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Spin Polarization Phases in Quark Matter: Interplay between Axial-vector and Tensor Mean-Fields

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The spontaneous spin polarization of strongly interacting matter due to axial-vector and tensor type interactions is studied at zero temperature and high baryon-number densities. We start with the mean-field Lagrangian for the axial-vector and tensor interaction channels, and find in the chiral limit that the spin polarization due to the tensor mean field (U) takes place first as the density increases for sufficiently strong coupling constants, and then that due to the axial-vector mean field (A) emerges in the region of finite tensor mean field. The mechanism for this result is presented in relation to the axial $U(1)$ symmetry breaking.

KEYWORDS: quark matter, spin polarization, axial vector feild, tensor field

1. Introduction

The discovery of the magnetars, which are the neutron stars with strong magnetic field of $O\left(10^{15}\right)$ G, revives the important question about the origin of the strong magnetic field [1]. The spontaneous spin polarization is one of the most possible candidates to explain such strong magnetic field. As an earlier work, Tatsumi [2] suggested a possibility of a ferromagnetic transition in quark matter interacting via one-gluon-exchange (OGE) force and showed that the maximum magnetic field can reach $B \sim O(10^{15-17}\mathrm{G})$ when the magnetar is a quark star.

In the relativistic framework we can consider two types of spinor bilinear form as the *spin* density operator [3]: One is a spatial component of the axial-vector (AV) current operator, $\psi^\dagger\Sigma_i\psi(\equiv -\bar{\psi}\gamma_5\gamma_i\psi)$, and the other is that of the tensor (T) operator, $\psi^\dagger\gamma^0\Sigma_i\psi$, with ψ being the Dirac field. These two become equivalent to each other in the non-relativistic limit, while they are quite different in the ultra-relativistic limit (massless limit) [3]. When the dynamical quark mass is zero, the chiral symmetry is restored, the

AV-type spin-polarized phase cannot appear, but the T-type one can do. In the NJL type effective models, it has been demonstrated that the AV-type spin-polarized phase can appear only in a narrow density region just inside the chiral condensed phase [4], while the T-type spin-polarized phase can exist in even higher density regions [5].

So far we have not known the spin-polarized phase of systems including both the AV- and T- type interactions simultaneously, which is expected to exhibit new features of the spin polarization (SP). In this work, thus, we investigate the interplay between them, and figure out the phase structure of the spin-ploarzed matter at zero temperature in the chiral limit.

2. Formaism

In this section we briefly explain our formalism, which holds the flavor $SU(2)$ and the color $SU(3)$ symmetry. In this work, we are interested in the high density region where the chiral condensation has already gone, so we take the quark mass ito be zero and neglect the Dirac sea contribution. In addition, we consider only the spin-isospin saturated quark matter.

We start with a Lagrangian density including the spatial parts of AV and T fields,

$$L = \bar{\psi} i\partial\!\!\!/ \psi + A_i \bar{\psi}\gamma_5\gamma_i\psi + U_{ij}\bar{\psi}\sigma_{ij}\psi - \frac{A_i^2}{g_A} - \frac{U_{ij}^2}{g_U}, \tag{1}$$

where $A_i = g_A\langle\bar{\psi}\gamma_5\gamma_i\psi\rangle$, $U_{ij} = g_U\langle\bar{\psi}\sigma_{ij}\psi\rangle$, and $g_{A,U}$ coupling constants of AV and T channels.

Here, we assume only the third components of the mean fields, $A_3(= A)$ and $U_{12}(= U)$, to be nonzero, and obtain the Dirac equation for the spinor $u(\boldsymbol{k}, s)$ with momentum $\boldsymbol{k} = (k_x, k_y, k_z)$ and spin s,

$$[\boldsymbol{\alpha}\cdot\boldsymbol{k} + \Sigma_z A + \beta\Sigma_z U]\, u(\boldsymbol{k}, s) = \varepsilon_{k,s} u(\boldsymbol{k}, s). \tag{2}$$

The single particle energy $\varepsilon_{k,s}$ becomes

$$\varepsilon_{p,s} = \sqrt{k_z^2 + k_t^2 + A^2 + U^2 + 2s\sqrt{k_z^2 A^2 + k_t^2 U^2 + A^2 U^2}}, \tag{3}$$

where $s = \pm 1$ and $k_t = \sqrt{k_x^2 + k_y^2}$. The thermodynamics potential is given by

$$\Omega[A, U, \mu] = N_d \sum_{s=\pm 1} \int \frac{d^3k}{(2\pi)^3} (\varepsilon_{k,s} - \mu)\,\theta(\mu - \varepsilon_{k,s}) + \frac{A^2}{g_A} + \frac{U^2}{g_U}, \tag{4}$$

where N_d is the degeneracy factor, and μ is the chemical potential.

3. Results

In Fig. 1, we exhibit the Fermi surfaces for $s = -1$, which have various shapes in the momentum space dependent on A, U, and μ,

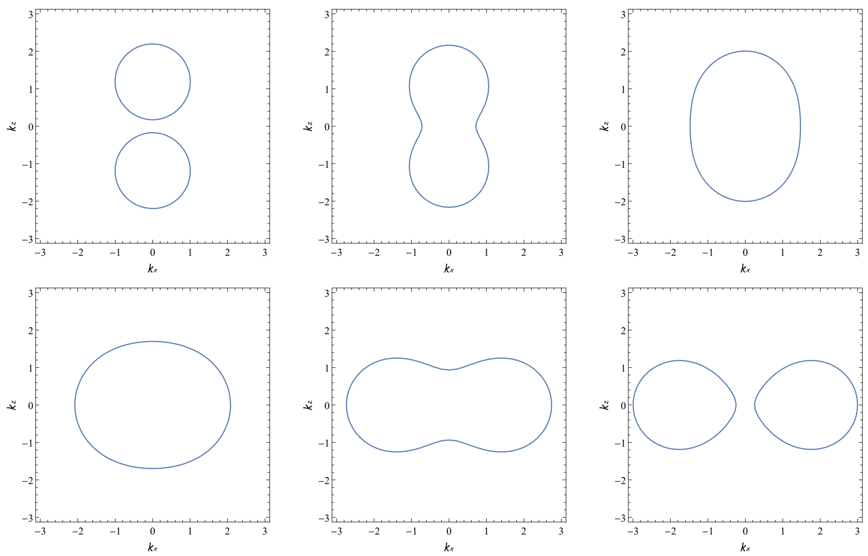

Fig. 1. Cross sections of Fermi surfaces in $k_x - k_z$ plane for $s = -1$. In the top row from left to right ① → ③, and in the bottom row from left to right ④ → ⑥. Note that the Fermi surface is symmetric under the rotation around k_z axis, so the last one is a doughnut in full three dimension.

In Fig. 2 we show the correspondence between the shape of the Fermi distributions and the values of A, U, and μ. The spin polarization by $A \neq 0$ never occurs at $U = 0$ in the chiral limit, that is, the $\Omega[A, U]$ is always stable against A fluctuations at the origin in the A-U space. This is because in general AV-type mean field, appearing in the form of a mean-field interaction term can be eliminated by a local $U_A(1)$ chiral transformation. A finite U breaks the time reversal symmetry as well as the $U_A(1)$ symmetry, and can invoke a finite A for sufficiently strong couplings.

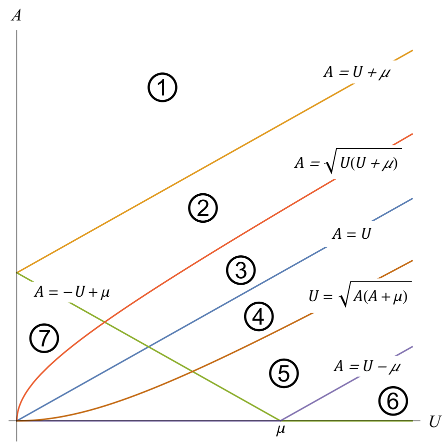

Fig. 2. ①- ⑥ correspond to different topologies of Fermi surfaces for $s = -1$ as shown in Fig. 1, and the Fermi surface for $s = 1$ becomes finite only in the region of ⑦: $A < -U + \mu$.

In Fig. 3 we show the phase diagram of the spin-polarized phases, which is described with the two parameters, $g_A\mu^2$ and $g_U\mu^2$. In this figuure $g_{U\mathrm{Crit}}$ is defied as the critical coupling for the spin-polarizastion. The normal phase ($A = U = 0$) appears for $g_U \leq g_{U\mathrm{Crit}}$, where in the chiral limit [5] while the T mean field becomes finite for $g_U > g_{U\mathrm{Crit}}$, The finiteness of the AV mean field depends on the strength of the coupling g_A. In strong coupling regions where $g_A, g_U \geq 12.5664/\mu^2$, the two spin-polarized phases are separated by the straight line $g_A = g_U$.

4. Summary

We have studied the interplay between the axial-vector (AV) and tensor (T) mean fields for a spontaneous spin polarization of the quark matter at zero temperature in the chiral limit. As the chemical potential increases the T mean field U becomes finite first at a critical point, and breaks the $U_A(1)$ chiral symmetries as well as the spatial rotation symmetries, while the AV mean field A becomes finite only in the region of a finite U because the finiteness of A requires the $U_A(1)$ chiral symmetry to be broken. All these phase boundaries correspond to continuous phase transitions.

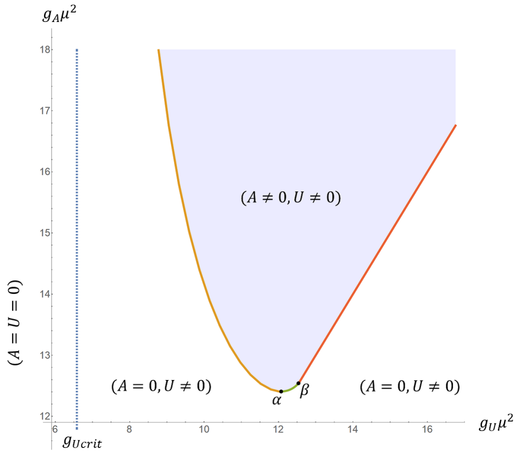

Fig. 3. Phase structure in the plane of axial-vector and tensor couplings, g_A and g_U, normalized by the chemical potential μ. The shaded region bounded corresponds to the $(A \neq 0, U \neq 0)$ phase.

Here, we note that the formalism holds $SU(2)$ chiral symmetry [5] though we do not show it apparently by treating the iso-spin saturated matter.

Although the spin polarization of the dense matter is not defined uniquely in the relativistic framework, we quantify it by AV and T mean fields in this study. The response to external stimulations may give another insight into the spin or magnetic properties of the strongly interacting matter, such as susceptibilities to external magnetic fields, spatial rotations and so on.

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