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Alignments and Gaps Between Multi-fold Universes And Loop Quantum Gravity

Stephane H. Maes¹

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Abstract:

Multi-fold universes and LQG have a lot in common as well as complementarities. A priori, both models are compatible with each other. The present paper analyses these touch points and suggests ways forward both in terms of multi-fold universes and of LQG. Also, considering the relationships between multi-fold universes results and superstring theories, there are many ways that the three approaches could use lessons learned or mechanisms from each other, in order to fill gaps or improve some of their own models. Maybe they could even converge towards new and more encompassing combined frameworks. We conclude with a call for such a collaboration.

1. Introduction

The new preprint [1] proposes contributions to several open problems in physics like the reconciliation of General Relativity with Quantum Physics, explaining the origin of gravity proposed as emerging from quantum (EPR-Einstein Podolsky Rosen) entanglement between particles, detailing contributions to dark matter and dark energy and explaining other Standard Model mysteries without requiring New Physics beyond the Standard Model other than the addition of gravity to the Standard Model Lagrangian. All this is achieved in a multi-fold universe that may well model our real universe, which remains to be validated.

With the proposed model of [1], spacetime and Physics are modeled from Planck scales to quantum and macroscopic scales and semi classical approaches appear valid till very small scales. In [1], it is argued that spacetime is discrete, with a random walk-based fractal structure, fractional and noncommutative at, and above Planck scales (with a 2-D behavior and Lorentz invariance preserved by random walks till the early moments of the universe). Spacetime results from past random walks of particles. Spacetime locations and particles can be modeled as microscopic blackholes (Schwarzschild for photons and spacetime coordinates, and metrics between Reisner Nordstrom [13] and Kerr Newman [14] for massive and possibly charged particles – the latter being possibly extremal). Although surprising, [1] recovers results consistent with other like [15], while also being able to justify the initial assumptions of black holes from the gravity or entanglement model. The resulting gravity model recovers General Relativity at larger scale, as a 4-D process, with massless gravity, but also with massive gravity contributions at very small scale that make gravity significant these scales. Semi-classical models also work well till way smaller scales than usually expected.

Loop Quantum Gravity (LQG) [5] is based on the initial work of Ashtekar to reformulate General Relativity (GR) in terms of new variables and Actions, with a formalism closer to Einstein Cartan (and (Yang Mills) Gauge Theories), and hence also able to model torsion in addition to curvature. The formalism is also known as canonical formulation of GR in terms of Ashtekar variables [2], leading to canonical quantum gravity.

¹ shmaes.physics@gmail.com

Throughout the analysis, [1] finds numerous touch points with LQG and its related derivatives (e.g. spin networks and spin foams), or related constructive approaches as, for example, enumerated and analyzed in [3], despite multi-folds coming from a complete different starting point, i.e.

- 1) not starting from General Relativity (GR) or the Hilbert Einstein Action, Hamiltonian (As in spin networks) or Lagrangian (as in spin foams) (with path integrals), and Action variants like the Palatini, Holtz (new formulations equivalent to Hilbert-Einstein Action but best suited for tetrahedron frames) and Holtz Immirzi Action [4].
- 2) not imposing, a priori, spacetime area and volume conservations (except to the extent that, at the end. the derived microscopic black holes come with their usual area laws; but these law have first been rediscovered in a multi-fold universe in the top down phase of [1] and this is post recovery of GR equation and Hilbert Einstein action by).

In the present paper, we remain at a high level of analysis. It makes the points accessible to a wider audience and keeps the door open to further papers or discussions devoted to details of interest. Yet, it requires the reader to review [1], as we do not revisit here all the details of the multi-fold mechanism or reconstruction of spacetime. The followings subsections are organized as a series of observations in [1] that relate to LQG and conversely. [1] did not focus on presenting these facts as dedicated comparisons or as lessons learned for and from LQG. Our analysis is by no means exhaustive. However, we hope that it will intrigue enough the reader to push him or her to dig deeper. Most of the more detailed (or entry point) references are provided in [1], and so every statement is not motivated here or presented with the most appropriate references. This paper is rather a story tale. “[1]” appears often, as a person or a model, to refer to the original arguments, analysis, mechanisms or proposals discussed in [1]. In general, we will use LQG to designate all the variations out there, unless if a specific approach needs to be called out.

[1] is about a multi-fold universe. We discovered that the framework and mechanism of [1] have many touch points with LQG, in terms of the resulting picture of the universe and specific properties or phenomena. [1] argues that it can model well our real space time and provides predictions and opportunities for falsifiability.

2. Selected Points of Comparison

2.1 Spacetime reconstruction

The second part of [1] describes a spacetime reconstruction phase (bottom up) based on the results obtained top down, in the first part. These top down approach justified the bottom up reconstruction choices. The justifications provide may help also with LQG lack of justification of its starting points. The bottom up reconstruction leads to a discrete, fractal, random and Lorentz invariant spacetime as well as a network of microscopic black hole models for spacetime locations and particles. The model recovers GR at larger scales and therefore the classical spacetime.

Spin networks and spin foams start with an a priori situation where there is no spacetime, but a discrete graph of points and principles like: conservation and quantization of angular momentum, of areas (or deficit angles à la Regge Calculus) and of volumes, for tetrahedrons [19]. While very generic and somehow “logical”, these steps are actually, from the onset, bringing in GR and the Hilbert Einstein Action, that also extremizes volumes and surfaces. As a result , LQG does not try to recover GR from its model, it rather uses GR to recover a quantized spacetime and its dynamics, using Action frameworks.

Graphs are built and evolve with kinematics and dynamics prescribed by the formalism. Yet, it never recovers (yet) a smooth or convincing macroscopic spacetime [18]. GR recovery is somehow handwaved, something surprising considering how the Hilbert Einstein Action is so deeply baked into everything related to LQG. It is fair to say that in our view, LQG is more focused on recovering quantum kinematics and dynamics of spacetime than recovering spacetime, with say measures (metrics) as in GR. LQG probably needs to feel to gaps to be more broadly accepted beyond its base of supporters.

[1] can probably help.

Besides direct recovery of Einstein fields equations of GR in [1], the microscopic black holes, introduced in [1] as a bottom up reconstruction, provide a way to ensure recovery of a smooth continuous spacetime at large scales (semi classical and classical), something that is still stomping LQG. They also justify the area and volume extremization starting point of LQG.

Passing over on how to formulate these concepts from [1] in the context of LQG, [1] can expand on the reconstruction scheme by providing a random walk model for the nodes of the graphs used by LQG. This could finally attach the spin networks or spin foams to an actual spacetime. More on these thoughts later, especially in terms of Lorentz invariance.

The problems of time, and its absence, in LQG is also puzzling (even if explained in terms of the nature of GR field equations, and their phase space outcome). At least from the point of view of [1], where reconstruction, and the fractal model due to the random walk, is fractal in space and also in time, a key feature – not a detail. The suggestions, already made about fractals and random walks, may clarify these aspects, assuming that one wants to, as there seems to be no clear agreement, on the LQG side, if time is fundamental or emergent. In [1], time and space cells are created hand in hand by random walk.

All the inferences and reconstructions, in [1], are based on the multi-fold mechanisms that recover GR without assuming or postulating anything about it. It is to be compared to what string theories can claim when it is argued that strings recover the graviton without looking for it. As explained in [1,9], the superstring graviton is baked in the string Actions and in the Physics of minimizing the surface of the world sheet evolutions just like soap bubbles between supports. There is nothing more profound or mysterious. Of course, gravitons and GR are modeled in strings! No need to compute or model anything else, it's obvious. It does not mean anything much more profound about being automatically a ToE, as selecting such models was purely the result of a lucky fit of scattering amplitudes with a formula that worked well and matched the physics of strings. Indeed, it modeled, before QCD, mesons as stretchable strings. The work is fantastic and was not trivial; but discovering gravitons this way may not have warranted the sudden quasi new religion status that was bestowed over strings, then superstrings and then M-theory etc. [18] argue the same from a different angle. It does not seem that this immediate consequence of the string action has been really discussed before... Note, that this paragraph is not at all about dissing superstrings, just understanding what is established (e.g. as validation of superstrings) and what is not. In fact [9], shows many congruent results between superstring theory and multi-folds universes as introduced in [1].

2.2. Background independence

As we just discussed, LQG is often accused to never suitably recovering spacetime, only dynamics through networks or graphs of nodes. To some extent, it is taken positively by LQG proponents, as a consequence of being background independent; just as GR is through its covariant principle.

In fact, LQG proponents often promote background independence as one of the main reasons why QFT and strings (e.g. superstrings) encounter so much complexity and so many challenges and why QFT methods of quantizing GR systematically fail with divergences and non-renormalizability [6]. We agree with that view and it aligns with what we observed in [1]. So LQG is background independent because it does not assume any background. Unfortunately, somehow, it also never recovers any spacetime connected to our macroscopic reality, because of the approach followed so far. It does not have to be like that. Background independence does not have to leave us midway on the spacetime reconstruction path. Look at multi-fold universes for example.

[1] is also background independent because it is covariant, by design as it is relying on Path Integrals in proper particle frameworks. Its model can support different choices of initially flat or curved spacetime, as background spacetime (and hence as initial conditions) with the effects of entanglement (and gravity), due to additional matter or energy, captured on top. It is another way to assess that background independence has been achieved. Gravity

(and gravitons when quantized), via the multi-folds that live in $AdS(5)$ - outside our spacetime, lives also outside our spacetime. Its effects result into attractive effective potentials felt in spacetime, or effective curvature contributions. The effects are propagated back to spacetime through the multifold mappings, which reminds of holographic principles of the superstrings AdS/CFT correspondence conjecture. This way, in our spacetime, there are no divergence, no renormalizability problems. The fact that spacetimes ends up being discrete further addresses these issue; more on this later. But this good behavior gels very well and support the contention that background independence is needed if we want well behaving (quantum) gravity models [6]: QFT [1] and superstrings [9] should really pay attention to this.

2.2. Spacetime discreteness

LQG, and its variants, do not really motivate that spacetime is discrete. As a result, objections are raised that argue that the starting point is wrong. The main argument of LQG is that the vectors normal to tetrahedron surfaces (and of norm equal to their areas) share the same commutator relationships as the angular momentum and therefore can be seen respectively as equivalent representations of a rotation group. Therefore they should similarly be quantized, hence the discrete spacetime. This is a correct argument, but discreteness and suitability of the tetrahedron are where there is a need for a leap of faith for many.

As many, despite the irrefutable arguments presented in [7,8], still do not believe that spacetime is discrete, or have objections to it. LQG has only weak justifications beyond the above. For LQG, discretization is a fact and the starting point [19]. The fact that discrete spacetime is perceived as imply the lack of Lorentz invariance (which is certainly true with if spacetime is discrete with only a few points – there is no mystery there and noncommutative geometrical trick and randomness can only play when spacetime is a bit larger...). It is a valid concern addressed below by [1]. Others have stated that because LQG violates Lorentz invariance and does not seem to recover it, it can not be correct. LQG has again limited arguments to provide in answer. We will discuss below how with [1] this can be addressed.

[1] can provide better arguments as it not only manages to infer discreteness, but also the fact that it must come with a random walk structure and therefore is non-commutative (inferred not just from [7] but, more surprising, from the dynamics of the multi-fold mechanisms that model EPR entanglement), and Lorentz invariant (ensured by randomness of the structure and formalized by the non-commutative geometry of spacetime) till very smalls scales or very early moments of the universe (that's the statement about few points in finite discrete spacetime).

The spacetime discreteness in [1] does not just come from considerations like [8], but from the actual kinematics and dynamics of the multi-folds that imply non-commutativity of $AdS(5)$, and therefore of our spacetime. These are powerful justifications for discreteness!

Non-commutative geometry is also encountered in LQG approaches, and comes, from the beginning, from the commutators of vectors normal to tetrahedrons surfaces (with area as length) matching angular momentum commutators.

It seems of interest to consider adding the notion of random walk or fractal structure to LQG. It could provide a path to a macroscopic spacetime à la GR. It would also help with Lorentz invariance as discussed below.

2.3 Matter and Energy

With its focus on spacetime, LQG has little to say on matter, and energy, other than reusing the outcome of Quantum Physics in terms of matter and spin coupling to gravity, and matter Actions. So, in our view, LQG rather reconstructs well the dynamics of an empty spacetime from first principles (that are already based on the Hilbert Einstein action), then dumps in conventional matter models to add matter and energy. Depending on where the focus is, this is fine or a gap to be filled.

The top down derivation in [1], essentially ignores spacetime dynamics (another way to see its background independence). Instead, it focuses on reusing conventional physics for matter (That includes energy and radiation for the purpose of this discussion. We do not try to mean Fermions only by saying matter.) and on modeling EPR entanglement of real and virtual particles, in ways that address the EPR paradox. It leads to the multi-folds mechanisms that recovers gravity like effects between entangled particles (Attractive effective potential and effective curvature), real or virtual, and can recover GR at larger scales. From this analysis, [1] discovers that AdS(5) surrounds every particles (and is therefore tangent to our spacetime) and that microscopic black holes seem to form around them. It is the basis for the bottom up reconstruction of spacetime, starting from no spacetime at all. Particles are created and random walk to new points, following random fractal (relativistic) paths in path integrals. As they do, they concretize spacetime with a fractal structure that is random, and therefore can be Lorentz invariant (and non-commutative). The microscopic black holes recover the area laws (and area quantization) of LQG. With such a model, well known thermodynamics considerations and analysis also recover GR as was taught by [10].

It is puzzling, why LQG does not at least repeat the methodology of [10], to also recover GR. We are not clear why not. Thermodynamically, LQG must be recovering GR. Again it result from the Hilbert Einstein action backed in the spin networks Hamiltonians or spin foam Lagrangians.

It seems that it would be of interest to evaluate if tracking particles and entanglement as proposed in [1] may lead to a more complete way to handle matter / energy in LQG. Today, LQG, like QFT and Superstrings, has challenges modeling particles or entanglement between specific particles (versus statistically with entanglement entropy or entanglement Hamiltonians). Of course, reconciling these aspects is not trivial. Considerations of [1] should help and provide inspiration.

Conversely, it may be advantageous for [1] to investigate building an hybrid spin network or spin foam model where microscopic black holes are replaced by these constructs: the spacetime around the microscopic black holes (for particles and spacetime locations) could now fluctuate and settle using LQG dynamics models, instead of the black box (no pun intended) models provided by the black holes and their immediate surrounding. If it recovers the same type of space time and behaviors, it probably will offer insights both to multi-fold reconstruction models and to LQG, and it may provide a more compelling reconstruction of spacetime than what [1] has done so far, by passing the buck to microscopic black holes. In [1], the microscopic black hole model solves the problem of the smooth spacetime at larger scales that LQG has not been able to address so far. But yet again, with the top down approach, [1] already recovers GR. Also, in [1], the microscopic black holes are the smallest constructs. Therefore it is not sure if they can or should be decomposed into spin networks or spin foams. All this is for future work, but it warrants some investigation.

2.4 Lorentz Invariance

[1] argues for a strict respect of a non supra luminosity principle that implies, among other things, forbidding, in Path Integrals, space like paths, i.e. paths that have portions space like with respect to other portions. It is at the difference of what is done by most conventional physics, especially QFT. We believe that such new criteria has already the potential to remove some of the Lorentz violations effects as were tracked in [16]. This is to be validated.

During the reconstruction phase in [1], background independence is recovered through the random walks and the Lorentz invariance of the reconstructed spacetime. It is the randomness that ensures this invariance despite the discreteness of spacetime and requires non-commutative geometry to model it.

In general, LQG spacetime is not Lorentz invariant (with some exceptions for specific Lorentz networks) and we already discussed how this concern has led some to disregard it on that basis. Yet, it does not mean that Physics on this spacetime cannot be Lorentz invariant, if say the laws are correctly expressed as tensors etc. This Lorentz

invariance problem is recognized [17], often only blamed on discreteness, and even sometimes considered that the signs that it can be resolved is or would be a sign that superstrings and LQG can converge.

Adding the suggestions above in terms of random walk fractal structure can restore the Lorentz invariance (through randomness) to LQG, especially when combined with the strict no supra luminosity principle of [1]. Success with LQG recovering generically Lorentz invariance as a result, could be a great way to theoretically validate the importance of the strict supra luminosity principle and the approach of supra-luminosity filtered path integral proposed in [1].

2.5 Actions

As mentioned earlier, LQG relies on variations of the Hilbert Einstein Action [4,5]. [1] does not. It derives the Hilbert Einstein Action from the multi-fold mechanisms, in its top down phase. Entanglement and multi-fold mechanism have, a priori, nothing to do with area extremization. Well they do, that is the whole point of [1], but as starting point, it was absolutely not obvious that these concepts were in any way related.

Bottom line, the Actions in LQG and multi-fold universes are very similar, as [1] recovers the Hilbert Einstein Action (and variations can be accommodated on both side); the Physics is the same in both spaces. It is much more the case than with respect to superstrings, where most Physics do not happen in the same spaces[9].

2.6 Torsion, No Gravitational Singularity, No Divergence, (Re-)normalizability

LQG accommodates models that ensure the absence of singularities and can support big bounces model in cosmology [19]. In our view, this is a direct consequence of the discrete spacetime (and hence a minimum length) and the LQG formulation of the action convenient to expose torsion. In LQG, torsion, and spin to gravity couplings, appear when fermions are added. It is known that torsion ensure the absence of singularities [11]. In addition, it has been shown in many cases and argued that no divergence appear in LQG (Again because of discreteness and background independence – theorems also claim to bound the series, in many cases) and so the theory is believed to be normalized.

In [1], we have the same conclusions. The main enablers are:

- Discreteness of spacetime
- Background independence
- Torsion within matter resulting from the multi-fold mechanisms combined with uncertainties
- Positive cosmology constant and dark energy behaviors due to the multi-folds mechanisms combined with uncertainties
- Non perturbative approach (so no infinite series that would diverge)

As a result, both approaches are compatible, essentially for very similar reasons, with big bounce cosmology scenarios. Simulations have demonstrated such cosmological scenarios for LQG.

2.6 The AdS Saga

LQG has no notion or relations to an AdS space tangent to our spacetime. Nothing introduces it. Sure, proposal have been made to adapt LQG to AdS and higher dimensional spaces [20], and therefore in that context, exploit the AdS/CFT correspondence conjecture; but that is a different point. Our analysis in [1,9] questions the suitability of AdS(5), or AdS(5) (+ other dimensions), as the physical spacetime. Therefore, we do not believe that proposals to bring LQG to AdS are helping; other than at best maybe if they were used to model folds. But we emphasized a lot in [1,9], that folds do not have to follow GR. They may or they may not. Modeling folds with LQG probably correspond to the situation where they do follow GR.

[1] introduce AdS(5) where multi-folds, and gravitons post quantization, live. We can see in [1,9] how AdS(5)(+ more dimensions) is also the space where superstring theories find gravitons ,as well as other superstrings (e.g. super partners). By now, we also know that strings cannot live in a positive curvature spacetime (or spacetime with dark energy or positive cosmological constant) [12]; which makes [9] a new (because more concrete than the AdS/CFT correspondence conjecture) attractive positioning of superstrings and physical spacetime.

Based on [1,9] and on its amazing corroboration of the AdS/CFT correspondence met as a conjecture in superstrings, but as facts in a multi-fold universe, there could be value to see how to encounter AdS(5) in LQG. A most obvious option is to add multi-folds, to the graphs of LQG, possibly (but not necessarily – as discussed in [1], multi-folds may or may not follow GR) built by similar graphs. This also would allow tracking particles and modeling (EPR) entanglement at the nodes (spacetime entanglement or entanglements of particles at the nodes). It seems that making AdS appear is directly related to bringing more attention to matter in LQG, something still lagging as mentioned earlier.

3. Superstrings and LQG

It is not a stretch to say that LQG, along with its variations, and string theories are often seen as fiercely competing with each other. The popular scientific literature is full of stories of alleged misbehaviors towards each other, pointing mutual shortcomings or even presenting ones preferred theory as the only game in town. Yet recently, both their more or less apparent stalled states, at least from an external perspective, have finally led to calls for more collaboration and combinations of ideas and frameworks.

[9] shows how multi-fold universes and superstrings have many congruent results and how superstring could benefit from including aspects or lessons learned from multi-fold universes. It even shows how multi-fold universes might account for superstrings.

This paper does the same for LQG and some recommendations are the same (e.g. add particle modeling, add entanglement modeling between particles, add the random walk/ fractal reconstructions, consider the microscopic black hole models). Others are different and specific for superstrings and LQG but need to equally take place to converge or build on the two approaches (even without multi-fold considerations) like resolve background independence vs background dependence (e.g. for string to address problems of back reaction and LQG to relate to a concrete macroscopic spacetime), understand AdS/CFT correspondence in LQG, model matter, justify discreteness and build in Lorentz invariance etc.

4. Conclusions

We believe that [1] makes a compelling case for the consistency of its multi-fold proposal. The present paper shows how LQG and multi-fold universes relate: where they agree and complement each other versus some of the gaps. There are no real incompatibilities in our view, just facets of many similar concepts. It seems clear that concepts of multi-fold universes can help address challenges of LQG and all its variations.

When adding the considerations of superstrings, including aspects of the analysis in [9], we also showed many touch points with these three approaches.

[1] shows also significant impact on the Standard Model, when we add gravity (especially the short scale massive contributions). It could contribute explanations to several famous open issues.

At this stage, we believe that there is a way forward exploiting multi-fold universes, LQG and superstrings that can result into strong benefits for all approaches or maybe even a converged approach. Hopefully, this paper and [1]

helped. Maybe these approaches can progress more collaboratively together? We just showed that LQG and Multi-folds could benefit a lot for sure.

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