# The Chiral Bag

The chiral bag or the "little bag" as formulated in 1979 has in recent years undergone a dramatic evolution due to the revival and a deeper understanding of the topological (Skyrmion) description of baryons. In this Comment, we review the present status of the chiral bag as a possibly realistic candidate model of low-energy quantum chromodynamics, particularly suited to nuclear dynamics, possessing the unique virtue of interpolating between the long-distance Goldstone-mode regime and the short-distance asymptotically free regime. Being the only four-dimensional topological soliton *observed* so far in nature, the nucleon as described in the model offers a valuable laboratory to study the intricate way that topology enters into physics.

**Key Words:** *chiral symmetry, Skyrmion, bag models, QCD, nonlinear σ-model*

## 1. INTRODUCTION: THEORY

In a previous Comments article,[1] we discussed the role that chiral symmetry plays in *nuclear* structure. In this Comment, we address a related—and potentially more fundamental—issue in hadron structure, namely the role that chiral symmetry plays in the structure of the *nucleon* and other baryons. A close relation is believed to exist between the two: a logically consistent picture arises from considerations based on the common features of chiral invariance.

When it was first suggested that the chiral bag (or the little bag)[2] be *seriously* considered as a model for the nucleon, the primary motivation was to render the bag description of the quark–gluon structure of the nucleon compatible with observations in nuclei,

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namely success of the independent-particle description and the meson exchange phenomena in nuclear processes. The M.I.T. bag model was far from satisfactory in this respect when considered from the nuclear physics point of view. In the original description of the chiral bag, as in subsequent developments, pions as the Goldstone bosons of chiral SU(2) × SU(2) symmetry played a crucial role, but the pion-cloud effect was treated only perturbatively. Based on second-order perturbation theory, it was argued that the pion cloud surrounding the quark bag exerted a strong pressure on the quark core, squeezing the quarks into a "little bag" whose radius turns out to be considerably smaller than what was predicted by the M.I.T. model or from the simple QCD order-of-magnitude estimate $\Lambda_{QCD}^{-1}$, both of which gave radii $R \simeq 1$ fm. It was quickly realized, however, accelerated by the work of Witten,[3] that nonperturbative aspects of the pion could not be ignored and that the chiral bag, particularly if the bag is small, was a topological object.[4] In recent years, the concept of the pion cloud has undergone a drastic change, thanks to the rediscovery of Skyrme's prescient idea[5] of the baryons as topological solitons. In view of the recent intense activities which have shed considerable new light on Skyrme's picture (Skyrmion) as well as quark-bag models with pion clouds, the chiral bag of today looks much different—and more promising as a simple but realistic model (if not a theory) of the nucleon—than the older version. In this Comment, we wish to summarize the present status of the model, focusing on what is understood and which problems are still to be resolved.

The basic idea of the chiral bag is to hybridize two possible phases of chiral symmetry, i.e., Wigner mode and Goldstone mode. In the former, the description is in terms of quarks and gluons; in the latter, in terms of mesons. The motivation for this is to incorporate, to the extent that it is feasible, two elements of quantum chromodynamics—asymptotic freedom and spontaneously broken chiral symmetry. One hopes in this way to take into account, simultaneously, both short- and long-distance strong-interaction properties. There are probably several, perhaps equivalent, ways of hybridization.[6] The chiral bag model is constructed in one particular way that seems to offer the simplest conceptual framework. The model consists of a bag in which quarks propagate almost freely within, but confined by a boundary condition on the surface.

The exterior region is populated by meson clouds, in general pions, vector mesons, etc. The boundary condition is to provide a consistent connection between the two regions.

We will consider first, for concreteness, the nonstrange quark systems. Imagine further that the $u$- and $d$-quark masses are zero. (The nonzero masses needed for current algebra do not make qualitative changes.) There is then $SU(2)_L \times SU(2)_R$ chiral symmetry. (The extension to the strange quark sector will be discussed later.) We will start with the simplest hybridization scheme: free quarks inside and massless pions outside, corresponding to the longest wavelength oscillation. The dynamics are then given by the action $\Gamma$ involving quark field $\psi$ and pion field $\pi^i$ ($i = 1, 2, 3$):

$$\Gamma_{\text{in}} = \int_{V_i} d^4x\, \bar{\psi} i\gamma \cdot \partial\psi \tag{1}$$

$$\Gamma_{\text{out}} = \int_{V_o} d^4x\, \frac{F_\pi^2}{4} \operatorname{Tr}(\partial_\mu U \partial^\mu U^+) + \cdots \tag{2}$$

where $V_i$, $V_o$ are the volume inside and the volume outside, respectively, $F_\pi$ is the pion decay constant ($\approx$93 MeV) and $U$ the chiral field taking values in SU(2),

$$U(\mathbf{r}) = \exp(i\boldsymbol{\tau} \cdot \boldsymbol{\pi}(\mathbf{r})/F_\pi). \tag{3}$$

$\Gamma_{\text{out}}$ may contain, as we mention in the Appendix, higher-order derivatives represented by $\cdots$ in Eq. (2) in addition to the low-energy current algebra term. For the moment, we will ignore them. [The notation involving $U$ may be unfamiliar to some readers. The action (2) is just that obtained from the Lagrangian of the nonlinear $\sigma$ model

$$\mathcal{L}_o = \frac{1}{2}[(\partial_\mu \boldsymbol{\pi})^2 + (\partial_\mu \sigma)^2] \tag{3a}$$

where the scalar field $\sigma$ is given by

$$\sigma^2 + \boldsymbol{\pi}^2 = F_\pi^2. \tag{3b}$$

The notation in terms of $U$ is economical, the Skyrmion Lagrangian offering a convenient nonlinear representation of chiral symmetry.] The $\psi$ and $U$ fields communicate with each other through (classically) chirally invariant boundary conditions

$$\left.\begin{aligned} -in \cdot \gamma\psi &= U_5^+ \psi \qquad (4) \\ \frac{iF_\pi^2}{2} n^\mu \operatorname{Tr}[\tau_i(\partial_\mu \cup U^+ + U^+\partial_\mu U)] & \\ = i\bar{\psi}n \cdot \gamma\gamma_5\tau_i\psi & \qquad (5) \end{aligned}\right\} \text{on the surface } S$$

with $U_5 \equiv \exp(i\boldsymbol{\tau} \cdot \boldsymbol{\pi}\gamma_5/F_\pi)$.

Equation (4) is the usual confinement boundary condition and Eq. (5) is the condition on the boundary for axial-current conservation. The right-hand side of Eq. (5) is clearly just the normal component of the axial current, realized in the internal quark variables, at the bag surface. The left-hand side is the axial current realized in meson variables, again at the bag boundary. For the special case of the hedgehog configuration which we shall introduce in Eq. (7), the left-hand side is just $F_\pi^2 d\theta/dr$, where $\theta$ is the chiral angle. [Within the respective space, the actions $\Gamma_{in}$ and $\Gamma_{out}$ are invariant under (global) chiral transformation.] Equations (1)–(5) define, when supplemented by a pressure balance relation* (or energy-momentum tensor conservation), the entire content of the chiral bag model.

Before addressing what physics this model represents, we first describe an intriguing phenomenon that is known about this set of equations. Consider first the Dirac particles (quarks) satisfying**

$$i\gamma \cdot \partial\psi = 0 \qquad r < R \tag{6}$$

subject to Eq. (4) on the boundary $r = R$. If the pion field describes a mode built on a *topologically trivial vacuum*, then Eqs. (1) or (6) and (4) are C (charge conjugation) invariant, so the chiral field

*This may not be needed as an additional condition if, for instance, the Cheshire Cat picture (described later) holds.

**From now on, we will restrict ourselves to a spherical bag of radius $R$.

$U$ leaves the Dirac sea in the bag undisturbed and nothing strange happens. However, suppose that the pion field is of topologically nontrivial configuration, i.e., soliton; then a surprising thing happens.[7,8] Consider the hedgehog configuration

$$U_o(x) = \exp(i\boldsymbol{\tau} \cdot \hat{r}\theta(r)),$$

a configuration first studied by Skyrme.[5]

If at spatial infinity ($r \rightarrow \infty$), $U_o$ approaches the trivial configuration, $U_o(\infty) = 1$, then it corresponds to a mapping of a three-sphere ($S^3$) into the internal space SU(2) (isospin). Because of this interlocking of the ordinary space and the internal space, while the equation of motion (6) remains invariant, the boundary condition (4)* is no longer symmetric under C or CP for angles $\theta \neq n\pi/2$, $n = 0, 1, 2, \ldots$ . What this implies is that when $U_o$ is considered as a background field in Eq. (4), the solution of the Dirac equation is, for chiral angles different from $n\pi/2$, asymmetric between positive and negative levels. This spectral asymmetry influences all the physical observables one wants to calculate. [Said more prosaically, the bag boundary conditions (4) and (5) affect not only the valence quark states, but also all of the filled negative energy states, so that contributions from this sort of vacuum polarization pertain to essentially all physical observables. For example, there are Casimir effects on the energy.]

The first quantity studied, which is also the simplest, is the baryon charge.[7,8] In terms of the quark field $\psi$, the baryon number operator is

$$\frac{1}{2N_c}\int d^3x[\psi^+(x), \psi(x)].$$

We will first consider an empty bag (that is, without valence quarks) for which, naively, the baryon number $B_{\text{vac}}$ would be

$$-\frac{1}{2N_c}\left[\sum_{\omega<0} 1 - \sum_{\omega>0} 1\right]$$

*Note that $U_5^+\psi = U^+\psi_R + U\psi_L$, with $\psi_{R-L} = (1/2)(1 \pm \gamma_5)\psi$; so what is relevant is the $U$ field, Eq. (7).

where the sum goes over all negative-energy and positive-energy Dirac levels. Involving infinite sums, this quantity is not mathematically well defined, so it requires a suitable regularization. A convenient procedure is the so-called heat-kernel regularization,

$$B_{\text{vac}} = \lim_{s \to 0} \left[ -\frac{1}{2} \sum_{\omega} \sin(\omega)\, e^{-s|\omega|} \right]. \tag{7}$$

The exponential factor cuts down contributions from large $|\omega|$. (Mathematicians have shown that the quantity in square brackets is a well-defined quantity and that the limit exists). It is clear from Eq. (7) that any asymmetry in the Dirac spectra $\omega$ would induce a nonvanishing baryon charge $B_{\text{vac}}$. Indeed, calculations[7–10] show that with $\theta = \theta(R)$

$$B_{\text{vac}}(\theta) = -\frac{1}{\pi} \left[ \theta - \frac{1}{2} \sin 2\theta \right], \tag{8}$$

a surprising result that a bag is *never* empty in the presence of topologically nontrivial background. When $N_c$ quarks are introduced into the bag, one discovers* that the baryon charge is $1 - (1/\pi)(\theta - (1/2) \sin 2\theta)$.

What happened can be easily understood as follows. Classically quarks are absolutely confined within the bag by the boundary condition (4). However, quantum effects involving quarks break some of the classical symmetries, generating anomalies. In the case of Eq. (4), the flavor singlet vector current is no longer conserved, as a consequence of which the baryon charge "leaks" out by the amount given by Eq. (8). This is an example of chiral anomalies, a subject at the core of the recent exciting developments in theoretical physics.[10]

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*This baryon charge fractionization was first noticed at $\theta = \pi/2$ as follows.[7] At this angle (called the "magic angle"), the Dirac spectrum is CP symmetric; however, there is a CP self-conjugate zero-energy level, so as in the magnetic monopole case,[11] the baryon (or fermion) charge is $\pm 1/2$. The generalization (8) was deduced from the baryon charge lodged outside.

We have thus learned that, quantum mechanically, baryon charge is not confined within the bag. Where does it leak to? The model as a whole, if it models QCD, *cannot have* a vector current anomaly. Therefore the anomaly must be a phenomenon *localized* in the subsystem, namely, in the bag, and the leaked baryon charge must reside in the meson cloud. In fact, if we identify the $U_o$ field with a Skyrmion, we obtain a consistent description. Indeed we have, in the meson-cloud sector,

$$B_{\text{cloud}} = \int_{V_0} d^3x J_o(x) = \frac{1}{\pi}\left[\theta(R) - \frac{1}{2}\sin 2\theta(R)\right] \qquad (9)$$

where $J_\mu$ is Skyrme's topological baryon current $(1/24\pi^2)\epsilon_{\mu\nu\lambda\rho}\mathrm{Tr}(U^+\partial^\nu U\, U^+\partial^\lambda U\, U^+\partial^\rho U]$, so the vacuum as a whole does have zero baryon charge as expected. These results are unmodified when quark mass, gluon and other effects ignored so far are taken into account.[12] The reason is simply that they are topological and hence unaffected by detailed dynamics.

We now state what physics the chiral bag represents.

The most attractive and perhaps most fruitful way of viewing the Skyrmion picture is to consider it as a bosonized (albeit approximate) version of quantum chromodynamics. It is well known that in (1 + 1) dimensions, some fermion theories can be completely bosonized. For instance, the massive Thirring model (fermion theory) is known to be identical to the Sine–Gordon model (boson theory),[13] a free Dirac system with non-Abelian flavor symmetry is the same as a nonlinear $\sigma$-model with a Wess–Zumino term,[14] to which two-dimensional QCD also reduces in the limit of large number of colors[15] and so on. In these examples, solitons in the boson theories correspond to fermions in the fermion theories. Application of this bose–fermi correspondence to bag models leads to the so-called Cheshire Cat model,[16] which we describe briefly.

Consider massless Dirac fermions living in the one-dimensional space $-R \leq x \leq R$, confined by an M.I.T.-like boundary condition. Let $y_B$ be a point within the range, and bosonize the fermion field $\psi(x)$ for $y_B \leq x \leq R$. If we let $y_B = -R$, then the theory is equivalent to a *bosonized* one, and if $y_B = R$, then it is said to be completely *fermionized*. For any other values of $y_B$, it is a hybrid.

The boundary condition to be imposed at $y_B$ for a boson–fermion correspondence is precisely the analog to Eq. (5)

$$n \cdot \partial\phi = \sqrt{\pi}\, \bar{\psi}\, n \cdot \gamma\gamma_5\psi, \qquad x = y_B \tag{10}$$

and the most general condition[16] invariant under P, C, T that *physics be independent of* $y_B$ (called the "Cheshire Cat principle") is

$$-in \cdot \gamma\psi = e^{i\sqrt{4\pi}\gamma_5\phi}\psi, \qquad x = y_B \tag{11}$$

which is a precise analog to Eq. (4). [In (1 + 1) dimensions, $F_\pi$ is dimensionless, equal to $1/\sqrt{4\pi}$]. The Cheshire Cat phenomenon can be readily demonstrated explicitly for ground-state and low-excited state properties of the system by choosing $\phi$ to be a soliton.[17]

Although bosonization may also be feasible in (3 + 1) dimensions,[18] no workable method has yet been developed. A few cases so far studied are: the monopole–fermion system—the monopole catalysis of the proton decay (Rubakov–Callan effect[19])—where the problem reduces by a physical condition to an effective two-dimensional one and the free Dirac system with a boundary condition at the origin.[20] In the latter case, one can show that a free massless Dirac field can be completely bosonized provided an infinite set of boson fields consisting of a tower of angular momentum states is introduced. Limited though they are, both of these cases are seen to be highly relevant to the chiral bag model.

In the absence of an exact bosonization, then, the chiral bag may then be viewed as a *partial* bosonization scheme, with the boundary conditions (4) and (5) playing the roles of both bose–fermi correspondence *and* an approximate Cheshire Cat principle. The latter is inevitable, because in (3 + 1) dimensions, an exact Cheshire Cat principle cannot be established unless an exact solution of QCD is known. In the spirit just stated, suppose one starts with a big bag (radius $R > 1$ fm, say) with confined quarks and gluons. Short wavelength properties of the hadrons may be treated perturbatively in terms of quark–gluon variables, but not long wavelength properties. To describe the latter, consider bosonizing long wavelength quark–gluon degrees of freedom. The longest wavelength mode thus bosonized is just the pion; at the next wavelength scale appear the vector mesons, ρ and ω. Since for a large bag, $R > 1$ fm, little baryon charge is expected to be lodged in the bosonized sector (call it "meson cloud"), the meson cloud will primarily consist of fluctuating pion fields (hence non-topological) which couple to the bag via the given boundary conditions. One expects soft-pion–baryon phenomena (i.e., soft-pion theorems) to be correctly described in this picture. This has been extensively verified in pion–nucleon processes.[21] As one reduces the bag further by integrating out more quark–gluon degrees of freedom, the topological chiral field will become non-negligible. One may still have the boundary conditions (4) and (5), but the pion field will now consist of a soliton configuration in addition to the fluctuating field. (As we will see later, this picture seems to hold up to $\theta = \pi/2$.) If one integrates out more and more of shorter-wavelength degrees of freedom, an ever-increasing number of meson fields, beginning with the vector mesons, will have to be introduced, with an appropriate modification of the boundary conditions. When all the quark–gluon degrees of freedom are integrated out, the Skyrmion description will presumably result. If this picture is to be valid when *probed* at a very short distance, an infinite number of meson fields will be needed, as suggested by Rubakov's result.[20] A similar conclusion is reached by a large-$N_c$ QCD.[3]

Two consequences emerge from this discussion: one, the bag radius $R$ in the chiral bag is an optimal radius at which a partial bosonization *best* approximates nature, hence not a genuine physical quantity, and second, $R$ can be probe-dependent; different currents with different kinematics may require different values of $R$. The crucial element in this hybrid description is that some properties are better described in boson language, others in fermion language. (This is known in some (1 + 1) dimensional models. For instance highly nonperturbative properties in fermion fields can be described simply in semiclassical approximation in boson fields and vice versa.)

This somewhat eclectic way of choosing $R$ is familiar to nuclear physicists who work with the Wigner $R$-matrix. There the radius is generally chosen so as to minimize higher-order corrections in the problem one is considering. Whatever value is chosen for $R$, one would, in this theory, obtain the same final results, were one able to calculate all higher-order corrections.

When calculating the properties of single baryons, it may well be that the hybrid description is not much superior in details to the M.I.T. bag or the Skyrmion: there is clearly the danger of losing some accuracies inherent in the two pictures when two extremes are *approximately* hybridized. However, when applied to nuclei where nucleons interact in configurations ranging from asymptotic freedom (when two nucleons overlap) to Goldstone mode (when they are apart exchanging one pion), the chiral bag picture will definitely be more appropriate and more predictive. It also has the virtue of providing links to the nuclear phenomena (manifesting chiral symmetry) reviewed in Ref. 1.

A deep theoretical issue that has attracted some attention recently is whether or not the chiral bag is *implied* by QCD. A fully satisfactory answer may not be available until QCD is completely solved. (Presumably future lattice gauge calculations will provide some hints.) At present, there is *no* derivation of the chiral bag. Nonetheless there is a strong indication for it in effective theories obtained by integrating out gluons and quarks. For instance, in an attempt to derive a long-wavelength effective Skyrmion Lagrangian from QCD, Simić[22] finds that when heavy-meson degrees of freedom are ignored, the resulting theory "resembles" the chiral bag with a rapid delineation between the nonperturbative vacuum (Skyrmion sector) and the perturbative vacuum (bag). Other authors arrive at a similar qualitative result,[23] based on different considerations. To the best of our knowledge, there is no theoretical argument against the chiral bag structure.

## 2. PHENOMENOLOGY

We now turn to some phenomenological consequences of the baryon number fractionation inherent in the chiral anomaly structure of the chiral bag. The question is: What happens to physical observables when the baryon number fractionates, as predicted, into the quark and Skyrmion sectors? Some interesting answers have been obtained by the workers of Ref. 24.

The obvious quantity to look at is the *n*th moment of baryon number distribution of the nucleon, $M_n$,[24]

$$M_n = M_n^{\text{bag}} + M_n^{\text{Skyrmion}}$$

with (12)

$$M_n^{\text{bag}} = M_n^{\text{vac}} + M_n^{\text{val}}.$$

The nontrivial quantity is the contribution due to the polarized vacuum

$$R^n M_n^{\text{vac}}(\theta_R) = -\frac{1}{2}\lim_{s\to 0^+}\sum_\omega \sin(\omega)\exp[-s|\omega|] \times \int_0^R dr\, r^{n+2}\int d\Omega\, q_\omega^+ q_\omega \qquad (13)$$

where $q_\omega$ is a normalized quark solution with eigenenergy $\omega$; $M_n^{\text{val}}$, the valence quark contribution, and $M_n^{\text{SK}}$, the meson-cloud contribution, are calculated straightforwardly with their known charge density. For $n = 0$, we recover Eq. (8), namely, $M_0^{\text{vac}}(\theta_R) = B_{\text{vac}}(\theta_R)$. For $n = 2$, we get $\langle r^2\rangle_{I=0}^{\text{vac}}$, the mean square isoscalar charge radius. As noted by Vepstas *et al.* and Heller *et al.*, all the moments (13) are well defined, free of singularities.[24]

Of immediate relevance to experiments is the isoscalar charge radius $\sqrt{\langle r^2\rangle_{I=0}}$. This has been investigated in Ref. 24; some of the results are given in Fig. 1. What is found is that for reasonable values of parameters, the charge radius is independent of $R$ *provided* the coefficient $\epsilon$ of Skyrme's quartic term (see Appendix) is a constant for $\theta > \pi/2$ (see Fig. 1(b)). As we will elaborate later, this feature is qualitatively consistent with what one expects in effective Lagrangian theories "derived" from QCD.[22]

One cannot extend the analysis to too large a bag radius (say, $R \gtrsim 1$ fm) or smaller $\theta_R$, since for larger bags, nonperturbative corrections in the quark–gluon sector can become very important.* This may induce a large deviation from the Cheshire Cat picture for large bag size.

*For example, in the case of the MIT bag model ($R \simeq 1$ fm) $\alpha_s = 2.2$. It is then found[25] that the Lamb shift, which is of order $\alpha_s$ higher than the kinetic energy, is just as large as the latter.

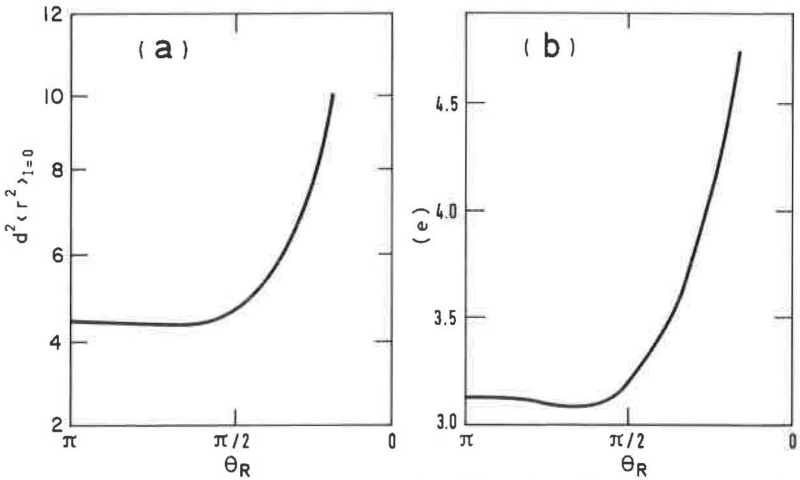


FIGURE 1 The isoscalar mean square radius $\langle r^2\rangle_{I=0}$ times $d^2 = F_\pi^2/8\epsilon^2$ vs. chiral angle $\theta_R$ on the surface (Fig. 1(a)) and the value of $e \equiv (\sqrt{8}\epsilon)^{-1}$ *required* to keep $\langle r^2\rangle_{I=0}$ at the empirical value $(0.72\ \text{fm})^2$ for $0 \leq \theta_R \leq \pi$ (Fig. 1(b)). These results, taken from Heller *et al.* (Ref. 24), are similar to those obtained by Vepstas *et al.* (Ref. 24).

Consider now the vacuum energy inside the bag

$$E^v(\theta) = \lim_{\tau\to 0} \sum_{m\in\text{sea}} \omega_m \exp[-\tau|\Omega_m|] \tag{14}$$

with $\Omega_m = \omega_m R$ the quark eigenfrequency. The "axial flux" $\Phi(\theta)$ on the bag surface is given by[9]

$$\Phi(\theta) = \int d^2S\, n_i n^\alpha \langle A_\alpha^i\rangle_{\text{vac}} \equiv \lim_{\tau\to 0} \Phi(\theta, \tau) \tag{15}$$

with

$$\Phi(\theta, \tau) = \sum_{m\in\text{sea}} \frac{d\omega_m}{d\theta} \exp[-\tau|\Omega_m|].$$

Clearly

$$\frac{dE^v(\theta)}{d\theta} = \lim_{\tau\to 0} \left(1 + \tau \frac{\partial}{\partial\tau}\right)\Phi(\theta, \tau). \tag{16}$$

Here we encounter divergences that are not present in the baryon density distribution. It has been established[9,26] that as $\tau \rightarrow 0$, $[E^v(\theta, \tau) - E^v(0, \tau)]$ contains $\ln\tau$ divergence and $\Phi(\theta, \tau)$ both $\tau^{-1}$ and $\ln\tau$ divergences. The logarithmic divergence is harmless, since one can presumably eliminate it by a suitable renormalization prescription. However, the linear divergence* looks at first sight disastrous and has led the authors of Ref. 26 to conclude that the model is intrinsically sick. We do not share this opinion, for the following reason: the chiral bag is not a complete theory on its own but an effective one. As such, we cannot expect it to be valid over *all* ranges of parameters. Now the $\tau^{-1}$ term comes multiplied by $\sin 2\theta$, so for the chiral angles $\theta = 0, \pi/2, \pi$, the linear divergence is absent. At this point, one can take either one of the two options. One, it may be that the model (and its regularization) is well defined *only* for these three angles; two, if one wants to adhere to the Cheshire Cat principle, one should devise subtraction (and regularization) procedures such that one can continue smoothly between the three chiral angles. The latter is the procedure adopted by Vepstas, Jackson and Goldhaber[9] and applied in Ref. 27. (This procedure was recently shown to be the *correct* one in the (1 + 1) dimensional Cheshire Cat model[17].) Specifically, the subtraction is made such that in the limit $\theta \rightarrow \pi$, the axial flux relation (5) holds, involving only finite quantities. Effectively this corresponds to demanding that as $R \rightarrow 0$,

$$\frac{d^2E^v(\theta)}{d\theta^2} = \frac{d\Phi(\theta)}{d\theta} = 0. \tag{17}$$

This procedure may still leave some nonuniqueness in finite subtraction terms, while all the divergences in $E^v$ and $\Phi$ are taken care of. Whether this is a serious cause for worry is not known, but the presently available numerical results indicate that the error can be at a $\lesssim 10\%$ level.

Evidently a sensible thing to do is to focus on the chiral angle $\pi/2$, acceptable in both options discussed above. We will indeed find some remarkable results, working at this angle. [While this

*This may also be an artifact of the regularization chosen and may be absent in a different regularization scheme. We thank L. Vepstas for a comment on this.

may appear to the readers to be overly restrictive, let us remind them that in the fermion–plus–monopole problems[28] it was thought for many years that one should work with chiral angle 0 or $\pi$ (the latter corresponds to $\pi/2$ in our case, due to the higher degeneracy of $K = 0$ states). Only at these chiral angles is the spectrum of positive and negative energy states symmetrical. Of course, later[29] the problem of fermion number fractionation for arbitrary chiral angle was worked out.]

In Ref. 27, the energy and static properties of the SU(2) baryons have been investigated following the subtraction procedure discussed above. When the contributions from the vacuum and valence quarks of the bag and from the Skyrmion sector are summed, the results are again found to be remarkably independent of the bag radius for $0 \leq R \lesssim 1$ fm. For instance, the centroid of the masses of the $N$ and $\Delta$ comes out to be in the range 1.1–1.5 GeV for a wide range of radii. (For precise conditions on which this result is based, we refer to the literature.[27]) Similar behavior is found for the axial charge $g_A$, magnetic moments and other static properties. It is tempting to regard these results as a confirmation of an approximate Cheshire Cat picture.

A particularly interesting qualitative result of Refs. 27 and 24 is that as the bag size is increased, Skyrme's quartic term—and presumably any other higher derivative terms—gets increasingly suppressed. In fact, at a chiral angle near $\pi/2$, it vanishes rapidly. This is quite consistent with the observation that Skyrme's quartic term and higher derivatives arise when quark degrees of freedom are integrated out. Simić finds a qualitatively similar result by integrating out fermions from a QCD action[22]; specifically the quartic terms occur multiplied by a factor that vanishes at some short-wavelength scale, simulating an effect similar to what we discussed above. The latter is what seems to distinguish a QCD-based calculation from a $\sigma$-model,[30] as far as the quartic terms are concerned. We think that this is one of the crucial features that are needed in thinking of nuclear interactions in terms of quarks and gluons.

Detailed analyses made so far with some obvious refinements on the model (Eqs. (1)–(5)), such as inclusion of the pion mass, the bag constant $B$, gluon radiative corrections in the bag, pionic fluctuations in the cloud etc., lead one to conclude (albeit tentatively) that the low-energy baryon properties such as energies and static moments are insensitive to the bag radius; the chiral bag provides an interpolation between the two extreme descriptions, the Skyrmion and the quark bag. Clearly, then, as far as these properties are concerned, there is no one particular model that is to be preferred over others. (Of course, one can always get a better fit with experiments with a particular model by a fine-tuning of parameters, but this cannot be construed as a sign of superiority.) The reason for this peculiar (and perhaps unsatisfactory) situation is not difficult to find: as stated elsewhere,[31] what really matters is the broken flavor SU(6) symmetry which figures in all the models in question.

Where can one see the differences; in particular, where can one see that the chiral bag has the virtue of interpolating the two extreme regimes—asymptotic freedom and Goldstone mode?

The first case we will discuss is the axial-vector coupling constant $g_A$.

One of the prominent failures in the Skyrmion phenomenology[32] is that the $g_A$ comes out too low (by a factor of about 2). This is presumably related, in the way the calculation is done in Ref. 32, to a smaller value of $F_\pi$ that results from the fit. (This gets worse when the flavor group is SU(3), as we shall discuss later.) On the other hand, in the M.I.T. (large bag) limit, when the fluctuating pion field is introduced to restore axial-current conservation, the $g_A$ comes out too big,[4,33] $g_A^{\text{bag}} = 1.635$. This indicates that even though the $g_A$ is a static quantity, it can be sensitive to short-distance phenomena. We are thus led to consider the "halfway house," the magic angle $\pi/2$. Here the calculation is extremely simple.[7] At this angle, there are no divergence difficulties (as we discussed above), and furthermore we need to consider only the $K^\pi = 0^+$ zero mode (where $\mathbf{K} = \mathbf{J} + \mathbf{T}$). The $g_A$ is directly proportional to the net baryon charge inside the bag. With a suitable angular momentum projection onto the nucleon sector, one finds[7,34] that $g_A \approx 5/4$. (There is a caveat here. In the Skyrmion case the collective coordinate quantization relies on large-$N_c$ limit, so that one is forced to take $(N_c + 2)/N_c \to 1$. In the quark model, such a factor is taken to be 5/3 since $N_c = 3$. The result $g_A = 5/4$ corresponds to this way of calculating the quark contribution. Were we to impose the large-$N_c$ constraint as in the Skyrmion case, we would find instead $g_A = 5/4 \cdot 3/5 = 3/4$. The point is that in quark models, you "see" the quarks, while in the Skyrmion, you do not.)

The next case is the tensor coupling of a ρ meson to a nucleon (denoted as $\kappa_\rho^v$)[35] *

$$\delta\mathscr{L}_\rho = f_{\rho NN}\left\{\bar{\psi}_N\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu\psi_N + i\bar{\psi}_N\frac{\kappa_\rho^v}{2m_N}\sigma_{\mu\nu}k_\nu\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu\psi_N\right\} \quad (18)$$

where $\psi_N$ is the nucleon field operator, $\boldsymbol{\rho}_\mu$ the isovector ρ field, $m_N$ the nucleon mass and $k_\mu$ the momentum transfer. In the vector-dominance model (VDM), $\kappa_\rho^v$ is equal to $\kappa^v$ of the nucleon electromagnetic current

$$J_\mu = e\left\{\bar{\psi}_N\frac{1+\tau_3}{2}\gamma_\mu\psi_N + i\bar{\psi}_N\frac{1}{2m_N}\frac{\tau_3\kappa^v+\kappa^s}{2}\sigma_{\mu\nu}k_\nu\psi_N\right\}. \quad (19)$$

Since the vector dominance arises naturally in the Skyrmion picture,[36] the Skyrmion model should have

$$\kappa_\rho^v = \kappa^v = 3.5, \quad (20)$$

the last equality being empirical. Experimentally, however, $\kappa_\rho^v$ is not equal to $\kappa^v$; instead[37]

$$\kappa_\rho^v = 6.6 \simeq 2\kappa^v. \quad (21)$$

This deviation from the VDM has been a long-standing puzzle in hadron physics.

In nuclear physics, the magnitude of $\kappa_\rho^v$ is believed to play an extremely important role in processes in nuclei involving spin–isospin modes. For instance, the large value (21) has a dramatic influence on the suppression of would-be collective modes in the pionic channels.[38] The smaller value (20) would have produced in nuclei some spectacular phenomena that have not been observed. That the Skyrmion picture corresponds to (20) rather than to (21)

*This was discussed in a previous Comments article[35]; here it illustrates in a clear way a possible deviation from the Cheshire Cat phenomenon.

can also be seen in the $NN$ tensor potential predicted in the Skyrme model,* as compared with the $\rho$- and $\pi$-exchange prediction of the same potential with (21). Figure 2 exhibits these features. It seems that the Skyrmion fails here.

The chiral bag model offers a simple explanation of Eq. (21), based on the baryon number fractionation near the magic angle $\theta = \pi/2$. The argument goes as follows.[35] As we know now, the baryon charge is equally divided into the bag and the Skyrmion

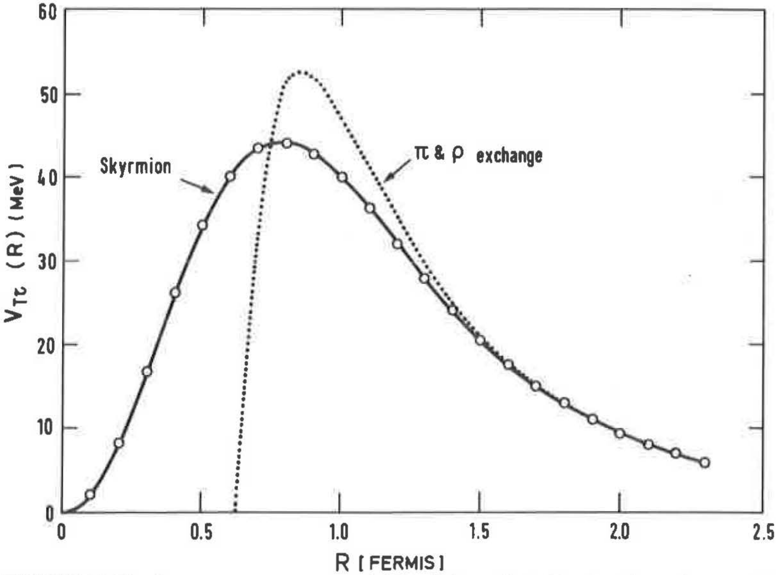


FIGURE 2 The isovector tensor potential $V_{T\tau}(R)$ predicted by the Skyrmion model (A. Jackson, private communication and Ref. 55) given in solid line end by the $\pi$- and $\rho$-exchange model with the $\rho$ tensor coupling $\kappa_\rho^v \simeq 6.6 \simeq 2\kappa^v$ given in dotted line. Plotted is $3V_{T\tau}(R)$ where

$$V_{T\tau}^{NN}(R) = \left(\frac{5}{9}\right)^2 S_{12}\tau_1 \cdot \tau_2 V_{T\tau}(R).$$

The characteristic feature of the strong $\rho$ tensor coupling is a sharp drop of the potential crossing zero at $R = 0.6$–$0.7$ fm. The detailed behavior inside of $R \simeq 0.6$ fm is probably irrelevant because of the repulsive core.

*This result was kindly supplied to us by A. Jackson. It is essentially equivalent to the published result of Jackson *et al.*[55]

cloud. One can show[35] that at $\theta = \pi/2$, the electric charge fractionizes equally just as the baryon charge does. Therefore the photon will interact with charge $e/2$ with the quarks in the bag and with $e/2$ (through the vector mesons) with the Skyrmion cloud, as shown in Fig. 3. Furthermore, at this magic angle, only the valence particles at the zero-energy $K^\pi = 0^+$ level ($K^\pi \neq 0^+$ levels do not contribute because of the CP symmetry) participate. However, the momenta of the valence quarks are zero and hence they contribute nothing to the magnetic coupling in Eq. (19). Thus $\kappa^{v,s}$ get their contributions solely from the meson sector (Fig. 3(b)). Therefrom follows the relation $\kappa^v \cong (1/2)\kappa_\rho^v$. It turns out that one can build a successful phenomenology of the nucleon electromagnetic form factors on this relation. (For details, see Ref. 35.)

Here we evidently encounter a case where there is no *obvious* Skyrmion or pure bag description equivalent to the chiral bag. Within the scheme involving pions and vector mesons, it does not appear possible to obtain Eq. (21) while preserving other successful results of VDM. It is not inconceivable that with a larger number of meson fields and derivative terms, one can arrive at Eq. (21); but the mechanism cannot be simple. It is thus highly appealing to think that the simple explanation of Eq. (21) provided by the chiral bag reflects a manifestation of explicit quark–gluon degrees of freedom. If this is borne out, it will represent the very *first* signature of quarks in the low-energy domain. It is amusing that the effect of the quark/gluon core is here manifested in a factor

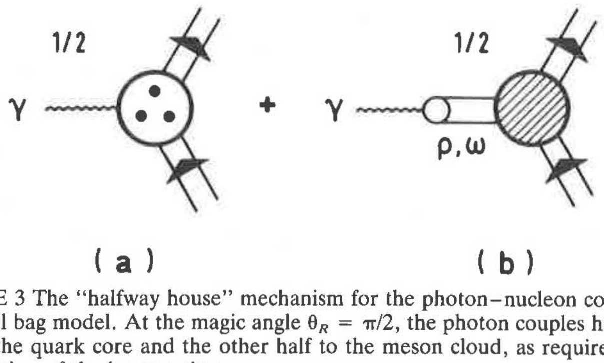


FIGURE 3 The "halfway house" mechanism for the photon–nucleon coupling in the chrial bag model. At the magic angle $\theta_R = \pi/2$, the photon couples half of the time to the quark core and the other half to the meson cloud, as required by the fractionation of the baryon charge.

~2 change in $\kappa_\rho^v$, or a nearly 100% effect. This should be compared with the ~10% correction in the *np* capture, explained by Riska and Brown,[39] which led to the reconstruction of the explicit meson presence in nuclei through the study of exchange currents (see Ref. 1 for the development). The puzzle will remain, however: what is so special about $\kappa_\rho^v$ *and* the magic angle?

## 3. STRANGENESS

So far our discussion has been focused on the flavor SU(2) system. Most of the low-energy properties of the baryons are insensitive to specific features of quark–gluon degrees of freedom, although some characteristics are rendered simpler explanations if quarks are explicitly included in the form of the chiral bag model. There are strong indications, however, that when strange quarks are introduced into the baryons, there occur qualitative changes in the role that the Goldstone mode of chiral symmetry plays. We are unclear as to what actually happens, but the subject is so intriguing that we think it deserves comment, particularly regarding the potential advantage of the chiral bag.

Whereas the SU(2) Skyrmion predicts energies, static properties, and also pion–baryon scattering amplitudes in fair agreement with experiments (within, roughly, 20–30% accuracy),* the situation is thus far very different for the strange quark system.[40] With the standard Skyrme Lagrangian with the flavor SU(3), a reasonable fit can be obtained *only if* one takes $F_\pi \sim (1/4)F_\pi^{\exp}$ (compare this to the SU(2) case[32] where the required $F_\pi \sim 0.7F_\pi^{\exp}$). This signals a clear failure of the Skyrmion picture. Furthermore, model-dependent relations are in poor agreement with experiments.[41]

One might argue that being an effective Lagrangian, the necessity of a small effective $F_\pi$ is not to be taken as a serious blow to the model. However, such a low $F_\pi$ would predict[42] a disastrously low-mass dibaryon *H*. (*H* would be predicted at a mass comparable to a single nucleon, while the mass predicted in the M.I.T. bag model is more than twice the nucleon mass.)

*Model-independent relations are verified with a better accuracy (a few percent), while some (such as $g_A$ as mentioned before) are worse.

The reason for the failure of the model could be complex. Prominent possible causes are: the flavor SU(3) breaking (i.e., the $s$-quark or kaon mass) cannot be treated perturbatively; higher derivative terms may be more important in the strange sector than in the nonstrange sector.

In the standard treatment of the SU(3) Skyrmion,[43] flavor quantum numbers are associated with collective coordinate rotations of the soliton, and the collective coordinates used in quantization arise from *unbroken* flavor symmetries. As a consequence, the kaon mass can be treated *only* as a perturbation. That treating the kaon mass as a perturbation is not a good approximation has been shown by Callan and Klebanov.[44] Their arguments are subtle and somewhat complex and we do not have space to cover them here. We will just point out the salient features that are particularly relevant to the flavor SU(3) chiral bag. Details will be given elsewhere.

In the Callan–Klebanov scheme, the effect of the strange quark mass is fully taken into account by considering explicitly kaon–soliton bound states, avoiding use of SU(3) collective coordinates. Thus the mass problem is resolved. Consequently, mass relations valid to O(1) in large-$N_c$ expansion are found to work well. However, grave problems arise at the order $O(1/N_c)$; this is the order in which gluon effects are known to play important roles in quark models. There is good reason to believe that unlike in the SU(2) case, higher derivatives in the effective Lagrangian (see Appendix) play a much more important role in the strange sector, and that the disastrous result obtained for hyperfine splitting (e.g., the mass difference $\Sigma^* - \Sigma < 0$) is caused by this defect.

What this implies in terms of the chiral bag is that as the bag is shrunk, a lot more shorter wavelength degrees of freedom in the meson sector than in the case of flavor SU(2) will be required to obtain the Cheshire Cat phenomenon. It is thus preferable to have a larger bag with only a small meson cloud, the bulk of dynamics being given by gluon-exchange interactions between the strange and nonstrange quarks. The boundary condition (4), suitably generalized to the flavor SU(3) for $U$, is expected to provide this "breaking" of a Cheshire Cat symmetry. There is a compelling indication that this might be the case in nature: the hyperon magnetic moments are best described with a bigger bag radius than that of nonstrange baryons.[45]

## 4. PROBLEMS

In this section, we discuss a few important problems that are being resolved at the moment or will be worked out more fully in the near future.

Although it is now well established that the baryon charge fractionates due to chiral anomalies, the "fractionation" of other quantum numbers has not yet been fully understood. In discussing the tensor coupling of the $\rho$ meson, we discussed how isospin fractionates for $\theta = \pi/2$; however, no such proof exists for $\theta \neq \pi/2$. What about the angular momentum? As discussed before, using an argument made by Niemi,[46] one can establish that the spin fractionates as does the baryon charge for $\theta = \pi/2$, but so far no general proof exists for $\theta \neq \pi/2$. As we alluded to in the previous section, strangeness need not be associated with topology and hence may not fractionate. (Understanding strangeness in the context of the chiral bag is an open problem.)

In order to answer these questions, and to calculate physical observables, the model must be quantized. As we mentioned before, there still remains the question of how to unambiguously eliminate the infinities that arise due to the polarization of the Dirac sea inside the bag (except when $\theta = 0, \pi/2, \pi$); there is also the basic issue as to whether the quantization of the hybrid—bag plus meson cloud—can be done in a unique and completely consistent way. Related to this is the interpretation of the boundary conditions: are the meson fields that appear exponentiated in Eq. (4) *full* quantum fields? So far, we have treated them as classical (soliton) fields, but if the boundary conditions are bosonization relations, then full quantized fields may have to be considered (particularly if temperature effects are considered).

Significant progress has recently been made on one of these issues, namely the $\Delta - N$ mass difference. In the pure Skyrmion description, the mass difference $\Delta M$ arises when the soliton is quantized by the collective coordinates; it is given by $3/2\mathcal{I}$,* where $\mathcal{I}$ is the moment of inertia. In the pure bag description, $\Delta M$ is described by one-gluon exchange; it is given by $3C\alpha_s/R$* where $C$ involves an integral of quark wave functions and $\alpha_s$ is the color fine-structure constant. When considering the hybrid system, the hedgehog quark system of the bag has to be rotated in a way consistent with the rotation of the Skyrmion cloud. Recently the collective coordinate quantization of the hybrid system has been satisfactorily worked out.[47] The result is a "cranking" formula familiar in nuclear physics. (The cranking of the Dirac sea has not been worked out yet.) One finds, ignoring the contribution from the cranked Dirac sea which is expected to be small, that the combined rotation of the soliton cloud and the valence quarks accounts successfully for the observed $\Delta - N$ splitting over a wide range of radii, $0 \lesssim R \lesssim 0.6$ fm: the moment of inertia is surprisingly flat within this region. For $R > 0.6$ fm, the quark moment of inertia (and hence the total) increases rapidly, with the consequent decrease of the $\Delta - N$ splitting. It is reasonable to infer from this that down to chiral angle $\theta \sim \pi/2$, the gluon exchange, not accounted for in Ref. 47, is negligible or $\alpha_s \ll 1$ and that as $\theta$ becomes smaller ($R$ becoming larger), the gluon effect starts dominating or $\alpha_s$ becomes large, say, the M.I.T. value $\alpha_s \approx 2$. This is another indication of an approximate Cheshire Cat phenomenon operative in $(3 + 1)$ dimensions.

## 5. CONCLUSIONS

The chiral bag is the simplest possible modeling of QCD that has the potential virtue of encompassing the extreme situations: the Skyrmion and the M.I.T. bag. The extreme simplicity may not allow a detailed quantitative fit with experiments (although with some obvious improvements on the model, one could hope to do better even quantitatively than other models), but offers an economic way to study the physics of strong interactions that take place in nuclei. In particular, boson-exchange models for the interaction are only slightly changed by the quark bag, modification

*$\Delta[J(J + 1)] = 3$ for $J_\Delta = 3/2$ and $J_N = 1/2$.

occurring in the regularization of these at short distances. One can thus understand the success of the shell model, because the volume occupied by the quark–gluon core can be effectively taken to be small, and it is reasonable for the nucleons to carry out nearly independent motion. This contrasts with the hyperons which are predicted to be big by the model. There are other successes here and there, and some of the conceptual problems that one faces when one wants to think about nuclear processes in terms of quark–gluon degrees of freedom are resolved in a natural way. Even so, only a small part of the structure of the chiral bag, simple though it may be, is understood. The main reason for this is that there is a deep connection, not yet fully understood, between the structure of the chiral bag and chiral anomalies. To the extent that the problem is topological, associated with chiral symmetry, we suspect that it is a general problem not confined to the chiral bag per se. Thus whatever sensible model one constructs (independently of details)—and there are in the literature a plethora of models that purport to be sensible—must have features in common with the chiral bag model.

One reason to believe that the chiral bag can be a useful model is that it shares common features, through chiral anomalies, with other systems such as the monopole–fermion system. In fact, the chiral bag can be thought of as an *inside-out* version of the latter. This is not so surprising if one realizes that a Skyrmion is like a particle moving on a sphere in the presence of a Dirac magnetic monopole.[48] In fact, there is an interesting development to treat both problems in complete parallel with some surprising results.[49] We find this exciting.

The most urgent problem, in our opinion, is *not* to fit other experimental data on single-baryon properties, *but* to make qualitative predictions based on the striking topological and quark–gluon properties in a baryon-rich environment. For instance, it would be interesting to know what happens to the chiral bag structure or the Cheshire Cat structure when nuclear matter is heated or compressed.[50] How the strong-interaction vacuum changes in a baryon-rich environment is the most interesting problem in nuclear physics, and the chiral bag picture should provide a useful tool to work with.

## APPENDIX: SKYRMIONS*

We briefly summarize the essentials of the Skyrmion, chiefly in order to define terminologies used in the main text.

The effective action is of the form

$$\Gamma = \Gamma^N + \Gamma^A$$

$$\Gamma^N = \frac{F_\pi^2}{4} \int d^4x \, \mathrm{Tr}(\partial_\mu U \partial^\mu U^+)$$

$$+ \frac{\epsilon^2}{4} \int d^4x \, \mathrm{Tr}[U^+\partial_\mu U, U^+\partial_\nu U]^2 + \cdots \qquad \text{(A1)}$$

$$\Gamma^A = \frac{N_c}{240\pi^2} \int d^5x \, \epsilon_{ijklm}$$

$$\times \, \mathrm{Tr}[U^+\partial^i U U^+ \partial^j U U^+ \partial^k U U^+ \partial^l U U^+ \partial^m U].$$

For SU(2) × SU(2) flavor symmetry, $U$ is given by Eq. (3). For three flavors, $\tau^a \to \lambda^a$, where the $\lambda^a$ are the SU(3) matrices, etc. Here $\Gamma^N$ is the normal-parity component. As noted in the text, the first term corresponds to the nonlinear $\sigma$-model. It is the current algebra term. The second term, Skyrme's quartic term, is often called the "Skyrme term" for short. It is now understood as summarizing effects from $\rho$-mesons. Effects from $\omega$-mesons would be described in a term with six derivatives, etc.

The $\Gamma^A$ is the abnormal-parity ("anomalous") component known as the Wess–Zumino term.[52] It has its origin in the chiral anomalies. Naively this term vanishes for SU(2) × SU(2), but it does not in general[53]: it plays a similar role as in the flavor SU(3) case. Baryons arise as solitons, static solutions to the equation of motion for the $U$ field. The spin-statistics of the solitons are signalled by the W-Z term, the coefficient of which is quantized in a manner analogous to the Dirac quantization of the magnetic monopole.

*For an extension review of Skyrmions, see Ref. 51.

The soliton solution is sought in the hedgehog form[5]

$$U_s(r) = \exp(i\boldsymbol{\tau} \cdot \hat{r}\, \theta(r)) \qquad \text{for SU(2)} \tag{A2}$$

and quantization is made through the collective coordinate $A(t)$, which is an SU(2) matrix, by

$$U(\mathbf{x}, t) = A(t)U_s(\mathbf{x})A(t)^{-1}. \tag{A3}$$

Similar to cranking in nuclear physics, this quantization projects out correct spin and isospin states; in this case $I = J = 1/2, \ldots, N_c/2$.

In flavor SU(3), there are two spherically symmetric embeddings. The SU(2) embedding, namely putting Eq. (A2) in the upper left corner of an SU(3) matrix, gives $B = 1$ systems, i.e., *8* for $J = 1/2$, *10* for $J = 3/2$. Here the Wess–Zumino term plays an essential role. The SO(3) embedding which uses $\lambda_7$, $-\lambda_5$, $\lambda_2$ gives $B = 2$ systems,[54] i.e., the dibaryons among which the $H$ dibaryon with $S = -2$ has attracted the greatest attention. Here the Wess–Zumino term provides only the triality zero condition and has no other role. Very little is understood and no phenomenological success has been obtained in the SU(3) sector. The Callan–Klebanov scheme[44] and the chiral bag scheme discussed above offer a more promising avenue.

Application of the Skyrme model to nucleon–nucleon[55] and pion–nucleon[56] systems has been rather successful. A very exciting recent development in this connection is an attempt to derive exchange currents (as described in Ref. 1) from the Skyrmion Lagrangian which has led to a remarkable result[57]: the isoscalar exchange current can be obtained *free of model dependence* through the Wess–Zumino term, resolving a long-standing problem[1] in nuclear physics.

The vector mesons ($\rho$, $\omega$, . . .) that constitute the next wavelength scale to the pion can be incorporated into Skyrme's effective Lagrangian in two different ways. The obvious way is to gauge[58] partly or wholly the flavor symmetry, with the constraint that vector dominance emerges correctly. There is another, more subtle way: vector mesons arise as guage particles of a hidden gauge symmetry through (assumed) quantum effects.[36] At low energies, the two ways most probably give the same results: current algebra, vector dominance, etc. However, the second is more attractive for several reasons. While there is no conceivable reason why $SU(N) \times SU(N)$ chiral symmetry should be gauged (so the first method is unnatural in this sense) a gauged *hidden* symmetry does not suffer from this problem. In fact, the electroweak $SU(2)_L \times U(1)_Y$ can easily be incorporated into the latter by gauging a subgroup of explicit chiral symmetry which the former cannot. The basic idea is the following: starting with the current algebra term in (A1) that respects the longest wavelength physics, one imagines that the next wavelength excitation can also be deduced from chiral symmetry alone. The point is that the current algebra term in (A1) is just the nonlinear $\sigma$-model defined in the quotient space $SU(N)_L \times SU(N)_R/SU(N)_v$ since the chiral symmetry is spontaneously broken down to the diagonal subgroup $SU(N)_v$ (for instance, the unbroken group is the isospin for $N = 2$, or the eight-fold way for $N = 3$). Hence it can be formulated as a linear theory provided one introduces explicit gauge boson degrees of freedom, corresponding to a *local* $SU(N)_v$ symmetry.[59] At the classical level, such new gauge bosons are just auxiliary fields and can be algebraically eliminated, recovering the original Lagrangian. However, it is known in two-dimensional $\sigma$-models[59] that quantum fluctuations can produce a kinetic energy term, transforming the gauge fields into propagating, dynamical fields. In fact, it is claimed[60] that the kinetic energy term is already present in the extended Wess–Zumino Lagrangian. Suppose it does happen. Then one can show[36] that vector mesons whose masses are generated dynamically (by a Higgs mechanism) emerge *naturally*, with gauge couplings consistent with the vector dominance picture. The vector mesons $\rho$ and $\omega$ in particular have been discussed in this context.[35,36] Furthermore, in the limit that the vector-meson masses are infinite with the gauge coupling $g$ held fixed, one obtains in the Lagrangian*

$$\frac{1}{32g^2}\,\mathrm{Tr}[U^+\partial_\mu U,\ U^+\partial_\nu U]^2$$

which is just Skyrme's quartic term.[61]

*This is the coefficient of the term used in Ref. 32 with $e$ replacing $g$. Note that in this limit, the $\omega$ meson does not contribute.

The meson sector (trivial topological sector) of this theory is equivalent to Weinberg's nonlinear Lagrangian[62] written down in 1968. This suggests that up to a mass scale ~1 GeV, the physics of QCD may be dictated predominantly by chiral symmetry, a qualitative conclusion also reached from nuclear physics.[1]

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