FERMION MASS IN E_6 GUT WITH DISCRETE FAMILY PERMUTATION SYMMETRY S_3

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A discrete symmetry S3 easily explains all neutrino data. However it is not obvious the embedding of S3 in GUT where all fermions live in the same representation. We show that embedding S3 in E6 it is possible to make distinction between neutrinos and the rest of matter fermions.

1 Introduction

So far it is not clear how to extend the standard model to include fermion masses. In general the mass terms are arbitrary $N_f \times N_f$ complex matrices M_f where N_f is the number of generations. M_f are not univocally fixed by experimental data. To reduce the remaining arbitrariness flavour and gauge symmetry are used in the model building. Gauge coupling unification, anomaly cancellation and charge quantization are hints to consider Grand Unification (GU) models. We assume a gauge symmetry G_g acting vertically within each generation and a flavour symmetry G_f acting horizontally between different generations and we study the group $G_g \times G_f$.

We can get information about the flavour symmetry G_f from the observed mass and mixing fermions hierarchies. First we consider the *lepton sector*. The three neutrino analysis[a] [1] is well compatible with the following mixing matrix [2]

$$U_{HPS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{1}$$

called tri-bimaximal, where the heaviest third neutrino is maximally mixed between μ and τ flavours and $\theta_{13} = 0$ while the second neutrino is equally mixed between e, μ and τ. Recently discrete symmetry are studied to explain neutrino mixing (1), in particular S_3 [2 3 4 10 11], $A(4)$ [4 5] and S_4[6]. In *quark sector* the three mixing angles are small, the only relevant angle is the 1-2 Cabibbo angle which is smaller than the 1-2 and 2-3 leptonic angles and the mass hierarchy is strong. Since quarks mixing is very small we have more information and constrains on flavour symmetry G_f from leptons, where the mixing is larger than quarks. Quarks and leptons mixing and mass hierarchies are very different. The neutrino masses can be degenerate and the mixing is large (1), while quark and charged lepton masses are strong hierarchy and the CKM mixing matrix is very close to the identity. Very different hierarchy in quark and lepton sectors, could be a problem in GU models where in general quark and lepton Yukawa couplings are related.

[a] If MINIBOONE will not confirm LSND results, neutrino data are compatible with only two mass difference and the number of neutrinos is three.

We consider the problem to reconcile different mass and mixing hierarchy with lepton-quark symmetry in unified models. In literature there are at least two class of solutions. One possibility is to extend the observed lepton symmetry to all fermions[7]. Another possibility is that scalars couple differently to charged fermion Yukawa from neutrino Yukawa, but there are very interesting different possibility like the *screening* mechanism[8]. We have studied[10] the possibility to make a difference between neutrino and charged fermions selecting the gauge group G_g and its scalar sector.

2 Leptonic flavour symmetry

In this section we study the flavour symmetry G_f that follows from lepton sector explaining very well neutrino data, then we will select the gauge group in the next section. Neutrino mass matrices M_ν is $\mu \leftrightarrow \tau$ invariant (S_2 symmetric) only if it commutes with the matrix P

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad P^{-1}\, M_\nu\, P = M_\nu \quad \Rightarrow \quad M_\nu = \begin{pmatrix} a & d & d \\ d & b & c \\ d & c & b \end{pmatrix}.$$

Neutrino mass matrix M_ν and P are diagonalized by the same unitary matrix O which is

$$O(\theta) = \begin{pmatrix} -\cos\theta & \sin\theta & 0 \\ \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{\sqrt{2}}\cos\theta & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{\sqrt{2}}\cos\theta & \frac{1}{\sqrt{2}} \end{pmatrix} \qquad \Leftrightarrow \qquad \begin{pmatrix} \sqrt{2/3} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

Assuming the charged leptons mass matrix diagonal, O is the PMNS leptonic mixing matrix where the angle θ is the solar angle and it is not fixed by the $\mu \leftrightarrow \tau$ symmetry while the atmospheric and θ_{13} angles are the same of the tri-bimaximal (1). To obtain the solar angle we need that the singlet eigenstate (1, 1, 1) is and eigenvector of the mass matrix M_ν and there are two possible solutions: *i)* M_ν is S_3 invariant (S_3 is the permutation group of three objects) or *ii)* the parameters in M_ν are constrained by $a = b + c - d$ which can follow directly from A_4 symmetry[5]. We are interested in the first case. The S_2 permutation group is contained into S_3 and in general S_3 breaks spontaneously into S_2. We have shown[11] that in case $S_3 \supset S_2$ breaking is *soft*, the solar angle is $\sin^2\theta_{sol} = 1/3$ that agree very well with the experimental value.

3 A grand unification model for fermion masses

In previous section we have said that in case S_3 symmetry is softly broken into S_2 ($\mu \leftrightarrow \tau$), we can explain very well neutrino mass and mixing hierarchies. Differently S_2 symmetry is strongly broken in charged fermion sector. The issue is how to embed a neutrino mass matrix in the same unified gauge group where leptons and quarks Yukawa are equal (lepton-quark unification). In SU(5) neutrinos and charged leptons Yukawa couplings are distinct since SU(5) does not contain right-handed neutrino and we must introduce an additional singlet S

$$L = g_u\, T^{\alpha\beta}\, T^{\gamma\delta}\, H^\sigma\, \varepsilon_{\alpha\beta\gamma\delta\sigma} + g_d\, T^{\alpha\beta} F_\alpha\, \bar{H}_\beta + g_v\;\; F_\alpha\, S\, H^\alpha + M S\;S \qquad (2)$$

where T and F are the weyl fermions belonging to the 10 and $\bar{5}$ representation of SU(5). However we are interested in model beyond SU(5) since we want to explain why Yukawa are proportional to different vevs embedding SU(5) in bigger groups and we are interested in non supersymmetric extension of the Standard Model, but in such case gauge couplings do not unify in SU(5). Besides these general motivations, there is one strong reason to consider other gauge groups than SU(5). Even if Yukawa couplings are distinct in SU(5), if we embed S_3 in SU(5) requiring only one

Higgs doublet [b], we obtain wrong prediction. In fact taking the Higgs as a S_3 singlet the only S_3 invariant renormalizable Yukawa operators in SU(5) that give up mass terms are

$$\lambda_1\, T_i\, T_i\, H\ , \quad \lambda_2\, T_i\, T_j\, H$$

from which we obtain respectively the following up quark mass matrices

$$\lambda_1\, v \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \lambda_2\, v \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{3}$$

where λ_1 and λ_2 are arbitrary couplings and v is the Higgs vev. The matrices (3) give wrong masses ($m_u = m_c = m_t$) and mixings. The S_3 permutation symmetry is approximatively exact only for neutrinos [c], while the permutation symmetry is strongly broken in charged fermion sector. Thus assuming only one Higgs doublet and the permutation symmetry, the unifying gauge group choice is constrained by the fact that the tree level Yukawa interactions are zero for charged fermions, but this is not true for the SU(5) unification gauge group. In the following we study one possible choice for the unifying group so that only Dirac neutrino gets Yukawa coupling at tree level and neutrino sector is distinct from charged fermion sector. If we embed SU(5) into SO(10) we have an additional $U_r(1)$ gauge group that commutes with the full SU(5). The $U_r(1)$ charges for the representation above are $H(+q), \bar{H}(-q)$, $T(-1)$ $F(+3)$ and $\nu_R(-5)$ [9]. From these charges we derive that each mass operators have $U_r(1)$ charges

SU(5) mass operator	$U_r(1)$
$T^{\alpha\beta} F_\alpha$	+2
$F_\alpha\, \nu_R$	-2
$\nu_R^t\, \nu_R$	-10
$T^{\alpha\beta}\, T^{\gamma\delta}$	-2

(4)

We observe that the mass operators $T^{\alpha\beta}\, T^{\gamma\delta}$ and $F_\alpha\, \nu_R$ have the same charges, thus we expect that the same SU(5) singlet is at the origin of their Yukawa interaction. As said, while neutrinos have an approximate S_3 symmetry [11], the same symmetry is not observed in the up sector. If we embed SU(5) into E_6 we have an additional $U_t(1)$, $E_6 \supset SO(10) \times U_t(1) \supset SU(5) \times U_r(1) \times U_t(1)$ where the 27 fundamental representation of E_6 contains an extra Standard Model singlet (27=1(4)+10(-2)+16(1)) and the table above becomes

SU(5) mass operator	$U_r(1)$	$U_t(1)$
$T^{\alpha\beta} F_\alpha$	+2	+2
$F_\alpha\, \nu_R$	-2	+2
$\nu_R^t\, \nu_R$	-10	+2
$T^{\alpha\beta}\, T^{\gamma\delta}$	-2	+2
$F_\alpha\, x_L$	+3	+5
$\nu_R^t\, x_L$	-5	+5
$x_L^t\, x_L$	0	+8

(5)

The advantage here is that the 27 contains two standard model singlets that will play the role of right-handed neutrinos ν_R and x_L, and the Dirac mass operator $F_\alpha\, x_L$ has different quantum

[b] We prefer to keep just one Higgs doublet , that will give mass both for the up and the down sector. This is because we want to have the Standard Model with just one higgs at the weak scale where the FCNC are strongly suppressed due to the GIM mechanism.

[c] We have proposed a phenomenological model [11] where the Dirac neutrino mass matrix is proportional to the identity and mixing and mass hierarchies come from Majorana mass terms that break softly S_3 into S_2 in the 2-3 direction through a seesaw mechanism.

numbers from all the others and in particular is different from $T^{\alpha\beta}T^{\gamma\delta}$ giving mass to the up sector. Thus we explore the possibility that the fundamental lagrangian has a E_6 unifying gauge symmetry times a S_3 permutation symmetry of the three fermion families that belong to the 27 of E_6. Now we have to choose the representation for the Higgs $SU(2)_W$ doublet which is a S_3 singlet. Now we have to decide to which E_6 representation we have to assign the Higgs doublet. The $351'$ contains a $SU(2)_W$ doublet with (-3,-5) charges with respect the $U_r(1)\times U_t(1)$ [9]. Thus, if we put the Higgs doublet in the $351'$, the Yukawa interaction for fermions at the tree level can be

$$27_i^{\alpha}\ 27_i^{\beta}\ 351'_{\alpha\beta} \tag{6}$$

where i=1,2,3 are family index and the $351'$ is symmetric under the exchange of α and β the gauge symmetry indices. At the tree level of the fundamental high energy lagrangian, we have just one Yukawa interaction $g\ x_{iL}^t\ \nu_{iL} v$ that comes from (6) since this is the unique $U_r(1)\times U_t(1)$ gauge invariant operator. Thus there is only one Yukawa interaction in the fundamental E_6 symmetric renormalizable Lagrangian that gives the Dirac neutrino mass. The operator (6) does not introduce any mass neither for quarks nor for charged leptons. We remember that this result is important as explained above, since a Yukawa interaction $u^c_{R\ i}\ u_{L\ i} h_0$ (symmetric under S_3 family permutations) would give $m_t = m_c = m_u$ that is clearly unacceptable. So, before the E_6 symmetry breaking, quark and charged lepton yukawa couplings are zero, since they do not form a gauge invariant operator with the Standard Model Higgs. The up quark yukawa operator is $T^{\alpha\beta}T^{\gamma\delta}H^{\sigma}\varepsilon_{\alpha\beta\gamma\delta\sigma}$ and its charges are (+5, +3). We need a SU(5) singlet with opposite $U_r(1)\times U_t(1)$ to make an invariant operator. At first sight such a singlet is contained both in the 78 and in the 650, it has the correct U(1) charges. But we have shown [10] that in order to give a Yukawa coupling to the up quarks we have to write an interaction $27^{\alpha}\,27^{\beta}351'_{\gamma\sigma}\Sigma^{\gamma\sigma}_{\alpha\beta}$ where $\Sigma^{\gamma\sigma}_{\alpha\beta}$ is the irrep 2430.

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