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# STRING/GAUGE DUALITY AND SOFT–HARD POMERON* **

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We provide a brief review for the motivation of Maldecena duality conjecture, leading to the calculation for the glueball spectrum and the Pomeron intercept in the strong coupling limit. We next turns to the question of hard scattering. Using Maldacena's gauge/string duality, a unified description is provided for both the soft and the hard Pomeron in the strong coupling limit.

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## 1. Introduction

It has been a long held belief that QCD in a non-perturbative sitting can be described by a string theory. In such a framework, Pomeron should emerge as a closed string excitation. The Maldacena conjecture [1] and its further extensions [2, 3] state that Yang–Mills theory is exactly dual to a critical string theory in a non-trivial gravitational background. The particular target Yang–Mills theory depends on the geometry and symmetries of the string/gravity dual. By an appropriate choice, one is led to a specific suggestion as to how quarkless QCD (or SU($N$) Yang–Mills theory) may be represented by a theory of closed strings.

Let us recall that in the early days of string theory, (or the "dual resonance model" to use the nomenclature that predates both string theory and QCD), one observed that it was reasonable to represent the hadronic spectrum beginning with zero width "resonances" on exactly linear trajectories. With the advent of QCD this approach was reformulated as the $1/N$ expansion at fixed 't Hooft coupling, $g^2_{\rm YM}N$. States with vacuum quantum

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numbers could be assigned to closed-strings, including a massive $2^{++}$ tensor glueball on the leading Pomeron trajectory, which can be parameterized by

$$\alpha_P(t) = \alpha_P(0) + \alpha'_P\, t\,. \tag{1}$$

Soon a two-fold crisis appeared — zero-mass states and extra dimensions. A careful study of negative norm states (*i.e.* ghosts), tachyon cancellation and the consistency of the perturbative expansion at the one loop level led to supersymmetric string theories in 10 space-time dimensions. At the one loop level unitarity required that pair creation of two open strings, each contains "zero-mass" spin-1 states, is dual to a vacuum exchange with a "zero-mass" $2^{++}$ state. That is, we have a "Pomeron" trajectory with an intercept $\alpha_P(0) = 2$. In fact the low energy theory was clearly not QCD but rather **supergravity in 10 dimensions**!

What is the mechanism which restores the 4-$d$ space/time and generates a non-zero mass gap? How can one "lower" the Pomeron intercept so that $\alpha_P(0)$ takes on its phenomenological value [4] of $1.1 \sim 1.2$? Without getting into technical details, let us demonstrate how the incorporation of extra-dimensions with a nontrivial background provides a natural mechanism for mass-generation. Consider a situation where the addition of a fifth-dimension, $r$, leads to a metric, $ds^2 = r^2 \sum_{i=1,2,3,4} dx_i^2 + w(r)^{-2} dr^2$, where $w(r)$ takes on a non-trivial form, e.g., $w(r)^2 = r^2 - r^{-4}$. A minimally coupled massless scalar field, $\phi(x_\mu, r)$, then satisfies a simple wave equation $\{\partial_\mu^2 + \sigma(r)^{-1}\partial_r \tau(r)\partial_r\}\phi(x_\mu, r) = 0$, where $\tau(r) = r^4 w$ and $\sigma(r) = r^2 w^{-1}$. If one attempts to send a 4-$d$ plane-wave with 4-momentum $p_\mu$, *i.e.*, $\phi(x, r) = e^{ip\cdot x}\phi(r)$, one winds up having a "massive" propagation, where $-p^2 = m^2 > 0$, and

$$-\partial_r \tau(r)\partial_r \phi(r) = m^2 \sigma(r)\phi(r). \tag{2}$$

The discrete mass-gap, $m$, can be found by treating this as an eigenvalue problem.

## 2. AdS/CFT for QCD

One example of this approach begins with type-IIB string theory leading to $QCD_3$ strings. We know that there are many extended solitonic objects beyond the perturbative string expansion. In particular in IIB, there are 3 + 1 dimensional object called Dirichlet 3-branes, (D3-branes), and the low-energy dynamics of a set of $N$ parallel D3-branes is described by open strings with end-points restricted on these branes. Indeed the effective theory (or Born–Infeld action) reduces exactly to SU($N$) Yang–Mills theory at weak coupling. (To be exact in the present context the full target gauge theory is $N = 4$ SUSY YM in 4-$d$.) Note that one set of zero-mass states, the gauge bosons, is no longer an embarrassment as they represent the weakly coupled gluonic modes. The Maldacena conjecture states that both the open string/Yang–Mills and the closed string/gravity descriptions are **simultaneously** true or equivalent in the near horizon limit, where the above metric is $AdS^5 \times S^5$.

However this background is so symmetric that the resultant target Yang–Mills theory is conformal with $N = 4$ supersymmetries. Witten [2] has suggested a further modification to remove these unwanted symmetries. To arrive at $QCD_4$ as the target theory, one begins with the eleven dimensional M theory on $AdS^7 \times S^4$ or 10-$d$ type-IIA string theory. After appropriate compactification, one is led to a $AdS^7$ black hole metric,

$$ds^2 = \left(r^2 - \frac{1}{r^4}\right) d\tau^2 + r^2 \sum_{i=1,2,3,4,11} dx_i^2 + \left(r^2 - \frac{1}{r^4}\right)^{-1} dr^2 + d\Omega_4^2 \,. \quad (3)$$

To compute the glueball excitations for $QCD_4$ in the extreme strong coupling limit, one simply needs to find the spectrum of harmonic fluctuations for the bosonic supergravity fields around these AdS black hole backgrounds. [5–7] We have noted [8] that there is indeed a rather remarkable correspondence of the overall mass and spin structure captured by the lowest states on the type-IIA supergravity bosonic multiplet when capered to lattice calculations [9]. In addition the exact AdS $2^{++}/0^{++}$ degeneracy corresponds to a relatively small splitting in the lattice calculations. Finally, there is a radial excitation of the pseudoscalar $0^{*-+}$ that suggests that even this effect is approximated.

We next turn to a discussion on the slope of the leading glueball trajectory. The Pomeron is the leading Regge trajectory passing through the lightest glueball state with $J^{PC} = 2^{++}$. In a linear approximation, it can be parameterized by $\alpha_P(t) = 2 + \alpha'_P(t - m_{\rm T}^2)$, where we can use the strong coupling estimate for the lightest tensor mass, $m_{\rm T} \simeq [9.86 + 0(\frac{1}{g^2N})]\, \beta^{-1}$. The Pomeron slope can be related to the QCD string tension, which can also be calculated in this approach [10]. One finds that $\alpha'_P \simeq [\frac{27}{32\pi^2 g^2 N} + 0(\frac{1}{g^4N^2})]\, \beta^2$, if we make the standard assumption that the closed string tension is twice that between two static quark sources. Putting these together, we obtain a strong coupling expansion for the Pomeron intercept

$$\alpha_P(0) \simeq 2 - 0.66 \left(\frac{4\pi}{g^2N}\right) + 0 \left(\frac{1}{g^4N^2}\right) . \quad (4)$$

Turning this argument around to estimate a crossover value between the strong and weak coupling regimes by fixing $\alpha_P(0) \simeq 1.2$ at its phenomenological value [4]. In fact this yields for $N = 3$ QCD, a reasonable value for the fine structure constant: $g^2/4\pi = 0.176$ at a characteristic confinement scale, $\Lambda_{\rm QCD}$. Such estimates have proven sensible in the lattice approach to strong coupling QCD. Much more experience with this new approach to strong coupling must be gained before such numerology can be taken seriously.

However, in this estimate, we have not properly incorporated the effect of hard scattering. This will be done after we understand the role of BFKL Pomeron in the strong coupling limit.

## 3. Hard scattering

It is generally acknowledged that 't Hooft's $1/N_c$ expansion for QCD perturbation theory should map order by order onto the topological expansion for some kind of "QCD string". However, until the Maldacena's explicit examples of String/Gauge duality, a direct relationship between the fundamental superstring and a possible "QCD string" was lacking. Most problematic for such a connection is the conflict between the exponentially soft properties of superstrings and partonic behavior of leading $1/N_c$ diagrams of QCD.

Specifically, at wide angles QCD amplitudes scale as

$$T_{\rm QCD}(s,t) \sim \left(\sqrt{\alpha'_{\rm QCD}}\,p\right)^{4-n},$$

where $p \sim \sqrt{s} = \sqrt{-t}$ and $n = \sum_i n_i$ is sum of the minimal number of "partons" (or twist) in the external bound-state wavefunctions. (The small logarithmic corrections due to asymptotic freedom are beyond the present analysis.) In contrast the closed string scattering amplitude with linear trajectories $\alpha_s = \alpha'_{\rm s} s + \alpha_0$, is exponentially suppressed, $A_{\rm string}(s,t) \sim \exp[-\alpha'_{\rm s}(s\ln s + t\ln t + u\ln u)]$, indicative of its lack of point-like constituents. Although this soft behavior is a virtue in graviton scattering amplitudes allowing for a finite quantum theory, it is a disaster for a QCD string.

In a recent interesting paper, Polchinski and Strassler [11] may have begun to resolve this fundamental difficulty for a QCD string. They consider IIB strings scattering in an $\mathrm{AdS}^5$ background, breaking conformal invariance with an IR cut-off in the "radial" coordinate as a string model for hadrons. We have been able to extend their idea to the $\mathrm{AdS}^7 \times \mathrm{S}^4$ Black Hole background of M theory [8, 12].

## 4. Heterotic Pomeron in AdS background

In spite of this progress in seeing some partonic effects in the string picture, there is much more to be understood. For instance, we note that, consistent with the known spectrum of glueballs at strong coupling, the IR-region must in addition give a factorizable Regge pole contribution,

$$T(s,t) \sim A(s,t,r_{\min}) \sim (\alpha'_{\rm QCD} s)^{\alpha_P(0)+\alpha'_{\rm QCD} t}\,. \tag{5}$$

In our M theory construct [8] , $\alpha_P(0) = 2 - 0(1/g^2N)$, whereas in an AdS$_5$ setting, one expects $\alpha_P(0) = 2 - 0(1/\sqrt{g^2N})$. Of course, this "soft" Pomeron must mix with the corresponding hard component, leading to a single Pomeron singularity in the large $N$ limit. Mixing soft and hard components in a consistent fashion has led to the notation of a Heterotic Pomeron [13, 14]. It is then natural to ask how this mixing can take place also in an AdS-dual approach.

Regge factorization intrinsically requires non-local interaction, which emerges naturally in a stringy setting. It is therefore natural to ask, starting from a dual-string amplitude, how best can one characterize its asymptotic Regge behavior.

It is not difficult to show that the high energy limit of a dual amplitude satisfies a light-cone diffusion equation. Using a LC path integral representation for a long thin closed string, we can derive a diffusion equation for a propagator, which in flat space, takes on the form

$$(\partial_y - \alpha' \partial_b^2 - 2) G(y, \vec{b}; y_0, \vec{b}_0) = \delta(y - y_0)\delta^{(d-2)}(\vec{b} - \vec{b}_0)\,. \tag{6}$$

Here $y$ is the longitudinal rapidity, $\vec{b}$ is the transverse impact parameter. Generalizing to a deformed AdS background with confinement, we propose a generalized Master LC Diffusion Equation,

$$(\partial_y - \alpha' D_{\rm LC} - 2) G(y, r, \vec{b}; y_0, r_0, \vec{b}_0) = \delta(y - y_0)\delta(r - r_0)\delta^2(\vec{b} - \vec{b}_0)\,, \tag{7}$$

where $D_{\rm LC}$ is the light-cone Laplacian, with $r$ being the AdS-direction. The longitudinal boost can be diagonalized by going to the $J$-plane. Similarly, due to translational and rotational invariance in $\vec{b}$, this equation can also be diagonalized in terms of transverse momenta. In terms of hadronic scales, $\Big($glueball mass scale $\Lambda = \frac{r_{\min}}{R^2}$ and QCD (inverse) string tension $\alpha'_{\rm QCD} = \alpha' \frac{R^2}{r^2_{\min}} = \frac{1}{\Lambda^2 \sqrt{g^2 N}}\Big)$, one finds that the LC Master Diffusion Equation can be written as

$$\Big\{ - D_r - \frac{r^2_{\min} t}{r^2 \Lambda^2} \Big\} \Psi(r, J, t) = \sqrt{g^2 N}(2 - J)\,\Psi(r, J, t). \tag{8}$$

$D_r$ is the Laplacian in $r$, which for an AdS$_5$ motivated black-hole background, is

$$D_r = r^{-3} \partial_r r (r^4 - r^4_{\min}) \partial_r\,. \tag{9}$$

It is instructive to consider this Master Diffusion Equation in various limits.

- **Tensor Glueball:** With $J = 2$, the right hand side of the equation vanishes. It can be seen this truns into an eigenvalue equation for $t = m^2_{\rm gb}$, which yields the tensor glueball masses. [7, 8]
- **Conformal Limit at $t = 0$–BFKL Pomeron:** Consider $t = 0$ and assume that $D_r$ is conformal, *i.e.*, as $r \to \lambda r$, $D_r$ is invariant, *i.e.*,

$$D_r(\lambda r) = D_r(r)\,. \tag{10}$$

(In strong coupling, $D_r = r^{-3}\partial_r r^5 \partial_r$ is conformal.) In this case, the equation can be solved by a Fourier transform in $u \equiv \log r$. One can show that $e^{i\nu u}$ is an improper eigenvector, with eigenvalue

$$D_r e^{i\nu u} = \chi(\nu) e^{i\nu u}\,, \tag{11}$$

which corresponds to the anomalous dimension. One can then show that this leads to a BFKL Pomeron, located at

$$J = 2 - \chi(\nu^*)\,, \tag{12}$$

where $\nu^*$ is a saddle point where $\chi'(\nu^*) = 0$.

- **Pomeron Intercept:** This again can be found by solving our Master Equation at $t = 0$. It follows $\alpha_{\rm Pom}(0)$ can be determined by the following eigenvalue equation:

$$-D_r \Psi = \sqrt{g^2 N}(2 - \alpha_{\rm Pom}(0))\,\Psi\,, \tag{13}$$

where one must use a confining background. It follows that $\alpha_{\rm Pom}(0) = 2 - 0(1/\sqrt{g^2 N})$, as indicated earlier.

Clearly, the Pomeron Intercept depends on the confinement deformation. It remains to clarify if, at $t = 0$, whether the pole has moved below the BFKL cut or not. At any rate, what we have found is nothing but the strong coupling counter part of a Heterotic Pomeron [13, 14], where the Pomeron trajectory is approximately linear for positive $t$, and levels off as $t$ is moved to negative value approaching the BFKL cut. Much more can be said about this Master Equation, and this will be done in a separate publication [15].

## 5. Discussion

As one might have anticipated, this proposal itself leads to a difficult quantum string theory, even in the large $N$ (or string perturbative) limit. To date the most direct evidence supporting AdS/CFT duality has come from the strong coupling limit (*i.e.*, both large $N$ and large 't Hooft coupling $g^2N$) where the string theory reduces to a classical theory of gravity in a curved background metric. Moreover even in this restricted limit, most comparisons involve various supersymmetric variants of QCD. Nonetheless the situation is still very provocative. In principle, we now have a new approach to the confined phase of QCD in terms of color singlet closed-strings. The task remains to formulate the dual theory more precisely and to find reliable approximations that give it predictive power.

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## REFERENCES

[1] J. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231 (1998).

[2] E. Witten, *Adv. Theor. Math. Phys.* **2**, 505 (1998); *Adv. Theor. Math. Phys.* **2**, 253 (1998).

[3] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, *Phys. Lett.* **B428**, 105 (1998).

[4] C-I Tan, *Phys. Rep.* **315**, 175 (1999). For a review on experimental support for identifying the Pomeron Regge trajectory as a closed-string exchange in large-$N$ QCD, see: A. Capella, U. Sukhatme, C-I Tan, J.T.V. Tran, *Phys. Rep.* **236**, 225 (1994).

[5] C. Csáki, H. Ooguri, Y. Oz, J. Terning, *J. High Energy Phys.* **9901**, 017 (1999). [hep-th/9806021].

[6] R. De Mello Koch, A. Jevicki, M. Mihailescu, J. Nunes, *Phys.Rev.* **D58**, 105009 (1998) [hep-th/9806125].

[7] R.C. Brower, S. Mathur, C-I Tan, *Nucl. Phys.* **B574**, 219 (2000) [hep-th/9908196].

[8] R.C. Brower, S. Mathur, C-I Tan, *Nucl. Phys.* **B587**, 249 (2000) [hep-th/0003115].

[9] C.J. Morningstar, M. Pearson, *Phys. Rev.* **D60**, 034509 (1999).

[10] A. Brandhuber, N. Itzhaki, J. Sonnenschein, S. Yankielowicz, *J. High Energy Phys.* **9806**, 001 (1998) [hep-th/9803263 v3].

[11] J. Polchinski, M. Strassler, *Phys. Rev. Lett.* **88**, 031601 (2002) [hep-th/0109174].

[12] R.C. Brower, C-I Tan, *Nucl. Phys.* **B662**, 393 (2003) [hep-th/0207144].

[13] E.M. Levin, C-I Tan, [hep-ph/9302308]. A brief description can also be found in J.R. Forshaw, D.A. Ross, *Quantum Chromodynamics and the Pomeron*, Sect. 5,6, Cambridge University Press, 1997.

[14] S. Bondarenko, E. Levin, C-I Tan, hep-ph/0306231; *Nucl. Phys.* **A732**, 73 (2004).

[15] R.C. Brower, J. Polchinski, M. Strassler, C-I Tan, in preparation.