^2\text{-fixed}} = 0$$

$\alpha \in \mathscr{R}$ (3.15)

$$\int_{\mu_0^2}^{\infty} d\mu^2\left(\hat{g}^{1,0,0}_{16,\gamma} + \hat{g}^{1,0,0}_{17,\gamma}\right)^{\bar{\nu}+\nu}\Bigg|_{q^2\text{-fixed}} = 0 \quad ,$$

$0 < \gamma \notin \{1\} \cup \mathscr{R}$, (3.16)

$$f_{17,0}^{\bar{\nu}+\nu} = 0 \quad , \tag{3.17}$$

$$- \int_0^\infty d\nu\, \tilde{G}_2^{\bar{\nu}+\nu}\Big|_{q^2\text{-fixed}} + \frac{1}{2}\int_0^\infty ds\, \frac{1}{\sqrt{s-q^2}} \left[\tilde{C}_{16,0}(s) - \frac{1}{\sqrt{s-q^2}}\, C_{17,0}(s)\right]^{\bar{\nu}+\nu}$$

$$+ \frac{1}{2} f_{16,0}^{\bar{\nu}+\nu} B\left(1, -\frac{1}{2}\right)\sqrt{-q^2} - \int_0^\infty \frac{dx}{x} \left(2x\, \tilde{F}_{16} - \tilde{F}_{17}(x)\right)^{\bar{\nu}+\nu}$$

$$+ \frac{1}{2M} \int_{\mu_0^2}^\infty d\mu^2 \left\{ M\left[\hat{g}_{16,0}^{1,0,0} - M\hat{g}_{16,0}^{2,0,0}\right]^{\bar{\nu}+\nu} - \left[\hat{g}_{17,0}^{0,0,0} - M\,\hat{g}_{17,0}^{1,0,0}\right]^{\bar{\nu}+\nu} \right\}\Bigg|_{q^2\text{-fixed}} = 2g_A^{2B+Y} \tag{3.18}$$

$$\frac{1}{M} \int_{q^2}^\infty ds\, C_{17,\alpha}^{\bar{\nu}+\nu} \left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-2} + \int_{\mu_0^2}^\infty d\mu^2 \left[M^2 \hat{g}_{16,\alpha}^{2,0,0} + \hat{g}_{17,\alpha}^{0,0,0}\right]^{\bar{\nu}+\nu}\Bigg|_{q^2\text{-fixed}} = 0 \quad ,$$

$$\alpha \in \mathscr{R} \tag{3.19}$$

$$\int_{\mu_0^2}^\infty d\mu^2 \left[M^2 \hat{g}_{16,\gamma}^{2,0,0} + \hat{g}_{17,\gamma}^{0,0,0}\right]^{\bar{\nu}+\nu}\Bigg|_{q^2\text{-fixed}} = 0, \quad 0 < \gamma \notin \mathscr{R} \tag{3.20}$$

Here, we have defined

$$\tilde{C}_{i,\alpha}^{\bar{\nu}+\nu}(s) = C_{i,\alpha}^{\bar{\nu}+\nu}(s) - f_{16,\alpha}^{\bar{\nu}+\nu}\left(\frac{M}{s}\right)^\alpha \theta(s), \quad i=3,16 \tag{3.21}$$

$$\tilde{G}_2(\nu, q^2) = G_2(\nu, q^2) - \sum_{\alpha\in\mathscr{R}} C_{17,\alpha}(q^2)\nu^{\alpha-1} \tag{3.22}$$

$$\tilde{F}_{16}^{\bar{\nu}+\nu} = \theta(1-x)\, F_{16}^{\bar{\nu}+\nu}(x) \tag{3.23}$$

and

$$\frac{1}{x}\tilde{F}_{17}^{\bar{\nu}+\nu} = \frac{\theta(1-x)}{x}\, F_{17}^{\bar{\nu}+\nu} + \sum_{\alpha\in\mathscr{R}} f_{17,\alpha}^{\bar{\nu}+\nu}\, \frac{x^{-\alpha-1}}{2^\alpha} \quad , \tag{3.24}$$

where the $f_{j,\alpha}$ are given in (AI.8); and, $B(\rho, \delta)$ is the Eulerian integral of the first kind.

In obtaining (3.11)-(3.20) we have invoked the standard Regge ideas[12] to conclude that the relevant sets $\mathscr{R}$ of leading Regge singularities also satisfy

$$\max \mathscr{R} < 1 \quad ,$$

just as for the isospin antisymmetric cases discussed above.

The apparent validity of the sum rule of Gross and Llewellyn Smith implies

$$\int_{\mu_0^2}^{\infty} d\mu^2 \left[\hat{g}_{3,0}^{1,1,0}\right]^{\bar{\nu}+\nu} \Big|_{q^2\text{-fixed}} \approx 0 \tag{3.25}$$

The polarization sum rules (3.14) and (3.18), with $\widetilde{C}_{16,0}$, $\widetilde{C}_{17,0}$ and the respective $\hat{g}$'s set to zero, are recognized to be the familiar (formal) polarization results of Bjorken[10] and Dicus, Jackiw, and Teplitz.[13]

Considering finally the i=j and 0-i aspects of current algebra we obtain, for $q^2 < 0$, the results in Appendix II in addition to

$$\int_0^1 dx\, F_1^{\bar{\nu}-\nu}(x) + \frac{1}{4}\int_{\mu_0^2}^{\infty} d\mu^2 \left[\hat{g}_{1,0}^{1,0,0}\right]^{\bar{\nu}-\nu} \Big|_{q^2\text{-fixed}} = 2g_V^{I_3} \tag{3.26}$$

$$Mf_{1,\alpha}^{\bar{\nu}-\nu}\, B\left(1-\alpha, \frac{\alpha-1}{2}\right)\left(-q^2\right)^{\frac{1-\alpha}{2}} + \int_{q^2}^{\infty} ds\, \widetilde{C}_{1,\alpha}^{\bar{\nu}-\nu}(s)\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-1} + M\int_{\mu_0^2}^{\infty} d\mu^2 \left[\hat{g}_{1,\alpha}^{1,0,0}\right]^{\bar{\nu}-\nu} \Big|_{q^2\text{-fixed}} = 0$$

$$\alpha \in \mathscr{R} \tag{3.27}$$

$$\int_{\mu_0^2}^{\infty} d\mu^2 \left[\hat{g}_{1,\gamma}^{1,0,0}\right]^{\bar{\nu}-\nu} \Big|_{q^2\text{-fixed}} = 0 \quad , \qquad 0 < \gamma \notin \{1\} \cup \mathscr{R} \tag{3.28}$$

$$C_{3,0}^{\bar{\nu}-\nu}(q^2) = -f_{3,0}^{\bar{\nu}-\nu} \tag{3.29}$$

$$\int_0^\infty d\nu\ \widetilde{W}_3^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} + \frac{1}{2}\int_{q^2}^\infty ds\ \frac{\widetilde{C}_{3,0}^{\bar{\nu}-\nu}(s)}{s-q^2} - \int_0^{x_0}\frac{dx}{x}\ \widetilde{F}_3^{\bar{\nu}-\nu}$$

$$+ \sum_{\alpha>0}\frac{1}{\alpha 2^\alpha}\ f_{3,\alpha}^{\bar{\nu}-\nu}\ x_0^{-\alpha} + \frac{1}{2M}\int_{\mu_0^2}^\infty d\mu^2\left[\hat{g}'^{0,0,0}_{3,0}\right]^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} = 0$$

(3.30)

$$\int_{q^2}^\infty ds\ C_{3,\alpha}^{\bar{\nu}-\nu}(s)\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-2} + M\int_{\mu_0^2}^\infty d\mu^2\left[\hat{g}'^{0,0,0}_{3,\alpha}\right]^{\bar{\nu}-\nu} = 0\ ,\quad 0<\alpha\in\mathscr{R} \qquad (3.31)$$

$$\int_{\mu_0^2}^\infty d\mu^2\left[\hat{g}'^{0,0,0}_{3,\gamma}\right]^{\bar{\nu}-\nu} = 0\ ,\qquad 0<\gamma\notin\mathscr{R} \qquad (3.32)$$

where $x_0 \geq 1$,

$$\widetilde{C}_{1,\alpha}(s) = C_{1,\alpha}(s) - \theta(s)\ f_{1,\alpha}\left(\frac{M}{s}\right)^\alpha\ ,$$

$$\widetilde{C}_{3,0}(s) = C_{3,0}(s) - \theta(-s)\ C_{3,0}(q^2) - \theta(s)\ f_{3,0}\ ,$$

(3.33)

$$\frac{1}{x}\widetilde{F}_3^{\bar{\nu}-\nu} = \theta(1-x)\frac{F_3^{\bar{\nu}-\nu}(x)}{x} + \sum_{\alpha\geq 0}\frac{1}{2^\alpha}\ f_{3,\alpha}^{\bar{\nu}-\nu}\ x^{-\alpha-1}\ ,$$

and

$$\widetilde{W}_3 = W_3 - \sum_{0<\alpha\in\mathscr{R}} C_{3,\alpha}(q^2)\ \nu^{\alpha-1} - \theta\left(\nu + \frac{q^2}{2Mx_0}\right)\ C_{3,0}(q^2)\ \nu^{-1}\ .$$

Of course, (3.26) with $\hat{g}_{1,0}=0$ is Bjorken's[10] sum rule for F_1 and (3.30) is a properly truncated version of a formal sum rule for W_3 first given by Adler.[8]

The reader will notice that throughout our discussion we have implicitly taken the respective functions $\hat{g}_{i,\gamma}^{a,b,c}$ to fall off as $\mu^2\to\infty$ fast enough that, for $\gamma \geq 0$,

$$\int_{\mu_0^2}^{P_0^2} d\mu^2 \, \hat{g}_{i,\gamma}^{a,b,c} \tag{3.34}$$

converges faster than $1/P_0^{\gamma}$ as $P_0 \to \infty$. The DGS representation[6] taken together with the absence of terms like

$$(1+x)^{-m} , \qquad m > 0 ,$$

in the scaling limits for $x \to -1$, for example, guarantees the validity of this assumption for $i \neq 1, 16$ whenever

$$a-b + \frac{1}{2} c \geq 0 \tag{3.35}$$

and for $i = 1, 16$ whenever

$$a-b + \frac{1}{2} c \geq 2 \tag{3.36}$$

For all other cases, we make this assumption for convenience; it is trivial to relax.

We should also point out that in truncating the residues as in (3.27), we are taking, without loss of content, for example,

$$C_{1,\alpha}(s) \underset{s \to \infty}{\longrightarrow} \frac{M^{\alpha}}{s^{\alpha}} \sum_{n=0}^{\infty} s^{-n} f_n \tag{3.37}$$

with $f_0 = f_{1,\alpha}$. This assures the appropriate convergence of the integrals over the $\widetilde{C}_{i,\alpha}$.

This completes our derivation of residue sum rules. The results presented above, taken together with those listed in Appendix II, represent a complete discussion of such constraints.

IV. DISCUSSION

The analysis in III may be taken to represent a strict interpretation of equal time current algebra sum rules. To connect with the work of previous authors, we see that we must set the contributions of the $g_{i,\alpha}$'s to zero, even after subtracting the Regge contributions. Although theoretical prejudice would suggest that the $g_{i,\alpha}$'s are all indeed trivial, we see that the residue sum rules would clearly suggest otherwise in many cases. As we have pointed out above, the correctness of the famous sum rules such as Adler's for W_2 may be interpreted as implying the triviality of integrals over the respective $g_{i,\alpha}$. Of course, this is well known.[6]

Throughout our discussion, we have for the most part only isolated the nonvanishing functions of (P_0, η) in the vicinity of (∞, ∞) in the current algebra identities. However, in principle, all independent functions of these variables may be so isolated. Specifically, the current algebra identity, for $\vec{q} \cdot \vec{p} = 0$,

$$\int_0^\infty d\nu \; I = D \quad , \tag{2.4}$$

where D is a number, and takes the form , as $P_0 \to \infty$,

$$\sum_\gamma \Gamma_\gamma \, P_0^\gamma + \Gamma' \, \log P_0 = D \tag{4.1}$$

where the Γ_γ and Γ' are independent of P_0. (We are using (AI.1), (AI.2), and (AI.3).) Thus, we may conclude

$$\Gamma_0 = D \quad , \tag{4.2a}$$

$$\Gamma' = 0 \quad , \tag{4.2b}$$

$$\Gamma_\gamma = 0 \quad , \quad \gamma \neq 0 \quad . \tag{4.2c}$$

Equation (4.2a) is the (truncated) fixed-q^2-scaling sum rule associated with (2.4). The results (4.2b) and (4.2c) are residue constraints. In the text above, we have only discussed the cases $\Gamma' = 0$ and $\Gamma_\gamma = 0$ for $\gamma > 0$. The results for $\gamma < 0$ would in general yield relations involving the derivatives of the structure functions and, therefore, would often times require additional dynamical assumptions. For this reason, we have not discussed such constraints here, although they would be straightforward to consider. Indeed, at several points in III above we have already implicitly assumed (for convenience) the structure functions to be differentiable in q^2 almost everywhere.

The development presented here clearly provides a basis for confronting all aspects of equal-time current algebra with experiment, since it has naturally given truncated (finite) versions of all conventional divergent formal sum rules. Perhaps the prediction of our analysis which is easiest to check experimentally is the constancy in q^2 of the residues $C_{3,0}^{\bar{\nu}-\nu}$ and $C_{16,0}^{\bar{\nu}-\nu}$ for $q^2 < 0$. Unfortunately, there appear to be no candidates for the implied trajectories. In view of the work of previous authors, in comparing the truncated fixed-q^2 and scaling sum rules presented here with experiment, one should clearly interpret any violations as evidence against the trivially of all $g_{i,\alpha}$.

We end by re-emphasizing that the residue constraints discussed here, specialized to situations where the constant S-limit residues are trivial, are very restrictive insofar as the form of the Regge residues is concerned. To repeat, it is this fact that gives the results in III immediate applicability, even though they are experimentally somewhat intractable. The analytic models presently in the literature have not been constructed with such restrictions in mind and, in general, are not to be examined seriously in this context. However, clearly, any detailed model of inelastic scattering satisfying equal-time current algebra and assumptions (1)-(4) in II above will be subject to these restrictions. Such detailed models are under investigation.

ACKNOWLEDGEMENT

I find it most pleasing to acknowledge helpful discussions with J. D. Bjorken, S. J. Brodsky, F. J. Gilman, and S. B. Treiman.

APPENDIX I: ASYMPTOTIC FORMS OF THE STRUCTURE FUNCTIONS

In this appendix we shall for convenience list the forms we shall employ for the Bjorken, Regge and fixed-μ^2, $\nu \to -\infty$ limits of the structure functions. For brevity, we shall occasionally refer to the latter of these limits as the constant S-limit.[6] We list first the scaling behavior, which is well known: In the $\lim_{bj}$,

$$M(W_1, G_1) \to (F_1, F_{16})$$

$$\nu(W_2, W_3, V_1, V_3, V_5, V_6, V_8, G_2, G_4) \to (F_2, F_3, F_7, F_9, F_{11}, F_{12}, F_{14}, F_{17}, F_{19})$$

$$(\nu^2/M)(W_4, W_5, V_7) \to (F_4, F_5, F_{13}) \tag{AI. 1a}$$

For convenience, we also take each structure function T to have a Bjorken expansion of the form

$$T \underset{\lim_{bj}}{\longrightarrow} \sum_{n=0}^{\infty} t_n(x)\, \nu^{-n} \tag{AI. 1b}$$

Let us note that (AI. 1a) deviates slightly from the convention of paper (A) for $F_{16}, \ldots, F_{20}$.

We list next the presumed Regge asymptotic behavior, which is, of course, also familiar. In the Regge limit we have the standard forms

$$W_i \to \sum_{\alpha \geq 0} C_{i,\alpha}(q^2)\, \nu^{\alpha} \quad , \qquad i = 1, 4$$

$$W_2 \to \sum_{\alpha \geq 0} C_{2,\alpha}(q^2)\, \nu^{\alpha-2}$$

$$W_i \to \sum_{\alpha \geq 0} C_{i,\alpha}(q^2)\, \nu^{\alpha-1} \qquad i = 3, 5$$

$$V_i \to \sum_{\alpha \geq 0} C_{i+6,\alpha}(q^2)\, \nu^{\alpha-1} \qquad i=3,5,7$$

$$V_i \to \sum_{\alpha \geq 0} C_{i+6,\alpha}(q^2)\, \nu^{\alpha-2} \qquad i=1,6,8$$

$$G_1 \to \sum_{\alpha \geq 0} C_{16,\alpha}(q^2)\, \nu^{\alpha}$$

$$G_k \to \sum_{\alpha \geq 0} C_{15+k,\alpha}(q^2)\, \nu^{\alpha-1} \qquad k=2,4 \qquad \text{(AI. 2)}$$

In the fixed hadronic mass, $\nu \to -\infty$ limit, we first define[6]

$$W_1 \to \sum_{\gamma} g_{1,\gamma}(\mu^2)\, |\nu|^{\gamma}$$

$$W_i \to \sum_{\gamma} g_{i,\gamma}(\mu^2)\, |\nu|^{\gamma-1} \qquad i=2,3$$

$$W_i \to \sum_{\gamma} g_{i,\gamma}(\mu^2)\, |\nu|^{\gamma-2} \qquad i=4,5$$

$$V_j \to \sum_{\gamma} g_{j+6,\gamma}(\mu^2)\, |\nu|^{\gamma-1} \qquad j=1,3,5,6,8$$

$$V_7 \to \sum_{\gamma} g_{13,\gamma}(\mu^2)\, |\nu|^{\gamma-2}$$

$$G_1 \to \sum_{\gamma} g_{16,\gamma}(\mu^2)\, |\nu|^{\gamma}$$

$$G_k \to \sum_{\gamma} g_{15+k,\gamma}(\mu^2)\, |\nu|^{\gamma-1} \qquad k=2,4 \qquad \text{(AI. 3)}$$

For the derivation of the sum rules it is then convenient to introduce the functions $\{\hat{g}_{k,\lambda}^{\ell,m,n}(\mu^2, -\vec{q}^{\,2})\}$ defined by

$$\frac{P_0^{-\ell} q_0^m q^{-2n}}{\sqrt{1+\frac{\vec{q}^{\,2}+\mu^2-M^2}{P_0^2}}} \sum_{\lambda} g_{k,\lambda}(\mu^2)\,|\nu_+|^{\beta(k,\lambda)} \underset{P_0\to\infty}{\longrightarrow} \sum_{\lambda} P_0^{\lambda} \hat{g}_{k,\lambda}^{\ell,m,n}(\mu^2, -\vec{q}^{\,2}) \tag{AI. 4}$$

where $\beta(k,\lambda)$ is taken from (AI. 2):

$$\beta(k,\lambda) = \begin{cases} \lambda\ , & k=1,16 \\ \lambda-1\ , & k=2,3,7,9,11,12,14,17,19 \\ \lambda-2\ , & k=4,5,13 \end{cases} \tag{AI. 5}$$

We shall also use on some occasions the functions $\hat{g}'^{\,\ell,m,n}_{i,\gamma}$ defined by

$$\left(1-M^2/P_0^2\right)^{1/2} \sum P_0^{\gamma} \hat{g}_{i,\gamma}^{\ell,m,n} = \sum P_0^{\gamma} \hat{g}'^{\,\ell,m,n}_{i,\gamma} \tag{AI. 6}$$

In (AI. 3) we are taking $\vec{q}\cdot\vec{p}=0$ and q_0, q^2, and ν are all on the q_0^+-branch of $q_0(x,\vec{q},p_0)$ defined by

$$q_0^+(x,\vec{q},P_0) = \left[-x-\left(x^2+\vec{q}^{\,2}/P_0^2\right)^{1/2}\right] P_0 \tag{AI. 7}$$

Of course, we are presuming that (AI.4) and (AI. 6) make sense.

Finally, it is also convenient to introduce as $q^2 \to \infty$

$$\begin{aligned} C_{i,\alpha}(q^2) &\to f_{i,\alpha}\left(\frac{M}{q^2}\right)^{\alpha}\ , && i=1,3,9,11,16,17,19 \\ C_{i,\alpha}(q^2) &\to f_{i,\alpha}\left(\frac{M}{q^2}\right)^{\alpha-1}\ , && i=2,7,12,14 \\ C_{i,\alpha}(q^2) &\to f_{i,\alpha}\left(\frac{M}{q^2}\right)^{\alpha+1}\ , && i=5,13 \\ C_{4,\alpha}(q^2) &\to f_{4,\alpha}\left(\frac{M}{q^2}\right)^{\alpha+2}\ . && \end{aligned} \tag{AI. 8}$$

APPENDIX II: CURRENT ALGEBRA RESIDUE CONSTRAINTS

In this appendix we list the complement of the residue contraints presented in Section III of the text.

From the i-j aspect of (2.2) we have, for $q^2 < 0$,

$$\int_0^\infty d\nu \, V_1^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} + \frac{1}{2M}\int_{\mu_0^2}^\infty d\mu^2 \left[\hat{g}'^{-1,0,0}_{7,1} - M^2 \hat{g}'^{1,0,0}_{7,1}\right]^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}}$$

$$- \int_0^1 \frac{dx}{x} F_7^{\bar{\nu}-\nu} = 0 \tag{AII. 1}$$

$$\int_0^\infty d\nu \, V_6^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} + \frac{1}{2M}\int_{\mu_0^2}^\infty d\mu^2 \left[\hat{g}'^{-1,0,0}_{12,1} - M^2 \hat{g}'^{+1,0,0}_{12,1}\right]^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}}$$

$$- \int_0^1 \frac{dx}{x} F_{12}^{\bar{\nu}-\nu} = 0 \tag{AII. 2}$$

$$\int_{q^2}^\infty ds \, \tilde{\tilde{C}}^{\bar{\nu}-\nu}_{i,\alpha}(s)\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-3} + M^2 \, f^{\bar{\nu}-\nu}_{i,\alpha} \, (-q^2)^{(1-\alpha)/2} \, B\left(2-\alpha, \frac{\alpha-1}{2}\right)$$

$$+ \frac{2}{(\alpha-1)M^{\alpha-3}} \, C^{\bar{\nu}-\nu}_{i,\alpha}(q^2)\,(-q^2)^{(\alpha-1)/2}$$

$$+ M\int_{\mu_0^2}^\infty d\mu^2 \left[\hat{g}'^{-1,0,0}_{i,\alpha} - M^2 \hat{g}'^{1,0,0}_{i,\alpha}\right]^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} = 0 ,$$

$$\alpha \in \mathscr{R} , \qquad i = 7, 12 \tag{AII. 3}$$

$$\int_{\mu_0^2}^\infty d\mu^2 \left[\hat{g}^{-1,0,0}_{7,\gamma} - M^2 \hat{g}^{1,0,0}_{7,\gamma}\right]^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} = 0 , \quad 0 \le \gamma \notin \{1\} \cup \mathscr{R} \tag{AII. 4}$$

$$\int_{\mu_0^2}^{\infty} d\mu^2 \left[\hat{g}'^{-1,0,0}_{12,\gamma} - M^2\, \hat{g}'^{+1,0,0}_{12,\gamma}\right]^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} = 0\,, \quad 0 \le \gamma \notin \{1\} \cup \mathscr{R} \qquad \text{(AII. 5)}$$

$$2\int_0^1 dx\, F_{11}^{\bar{\nu}-\nu}(x) + \frac{1}{2M}\int_{\mu_0^2}^{\infty} d\mu^2 \left[\hat{g}'^{0,1,0}_{11,1}\right]^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} = 4g_A^{I_3} \qquad \text{(AII. 6)}$$

$$\int_{q^2}^{\infty} ds\, \tilde{C}_{11,\alpha}^{\bar{\nu}-\nu}(s)\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-1} + M\, f_{11,\alpha}^{\bar{\nu}-\nu}\, B\left(1-\alpha, \frac{\alpha-1}{2}\right)(-q^2)^{\frac{1-\alpha}{2}}$$

$$+ \int_{\mu_0^2}^{\infty} d\mu^2 \left[\hat{g}'^{0,1,0}_{11,\alpha}\right]^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} = 0\,, \quad \alpha \in \mathscr{R} \qquad \text{(AII. 7)}$$

$$\int_{\mu_0^2}^{\infty} d\mu^2 \left[\hat{g}'^{0,1,0}_{11,\gamma}\right]^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} = 0, \qquad 0 \le \gamma \notin \{1\} \cup \mathscr{R} \qquad \text{(AII. 8)}$$

where

$$\tilde{C}_{11,\alpha}(s) = C_{11,\alpha}(s) - \theta(s)\, f_{11,\alpha}\left(\frac{M}{s}\right)^{\alpha} \qquad \text{(AII. 9)}$$

$$\tilde{\tilde{C}}_{i,\alpha} = C_{i,\alpha}(s) - \theta(-s)\, C_{i,\alpha}(q^2) - \theta(s)\, f_{i,\alpha}\left(\frac{M}{s}\right)^{\alpha-1}, \quad i=7,12 \qquad \text{(AII. 10)}$$

From the 0-i aspect of current algebra we have the results in B and, in addition,

$$\int_0^{\infty} d\nu\, W_2^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} + \frac{1}{2M}\int_{\mu_0^2}^{\infty} d\mu^2 \left[q^2\, \hat{g}'^{-1,0,1}_{2,1} + \hat{g}'^{0,1,0}_{5,1}\right]^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} = 4g_V^{I_3}$$

(AII. 11)

$$\frac{1}{2M}\int_{q^2}^{\infty}\frac{ds}{\sqrt{s-q^2}}\left[\frac{q^2}{s}\;\bar{C}_{2,\alpha}(s)\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-2}+M\,C_{5,\alpha}(s)\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha}\right]^{\bar{\nu}-\nu}$$

$$-\frac{2\,\phi(\alpha)}{\alpha-1}\,(-q^2/2M^2)^{\frac{\alpha-1}{2}}\;C_{2,\alpha}^{\bar{\nu}-\nu}\;(q^2)$$

$$+\frac{1}{2M}\int_{\mu_0^2}^{\infty}d\mu^2\left[q^2\,\hat{g}'^{-1,0,1}_{2,\alpha}+\hat{g}'^{0,1,0}_{5,\alpha}\right]^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}}=0\;,\quad\alpha\in\mathscr{R}\;,$$

(AII. 12)

$$\int_{\mu_0^2}^{\infty}d\mu^2\left[q^2\,g'^{-1,0,1}_{2,\gamma}+g'^{0,1,0}_{5,\gamma}\right]^{\bar{\nu}-\nu}=0,\qquad 0<\gamma\notin\mathscr{R}\;, \tag{AII. 13}$$

where

$$\bar{C}_{2,\alpha}(s)\;=\;C_{2,\alpha}(s)\;-\;\theta(q^2/2-s)\,C_{2,\alpha}(q^2)\qquad,\text{ and}$$

$$\phi(\alpha)\;=\;\int_0^1 d\rho\;\frac{\rho^{(\alpha-1)/2}}{(\rho-2)^2}-\;1\qquad; \tag{AII. 14}$$

$$f_{11,0}^{\bar{\nu}+\nu}=0 \tag{AII. 15}$$

$$\int_0^{\infty}d\nu\quad\frac{1}{2x}(2V_1+2x\,V_3+V_6-V_8)\Big|_{q^2\text{-fixed}}^{\bar{\nu}+\nu}-\int_0^{\infty}dx\;\frac{\widetilde{F}_{11}^{\bar{\nu}+\nu}}{x}$$

$$+\frac{1}{2M}\int_0^{\infty}ds\left\{\left[\frac{-q^2}{s^2}\;(2C_{7,0}(s)+C_{12,0}(s))+\frac{1}{M}\,C_{9,\alpha}(s)+\frac{1}{s}\,C_{14,0}(s)\right]\left(\frac{M^2}{s-q^2}\right)\right.$$

$$\left.+\;\frac{M}{s}\;(C_{11,0}(s)+C_{13,0}(s))\right\}^{\bar{\nu}+\nu}$$

$$+\frac{1}{2M}\int_{\mu_0^2}^{\infty}d\mu^2\left\{-q^2\left[2\hat{g}^{-1,1,2}_{7,0}+\hat{g}^{-1,1,2}_{12,0}\right]+\hat{g}^{0,0,0}_{9,0}+\hat{g}^{0,2,1}_{11,0}+\hat{g}^{0,2,1}_{13,0}\right.$$

$$\left.+\;\hat{g}^{-1,1,1}_{14,0}\right\}\Big|_{q^2\text{-fixed}}^{\bar{\nu}+\nu}=\;0\quad, \tag{AII. 16}$$

where

$$\frac{1}{x}\,\widetilde{F}_{11}^{\bar{\nu}+\nu}(x) = \theta(1-x)\,\frac{F_{11}^{\bar{\nu}+\nu}(x)}{x} + \sum_{\alpha}\frac{1}{2^{\alpha}}\,f_{11,\alpha}^{\bar{\nu}+\nu}\,x^{-\alpha-1}\ ,$$

$$\int_{q^2}^{\infty} ds \left\{\left[\frac{-q^2}{s^2}\left(2C_{7,\alpha}(s) + C_{12,\alpha}(s)\right) + \frac{1}{M}\,C_{9,\alpha}(s) + \frac{1}{s}\,C_{14,\alpha}(s)\right]\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-2}\right.$$

$$\left. + \frac{M}{s}\left(C_{11,\alpha}(s) + C_{13,\alpha}(s)\right)\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha}\right\}^{\bar{\nu}+\nu}$$

$$+ \int_{\mu_0^2}^{\infty} d\mu^2 \left\{ -q^2\left[2\hat{g}_{7,\alpha}^{-1,1,2} + \hat{g}_{12,\alpha}^{-1,1,2}\right] \quad \hat{g}_{9,\alpha}^{0,0,0} + \hat{g}_{11,\alpha}^{0,2,1} + \hat{g}_{13,\alpha}^{0,2,1}\right.$$

$$\left. + \hat{g}_{14,\alpha}^{-1,1,1}\right\}^{\bar{\nu}+\nu}\Bigg|_{q^2\text{-fixed}} = 0\ , \qquad 0 < \alpha \in \mathscr{R} \qquad \text{(AII. 17)}$$

$$\int_{\mu_0^2}^{\infty} d\mu^2 \left\{ -q^2\left(2\hat{g}_{7,\gamma}^{-1,1,2} + \hat{g}_{12,\gamma}^{-1,1,2}\right) + \hat{g}_{9,\gamma}^{0,0,0} + \hat{g}_{11,\gamma}^{0,2,1} + \hat{g}_{13,\gamma}^{0,2,1} + \hat{g}_{14,\gamma}^{-1,1,1}\right\}^{\bar{\nu}+\nu}\Bigg|_{q^2\text{-fixed}} = 0\ ,$$

$$0 < \gamma \notin \mathscr{R}, \qquad \text{(AII. 18)}$$

$$\int_0^{\infty} d\nu\ \frac{MV_6}{\nu}\Bigg|_{q^2\text{-fixed}}^{\bar{\nu}+\nu} - \frac{M}{2}\int_{\mu_0^2}^{\infty} d\mu^2\left[\hat{g}_{7,0}^{1,1,1} - q^2\,\hat{g}_{12,0}^{1,-1,1} - \hat{g}_{14,0}^{1,1,1}\right]^{\bar{\nu}+\nu}\Bigg|_{q^2\text{-fixed}} = \sigma^5\ ,$$

(AII. 19)

where σ^5 is the Schwinger term defined in Eq. (59) of paper A, for example,

$$\int_{\mu_0^2}^{\infty} d\mu^2\left[g_{7,\gamma}^{1,1,1} - q^2 g_{12,\gamma}^{1,-1,1} - g_{14,\gamma}^{1,1,1}\right]^{\bar{\nu}+\nu}\Bigg|_{q^2\text{-fixed}} = 0\ , \quad 0 < \gamma\ , \quad \text{(AII. 20)}$$

$$f_{19,0}^{\bar{\nu}+\nu} = 0 \qquad \text{(AII. 21)}$$

$$\int_0^\infty d\nu\ \widetilde{G}_2^{\bar{\nu}+\nu}\Big|_{q^2\text{-fixed}} + \frac{1}{2}\int_0^\infty \frac{ds}{s-q^2}\left[\frac{q^2}{s}\ C_{17,\alpha} - \frac{s-q^2}{s}\ C_{19,\alpha}\right]^{\bar{\nu}+\nu}$$

$$+\int_0^\infty dx\ \frac{\widetilde{F}_{19}^{\bar{\nu}+\nu}}{x} + \frac{1}{2M}\int_{\mu_0^2}^\infty d\mu^2\left[q^2\ \hat{g}'^{\,0,0,1}_{17,0} - \hat{g}'^{\,0,2,1}_{19,0}\right]^{\bar{\nu}+\nu}\Big|_{q^2\text{-fixed}} = 0,$$

$$\text{(AII. 22)}$$

$$\frac{1}{M}\int_{q^2}^\infty \frac{ds}{s}\left[q^2\ C_{17,\alpha} - (s-q^2)\ C_{19,\alpha}\right]^{\bar{\nu}+\nu}\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-2}$$

$$+\int_{\mu_0^2}^\infty d\mu^2\left[q^2\ \hat{g}'^{\,0,0,1}_{17,\alpha} - \hat{g}'^{\,0,2,1}_{19,\alpha}\right]^{\bar{\nu}+\nu}\Big|_{q^2\text{-fixed}} = 0, \qquad \text{(AII. 23)}$$

$$\int_{\mu_0^2}^\infty d\mu^2\ \left[q^2\ \hat{g}'^{\,0,0,1}_{17,\gamma} - \hat{g}'^{\,0,2,1}_{19,\gamma}\right]^{\bar{\nu}+\nu}\Big|_{q^2\text{-fixed}} = 0, \quad 0<\gamma \notin \mathscr{R} \qquad \text{(AII. 24)}$$

where

$$\widetilde{G}_2 = G_2 - \sum_{\alpha\in\mathscr{R}} C_{17,\alpha}(q^2)\ \nu^{\alpha-1}\ , \qquad \text{(AII. 25)}$$

and

$$\frac{1}{x}\ \widetilde{F}_{19}^{\bar{\nu}+\nu} = \ \theta(1-x)\ \frac{F_{19}^{\bar{\nu}+\nu}}{x} + \sum_{\alpha\in\mathscr{R}} \frac{1}{2^\alpha}\ f_{19,\alpha}^{\bar{\nu}+\nu}\ x^{-\alpha-1}; \qquad \text{(AII. 26)}$$

$$\int_0^\infty d\nu\ \frac{M}{\nu}\ \widetilde{G}_1^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} + \frac{M}{2}\int_{q^2}^\infty d\nu\ \frac{\widetilde{C}_{16,0}^{\bar{\nu}-\nu}}{s-q^2} - \int_0^{x_0}\frac{dx}{x}\ \widetilde{F}_{16}^{\bar{\nu}-\nu}$$

$$+\sum_{\alpha>0}\frac{1}{\alpha 2^\alpha}\ f_{16,\alpha}^{\bar{\nu}-\nu}x_0^{-\alpha} + \frac{M}{2}\int_{\mu_0^2}^\infty d\mu^2\left[\hat{g}^{1,-1,0}_{16,0}\right]^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}} = 0\ , \qquad \text{(AII. 27)}$$

$$\lambda = 0 \quad , \tag{AII. 28}$$

where λ is the Schwinger term defined in Eq. (74) of paper (A), for example,

$$\int_{q^2}^{\infty} ds\, C_{16,\alpha}^{\bar{\nu}-\nu}(s) \left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-2} + M^2 \int_{\mu_0^2}^{\infty} d\mu^2 \left[\hat{g}_{16,\alpha}^{1,-1,0}\right]^{\bar{\nu}-\nu} \Bigg|_{q^2\text{-fixed}} = 0 \, ,$$

$$0 < \alpha \in \mathscr{R} \tag{AII. 29}$$

$$f_{16,0} = -C_{16,0}(q^2) \tag{AII. 30}$$

where $x_0 \geq 1$,

$$\tilde{G}_1 = G_1 - \sum_{\alpha > 0} C_{16,\alpha}(q^2)\, \nu^{\alpha} - \theta(\nu + q^2/2Mx_0)\, C_{16,0}(q^2) \quad , \tag{AII. 31}$$

$$\frac{1}{x} F_{16}^{\bar{\nu}-\nu} = \theta(1-x) \frac{F_{16}^{\bar{\nu}-\nu}}{x} + \sum_{\alpha} \frac{1}{2^{\alpha}} f_{16,\alpha}^{\bar{\nu}-\nu} x^{-\alpha-1} \quad , \tag{AII. 32}$$

and

$$\tilde{C}_{16,0}(s) = C_{16,0}(s) - \theta(-s)\, C_{16,0}(q^2) - \theta(s)\, f_{16,0} \; ; \tag{AII. 33}$$

$$\int_0^{\infty} d\nu\, (V_1 + V_6 - V_8) \Big|_{q^2\text{-fixed}}^{\bar{\nu}-\nu} - \frac{q^2}{2M} \int_{\mu_0^2}^{\infty} d\mu^2 \left[q^2 \left(\hat{g}_{7,1}^{-1,0,2} + \hat{g}_{12,1}^{-1,0,2}\right) - \right.$$

$$\left. \hat{g}_{11,1}^{0,1,1} - \hat{g}_{13,1}^{0,1,1} - \hat{g}_{14,1}^{-1,0,1} \right]^{\bar{\nu}-\nu} \Bigg|_{q^2\text{-fixed}} = 0 \tag{AII. 34}$$

$$\int_0^\infty \frac{ds}{s}\left[\frac{q^2}{s}\left(\bar{C}_{7,\alpha}(s)+\bar{C}_{12,\alpha}(s)-\frac{s}{q^2}\,\tilde{C}_{14,\alpha}(s)\right)^{\bar{\nu}-\nu}\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-3}-(C_{11}+C_{13})^{\bar{\nu}-\nu}\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-1}\right]$$

$$+\frac{2}{\alpha-1}\left(-q^2/2M^2\right)^{\frac{\alpha-3}{2}}\phi(\alpha)\left[C_{7,\alpha}(q^2)+C_{12,\alpha}(q^2)-C_{14,\alpha}(q^2)\right]^{\bar{\nu}-\nu}$$

$$+M\int_{\mu_0^2}^\infty d\mu^2\left[q^2\left(\hat{g}_{7,\alpha}^{-1,0,2}+\hat{g}_{12,\alpha}^{-1,0,2}\right)-\hat{g}_{11,\alpha}^{0,1,1}-\hat{g}_{13,\alpha}^{0,1,1}-\hat{g}_{14,\alpha}^{-1,0,1}\right]^{\bar{\nu}-\nu}\Bigg|_{q^2\text{-fixed}}=$$

$$0<\alpha\in\mathscr{R} \qquad \text{(AII. 35)}$$

$$\int_{\mu_0^2}^\infty d\mu^2\left[q^2\left(\hat{g}_{7,\gamma}^{-1,0,2}+\hat{g}_{12,\gamma}^{-1,0,2}\right)-\hat{g}_{11,\gamma}^{0,1,1}-\hat{g}_{13,\gamma}^{0,1,1}-\hat{g}_{14,\gamma}^{-1,0,1}\right]^{\bar{\nu}-\nu}\Bigg|_{q^2\text{-fixed}}=0,\quad 0<\gamma\notin\mathscr{R}\cup\{1\},$$

(AII. 36)

$$\int_0^\infty d\nu\,(V_1+V_6)^{\bar{\nu}-\nu}\Big|_{q^2\text{-fixed}}-\int_0^1\frac{dx}{x}F_{14}^{\bar{\nu}-\nu}+\frac{1}{2M}\int_{\mu_0^2}^\infty d\mu^2\left[q^2\left(\hat{g}_{7,1}^{-1,0,1}+\hat{g}_{12,1}^{-1,0,1}\right)\right.$$

$$\left.+\hat{g}_{14,1}^{-1,2,1}\right]^{\bar{\nu}-\nu}\Bigg|_{q^2\text{-fixed}}=0 \qquad \text{(AII. 37)}$$

$$\int_{q^2}^\infty ds\left[\frac{q^2}{s}\left(\tilde{C}_{7,\alpha}(s)+\tilde{C}_{12,\alpha}(s)\right)+\frac{s-q^2}{s}\,\tilde{\tilde{C}}_{14}(s)\right]^{\bar{\nu}-\nu}\left(\frac{\sqrt{s-q^2}}{M}\right)^{\alpha-3}-\frac{4M^2}{\alpha-1}\left(C_{7,\alpha}(q^2)+C_{14,\alpha}(q^2)\right)^{\bar{\nu}-\nu}$$

$$\phi(\alpha)(-q^2/2M^2)^{\frac{\alpha-1}{2}}+f_{14,\alpha}^{\bar{\nu}-\nu}M^2B\left(1-\alpha,\frac{\alpha-1}{2}\right)(-q^2)^{\frac{1-\alpha}{2}}$$

$$+M\int_{\mu_0^2}^\infty d\mu^2\left[q^2\left(\hat{g}_{7,\alpha}^{-1,0,1}+\hat{g}_{12,\alpha}^{-1,0,1}\right)+\hat{g}_{14,\alpha}^{-1,2,1}\right]^{\bar{\nu}-\nu}\Bigg|_{q^2\text{-fixed}}=0,\qquad \alpha\in\mathscr{R}$$

(AII. 38)

$$\int_{\mu_0^2}^{\infty} d\mu^2 \left[q^2 \left(\hat{g}_{7,\gamma}^{-1,0,1} + \hat{g}_{12,\gamma}^{-1,0,1} \right) + \hat{g}_{14,\gamma}^{-1,2,1} \right]^{\bar{\nu}-\nu} \Big|_{q^2\text{-fixed}} = 0, \quad 0<\gamma \notin \{1\} \cup \mathscr{R} \quad \text{(AII. 39)}$$

$$\int_0^{\infty} d\nu\, V_1^{\bar{\nu}-\nu} \Big|_{q^2\text{-fixed}} + 2 \int_0^1 dx\, F_9^{\bar{\nu}-\nu} + \frac{1}{2M} \int_{\mu_0^2}^{\infty} d\mu^2 \left[q^2\, \hat{g}_{7,1}^{-1,0,1} + \hat{g}_{9,1}^{0,1,0} \right]^{\bar{\nu}-\nu} \Big|_{q^2\text{-fixed}} = 4 g_A^{I_3}$$

(AII. 40)

$$\int_{q^2}^{\infty} ds \left[\frac{q^2}{Ms}\, \tilde{C}_{7,\alpha}(s) \left(\frac{\sqrt{s-q^2}}{M} \right)^{\alpha-3} + \tilde{C}_{9,\alpha}(s) \left(\frac{\sqrt{s-q^2}}{M} \right)^{\alpha-1} \right]^{\bar{\nu}-\nu} - \frac{4M}{(\alpha-1)}\, C_{7,\alpha}^{\bar{\nu}-\nu}(q^2)\, \phi(\alpha)\, (-q^2/2M^2)^{\frac{\alpha-1}{2}}$$

$$+ M f_{9,\alpha}^{\bar{\nu}-\nu}\, B\left(1-\alpha, \frac{\alpha-1}{2}\right) (-q^2)^{\frac{1-\alpha}{2}} + \int_{\mu_0^2}^{\infty} d\mu^2 \left[q^2\, \hat{g}_{7,\alpha}^{-1,0,1} + \hat{g}_{9,\alpha}^{0,1,0} \right]^{\bar{\nu}-\nu} \Big|_{q^2\text{-fixed}} = 0 ,$$

$\alpha \in \mathscr{R}$ (AII. 41)

$$\int_{\mu_0^2}^{\infty} d\mu^2 \left[q^2\, \hat{g}_{7,\gamma}^{-1,0,1} + \hat{g}_{9,\gamma}^{0,1,0} \right]^{\bar{\nu}-\nu} \Big|_{q^2\text{-fixed}} = 0 , \quad 0 < \gamma < \notin \{1\} \cup \mathscr{R} \quad \text{(AII. 42)}$$

$$\int_{q^2}^{\infty} \frac{ds}{s} \left(C_{17,\alpha}(s) + C_{19,\alpha}(s) \right)^{\bar{\nu}-\nu} \left(\frac{\sqrt{s-q^2}}{M} \right)^{\alpha-1} + \int_{\mu_0^2}^{\infty} d\mu^2 \left[\hat{g}_{17,\alpha}^{0,1,1} + \hat{g}_{19,\alpha}^{0,1,1} \right]^{\bar{\nu}-\nu} \Big|_{q^2\text{-fixed}} = 0 ,$$

$\alpha \in \mathscr{R}$ (AII. 43)

$$\int d\mu^2 \left[\hat{g}_{17,\gamma}^{0,1,1} + \hat{g}_{19,\gamma}^{0,1,1} \right]^{\bar{\nu}-\nu} \Big|_{q^2\text{-fixed}} = 0, \quad 0 < \gamma \notin \mathscr{R} \quad \text{(AII. 44)}$$

where

$$\bar{C}_{i,\alpha}(s) = C_{i,\alpha}(s) - \theta(q^2/2-s)\,\frac{s}{q^2}\,C_{i,\alpha}(q^2)\ , \qquad i=7,12$$

$$\tilde{C}_{i,\alpha}(s) = C_{i,\alpha}(s) - \theta(q^2/2-s)\,C_{i,\alpha}(q^2)\ , \quad i=7,12,14$$

$$\tilde{C}_{9,\alpha}(s) = C_{9,\alpha}(s) - \theta(s)\,f_{9,\alpha}\left(\frac{M}{s}\right)^{\alpha} \tag{AII. 45}$$

$$\bar{\tilde{C}}_{14,\alpha} = C_{14,\alpha}(s) - \theta(s)\,f_{14,\alpha}\left(\frac{M}{s}\right)^{\alpha-1}$$

and $\phi(\alpha)$ is defined in (AII. 14).

The results listed in this appendix, taken together with those listed in Section III, represent a complete discussion of current algebra residue sum rules.

REFERENCES

1. B. F. L. Ward, Nucl. Phys. B61, 1 (1973), hereafter referred to as A.
2. B. F. L. Ward, Stanford Linear Accelerator Center Report No. SLAC-PUB-1371, January 1974, hereafter referred to as B.
3. By "fixed hadronic mass limit residue" we mean the coefficients of $|\nu|^\gamma$ in the limit

$$\lim_{\substack{\nu \to -\infty \\ \mu^2=(p+q)^2\text{-fixed}}} W \to \sum_{\gamma \geq 0} G_\gamma(\mu^2)\, |\nu|^\gamma \quad ,$$

always assuming such an asymptotic expansion exists.
4. M. A. Keppel-Jones, Phys. Rev. D6, 1140 (1972), and M. O. Taha, ICTP preprint, August 1973, independently introduced the same approach to sum rules. However, as we pointed out in B, the states near x = 0, -1 were neither systematically considered in the discussions of these authors, nor the discussion of the present author in A. This latter situation was remedied in B.
5. By "leading" we mean that the respective contribution to the structure functions are non-vanishing as the appropriate asymptotic limit is approached.
6. J. M. Cornwall, D. Corrigan, and R. E. Norton, Phys. Rev. Letters 24, 1141 (1970); Phys. Rev. D3, 536 (1971); P. Vinciarelli, Phys. Rev. D8, 965 (1973).
7. Some of the spin dependent scaling and fixed $-q^2$ sum rules have been given by various other authors. For detailed references, see the reference list in A, for example.
8. S. L. Adler, Phys. Rev. 143, 1144 (1966).
9. D. J. Gross and C. H. Llewellyn Smith, Nucl. Phys. B14, 337 (1969).

10. J.D. Bjorken, Phys. Rev. 179, 1547 (1966); ibid. 163, 1868 (1967).

11. J.D. Bjorken and J. Kogut, Phys. Rev. D8, 1341 (1973).

12. See D. Dicus, R. Jackiw and V.L. Teplitz; Phys. Rev. D4, 1733 (1971); and G.C. Fox and E. Leader, Phys. Rev. Letters 18, 628 (1967).

13. D. Dicus, R. Jackiw, V.L. Teplitz, op. cit.

14. The formal version of (AII.27) is due to M.A. Bég, Phys. Rev. Letters 17, 333 (1966).