Interaction of Weak Coherent Light with a System of Two – Level Atoms in a Lossless Cavity

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1 Introduction

The Jaynes–Cummings model (JCM) of light–matter interaction despite its simplicity demonstrates a number of interesting phenomena such as collapses and revivals [1], sub–Poissonian photon statistics [2] and squeezing [3]. Early studies showed the appearance of the so-called Cummings collapse [4] at coherent quantum pumping. Eberly et al. [1] have later found a revival of the collapsed oscillations, in fact an infinite sequence of collapses and revivals with Gaussian decrease of the revival maxima. The origin of collapses and revivals in the JCM is connected with the photon number distribution which produces spread in Rabi frequencies. The Rabi oscillations, initially all in phase, periodically dephase and rephase which leads to collapses and revivals, respectively.

Barnett and Knight [5] studied numerically collective collapses and revivals for a group of two–level atoms. The atoms were assumed as initially unexcited or excited (*a maser case*). In general, there are two sources of spread in Rabi frequencies: the photon number distribution (*as previously*) and the collective atomic evolution. The origin of the collective collapses and revivals is related with a non–equidistant spectrum of the eigenfrequencies of the system.

Recently, a new solution to the problem of interaction of a system of N two–level atoms with a single quantized field mode has been proposed [6]. Strictly speaking, cooperative spontaneous emission of a small number s of initially excited atoms in the presence of a large number of $N-s$ unexcited atoms ($s \ll N$) has been considered in terms of the SU (2)–group representation. Our method consisted in construction of the perturbation theory with a small parameter $\epsilon_s = \left(N - \frac{s}{2} + \frac{1}{2}\right)^{-1}$. The results obtained in this way are valid for an arbitrary time t. In the first–order approximation in ϵ_s it was found that the atomic inversion evaluates as follows [6]:

$$E(t) = \frac{s}{2}\cos(2\Omega t) + \frac{\epsilon_s}{16}\, s\,(s-1)\,(1-\cos(4\Omega t))\ ,\ \Omega = g\sqrt{N - \frac{S}{2} + \frac{1}{2}}\ , \quad (1)$$

where g is the atom–field coupling. The time evolution of the system is truly periodic since the spectrum of the eigenfrequencies is equidistant in the linear approximation in ϵ_s. The second term in (1) appears for $s \neq 1$. The collectivity of the system adds in this approximation only the harmonic Rabi frequency 4Ω in comparison with the JCM. The above solution is suitable for the description of the dynamics of the system even for the moderate values of the ratio $\frac{s}{N}$; according to the computer calculations the agreement with the real behaviour is then particularly good for relatively short times. For the sufficiently small values of $\frac{s}{N}$ the dynamics of the system is almost exactly described for all times by the first term of the above formula.

The time evolution of $E(t)$, calculated within an accuracy of ϵ_s^2, may be aperiodic if $\frac{s}{N}$ is not small enough. This is because the eigenfrequencies are non-commensurate in this approximation. Then, depending on the magnitude of the ratio $\frac{s}{N}$ beatings between the terms with different frequencies, resulting in modulation of $E(t)$, appear sooner or later. So, the second-order approximation of our theory is responsible for the collective spread in Rabi frequencies.

In the present paper we discuss a system of N two-level initially unexcited atoms interacting in a high-Q cavity with a weak, initially coherent, single-mode field. We perform our calculations in terms of the SU (2)-group representations.

2 Results

The Hamiltonian for the model in the rotating wave approximation reads ($\hbar = 1$):

$$\mathrm{H} = \mathrm{H}_0 + V\,, \quad \mathrm{H}_0 = \omega_f a^+ a + \omega \sum_{j=1}^{N} S_3^{(j)}\,, \quad V = g \sum_{j=1}^{N} (S_+^{(j)} a + S_-^{(j)} a^+)\ . \quad (2)$$

$a(a^+)$ is the photon annihilation (creation) operator. The j-th atom is described by the pseudospin operators $\mathrm{S}_K^{(J)}$ ($k = 3, +, -$). Since we consider a small-sample approximation the coupling coefficient g is the same for all atoms. Moreover, it is implicit that the transition dipoles are aligned with the mode polarization. In what follows, we assume exact resonance (the field frequency ω_f is then equal to the transition frequency ω) and choose the scale such that $\omega_f = \omega = 1$.

Let us recall that the excitation number operator $\hat{N} : \hat{N} = a^+ a + \sum_{j=1}^{N} S_3^{(j)} + \frac{N}{2}$ is an integral of motion. Hence, if the initial state of the system belongs to the subspace with the fixed eigenvalue $\hat{N}$, the time evolution of the system is restricted to this subspace.

It is convenient to introduce the following basis vectors:

$$|n, m\rangle^{(0)} = |n - m\rangle_a \otimes |m\rangle_f\,, \quad \hat{N}|n, m\rangle^{(0)} = n|n, m\rangle^{(0)}\,, \quad 0 \leq m \leq n\ . \quad (3)$$

Here, $|m\rangle_f$ denotes the Fock state of the field, while $|n - m\rangle_a$ is the state of the atomic subsystem, symmetrical with respect to the permutations of the atoms. The dimension of the subspace corresponding to the eigenvalue n of the operator $\hat{N}$ is $n + 1$. The initial condition is $m = n$.

In general, the time evolution of the average photon number is calculated through formula

$$\bar{n}(t) = \sum_{n=0} P(n)^{(0)}\langle n,n|e^{i\mathrm{H}t}\, a^{+} a\, e^{-i\mathrm{H}t}|\, n,n\rangle^{(0)}$$

$$= \sum_{n=0} P(n) \sum_{p,q=0}^{n} A_{np}^{(n)} A_{nq}^{(n)} e^{i\,g\,t\,(\Lambda_{p,n}-\Lambda_{q,n})} \sum_{m=0}^{n} m\, A_{mp}^{(n)} A_{mq}^{(n)} \ . \qquad (4)$$

$P(n) = \exp(-\bar{n}_0)\frac{\bar{n}_0^{\,n}}{n!}$ is the Poissonian photon number distribution and $\bar{n}_0$ is the initial average number of coherent photons. $A_{mp}^{(n)}$ denotes the components of the eigenvector of the Hamiltonian (2), while $\Lambda_{p,n}$ is the eigenvalue corresponding to this eigenvector: $A_{mp}^{(n)} = {}^{(0)}\langle\, n, m \mid n, p\,\rangle\,,\, H\,|\,n,p\rangle = \Lambda_{p,n}|\,n,p\rangle$. The approximate forms of the quantities $A_{mp}^{(n)}$ and $\Lambda_{p,n}$ have been found by us in [6].

Here, we construct the perturbation theory with a small parameter ϵ_n:

$$\epsilon_n = (N - \frac{n}{2} + \frac{1}{2})^{-1} \ , \qquad (5)$$

i.e. the initial photon number is assumed to be much less than the total number of the atoms.

In particular, in the zeroth-order approximation in ϵ_n the spectrum of the eigenfrequencies is equidistant within each subspace with the fixed n and has the form:

$$\Lambda_{p,n} = (N - \frac{n}{2} + \frac{1}{2})^{\frac{1}{2}} \lambda_{p,n}^{(0)}\,, \quad \lambda_{p,n}^{(0)} = n - 2p\,, \quad 0 \le p \le n \ . \qquad (6)$$

It is worth noting that due to our choice of the form of the parameter ϵ_n the first–order corrections to the eigenfrequencies vanish, i.e. $\lambda_{p,n}^{(1)} = 0$. In consequence, the spectrum of the eigenfrequencies remains equidistant in this approximation as well.

Using the eigenvectors found by us in [6] and the properties of the matrix elements of the SU (2)–group representations, in the zeroth–order approximation from (4) we get that the mean photon number evaluates as follows:

$$\bar{n}(t) = \frac{\bar{n}_0}{2}\left[1 + \sum_{n=0} P(n)\cos\left(2\,g\,t\,\sqrt{N - \frac{n}{2}}\,\right)\right] \ . \qquad (7)$$

With respect to the assumed condition $\bar{n}_0 \ll N$ and to the properties of the Poissonian distribution we can abbreviate summation in (7) on n less than N. Hence, the term under the square root is always positive.

The spread in Rabi frequencies is solely related with the graininess of the quantized field mode. As in the case of the JCM we deal here with one series of revivals and in consequence the envelope of the quantum collapse of the mean photon number remains Gaussian in form in this order of approximation.

The above approximation is valid for the sufficiently small values of $\frac{\bar{n}_0}{N}$. The cooperavity of the system changes, in this approximation, the magnitude of the Rabi frequencies only.

The quasisteady-state values of the mean photon number (7) reached either at very long times or between collapse and revival is $\bar{n}(t) = \frac{\bar{n}_0}{2}$.

In turn, accurate to ϵ_n, we find:

$$\bar{n}(t) = \frac{\bar{n}_0}{2}\left\{1 + \sum_{n=0} P(n)\left[\cos\left(2gt\sqrt{N - \frac{n}{2}}\right) + \right.\right.$$
$$\left.\left. + \frac{n}{8\left(N - \frac{n}{2}\right)}\left(1 - \cos\left(4gt\sqrt{N - \frac{n}{2}}\right)\right)\right]\right\} . \qquad (8)$$

In this approximation a new collective term oscillating at double Rabi frequency $4\Omega = 4gt\sqrt{N - \frac{n}{2}}$ appears. However, as previously, its spread depends on the photon number distribution solely. We now deal with two series of revivals and both of term are due to the photon statistical mechanism [5,7]. Each series is Gaussian in form but they have different width. The width of the first series (2Ω) is obviously twice the width of the second series (4Ω). Moreover, both series contribute to the mean photon number with different weights. The amplitudes of the second series are seriously diminished by the factor $\frac{n}{8(N-\frac{n}{2})}$ in comparison with those for the first series. The total collapse, which is a linear superposition of these two series, is no longer Gaussian in form in this approximation in ϵ. Both series of revivals are observable in Fig.2. The second series of revivals (4Ω) leads to weak enhancement of the oscillation amplitudes between strong revivals of the first series (2Ω).

In order to include the collective mechanism of revivals we have to make calculations within an accuracy of ϵ^2. For this purpose it is sufficient to take into account the terms calculated in the zeroth-order approximation for the eigenvectors and in the second-order approximation for the eigenfrequencies. Then, obviously, only revivals of the significant first photon statistical series will be modulated by the collective mechanism. Thus instead of (7) we have

$$\bar{n}(t) = \frac{n_0}{2}\left\{1 + \sum_{n=0} P(n)\left[\frac{n!}{2^n\, p!\,(n-p)!}\cos gt\left(\Lambda_{p+1,n+1} - \Lambda_{p,n+1}\right) + \right.\right.$$
$$\left.\left. + \frac{n}{8\left(N - \frac{n}{2}\right)}\left(1 - \cos gt\sqrt{N - \frac{n}{2}}\right)\right]\right\} , \qquad (9)$$

where the eigenfrequency $\Lambda_{p,n}$ within an accuracy of ϵ^2 reads:

$$\Lambda_{p,n} = (N - \frac{n}{2} + \frac{1}{2})^{\frac{1}{2}}\left(\lambda^{(0)}_{p,n} + \epsilon_n^2\lambda^{(2)}_{p,n}\right) , \qquad (10)$$

$$\lambda^{(2)}_{p,n} = -\frac{(n-2p)}{16}\left[5\,p\,(n-p) - \frac{(n-1)(n-2)}{2}\right] , \qquad (11)$$

$\lambda^{(0)}_{p,n}$ is given by (6).

The time dependent part of the pure zeroth-order approximation (7) is certainly implicit in the formula (9). Namely, it is obtainable from the first term in square brackets of (9), if we put $\epsilon_n^2 = 0$ in (10). Then, in fact, the difference

$\Lambda_{p+1,n+1} - \Lambda_{p,n+1}$ becomes independent of p and after some simple algebra we find that this term goes over into $\cos\left(2gt\sqrt{N-\frac{n}{2}}\right)$ as it should be.

In general, it is seen from (10) and (11) that the spectrum of the eigenfrequencies is non-equidistant in the second-order approximation in ϵ. Since the eigenfrequencies are now non-commensurate beatings between the terms with different atomic frequencies, resulting in additional modulation of $\bar{n}(t)$, will occur. This is simply the collective mechanism of the spread in Rabi frequencies. The above calculated correction contributes to the collective mechanism with the highest weight and, as mentioned, is responsible for the saddles in the revival series 2Ω.

In Fig.2 the dynamics of $\bar{n}(t)$ for $N = 15$ and $n_0 = 4$ is presented. The agreement between the exact numerical solution and our analytical one is excellent. Both envelopes manifest saddle–like forms of the revival series 2Ω. The

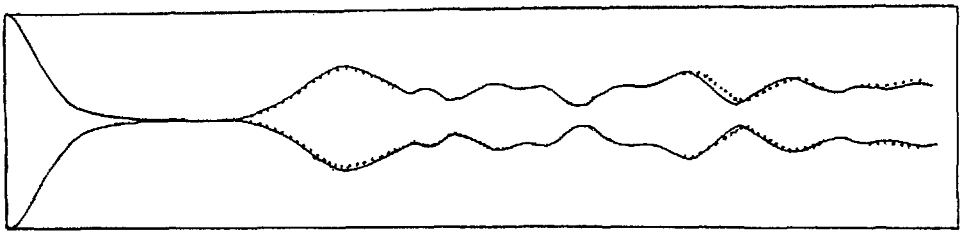

Fig. 1. The envelope of the mean photon number $n(t)$: $N = 15, n_0 = 4$.
——— exact (computer simulation)
o o oo from (9)
128 periods of oscillations are presented.

main results of this paper are contained in (7)-(9). Our comparative computer calculations allow us to conclude that (7) describes correctly the time evolution of $\bar{n}(t)$ for the extremely small values of the ratio $\frac{\bar{n}_0}{N}$. In turn, (9) is sufficient to describe the evolution of the system even for two moderate values of $\frac{\bar{n}_0}{N}$.

Finally we want to point out that the system under consideration may be viewed as possessing the approximate dynamical symmetry (in a sence of the works [8,9]) with SU(2) as the appropriate dynamical symmetry group. In the case $\frac{\bar{n}_0}{N} \to 0$ this approximate dynamical symmetry becomes an exact one and the Hamiltonian V from (2) becomes the generator of SU(2) group representation. Our example shows that the presence of approximate dynamical symmetry gives the possibilities for qualitative and quantitative description of the system dynamics.

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