

Running of Planck mass and Higgs VEV in holographic vs. 4-dimensional RG

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Abstract. We compute the scale dependence of the Planck mass and the Higgs VEV using two very different methods: a "holographic" procedure based on Einstein's equations in five dimensions, with scalar matter confined to a 3-brane, and a "direct" procedure based on the use of renormalization group equations in four dimensional gravity coupled to a nonlinear $O(N)$ scalar field theory. The two calculations lead to similar results, both suggesting that the coupled theory approaches a fixed point in the UV.

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1. Motivation

One of the most remarkable recent developments in quantum field theory is the realization that the coupling of a d -dimensional theory to gravity in $d + 1$ dimensions yields information about the renormalization group (RG) running of the couplings of that theory. This idea was already contained in the famous paper by Randall and Sundrum [1], and has been sharpened in a number of subsequent publications [2, 3, 5, 4]. While the notion of "holography" has come to have a rather specific meaning closely related to the AdS/CFT correspondence [6, 7], here I will generically call "holographic RG" the flow of couplings of a d -dimensional theory which is obtained by viewing it as living on a $(d - 1)$ -brane coupled to gravity in $(d + 1)$ dimensions, and identifying the transverse coordinate with the RG scale.

On a different field, there has been significant development in the use of entirely 4-dimensional "functional RG equations", *i.e.* equations which describe in a single stroke and directly the running of infinitely many couplings [8, 9]. The method has proven particularly helpful in the study of perturbatively non-renormalizable theories with the aim of establishing (or refuting) the existence of non-trivial UV fixed points (FPs) that could be used for a fundamental definition of the theory [10], a property that has become known as "Asymptotic Safety" [11].

To the extent that holographic and functional RG are equivalent descriptions of physics, they must be related in some way. There has been some work in this direction [16, 17]. In this article, instead of exploring this relation from first principles, I will evaluate similarities and differences of the two methods for a simple theory which incorporates some basic features of Nature.

The toy model to be considered is a $SO(N)$ non-linear sigma model coupled to gravity in 4d with an action of the form $S = S_g + S_m$, where

$$S_g = -m_P^2 \int d^4x \sqrt{g} R \quad (1)$$

is the gravitational action. The matter action S_m for the $SO(N)$ non-linear sigma model is obtained by a limiting procedure from the corresponding linear theory, which contains a multiplet of N real scalars ϕ^a with an action

$$S_m = \int d^4x \sqrt{g} \left(\frac{1}{2} \sum_{a=1}^N g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^a + \lambda(\rho^2 - v^2)^2 \right), \quad (2)$$

where $\rho^2 = \sum_{a=1}^N \phi^a \phi^a$. In a phase with spontaneous symmetry breaking, we have $v^2 = \langle \rho^2 \rangle > 0$. Without loss of generality we can assume that the background field is $\phi^\alpha = 0$ for $\alpha = 1 \dots N-1$ and $\phi^N = v$. The $N-1$ fields $\varphi^\alpha = \phi^\alpha$ are the massless Goldstone bosons, while the “radial” massive mode $\delta\rho = \phi^N - v$ corresponds to the physical Higgs field.

The non-linear sigma model is achieved by taking the limit $\lambda \rightarrow \infty$ with v constant. Then the potential for ρ becomes a constraint $\rho^2 = v^2$, which can be solved to eliminate one scalar field and describe the theory in terms of the remaining fields φ^α transforming non-linearly under $SO(N)$, the coordinates on the sphere. Higgs field $\delta\rho$ drops out becoming infinitely heavy in this limit. In an arbitrary coordinatization of the $(N-1)$ -sphere, the action becomes

$$S_m = \frac{1}{2} v^2 \int d^4x \sqrt{g} g^{\mu\nu} \partial_\mu \varphi^\alpha \partial_\nu \varphi^\beta h_{\alpha\beta}(\varphi). \quad (3)$$

Our toy model contains two dimensionful couplings m_P^2 and v^2 , which we identify with the square of the Planck mass and of the Higgs VEV. They appear in a very similar manner as prefactors of the respective terms (1),(3) in the action.

There are three main motivations for choosing this model as opposed to gravity coupled to model with linearly transforming scalars. The non-linear model in four dimension has a coupling constant with inverse mass dimension and is power-counting non-renormalizable, similar to gravity itself. It also suffers from violation of unitarity at high energy. Recent studies showed that it displays an UV fixed point [18], with very similar behaviour as found within pure Einstein gravity [12]. It has therefore been suggested that, quite independently of gravity, a strongly interacting Goldstone boson sector may exist, able to overcome its perturbative problems in a dynamical way [13, 14, 15].

Secondly the linear sigma model coupled to gravity displays “Gaussian matter FP”, where matter couplings are asymptotically free and gravity is safe [19, 20]. Given the existing evidence for asymptotic safety of the non-linear scalar theory and gravity separately, one may expect to find a non-trivially interacting FP also for the coupled theory.

The third motivation is of a more direct physical nature namely the non-linear theory is adequate, at least until the Higgs particle is detected experimentally [21].

2. Holographic RG

In this section we evaluate the running of the two dimensionful couplings m_P^2 and v^2 of the four-dimensional toy model using a holographic technique. This flow is obtained by putting our nonlinear sigma model on a flat 3-brane, coupling to 5d gravity and identification of the transverse coordinate with the RG scale. Following [1], I consider a 5-dimensional spacetime with coordinates $y^m = (x^\mu, t)$, $\mu = 1, 2, 3, 4$ and metric G_{mn} . In the bulk we have only gravitational part of the action

$$S_{\text{grav}} = \int d^5y \sqrt{-G} (2M^3 R - \Lambda), \quad (4)$$

where M is the 5-dimensional Planck mass and Λ is the bulk cosmological constant. We make an ansatz for the metric of the form

$$ds^2 = e^{2t} \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu + r_c^2 dt^2 . \quad (5)$$

We are looking for an AdS solution of the spacetime like in Randall-Sundrum model. Using the 5-dimensional Einstein equations we get the solution with $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$ (preserving 4-dimensional Poincaré symmetry), where we have identified the arbitrary length scale r_c with the AdS radius $\sqrt{24M^3/|\Lambda|}$ and a warping factor is identical to t . We can make the transformation to “conformal time” coordinates (z, x) defined by the following relation $t = -\log(z/r_c)$, which brings the metric to the form

$$ds^2 = \frac{r_c^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2) . \quad (6)$$

Note that the AdS “boundary” $z = 0$ corresponds to $t = \infty$. In the holographic interpretation of the 5-dimensional metric, the 5th dimension t is identified with the (logarithm of the) RG scale k . Following [5, 4], we make the identification $z = 1/k$, which implies $t = \log(kr_c)$. We choose the origin of t to correspond to the electroweak scale $k = v_0 = 246\text{GeV}$, which implies numeric value for the AdS radius $r_c = 1/v_0$.

To read off the β -functions of couplings in our model we imagine putting a test brane at a given value of t [22]. Except for dimensionless couplings which run logarithmically, all the natural mass parameters in the 4-dimensional matter theory are proportional to v , whose running is governed by the formula

$$v(t) = v_0 e^t . \quad (7)$$

The warping in AdS spacetime causes exponential running in t variable of Higgs field VEV v , which translates into linear running with RG momentum scale k . This is a manifestation of the quadratic divergences in the running of $(\text{mass})^2$ in the underlying field theory.

Inserting the ansatz (5) in the action (4), we find that the effective 4-dimensional gravitational action for the metric $\bar{g}_{\mu\nu}(x)$ is equal to

$$S_{\text{grav}} = 2M^3 r_c \int dt e^{2t} \int d^4x \sqrt{-\bar{g}} \bar{R} . \quad (8)$$

The relation connecting the effective 4-dimensional Planck mass m_P as seen by observer located at t and the 5-dimensional parameter M is obtained by performing the integral over t explicitly, leading to

$$m_P^2(t) = m_P^2(0) + \frac{M^3 r_c}{2} [e^{2t} - 1] . \quad (9)$$

This formula contains the unobservable five-dimensional Planck mass. We can rewrite it in terms of four-dimensional measurable quantities as follows. The Planck mass at the TeV scale $m_P(0)$ is not too different from the measured value at macroscopic scales $m_P(-\infty)$. Knowing the empirical values of v_0 and $m_P(0)$ we have $t_P \approx 38$. Furthermore we define the coefficient $c_P = \left(\frac{m_P(t_P)}{m_P(0)}\right)^2 - 1$ which measures the relative change of the t -dependent Planck mass between the TeV and Planck scale. The anti-screening nature of gravity implies that $c_P > 0$. From the definition of c_P and the assumption that $m_P \gg v_0$ we get the relation $M^3 r_c = 2 c_P v_0^2$, with the help of which we can rewrite formula (9) as

$$m_P^2(t) = m_P^2(0) + c_P v_0^2 [e^{2t} - 1] , \quad (10)$$

where we have replaced the 5-dimensional parameters by the Higgs VEV and the arbitrary constant c_P , which is expected to be of order one.

We observe that equation (7) describes a mass parameter that scales with the cutoff k exactly as dictated by dimensional analysis ($m(t) = m_0 e^t = m_0 \frac{k}{m_0} = k$), so the matter sector exhibits scale-invariance. Therefore, when the mass is measured in units of the cutoff, it is constant. If we regard this mass as the coupling constant of the non-linear sigma model (3), we are already at a RG fixed point. Likewise, when $t \rightarrow \infty$, also the Planck mass m_P scales asymptotically in the same way ($m_P^2(t) \rightarrow c_P v_0^2 e^{2t} = c_P k^2$), so if we regard it as the (inverse) gravitational coupling, (10) describes an RG trajectory for gravity that approaches a non-trivial FP.

3. Functional RG

In this section we evaluate the scale-dependence of m_P^2 and v^2 directly in the four-dimensional theory. Our starting point is the “quantum effective action” Γ_k , a coarse-grained version of the average effective action at some RG momentum scale k which interpolates between some classical action at $k = k_0$ and the full quantum effective action at $k = 0$. The RG momentum scale k -dependence is introduced at the level of the path integral by adding suitable momentum-dependent kernels $R_k(q^2)$ to the inverse propagators. They must decrease monotonically with k^2 , tend to 0 for $k^2/q^2 \rightarrow 0$ (in order to leave the propagation of large momentum modes intact), and tend to k^2 for $q^2/k^2 \rightarrow 0$ (in order to suppress the low momentum modes). In the following I am going to use logarithmic RG “time” defined by $t = \log(k/k_0)$. The change of Γ_k with it is given by a functional differential Wetterich equation [9]

$$\partial_t \Gamma_k = \frac{1}{2} \text{STr} \left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k. \quad (11)$$

Here, $\Gamma_k^{(2)}$ denotes the matrix of second functional derivatives with respect to all propagating fields, and the supertrace stands for a sum over all modes including a minus sign for fermionic fields.

The β -functions for the couplings are obtained from (11) by projection. For optimized choices of the momentum cutoff $R_k(q^2) = k^2 \theta(q^2 - k^2)$ the traces can be performed analytically [23]. We use the heat kernel techniques to expand polynomially functional RG flow and later evaluate traces around $R = 0$ and $\rho^2 = v^2$. This type of calculation was first described in [24, 25, 12] for pure gravity, and in [18] for the non-linear sigma model. For the details the reader is referred to the extended version of this article [28]. The flow for the inverse gravitational coupling m_P^2 and for the vacuum expectation value v^2 are then given by $\frac{d}{dR}(\partial_t \Gamma_k)$ and $-\frac{d}{d\rho^2} \partial_t \Gamma_k / (2\lambda)$ respectively.

Now we take the non-linear limit $\lambda \rightarrow \infty$ (or Higgs mass $\rightarrow \infty$) with v^2 held constant. In this limit the Goldstone bosons remain fully dynamical, in fact their action is completely unaffected by the limit. We end up with

$$\partial_t v^2 = B_H k^2; \quad B_H = \frac{N-1}{16\pi^2}, \quad (12)$$

$$\partial_t m_P^2 = B_P k^2; \quad B_P = \frac{N_g - N}{96\pi^2}, \quad (13)$$

where $N_g = 109/4$. The dependence of the result on the number of Goldstone modes is simple to understand. In (12), only the Goldstone modes contribute to the running of the VEV. In (13), the contribution from the modes originating from the graviton self-interaction takes over the Goldstone modes when $N_g > N$. Then the combined effect is to increase m_P ($B_P > 0$) with increasing RG time t .

For a better understanding of the system it is convenient to use the inverses $G = 1/(16\pi m_P^2)$, $f^2 = 1/v^2$, and to introduce dimensionless couplings $\tilde{v}^2 = v^2/k^2$, $\tilde{f}^2 = f^2 k^2$, $\tilde{m}_P^2 = m_P^2/k^2$, $\tilde{G} = G k^2$. This is because the perturbative analysis of the sigma model and gravity is an

expansion in the couplings \tilde{f} and \tilde{G} , respectively. Their β -functions are given by

$$\partial_t \tilde{G} = 2\tilde{G} - B_P \tilde{G}^2 \quad (14)$$

$$\partial_t \tilde{f}^2 = 2\tilde{f}^2 - B_H \tilde{f}^4. \quad (15)$$

Each one of these β -functions admits two fixed points: an IR FP at zero coupling and an UV FP at finite coupling $\tilde{f}^2 = 2/B_H$ and $\tilde{G} = 2/B_P$ respectively.

4. Comparison

For the sake of comparison with the results of the holographic procedure, we can write the general exact solutions of RG flow equations (12), (13) as:

$$v^2(t) = v_0^2 + \frac{1}{2} B_H (k^2 - k_0^2) = v_0^2 \left[1 + \frac{1}{2} B_H (e^{2t} - 1) \right], \quad (16)$$

$$m_P^2(t) = m_{P0}^2 + \frac{1}{2} B_P (k^2 - k_0^2) = m_{P0}^2 + \frac{1}{2} B_P v_0^2 (e^{2t} - 1), \quad (17)$$

where we have defined, in accordance with the definitions in section II, $k_0 = k(0) = v_0$.

The running of the two couplings have completely independent but very similar behavior. For $k \ll v$, \tilde{v} is close to the Gaussian FP. This is the domain where the dimensionful coupling v is nearly constant, the dimensionless \tilde{v} has an inversely linear “classical” running with energy, and perturbation theory is rigorously applicable. Then there is a regime where \tilde{v} is nearly constant and close to the non-trivial FP, while the dimensionful v scales linearly with energy. These considerations can be repeated verbatim for m_P , the sole difference being that the RG scale where the transition from “classical running” to non-classical behavior occurs, will be near the Planck scale. Thus, there are three regimes: the low energy regime $k \ll v \ll m_P$, where both G and f are constant, the intermediate regime where \tilde{f} has reached its FP value but G is still constant and the FP regime where both dimensionless couplings have reached the FP.

Strictly speaking, the only physical parameter of the theory is the ratio of mass scales

$$\alpha(t) \equiv \frac{m_P(t)}{v(t)}. \quad (18)$$

The plot of $\log \alpha(t)$ is shown in fig. 1 and illustrates the three regimes of the theory alluded before. For $t \rightarrow \infty$ the ratio tends, for all trajectories, to the constant value B_P/B_H , while for $t \rightarrow -\infty$ it tends to a number that depends on the initial conditions and is of order m_{P0}^2/v_0^2 .

Returning to equations (16) and (17), we see that if we could set $B_H = 2$ and $B_P = 2c_P$, they would agree with the flow obtained by the holographic method.

There is a difference here between the flows of v and m_P : whereas c_P is a free parameter in the holographic model, which can be adjusted to match the result of the functional RG, there is no corresponding free parameter for v . One is thus left with a prediction for the parameter B_H that does not seem to match the result of the functional RG. To clarify this difference further, we observe that if we set $B_H = 2$, as the AdS holographic RG seems to demand, v tends to zero in the IR and therefore α diverges linearly. This is shown by the dashed line in fig. 1. Therefore the holographic RG describes unique trajectory of the flow, whereas exact 4d approach allows for arbitrary initial values of the couplings. The holographic description agrees well with functional RG flow in the second and third regime, but fails to reproduce even at a qualitative level the generic low-energy regime of the theory. This is due to the fact that the holographic RG trajectory is such that v tends to zero in the IR, which is just one amongst infinitely many RG trajectories in (16) that would tend to different finite limits in the IR. In contrast, m_P can have an arbitrary limit in the IR: this is due to the freedom of choosing the parameter c_P .

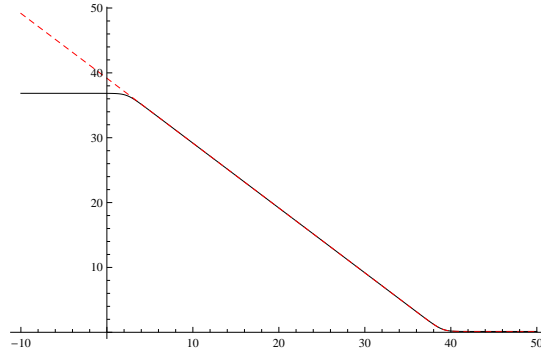


Figure 1. The running of the mass ratio $\alpha(t)$ defined in (18), for $N = 4$, on a logarithmic scale as a function of t . Solid curve: solution of the functional RG; dashed curve: solution of the holographic RG.

We can modify the holographic RG to resemble more closely the functional one by stopping sharply the flow of v at $k = v_0$. This can be achieved by putting a source 3-brane located at $t = 0$ with an action

$$\sqrt{6 M^3 |\Lambda|} \int d^5 y \delta(t) . \quad (19)$$

We generalize the ansatz (5) by replacing e^{2t} with new warping factor $e^{2\sigma(t)}$. Since $\sigma(t) = t$ for $t > 0$, we get $\sigma(t) = 0$ for $t < 0$ after solving five-dimensional Einstein equations. Thus, we have a solution where the brane at the origin joins continuously a flat space-time for $k r_c < 1$ with AdS space-time for $k r_c > 1$, where we recall that $t = \log(k r_c)$. For the running Planck mass the above construction implies a weak, logarithmic running for negative value of t coordinate. We conclude that the resulting five-dimensional spacetime has become very similar to the one used by Randall and Sundrum [1]. The behavior of the couplings for $t < 0$ is not exactly the same as the solution that we found from the functional RG, but it is qualitatively the same. The comparison could be improved further by making the model more realistic. Namely we can take into account threshold phenomena at low energy which basically switch off the running of v below v_0 [14].

5. Conclusion

There are two aspects of this work that need to be discussed: the physical meaning of a non-trivial FP for gravity coupled to a non-linear sigma model and the relation between holographic and functional RG.

We have shown that in the simplest approximation, retaining only terms with two derivatives of the fields, the non-linear sigma model minimally coupled to gravity exhibits a non-trivial, UV attractive FP, which could be used to define the theory non-perturbatively. This was proved in both ways: using holographic and functional approaches to RG flow. In the one of the realization of asymptotic safety scenario, each type of the interaction present in this model would be asymptotically safe by itself, and each coupling would reach the FP at a different energy scale: the TeV scale for electroweak interactions and the Planck scale for the gravitational interactions. This is the point of view that I am proposing in this work.

Taking this seriously, one is led to a non-standard picture of all interactions, where both electroweak and gravitational interactions would be in their respective “broken” phases, characterized by non-vanishing VEVs, and carrying non-linear realizations of the respective local symmetries. The theory as formulated does not admit the possibility of symmetry restoration

at high energy. In fact, rather than going to zero, the Higgs VEV goes to infinity at high energy. The behaviour of the ratio α , illustrated in fig.1, characterizes the three regimes of the theory, with the electroweak and gravitational interactions becoming scale-invariant above their characteristic mass scales. We noted good agreement of two methods at FP (high energy) regime and also at intermediate one. At low energy modification in the spirit of Randall-Sundrum in the holographic approach and presence of threshold phenomena in functional 4d approach was called to allow for better accordance in low-energetic behaviour of realistic theory.

We now come to the striking correspondence between the RG flows computed by holographic and functional methods. The holographic RG is based to a large extent on the AdS_5 solution. Given that the isometry group of AdS_5 is the group $SO(3,2)$, which can be interpreted as the conformal group in four dimensions, it is not so surprising that this space can be used to describe in geometric terms a theory with scale-invariance, so possessing nontrivial FP. My view here is therefore to interpret the five-dimensional metric as a geometrization of the four-dimensional RG flow. We do not claim to be describing a dynamical duality between a four-dimensional “boundary” theory and a five-dimensional “bulk” theory, which would typically relate different types of degrees of freedom. Here we obtained an example of a duality in a “kinematical” sense relating particular solution in 5d geometry to particular RG flow of 4-dimensional theory. Moreover it is claimed that the metric, near the AdS boundary at $z = 0$ (or $t \rightarrow \infty$), describes the RG running in the vicinity of a non-trivial FP. Secondly, if one views the graviton as a dynamical field propagating in a five dimensional spacetime, then graviton fluctuations that are nonzero at $z = 0$ are not normalizable. This is the reason why I stick to only kinematical meaning of this duality. This is in the same relation as analogue gravity models in fluids remain to fluid-gravity correspondence [27] derived from AdS/CFT . By this mean simple “kinematical” analogy between two RG flows was established.

The extended version of this work is contained in [28].

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