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Quantization and spectrum of RNS supersymmetric open 2-brane

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Abstract

We give the quantization and spectrum of an RNS supersymmetric open 2-brane described by a Polyakov-like action, the model is world-volume supersymmetric. We present the Hamiltonian of the system in terms of raising and lowering operators. We get a supersymmetric spectrum of excited states in a discrete form after a GSO-like projection, which may be useful for further exploration related to the continuous spectrum of supermembranes.

1. Introduction

String is a special case of p-branes. p-Branes are the extended p-dimensional objects moving in some D-dimensional spacetime, $D \geqslant p$. The case $p = 1$ refers to "strings". These are extended structures embedded in some higher-dimensional spacetime from which they inherit induced metrics [1,2]. Just like a string sweeps out a 2-dimensional world-sheet as it evolves with time, a p-brane sweeps out a d-dimensional world-volume and $d = p + 1$.

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The idea of relating different elementary particles to different vibrating modes of a membrane was put forward in 1962 by Dirac [3]. Later in 1986 there was a breakthrough in membrane theory when the work of Hughes, Liu and Polchinski [4] showed that supersymmetry could be incorporated into membrane theory, which was the birth of supermembrane theory.

The maximum allowed spacetime dimension is 11 [5,6] in a self-consistent brane-theory. And it is also possible [7] to perform a simultaneous dimensional reduction of both the spacetime and world-volume which leads to a theory of strings moving in $10D$. More precisely, such doing yields the Type-IIA superstring theory.

An undesired feature of bosonic string theories is that their spectrum does not contain fermions. Fermions play very fundamental roles in nature. In the standard model, they correspond to quarks and leptons. Hence fermions are unavoidable for a theory if it is to describe nature. The addition of supersymmetry to the usual bosonic string theory is crucial to achieve potentially realistic string theories [8,9]. Such string theories which include fermions are referred to as superstring theories. Moreover, there are different ways in which one can construct a superstring theory on a flat background and all of them require the critical spacetime dimension to be $D = 10$.

Compared to the works on the string quantization, works on the quantization of membranes are very few and are in the Green–Schwarz formalism and matrix regularization, [10,11]. However, quantizations using the Ramond–Neuver–Schwarz scheme are also possible and worthy of careful explorations. For the pure bosonic 2-branes, this question has been studied in the references [12,13]. It was shown that the Hamiltonian of the system could be constructed in terms of raising and lowering operators. The spectrum is shown to contain two kinds of tachyon states and some massless states such as graviton states, Kalb–Ramond fields, dilaton states and photon states which are all produced at the same level in the open 2-brane model.

In this paper we will add fermionic degrees of freedom to the open 2-brane [12] and focus on the resulting system's quantization and spectra in the RNS formulation. Since we fix the form of world volume metric and its supersymmetric partner, our work is a $2 + 1$ d analogue of the $1 + 1$ d RNS superstring theory.

The organization of this paper is: in Section 2, we follow reference [13] and write down the Polyakov-like action for the supersymmetric 2-brane, from which we derive the energy-momentum tensor and classical equations of motion. In Section 3, we provide the boundary conditions and mode expansions for the bosonic and fermionic world-volume fields. In Section 4, we derive the commutation/anticommutation relations for the oscillator operators from the commutation/anticommutation relations satisfied by the canonical field variables. In Section 5, we construct the Hamiltonian of the system in terms of raising and lowering operators separately for R-sector and NS-sector in two separate subsections. In Section 6, we present the physical spectrum of states for the quantized supersymmetric open 2-brane at the first few mass levels. Section 7 is summary and discussion.

2. Equations of motion for supersymmetric open 2-brane

The dynamics of 2-branes is described by the Nambu–Goto action which physically is nothing but the world-volume swept out by the brane when it evolves in the background spacetime. However the quantization of this action turns out to be quite awkward because of the presence of the square root computation. To avoid difficulties brought by this square root, we follow reference [13] and focus on the Polyakov-like action supplemented by a fermionic part

$$S_p = \frac{-1}{4\pi\acute{\alpha}} \int d^3\sigma(-h)^{\frac{1}{2}}\left[h^{ab}\partial_a X^\mu \partial_b X_\mu + h^{ab}\bar{\Psi}^\mu \gamma_b D_a \Psi_\mu + R - 2\Lambda - \overline{\chi}_\mu \gamma^{\mu\nu\rho} D_\nu \chi_\rho\right]$$

$$\text{where } D_\nu \chi_\rho = \partial_\nu \chi_\rho + \frac{1}{4}\omega_\nu^{mn}\gamma_{mn}\chi_\rho \tag{1}$$

where $\mu = 0, 1, \ldots, 9$, $d^3\sigma = d\tau d\sigma^1 d\sigma^2$, χ_ρ is the graviton and $\gamma^a, a = 0, 1, 2$ is the two dimensional representation of Dirac matrices, which obey the Clifford algebra

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab} \tag{2}$$

The action (1) is a simple generalization of that of superstrings. However, there are key differences between the superstring and supersymmetric 2-brane cases. Firstly, in the superstring case, supergravity plus X and Ψ captures all information about the system's dynamics. In the brane case, supergravity plus X and Ψ describes only part of the brane dynamics. What we wish to do in this paper is the quantization of 3 dimensional supergravity coupled to X and Ψ, instead of the supermembrane. Secondly, in contrast to the superstring case, we do not have enough symmetries in the brane case to fix the world volume metric h_{ab} (including its superpartner) completely. However, if we consider the full dynamics of h_{ab} at the very start, then we could almost do nothing except various arguments about a continuous spectrum. So, in this paper, we will fix the form of h_{ab} by hand and focus on the relative dynamics of the system. This is implemented just by removing the last three terms from the action (1) and changing the covariant derivative in the second term to partial derivative. Physically this corresponds to the fact that we go from a continuous spectrum space to a space consisting of infinite discrete points.

In 10-dimensional target space, the 2-brane has 7 transverse oscillating directions but the on shell Majorana–Weyl spinor has 8 degrees of freedom. However the supersymmetry of 2-brane is still possible when we include the gauge field degrees of freedom on the world-volume of the brane.

From the aspect of action, we can check that the action (1) is invariant under the infinitesimal world-volume supersymmetry transformations of the form

$$\delta X^\mu = \bar{\epsilon}\Psi^\mu \tag{3}$$

$$\delta\Psi^\mu = \gamma^a \partial_a X^\mu \epsilon \tag{4}$$

The free massless fermions Ψ^i are two component spinors, i.e., Ψ^μ is given by

$$\Psi^\mu = \begin{pmatrix} \psi_-^\mu \\ \psi_+^\mu \end{pmatrix} \tag{5}$$

Since the gamma matrices in 3 dimensions are purely real, they furnish a Majorana representation of the Clifford algebra. Using formalism of [14], the Majorana condition means $\Psi^{\mu *} = \Psi^\mu$.

As a 3-dimensional world-volume field theory, the energy-momentum tensor of X^μ, Ψ^μ fields can be derived from action (1) as follows

$$T_{ab} = \frac{-2\pi\acute{\alpha}}{\sqrt{-h}}\frac{\delta S_p}{\delta h^{ab}} = \frac{1}{2}\Big[\partial_a X^\mu \partial_b X_\mu - \frac{1}{2}h_{ab}h^{cd}\partial_c X^\mu \partial_d X_\mu - \frac{1}{2}h_{ab}h^{cd}\bar{\psi}^\mu \gamma_d \partial_c \psi_\mu + \frac{1}{2}\bar{\psi}^\mu \gamma_b \partial_a \psi_\mu + \frac{1}{2}\bar{\psi}^\mu \gamma_a \partial_b \psi_\mu\Big] \tag{6}$$

While h^{ab}'s equation of motion of action (1) requires

$$T_{ab} = 0 \tag{7}$$

This means that the energy–momentum tensor of the supersymmetric 2-brane system vanishes. While the 00-component of this equation when we choose the metric h_{ab} as $(-,+,+)$, reads

$$T_{00} = \left[\frac{1}{2}\partial_0 X^\mu \partial_0 X_\mu + \frac{1}{4}\bar{\psi}^\mu \gamma_0 \partial_0 \psi_\mu + \frac{1}{4}\bar{\psi}^\mu \gamma_0 \partial_0 \psi_\mu + \frac{1}{4}h^{cd}\partial_c X^\mu \partial_d X_\mu + \frac{1}{4}h^{cd}\bar{\psi}^\mu \gamma_d \partial_c \psi_\mu\right] = 0 \tag{8}$$

Using action (1), we write down the equation of motion for X^μ in the following form

$$\left(\partial_\tau^2 - \partial_1^2 - \partial_2^2\right) X^\mu\left(\tau, \sigma^1, \sigma^2\right) = 0 \tag{9}$$

This is the same as that in Ref. [12]. While that of the fermionic fields Ψ^μ in action (1), which are real two component spinors, can be deduced from the general Dirac lagrangian and by specializing to the case of $m = 0$ as follows [15,16],

$$\mathcal{L} = \bar{\psi}_\mu\left(\gamma^a \partial_a - m\right)\psi^\mu\left(\tau, \sigma^1, \sigma^2\right) \quad (m = 0) \tag{10}$$

From which we get

$$\left(\gamma^a \partial_a - m\right)\psi^\mu\left(\tau, \sigma^1, \sigma^2\right) = 0 \quad (m = 0) \tag{11}$$

3. Boundary conditions and mode expansions

The R-sector boundary conditions are given by

$$\psi_+^\mu(\tau, 0, 0) = \psi_-^\mu(\tau, 0, 0) \tag{12}$$
$$\psi_+^\mu(\tau, \pi, 0) = \psi_-^\mu(\tau, \pi, 0) \tag{13}$$
$$\psi_+^\mu(\tau, 0, \pi) = \psi_-^\mu(\tau, 0, \pi) \tag{14}$$

While the NS-sector boundary conditions read

$$\psi_+^\mu(\tau, 0, 0) = -\psi_-^\mu(\tau, 0, 0) \tag{15}$$
$$\psi_+^\mu(\tau, \pi, 0) = -\psi_-^\mu(\tau, \pi, 0) \tag{16}$$
$$\psi_+^\mu(\tau, 0, \pi) = -\psi_-^\mu(\tau, 0, \pi) \tag{17}$$

The corresponding boundary conditions for the bosonic fields are the following,

$$\partial_{\sigma^1} X^\mu\left(\tau, 0, \sigma^2\right) = \partial_{\sigma^1} X^\mu\left(\tau, \pi, \sigma^2\right) = 0 \tag{18}$$
$$\partial_{\sigma^2} X^\mu\left(\tau, \sigma^1, 0\right) = \partial_{\sigma^2} X^\mu\left(\tau, \sigma^1, \pi\right) = 0 \tag{19}$$

Now we can write the general solution to Eq. (11) for the R-sector as follows

$$\psi^\mu(\sigma) = \frac{1}{(2\pi)^2}\int_0^\infty \frac{d^2k}{\sqrt{2\omega}}\left(d_{\mathbf{ks}}^\mu e^{-ik_a\sigma^a} + d_{\mathbf{ks}}^{\mu\dagger} e^{ik_a\sigma^a}\right)u_{\mathbf{ks}} \tag{20}$$

$$\bar{\psi}^\mu(\sigma) = \frac{1}{(2\pi)^2}\int_0^\infty \frac{d^2k}{\sqrt{2\omega}}\left(d_{\mathbf{ks}}^{\mu\dagger} e^{ik_a\sigma^a} + d_{\mathbf{ks}}^\mu e^{-ik_a\sigma^a}\right)\bar{u}_{\mathbf{ks}} \tag{21}$$

and for the NS-sector

$$\psi^\mu(\sigma) = \frac{1}{(2\pi)^2}\int_0^\infty \frac{d^2k}{\sqrt{2\omega}}\big(b^\mu_{\mathbf{ks}} e^{-ik_a\sigma^a} + b^{\mu\dagger}_{\mathbf{ks}} e^{ik_a\sigma^a}\big)u_{\mathbf{ks}} \tag{22}$$

$$\bar\psi^\mu(\sigma) = \frac{1}{(2\pi)^2}\int_0^\infty \frac{d^2k}{\sqrt{2\omega}}\big(b^{\mu\dagger}_{\mathbf{ks}} e^{ik_a\sigma^a} + b^\mu_{\mathbf{ks}} e^{-ik_a\sigma^a}\big)\bar u_{\mathbf{ks}} \tag{23}$$

with the 2-component spinors u_{ks}, $\bar u_{ks}$ satisfying orthogonality conditions like

$$u^\dagger_{ks} u_{kt} = 2\omega\delta_{st} \tag{24}$$

while, to implement the Majorana condition one has to require (for fixed spin index)

$$d^{\mu\dagger}_{\mathbf{k}} = d^\mu_{-\mathbf{k}}, \qquad b^{\mu\dagger}_{\mathbf{k}} = b^\mu_{-\mathbf{k}} \tag{25}$$

Since the 2-brane has a finite size, thus later in the paper, after quantization we will change the oscillator index from continuous parameter to a discrete one by identifying $\mathbf{k}$ with "nm", i.e., the parameter $\mathbf{k}$ must be proportional to the discrete "nm".

For the R-sector oscillators $d^\mu_{\mathbf{k}}$ the index $\mathbf{k}$ takes integer values while for the NS-sector oscillators $b^\mu_{\mathbf{k}}$ the index $\mathbf{k}$ takes half integral values.

For the bosonic part of action (1), we get the mode expansion for the bosonic fields X^μ and the corresponding canonical momentum

$$\begin{aligned} X^\mu(\sigma) = {} & \frac{x^\mu}{\sqrt{\pi}} + \frac{2\acute\alpha p^\mu}{\sqrt{\pi}}\tau + i\sqrt{2\acute\alpha}\sum_{m,n=0}^{+\infty}\big(n^2+m^2\big)^{-\frac{1}{4}} \\ & \times \big(X^\mu_{nm} e^{i\tau\sqrt{n^2+m^2}} - X^{\dagger\mu}_{nm} e^{-i\tau\sqrt{n^2+m^2}}\big) \times \cos n\sigma^1 \cos m\sigma^2 \end{aligned} \tag{26}$$

and

$$\begin{aligned} P^\mu(\sigma) = {} & \frac{p^\mu}{\pi\sqrt{\pi}} + \frac{1}{\pi}\sqrt{\frac{2}{\acute\alpha}}\sum_{m,n=0}^{+\infty}\big(n^2+m^2\big)^{\frac{1}{4}} \\ & \times \big(P^{\mu\dagger}_{nm} e^{i\tau\sqrt{n^2+m^2}} + P^\mu_{nm} e^{-i\tau\sqrt{n^2+m^2}}\big) \times \cos n\sigma^1 \cos m\sigma^2 \end{aligned} \tag{27}$$

Later in this paper we will identify P^μ_{nm} with $\alpha^{\mu\dagger}_{nm}$, X^μ_{nm} with α^μ_{nm}.

4. Commutation and anticommutation relations

In order to quantize the model (1), we have to define the commutation and anticommutation relations for bosonic and fermionic world-volume fields respectively

$$\big\{\psi^\mu(\sigma), \psi^\nu(\sigma')\big\} = \pi\eta^{\mu\nu}\delta^2(\sigma-\sigma') \tag{28}$$

$$\big[X^\mu(\sigma), P^\nu(\sigma')\big] = \eta^{\mu\nu}\delta^2(\sigma-\sigma') \tag{29}$$

according to which we will get

$$\big\{d^{\mu\dagger}_{\mathbf{k}'s'}, d^\nu_{\mathbf{k}s}\big\} = \eta^{\mu\nu}\delta^2(\mathbf{k}-\mathbf{k}')\delta_{ss'} \tag{30}$$

$$\big\{b^{\mu\dagger}_{\mathbf{k}'s'}, b^\nu_{\mathbf{k}s}\big\} = \eta^{\mu\nu}\delta^2(\mathbf{k}-\mathbf{k}')\delta_{ss'} \tag{31}$$

and

$$\left[X^{\mu}_{nm}, P^{\nu}_{n'm'}\right] = \frac{\eta^{\mu\nu}}{2\pi}[2\delta_{nn'}\delta_{mm'} - \delta_{-n,n'}\delta_{mm'} - \delta_{nn'}\delta_{-m,m'}] \tag{32}$$

5. Hamiltonian of the system

Now we need a Hamiltonian of this system in terms of raising and lowering operators which would act on the Fock space. From the above mode expansion and anti/commutation relations, we can derive the Hamiltonian of the supersymmetric open 2-brane model for R-sector and NS-sector by the following formula,

$$\mathcal{H} = \int_0^{\pi} d\sigma^1 \int_0^{\pi} d\sigma^2 \left(P_{X^\mu}\dot{X}^\mu + P_{\psi^\mu}\dot{\psi}^\mu - \mathcal{L}\right) \tag{33}$$

Using the appropriate mode expansions for both fields (for R-sector) in the expression above, we achieve the Hamiltonian of the model in the following form after going to discrete space and identifying $X^{\dagger\mu}_{nm}$ with $\alpha^{\mu\dagger}_{nm}$ and X^{μ}_{nm} with α^{μ}_{nm} and noting that $\omega = \sqrt{n^2+m^2}$,

$$\begin{aligned}\mathcal{H} = {} & \eta_{\mu\nu}\sum_{n=1}^{\infty} n\left(\alpha^{\mu\dagger}_{n0}\alpha^{\nu}_{n0} + \frac{1}{2}\eta^{\mu\nu}\right) + \eta_{\mu\nu}\sum_{m=1}^{\infty} m\left(\alpha^{\mu\dagger}_{om}\alpha^{\nu}_{om} + \frac{1}{2}\eta^{\mu\nu}\right) \\ & + \eta_{\mu\nu}\sum_{n,m=1}^{\infty}\sqrt{n^2+m^2}\left(\alpha^{\mu\dagger}_{nm}\alpha^{\nu}_{nm} + \frac{1}{2}\eta^{\mu\nu}\right) - \alpha' M^2 \\ & + \eta_{\mu\nu}\sum_{n'=1}^{\infty} n'\left(d^{\mu\dagger}_{n'0}d^{\nu}_{n'0} - \frac{1}{2}\eta^{\mu\nu}\delta_{n'n'}\right) + \eta_{\mu\nu}\sum_{m'=1}^{\infty} m'\left(d^{\mu\dagger}_{m'0}d^{\nu}_{m'0} - \frac{1}{2}\eta^{\mu\nu}\delta_{m'm'}\right) \\ & + \eta_{\mu\nu}\sum_{n',m'=1}^{\infty}\sqrt{n'^2+m'^2}\left(d^{\mu\dagger}_{n'm'}d^{\nu}_{n'm'} - \frac{1}{2}\eta^{\mu\nu}\delta_{n'n'}\delta_{m'm'}\right)\end{aligned} \tag{34}$$

From here we can get the desired Hamiltonian for the RNS supersymmetric 2-brane which is discussed in the next subsections.

5.1. R-sector

In the R-sector, the normal ordering constants are exactly canceled due to the world-volume supersymmetry similar to the superstring case, after this we get the following form of the Hamiltonian

$$\begin{aligned}\mathcal{H} = {} & \sum_{n=1}^{\infty} n\eta_{\mu\nu}\left(\alpha^{\mu\dagger}_{n0}\alpha^{\nu}_{n0} + d^{\mu\dagger}_{n0}d^{\nu}_{n0}\right) + \sum_{m=1}^{\infty} m\eta_{\mu\nu}\left(\alpha^{\mu\dagger}_{0m}\alpha^{\nu}_{0m} + d^{\mu\dagger}_{0m}d^{\nu}_{0m}\right) \\ & + \sum_{n,m=1}^{\infty}\sqrt{n^2+m^2}\eta_{\mu\nu}\left(\alpha^{\mu\dagger}_{nm}\alpha^{\nu}_{nm} + d^{\mu\dagger}_{nm}d^{\nu}_{nm}\right) - \acute{\alpha} M^2\end{aligned} \tag{35}$$

We can interpret the particles created by $d^{\dagger}_{nms}$ as electrons of either spin up or spin down depending on whether s is up or down. Since we do not have any other kind of particles involved so this theory will be devoid of any antiparticles. This later fact will make our concept of Majorana spinors corresponding to particles with no antiparticles more reasonable. Using Eq. (8) we can get the mass squared operator of the system as follows,

$$\acute{\alpha} M^2 = N_n + N_m + N_{nm} \tag{36}$$

$$N_n = \sum_{n=1}^{\infty} n\eta_{\mu\nu}\left(\alpha_{n0}^{\mu\dagger}\alpha_{n0}^{\nu} + d_{n0}^{\mu\dagger}d_{n0}^{\nu}\right) \tag{37}$$

$$N_m = \sum_{m=1}^{\infty} m\eta_{\mu\nu}\left(\alpha_{0m}^{\mu\dagger}\alpha_{0m}^{\nu} + d_{0m}^{\mu\dagger}d_{0m}^{\nu}\right) \tag{38}$$

$$N_{nm} = \sum_{n,m=1}^{\infty} \sqrt{n^2+m^2}\,\eta_{\mu\nu}\left(\alpha_{nm}^{\mu\dagger}\alpha_{nm}^{\nu} + d_{nm}^{\mu\dagger}d_{nm}^{\nu}\right) \tag{39}$$

Because of the world-volume supersymmetry, we get a definitely positive mass formula.

5.2. NS-sector

While working in the NS-sector and using the NS-sector mode expansions and NS-boundary conditions, we have the following form of Hamiltonian

$$\begin{aligned}\mathcal{H} = {} & \sum_{n=1}^{\infty} n\left(\eta_{\mu\nu}\alpha_{n0}^{\mu\dagger}\alpha_{n0}^{\nu} + \frac{1}{2}\eta_{\mu}^{\mu}\right) + \sum_{n=\frac{1}{2}}^{\infty} n\left(\eta_{\mu\nu}b_{n0}^{\mu\dagger}b_{n0}^{\nu} - \frac{1}{2}\eta_{\mu}^{\mu}\delta_{nn}\right) \\ & + \sum_{m=1}^{\infty} m\left(\eta_{\mu\nu}\alpha_{0m}^{\mu\dagger}\alpha_{om}^{\nu} + \frac{1}{2}\eta_{\mu}^{\mu}\right) + \sum_{m=\frac{1}{2}}^{\infty} m\left(\eta_{\mu\nu}b_{0m}^{\mu\dagger}b_{0m}^{\nu} - \frac{1}{2}\eta_{\mu}^{\mu}\delta_{mm}\right) \\ & + \sum_{n,m=1}^{\infty} \sqrt{n^2+m^2}\left(\eta_{\mu\nu}\alpha_{nm}^{\mu\dagger}\alpha_{nm}^{\nu} + \frac{1}{2}\eta_{\mu}^{\mu}\right) \\ & + \sum_{n,m=\frac{1}{2}}^{\infty} \sqrt{n^2+m^2}\left(\eta_{\mu\nu}b_{nm}^{\mu\dagger}b_{nm}^{\nu} - \frac{1}{2}\eta_{\mu}^{\mu}\delta_{nn}\delta_{mm}\right) - \acute{\alpha} M^2\end{aligned} \tag{40}$$

The mass squared operator in NS-sector reads

$$\acute{\alpha} M^2 = N_n + N_m + N_{nm} + a_{\mathrm{NS}} + b_{\mathrm{NS}} \tag{41}$$

where

$$a_{\mathrm{NS}} = \eta_{\mu}^{\mu}\sum_{n=1}^{\infty} n - \eta_{\mu}^{\mu}\sum_{n=\frac{1}{2}}^{\infty} n\delta_{nn} \tag{42}$$

After a small computation using the Zeta function regularization scheme, the value of a_{NS} turns out to be "−2", while the expression for b_{NS} is given by

$$b_{\mathrm{NS}} = \frac{1}{2}\sum_{n,m=1}^{\infty} \sqrt{n^2+m^2}\,\eta_{\mu}^{\mu} - \frac{1}{2}\sum_{n,m=\frac{1}{2}}^{\infty} \sqrt{n^2+m^2}\,\eta_{\mu}^{\mu}\delta_{nn}\delta_{mm} \tag{43}$$

After regularization by making use of a special type of Epstein Zeta functions [17], the first sum in Eq. (43) gives the finite value of "0.0476873", however for the second sum we can get that some similar regularization scheme exists such that we get an overall value of $b_{\mathrm{NS}} = \frac{3}{2}$. Hence the mass squared operator for NS-sector reads

$$\acute{\alpha} M^2 = N_n + N_m + N_{nm} - \frac{1}{2} \tag{44}$$

6. Analysis of spectrum

The general form of a supersymmetric brane excitation state can be written as

$$|\Psi\rangle = (\alpha_{n_1 m_1})^{k_1} (d_{\nu_1 \mu_1})^{\kappa_1} \cdots \left|0, 0, k^\mu\right\rangle_R \tag{45}$$

or

$$|\Psi\rangle = (\alpha_{n_1 m_1})^{k_1} (b_{\nu_1 \mu_1})^{\kappa_1} \cdots \left|0, 0, k^\mu\right\rangle_{\mathrm{NS}} \tag{46}$$

To tell if a given form of a state is bosonic or fermionic, we first need to know whether the ground state is bosonic or fermionic. For example, if the ground state is bosonic, then any state of the above form involving odd number of "d" or "b" oscillators will be fermionic, while those states including even number of "d" or "b" oscillators will be bosonic. If the ground state is fermionic, things will be reversed.

Obviously the bosonic/fermionic property of the ground state is crucial for analysis of brane excitations.

6.1. R-sector

From Eq. (30) we can deduce that the zero modes of the fermionic fields obey the SO(1, 9) Clifford algebra [8,9,14].

$$\left\{d^\mu_{00s}, d^\nu_{00t}\right\} = \eta^{\mu\nu} \delta_{st} \tag{47}$$

The set of ground states in R-sector must furnish a representation of this algebra so we can say that they are spacetime fermions. Moreover, the ground state is degenerate because it satisfies a 10 D Dirac algebra [8]. From the viewpoint of target space symmetries, the ground state corresponds to a 32 component spinor. The Dirac equation takes away half of the degrees of freedom leaving behind only 16 degrees of freedom, we want to further reduce the degrees of freedom to 8 in order to match with the bosonic degrees of freedom. Therefore we impose Weyl condition on these spinors which further reduces the number of degrees of freedom to 8 at the cost of introducing chirality to these spinors.

By $|\zeta\rangle$ and $|\bar{\zeta}\rangle$ we denote the decomposition of two possible ground states with opposite chirality, where $\zeta, \bar{\zeta} = 1, \ldots, 8$ are the spinor indices labeling the two Majorana–Weyl spinor representations of SO(8). By Lorentz invariance in $D = 10$, the states must constitute a representation of SO(8) or SO(9) depending upon whether they are massless or massive, respectively. Therefore the ground state gives two SO(8) representations, a spinor and a conjugate spinor representation, whereas the states at the first massive level combine into SO(9) representations.

The first state $\alpha^i_{-1,0}|\zeta\rangle$ decomposes under the conjugate spinor representation of SO(9) as $8_{\mathbf{c}} \oplus 56_{\mathbf{c}}$, the second state $d^i_{-1,0}|\bar{\zeta}\rangle$ decomposes under the spinor representation of SO(9) as $8_{\mathbf{s}} \oplus 56_{\mathbf{s}}$. The state $\alpha^i_{-1,0}|\bar{\zeta}\rangle$ decomposes as $8_{\mathbf{s}} \oplus 56_{\mathbf{s}}$ and decomposition of $d^i_{0,-1}|\zeta\rangle$ is $8_{\mathbf{c}} \oplus 56_{\mathbf{c}}$.

Table 1
The ground and first excited states in R-sector.

$\acute{\alpha} M^2$	States	γ_{11}	Group	Representation
0	$\lvert\zeta\rangle$	+1	SO(8)	$8_{\mathbf{s}} \oplus 8_{\mathbf{c}}$
	$\lvert\bar{\zeta}\rangle$	−1		
+1	$\alpha^i_{-1,0}\lvert\zeta\rangle; d^i_{-1,0}\lvert\bar{\zeta}\rangle;$	+1	SO(9)	
	$\alpha^i_{0,-1}\lvert\zeta\rangle; d^i_{0,-1}\lvert\bar{\zeta}\rangle;$	+1		512
	$\alpha^i_{-1,0}\lvert\bar{\zeta}\rangle; d^i_{-1,0}\lvert\zeta\rangle;$	−1		
	$\alpha^i_{0,-1}\lvert\bar{\zeta}\rangle; d^i_{0,-1}\lvert\zeta\rangle$	−1		

Therefore the representation of this first massive multiplet is $128 \oplus 128 \oplus 128 \oplus 128 = 512$. The column "$\gamma_{11}$" in Table 1 contains the value of chirality of the corresponding spinors which is either positive or negative. This is summarized in Table 1.

6.2. NS-sector

The ground state in the NS-sector is a scalar, i.e., spacetime boson of spin zero. By looking at Eq. (46), we know that even when no oscillators are excited, we will still get a negative mass square of the sate because of the presence of an anomaly term in Eq. (44), therefore the ground state in the NS-sector is tachyonic. By using the selection rule similar to the GSO projections in the string theory, we can rule out this state from the physical spectrum of the brane excitation. Using such selection rule, we can also get rid of all states with half integer mass square. So, by matching the first excited state in NS-sector bosonic spectrum with the ground state in the fermionic spectrum of R-sector, we can form a supersymmetric vector-multiplet, and also at the first excited state level, we have a Bose/Fermi matching of states which implies that we can implement supersymmetry at least up to the ground and first excited state level.

Hence by looking at the bosonic and fermionic spectrum of this model we can say that we have achieved a necessary condition for the unbroken spacetime supersymmetry at least at the ground and first excited state level after truncating the NS-spectrum.

We have two massless vector bosons $b^i_{-\frac{1}{2},0}$ and $b^i_{0,-\frac{1}{2}}$ at the ground state level after the GSO condition, which decompose under the vector representation of SO(8) as $8_{\mathbf{v}} \oplus 8_{\mathbf{v}}$. At the massive level "+1" we have four states like $\alpha^i_{-1,0} b^j_{-\frac{1}{2},0}$ whose decomposition is $64 = 1 \oplus 28 \oplus 35$, and four states like $b^i_{-\frac{1}{2},0} b^j_{-\frac{1}{2},0} b^k_{-\frac{1}{2},0}$ which give $56_{\mathbf{v}}$ states, while the two states like $b^{i'\pm}_{-\frac{3}{2},0}$ with $i' = 1, \ldots, 7$ can be explicitly written like $b^{i'}_{-\frac{3}{2},0} + A^-; b^{i'}_{-\frac{3}{2},0} + A^+$ and $b^{i'}_{0,-\frac{3}{2}} + A^-; b^{i'}_{0,-\frac{3}{2}} + A^+$, where $A^\pm$ is the world-volume gauge field. In the massless case ($i = 1, \ldots, 8$) this is implicit as it has just one polarization, while at the massive level this needs to be treated specially because it has two polarizations, thus giving us a total of 32 such states. Since it is a massive multiplet, all these states should combine into an SO(9) representation. We get a total of 512 states at this level.

Combining previous analysis, we list the lowest lying states in the NS-sector spectrum in Table 2.

Table 2
Ground and lowest excited states in NS-sector.

$\acute{\alpha}M^2$	States	G	Group	Representation
$-\frac{1}{2}$	$\lvert 0\rangle$	-1	SO(9)	1
0	$b^i_{-\frac{1}{2},0}; b^i_{0,-\frac{1}{2}}$	$+1$	SO(8)	$8_{\mathbf{v}} \oplus 8_{\mathbf{v}}$
$+\frac{1}{2}$	$\alpha^i_{-1,0}; \alpha^i_{0,-1}$ $b^i_{-\frac{1}{2},0} b^j_{-\frac{1}{2},0}; b^i_{0,-\frac{1}{2}} b^j_{0,-\frac{1}{2}}$; $b^i_{-\frac{1}{2},0} b^j_{0,-\frac{1}{2}}$	-1	SO(9)	$8_{\mathbf{v}} \oplus 8_{\mathbf{v}} \oplus 84$
$+1$	$\alpha^i_{-1,0} b^j_{-\frac{1}{2},0}; \alpha^i_{0,-1} b^j_{-\frac{1}{2},0}$; $\alpha^i_{-1,0} b^j_{0,-\frac{1}{2}}; \alpha^i_{0,-1} b^j_{0,-\frac{1}{2}}$; $b^i_{-\frac{1}{2},0} b^j_{-\frac{1}{2},0} b^k_{-\frac{1}{2},0}$; $b^i_{0,-\frac{1}{2}} b^j_{0,-\frac{1}{2}} b^k_{0,-\frac{1}{2}}$; $b^i_{-\frac{1}{2},0} b^j_{0,-\frac{1}{2}} b^k_{-\frac{1}{2},0}$; $b^i_{-\frac{1}{2},0} b^j_{0,-\frac{1}{2}} b^k_{0,-\frac{1}{2}}$; $b^{i'\pm}_{-\frac{3}{2},0}; b^{i'\pm}_{0,-\frac{3}{2}}$	$+1$	SO(9)	512

7. Summary and conclusion

In this paper, we present the quantization and spectrum of an open supersymmetric 2-brane. The classical equations of motion for bosonic and fermionic fields are obtained directly from the action (1), their mode expansions are also given in the paper, and the fermionic fields have two different mode expansions depending on the periodic or anti-periodic boundary conditions for them. The quantization scheme that we adopt is something of a hybridization of canonical and lightcone quantization of RNS superstrings. However, different from the string case, we have to fix the form of the world volume metric. Physically, this means that we partly fix the phase space of the supersymmetric 2-brane dynamics.

After quantization, we get the Hamiltonian of the 2-brane in terms of raising and lowering operators, the normal ordering constants arising in the R-sector exactly cancel due to world-volume supersymmetry which is the same as in the superstring case. However for the NS-sector, some zeta function regularization scheme should be done such that the value of the second infinite sum in Eq. (43) is approximated to the value "-0.327313" so that the overall value of b_{NS} turns out to be $\frac{3}{2}$. This is the only way we can realize supersymmetry on the target spacetime at least at the massless and first massive level.

We give the physical spectrum of the open 2-brane in the lightcone gauge. We get more new states in both the R-sector and NS-sector as compared to the superstring case, as well as tachyon states in NS-sector spectrum. We resort to a GSO-like selection rule as in the superstring theory to remove not only the tachyons but also all half integer mass squared states in the NS-sector. We get a supersymmetric spectrum of excited states after a GSO-like projection. But we get twice as more states in the open 2-brane quantization as in the superstring case. The massless supermultiplet matches the characterization given in the brane scan of [18]. However, spacetime supersymmetry can be certainly achieved at the massless and first massive level after the projection conditions are imposed. Since, we can realize the Bose/Fermi matching at these two lowest levels. The presence of a tachyon implies that the 2-brane is unstable and will decay.

Since we partly fix the phase space of the supersymmetric 2-brane dynamics through fixing the form of the world volume metric, the spectrum we obtain is discrete. Nevertheless, the discrete part of the spectrum we obtained here may be a useful starting point for further investigation of the supermembrane dynamics through methods such as perturbation around these discrete points.

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