THE LIMIT OF APPLICABILITY OF THE THEORY OF WEAK INTERACTIONS

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We calculate those higher order corrections to the weak interactions which destroy the equality of β decay and μ decay coupling constants and from which the processes $\mu \to e+\gamma$ and $\mu \to 3e$ should arise. These calculations all depend on a cut-off parameter. An experimental upper limit for the cut-off is fixed by the experimental upper limits for the $\mu \to e+\gamma$ and $\mu \to 3e$ decay rates.

The local four-Fermi theory is assumed for the weak interaction. A way of checking the validity of the local four-field form is to consider the higher order effects which are particularly strong in this theory and give rise to the above-mentioned effects. The main contributions will be those involving only leptons in intermediate states, the baryon terms being supressed by form factors due to the strong interactions.

From dimensional considerations it is obvious that the weak interactions become strong at energies $E \approx \frac{1}{\sqrt{g}} \approx 10^3$ BeV. This means that in the calculation of radiative corrections the cut-off should be some number less than $\frac{1}{\sqrt{g}}$. In the present work we shall estimate the value of the cut-off, Λ, from experimental data.

The experimental coupling constants in β and μ decay agree with high accuracy. This equality, however, can be proved only if one neglects the electromagnetic and weak radiative corrections. The connection between the bare and renormalized charge in β decay and in μ decay differs due to the form factors for the strongly interacting particles. In β decay we have

$$g_\beta^2 = g_0^2 Z_{1\beta}^{-2} Z_2^2 (2-Z_2)^{-2} \tag{1}$$

and in μ decay,

$$g_\mu^2 = g_0^2 Z_{1\mu}^{-2} Z_2^4 (2-Z_2)^{-4}. \tag{2}$$

Here $Z_{1\mu}$ and $Z_{1\beta}$ are the vertex renormalizations for μ and β decay respectively. Z_2 is the wave function renormalization factor for the leptons. The Z_2 factors for the μ meson, the electron and the neutrino, are considered to be equal since we are interested in momenta of the virtual particles of the order of Λ, where one may neglect all masses.

The equality, $g_\beta^2 = g_\mu^2$, can be realized in the case when the ratio of the renormalization factors in Eqs. (1) and (2) equals one, i.e.,

$$\frac{Z_{1\mu}^{-2}}{Z_{1\beta}^{-2}} = Z_2^{-2}(2-Z_2)^2 \tag{3}$$

We shall retain only terms of the form $(g\Lambda^2)^n$ in our answer, dropping those of the form, $(g\Lambda^2)^n(gm^2)^q$.

Therefore in our matrix elements we may put the momentum of each particle equal to zero. As an example let us first consider the corrections to the vertex part in μ decay corresponding to Fig. 1. We shall prove that the highest order terms in the cut-off are missing.

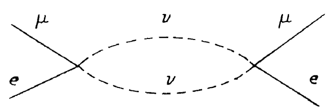

Fig. 1 Corrections to the vertex part in muon decay.

Since the muon and electron lines enter the diagram in Fig. 1, at a point, the (μe) combination of fields can be described formally as an external vector field,

$$A_\lambda = \bar{\psi}_\mu \gamma_\lambda (1+\gamma_5)\psi_e + \bar{\psi}_e \gamma_\lambda (1+\gamma_5)\psi_\mu ,$$

which is independent of the coordinates by virtue of the condition $p_\mu = p_e = 0$. The vacuum polarization (Fig. 2) by the field A_λ is given by

$$M = \mathrm{Tr}\, \gamma_\lambda G(x,x) \tag{4}$$

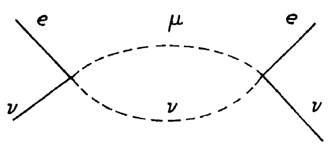

Fig. 2 Another diagram contributing to vertex parts.

where $G(\chi, y)$ is the Green's function of the neutrino in the external field. We can show from invariance under the transformation,

$$\psi'_\nu = e^{i\phi}\psi_\nu,\ \bar{\psi}'_\nu = e^{-i\phi}\psi_\nu,\ \phi = A_\lambda x_\lambda \tag{5}$$

that this Green's function is given by

$$G(xy) = \{\exp i[\phi(x) - \phi(y)]\} G_0(x,y) \tag{6}$$

where $G_0(xy)$ is the Green's function for the free neutrino field. Therefore $G(xx) = G_0(xx)$ and the contributions of Fig. 1 or of any series of neutrino loops, vanishes. This result applys to all diagrams with only neutrino loops and with all external electrons and muon lines coming in pairs from points. One can prove such a result also for diagrams of the type shown in Fig. 2.

We see, then, that corrections in first order in gA^2 to the vertex parts in β decay do not exist. In μ decay we obtain such corrections if we allow, in addition to the interaction, $(\bar{\mu}\nu)(\bar{\nu}e)$ interactions of the form $(\bar{e}\nu)(\bar{\nu}e)$ and $(\bar{\mu}\nu)(\bar{\nu}\nu)$, giving rise to the non-vanishing diagrams of Fig. 3. These diagrams give a vertex renormalization,

$$Z_{1\mu}^{-1} = 1 + \sqrt{2}\, gA^2/\pi^2 \tag{7}$$

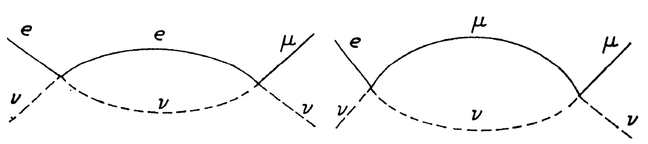

Fig. 3 Lowest order corrections to muon decay.

Since, to first order in gA^2, $Z_2 = 1$ and $Z_{1\beta} = 1$, we get from Eq. (3),

$$\frac{g_\beta^2}{g_\mu^2} = 1 - 2\sqrt{2}\, gA^2\pi^2 \tag{8}$$

From the experimental fact that g_β^2 and g_μ^2 differ by no more than 5% we obtain an upper limit for the cut-off, $A < 120$ BeV.

If the interactions $(\bar{e}\nu)(\bar{\nu}e)$ and $(\bar{\mu}\nu)(\bar{\nu}\mu)$ do not exist then we have charge renormalization only in the approximation g^2A^4. To calculate these corrections we note that in β decay there is a result analogous to Ward's identity, which in this case applies to all diagrams for vertex parts except the chains of Fig. 2,

$$\Gamma(0) = \bar{u}_e \left.\frac{\partial G^{-1}(p)}{\partial p_\lambda}\right|_{p=0} (1+\gamma_5) u_\nu \bar{u}_p \gamma_\lambda (1 + a\gamma_5) u_n \tag{10}$$

where $a = g_A/g_V$ and $G(p)$ is the Green's function of the electron. Therefore the correction for the vertex part of Fig. 4 can be obtained by differentiating the

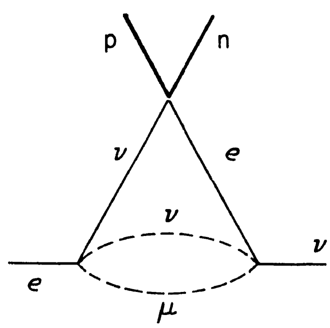

Fig. 4 A contribution in nuclear beta decay.

diagram (Fig. 5) for the mass operator. Chains of the type of Fig. 2 do not contribute since they do not contain the higher terms of order $(gA^2)^n$. Therefore one can say that Eq. (10) is valid for the whole vertex part.

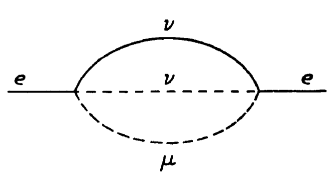

Fig. 5 The electron mass operator.

From Eq. (10) we obtain,

$$Z_{1\beta} = Z_2(2 - Z_2)^{-1}$$

and consequently from Eq. (1) the β decay constant is not renormalized by the weak interactions.

For the μ decay diagrams of Fig. 6, we have a relation analogous to Eq. (10). We find

$$Z_{1\mu}^2 = Z_2^2(2-Z_2)^{-2} . \tag{12}$$

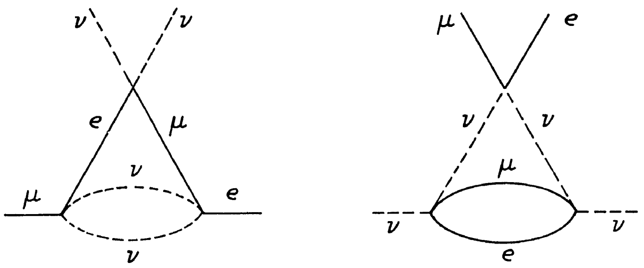

Fig. 6 Muon decay diagrams.

When the contribution of Fig. 7 is included we finally obtain

$$\frac{g_\beta^2}{g_\mu^2} = 1 - \frac{g^2 \Lambda^4}{\pi^4} \tag{13}$$

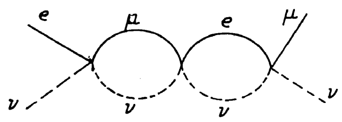

Fig. 7 Another muon decay diagram.

We obtain in this case another upper limit for the cut-off, $\Lambda < 400$ BeV.

In the process $\mu \to e + \gamma$ we calculate the diagrams of Figs. 8 and 9, obtaining

$$\frac{W_{e+\gamma}}{W_{\mu+\nu+\bar{\nu}}} = \frac{2}{3\pi^5} e^2 g^2 \Lambda^4 \left(\ln \frac{\Lambda^2}{\mu^2} \right)^2 .$$

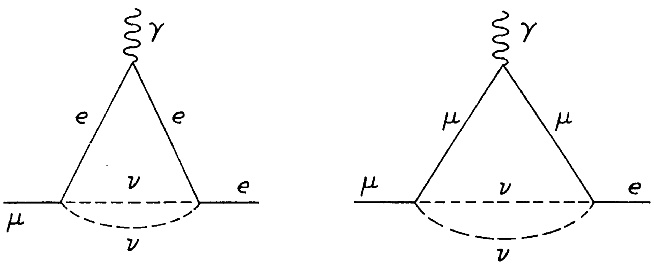

Fig. 8 Diagrams for $\mu \to e + \gamma$.

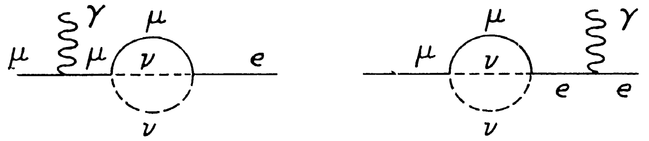

Fig. 9 Two other $\mu \to e + \gamma$ diagrams.

From this and the experimental upper limit of $\mu \to e + \gamma$ we find $\Lambda \lesssim 50$ Bev.

The ratio of the $\mu \to 3e$ probability to the usual decay has been calculated from the diagrams of Fig. 10 with the result, $\Lambda \lesssim 100$ BeV.

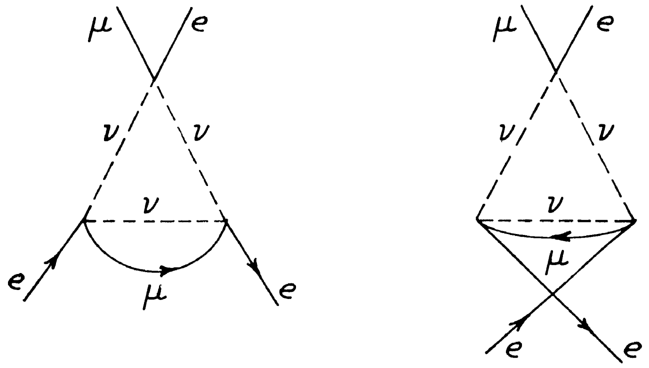

Fig. 10 The three-electron decay mode of the muon.