Superfluidity: Cold Atoms to Neutron Stars

S. Ramanan[1],* M. Urban[2], and Viswanathan Palaniappan[1]
[1] *Department of Physics, IIT Madras, Chennai, India and*
[2] *Université Paris-Saclay, CNRS-IN2P3, IJCLab, 91405 Orsay, France*

Introduction

Neutron stars, born as a result of a supernova explosion, presents a rich system of asymmetric nuclear matter, infinite in extent, that is held is together by gravity. The interaction between constituent particles, results in a diverse landscape of matter ranging from nuclei at terrestrial densities at the outermost layers, to neutron rich nuclei and neutron superfluid interspersed between a lattice of quasi-nuclei clusters to finally a core, whose composition is still under investigation. A complete understanding of the physics of the crust and the outer layers of the core of the neutron star remains far from satisfactory due to the uncertainties in the nuclear interaction that is strongly modified by the nuclear medium.

A good starting point to understand the physics the inner layers of the crust where the neutrons form a superfluid is to model the matter to be made of up of *only* neutrons and to ignore the effects of the lattice. At low-densities ($k_{\mathrm{F}} < 0.8\,\mathrm{fm}^{-1}$), the interaction between the neutrons is dominated by two-body interactions and the three-body effects usually become important at higher densities, although still relevant to the densities present in the inner crust. In addition, there are medium effects that result in important modification to the interaction between two neutrons.

The free-space two-neutron interaction is characterised by a large S-wave scattering length (corresponding to the nearly bound state at very low energies). While a large scattering length naturally occurs in nuclear systems, in a laboratory, it is possible to tune the scattering length of an atomic gas of ultracold fermions to very large values via Feshbach resonances and in this respect, the neutron gas and cold atomic fermions are very similar. The simplicity of atomic interactions makes for an ideal test bed of various theoretical approaches that are relevant to neutron star physics. In fact, ultracold fermi gas can be considered as a quantum simulator for a neutron star. It is in this context that Bertsch proposed the unitary fermi gas as a possible model for pure neutron matter [1].

In this talk, we present our results for the equation of state of superfluid fermi gas (atomic gas and gas of neutrons). For the interaction, we use the renormalization group based low-momentum effective interactions [2] and the equation of state is calculated using the Bogoliubov many-body perturbation theory (BMBPT) [3, 4]. In the following two sections, we outline the approach and highlight key results.

Formalism

The interaction between two-nucleons is determined by the one pion exchange potential for the long-distance attraction, and is augmented phenomenologically to include intermediate scale attraction and short range repulsion. The parameters in such a two-body interaction are fitted to reproduce the experimental scattering phase shifts and the binding energy of the deuteron. When such interactions are used as inputs in many-body calculations, strong model dependence arises due to the coupling between low and high momentum states. Over the last two decades, the renormalization group (RG) approach has been used to achieve the required decoupling thereby resulting in soft effective interactions that is valid at low-momenta (typically specified by a scale Λ). Such interactions yield better convergence when used as inputs in many-

*Electronic address: suna@physics.iitm.ac.in

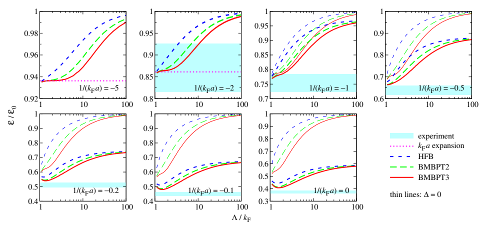

FIG. 1: Ultracold atomic gas: equation of state scaled with respect to the energy of the non-interacting ultracold gas as a function of the cutoff.

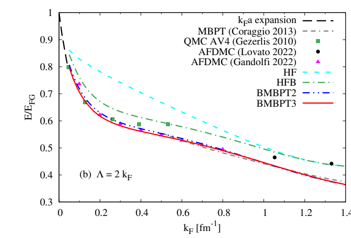

FIG. 2: Pure neutron matter: equation of state as a function of density.

body calculations. Physical quantities should be independent of RG cutoff Λ and any dependence can be used as a test of missing contributions in a many-body calculation.

On the other hand, in cold atomic systems, the effective range is several orders of magnitude smaller than the scattering length and as a result, it is customary to model these systems via a contact interaction. However, a contact interaction requires infinite resummation in a many-body system. To improve convergence in the many-body calculations for the cold atomic system, as we will see in the talk, it is beneficial to construct soft effective interactions using the RG approach.

The ground state of ultracold Fermi gases and neutron gas is a superfluid. Therefore, the equation of state is calculated using the Bogoliubov many-body perturbation theory, that builds perturbative corrections (up to third order) around the Hartree-Fock Bogoliubov ground state, thereby taking into account the superfluid nature of these fermionic systems.

Results

In this section, the main results are briefly highlighted. Fig. 1 shows the equation of state suitably scaled by the energy of the non-interacting gas for a system of ultracold fermions, where the panels represent the varying strengths of the scattering length, starting from the weak coupling regime ($1/(k_{\mathrm{F}}a) = -5$) to the unitary limit ($1/(k_{\mathrm{F}}a) = 0$) which is strongly correlated. The thick lines are the BMBPT results, while the thin lines (where visible) represent the equation of state using the ordinary many-body perturbation theory (MBPT). The BMPT results show convergence in perturbation theory for all strengths of interaction. In addition, in the weak coupling regime, one observes convergence at low values of the RG cutoff.

Fig. 2 presents the equation of state of pure neutron matter as a function of density within BMBPT as well as compares our main result with other calculations in the literature. It is worth noting that our results including third order BMBPT corrections reproduce the low-density $k_{\mathrm{F}}a$ expansions. In addition, the BMBPT results are good agreement with the latest Quantum Monte Carlo calculations.

The cutoff dependence of the results (both in Fig. 1 and Fig. 2) and subsequent work including three-body will be discussed in the talk.

Acknowledgments

We acknowledge the funding support through the Indo-French project IF-CAR/CEFIPRA 6304-4.

References

[1] G.A. Baker, Phys.Rev. C60 (1999) 054311.

[2] S. K. Bogner, R. J. Furnstahl and A. Schwenk, Prog. Part. Nucl. Phys. **65**, 94 (2010).

[3] M. Urban and S. Ramanan, Phys. Rev. A **103** (2021) 063306.

[4] V. Palaniappan, S. Ramanan and M. Urban, Phys. Rev. C **107** (2023) 025804.