Generic features of Einstein-Aether black holes

Takashi Tamaki[1] and Umpei Miyamoto[2]

[1]*Department of Physics, Waseda University, Okubo 3-4-1, Tokyo 169-8555, Japan*
[2]*Racah Institute of Physics, Kaplun Building, The Hebrew University of Jerusalem, Givat Ram, Jerusalem 91904, Israel*

Abstract

We reconsider spherically symmetric black hole solutions in Einstein-Aether theory with the condition that this theory has identical PPN parameters as those for general relativity, which is the main difference from the previous research.

1 Introduction

Identifying the contents of dark energy and dark matter (DE/DM) is one of the most important subjects in cosmology. It is frequently argued that gravitational theories are an alternative to DE/DM. Recently, tensor-vector-scalar (TeVeS) theories have attracted much attention since they do not only explain galaxy rotation curves but also satisfy many constraints from solar experiments [1].

However, it is nontrivial whether or not these theories satisfy the constraints by strong gravity tests. To study vector fields in a general form is difficult. Thus, as a first step, it is important to investigate a simplified model which is tractable and instructive for general cases. One such useful model would be Einstein-Aether (EA) theory [2], where all parameterized post-Newtonian (PPN) parameters can be the same as those in GR [3]. In EA theory, strong gravitational cases including black holes have been analyzed to some extent [4, 5, 6, 7]. Nevertheless, the analysis of black holes has been limited to the case in which the event horizon coincides with the spin-0 horizon [5], and this case does not necessarily satisfy weak fields tests. Thus, it is interesting to ask whether or not significant differences from the Schwarzschild black hole appear when weak fields tests are satisfied. For this reason, we argue black holes with the case in which the EA theory has identical PPN parameters as in GR [8]. We use units in which $c = 1$ and the sign convention $(-, +, +, +)$ for metrics.

2 Einstein-Aether theory

We consider the following action:

$$I = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\, R - K^{ab}{}_{cd}\nabla_a u^c \nabla_b u^d + \lambda(u^2+1)\ , \tag{1}$$

$$K^{ab}{}_{cd} := c_1 g^{ab}g_{cd} + c_2\delta^a_c\delta^b_d + c_3\delta^a_d\delta^b_c - c_4 u^a u^b g_{cd}\ , \tag{2}$$

where u^a is a vector field and $u^2 := u^a u_a$. c_i $(i = 1, 2, 3, 4)$ are theoretical parameters in EA theory. λ is a Lagrange multiplier ensuring the vector field u^a to be unit timelike vector everywhere.

Varying this action with respect to λ and u^a, we have

$$u^2 + 1 = 0\ ,\quad c_4\dot{u}^m\nabla_a u_m + \nabla_m J^m{}_a + \lambda u_a = 0\ , \tag{3}$$

where $J^a{}_m := K^{ab}{}_{mn}\nabla_b u^n,\ \dot{u}^b := u^a\nabla_a u^b$. Multiplying Eq. (3) by u_a, we have

$$\lambda = c_4\dot{u}^2 + u^a\nabla_m J^m{}_a\ . \tag{4}$$

Varying the action with respect to the metric, we have

$$G_{ab} = \nabla_m\left[J^m{}_{(a}u_{b)} - J_{(a}{}^m u_{b)} + J_{(ab)}u^m\right] + c_1\left(\nabla_a u_m\nabla_b u^m - \nabla_m u_a\nabla^m u_b\right) + c_4\dot{u}_a\dot{u}_b + \lambda u_a u_b - \frac{1}{2}g_{ab}\mathcal{L}_{\rm u}\ ,$$

[1]E-mail:tamaki@gravity.phys.waseda.ac.jp
[2]E-mail:umpei@phys.huji.ac.il

where $\mathcal{L}_{\mathrm{u}} := K^{ab}{}_{cd}\nabla_a u^c \nabla_b u^d$.

If we assume the weak field and slow-motion limits in EA theory [3], we have to take Newton's gravitational constant as $G_{\mathrm{N}} = \left(1 - \frac{c_1+c_4}{2}\right)^{-1} G$, to reproduce Newtonian gravity correctly. For all the PPN parameters to coincide with those in GR, we have

$$c_2 = \frac{-2c_1^2 - c_1c_3 + c_3^2}{3c_1} \ , \quad c_4 = -\frac{c_3^2}{c_1} \ . \tag{5}$$

From the maximum mass of neutron stars $\sim 2M_{\odot}$, we have $c_1 + c_4 \leq 0.5 \sim 1.6$ [7]. In [9], the sound modes are analyzed by expanding the metric and the Aether around the Minkowski metric. For these sound velocities to be equal to or larger than the photon velocity, or, to ensure stability against linear perturbation in Minkowski (or FRW) background and linearized energy positivity, we have [9, 10, 11]

$$0 < c_+ < 1 \ , \quad 0 < c_- := c_1 - c_3 < \frac{c_+}{3(1-c_+)} \ . \tag{6}$$

Radiation damping was also analyzed in [12], which almost restricts c_+ as a function of c_- based on the observation of, say, B1913+16.

3 Analysis in a single-null coordinate system

Our purpose in investigating black holes in EA theory is not to give a further restriction but to understand generic features of vector-tensor theories under the condition that weak gravity tests are satisfied. From this point of view, we take the following strategy. (i) We assume (5) since the constraints by the solar experiments are severe. (ii) We assume (6). Otherwise, a naked singularity appears outside the event horizon in general. Constraints from neutron stars and from radiation damping are related to strong gravity tests at least partially. For the above reasons, we do not impose these constraints. Thus, we have two theoretical parameters (c_+, c_-) with the condition (6).

We write a static and spherically symmetric line element in a single null coordinate system as,

$$ds^2 = -N(r)dv^2 + 2B(r)dvdr + r^2 d\Omega^2 \ . \tag{7}$$

In this coordinate, the vector field takes the form of $u = a(r)\partial_v + b(r)\partial_r$. $b(r) \neq 0$ means that the Aether is not aligned with the timelike Killing field, which is inevitable because of the event horizon. From Eq. (3), $-Na^2 + 2Bab = -1$. We can eliminate λ with Eq. (4). Then, we obtain basic equations, which can be written schematically as

$$N' = f_1(B, N, a, a') \ , \quad B' = f_2(B, N, a, a') \ , \quad a'' = f_3(B, N, a, a') \ , \tag{8}$$

where the prime denotes the derivative with respect to r.

The boundary condition at the horizon r_{h} is $N(r_{\mathrm{h}}) = 0$. We set $B(r_{\mathrm{h}}) = 1$. We can also set $r_{\mathrm{h}} = 1$ since there is no scale in the present theory. In this sense, it is assumed that the area coordinate r is normalized by the horizon radius below. If we use a rescaling freedom of v as $dv' = B(\infty)dv$, the asymptotic form of the metric is written as $ds^2 = -\frac{N(\infty)}{B(\infty)^2}dv'^2 + 2dv'dr + r^2 d\Omega^2$. Thus, the boundary condition at spatial infinity for the asymptotic flatness is $N(\infty) = B(\infty)^2$. We should require $b(\infty) = 0$, for the Aether to be aligned with the timelike Killing field. Then, we have $a(\infty) = -B(\infty)^{-1}$. We can determine the pair of $a_{\mathrm{h}} := a(r_{\mathrm{h}})$ and $a'_{\mathrm{h}} := a'(r_{\mathrm{h}})$ as shooting parameters, one of which is fixed by $a(\infty) = -B(\infty)^{-1}$. Thus, there remains one freedom. Fixing this freedom is done as follows. Even in the spherically symmetric case, there is a spin-0 mode. The freedom mentioned above is fixed by the requirement that the regularity at the spin-0 horizon which is inside the event horizon.

However, since the asymptotic observer is insensitive to the regularity at the spin-0 horizon, we permit the singularity at the spin-0 horizon. For this reason, we leave one freedom. In concrete terms, we obtain a_{h} iteratively for some a'_{h}, which is regarded as a free parameter.

4 Properties of solutions

We show several asymptotically flat solutions in Figs. 1 (a) and (c) for $c_+ = 0.4$ and $c_- = 0.1$. Figure 1 (a) shows that we can determine an a_h that satisfies the asymptotic condition for various values of a'_h. Figure 1 (c) shows a "mass" function. If we define the mass function $m(r)$ by $m(r) := \frac{r}{2G}\left(1 - \frac{N}{B^2}\right)$, we can interpret $m(\infty)$ as ADM mass M_ADM. As we can see, $m(r)$ monotonically decreases. This is not surprising since energy conditions are not necessarily satisfied in EA theory [11].

Since Figs. 1 show that the deviation from the Schwarzschild black hole is largest for the smallest value of a'_h, it is natural to ask whether or not there is a lower limit $a'_\mathrm{h,crit}$ below which there is no regular solution. We show the relation a'_h and M_ADM for various values of c_+ and c_- in Fig. 2 (a). Typically, M_ADM is smaller than that of a Schwarzschild black hole by about 10%, which is consistent with the result in [5]. For $a'_\mathrm{h} < a'_\mathrm{h,crit}$, we cannot find an appropriate value of a_h. $a'_\mathrm{h,crit}$ depends on c_+ and c_-. As a'_h approaches $a'_\mathrm{h,crit}$, $dM_\mathrm{ADM}/da'_\mathrm{h}$ tends to diverge.

We consider the possibility of distinguishing black holes in EA theory from Schwarzschild black hole by observation. In Ref. [7], the innermost stable circular orbit (ISCO) for neutron stars in EA theory was analyzed. The result is that the deviation from the Schwarzschild black hole is at most several percent. But this is not necessarily the case in the present situation, as shown below. The differences occur since we have the freedom parameterized by a'_h and the Aether is not static. These facts will be important if we consider observations such as Constellation-X, which tracks the motion of individual elements near black holes. We show the dependence of r_ISCO (normalized by r_h) on a'_h in Fig. 2 (b). Notice that $r_\mathrm{ISCO} = 3$ for the Schwarzschild black hole. Therefore, the difference is nearly 10% for $a'_\mathrm{h} \simeq a'_\mathrm{h,crit}$.

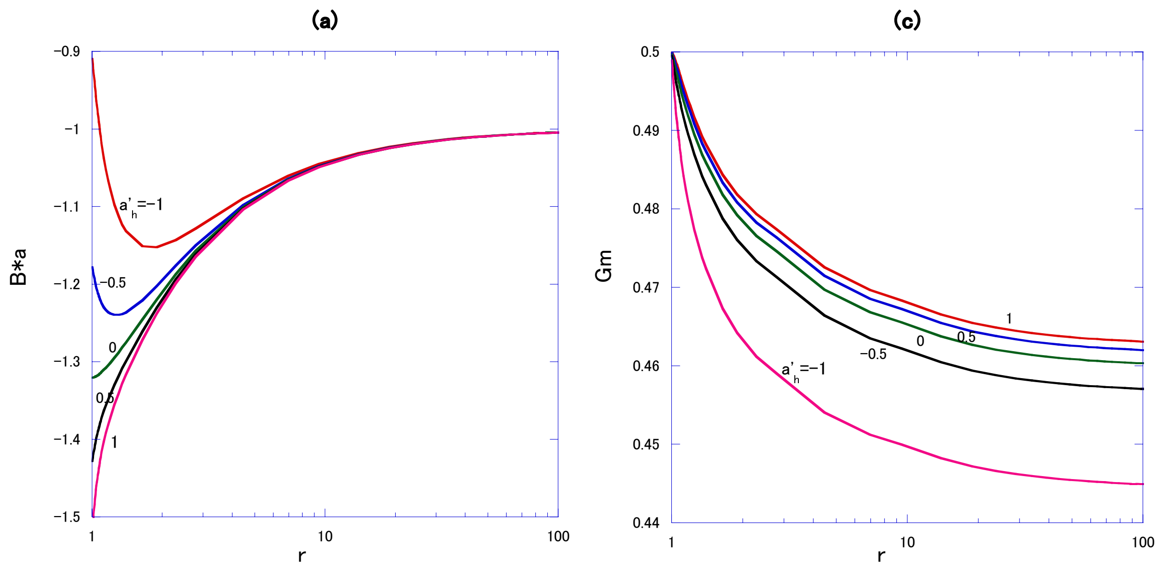

Figure 1: Field configurations for $c_+ = 0.4$ and $c_- = 0.1$.

5 Discussion

We have reanalyzed black hole solutions in EA theory while assuming that all the PPN parameters are the same as those for GR, resulting in two theoretical parameters c_+ and c_-. As another difference, we do not assume regularity at the spin-0 horizon since this is inside the event horizon. Interestingly, we find $a'_\mathrm{h,crit}$ below which there is no regular black hole solution. Near $a'_\mathrm{h,crit}$, the deviation of black hole mass M_ADM and ISCO r_ISCO from those for the Schwarzschild black hole become large.

These results are instructive for other cases. If we consider the case with rotation, freedom of the vector field is added. Then, it also contributes the kinetic term of the vector field, enhancing the differences

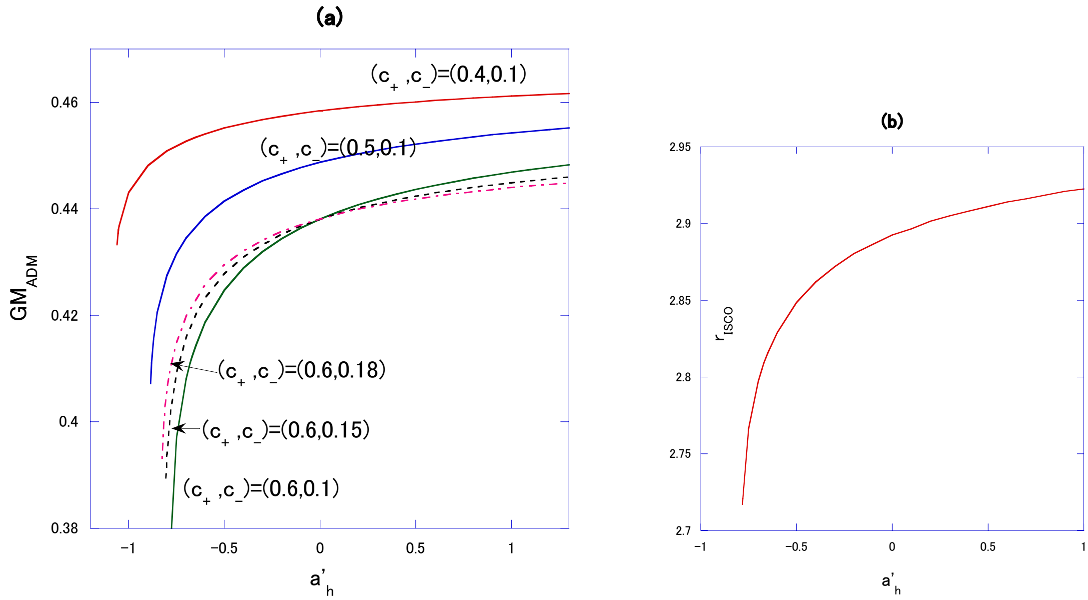

Figure 2: (a) $a'_{\rm h}$ v.s. $GM_{\rm ADM}$ and (b) $a'_{\rm h}$ v.s. $r_{\rm ISCO}$.

from the vacuum solution. This would also be true in other vector-tensor theories. For this reason, it is important to consider rotational black holes in vector-tensor theories, if we are to constrain them.

References

[1] J. D. Bekenstein, Phys. Rev. D **70**, 083509 (2004); *ibid.,* 069901 (E) (2005).

[2] T. Jacobson and D. Mattingly, Phys. Rev. D **64**, 024028 (2001).

[3] B. Z. Foster and T. Jacobson, Phys. Rev. D **73**, 064015 (2006).

[4] C. Eling and T. Jacobson, Class. Quant. Grav. **23**, 5625 (2006).

[5] C. Eling and T. Jacobson, Class. Quant. Grav. **23**, 5643 (2006).

[6] R. A. Konoplya and A. Zhidenko, Phys. Lett. B **644**, 186 (2007).

[7] C. Eling, T. Jacobson, and M. C. Miller, Phys. Rev. D **76**, 042003 (2007).

[8] T. Tamaki and U. Miyamoto, arXiv: 0709.1011 [gr-qc], to appear in Physical Review D.

[9] T. Jacobson and D. Mattingly, Phys. Rev. D **70**, 024003 (2004).

[10] J. W. Elliott, G. D. Moore, and H. Stoica, JHEP **0508**, 066 (2005).

[11] C. Eling, Phys. Rev. D **73**, 084026 (2006).

[12] B. Z. Foster, Phys. Rev. D **73**, 104012 (2006); Erratum *ibid.*, **75**, 129904 (2007).