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PARTICLE PRODUCTION SPECTRA FOR NEUTRINO BEAM DESIGN

NEUTRINO TASK FORCE REPORT NO. 1

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Introduction:

The spectra of charged pions and kaons produced at the Area I target will dictate the focusing procedures and shielding requirements of the neutrino beam, and also the neutrino flux and distribution at the detector. The angular distribution of secondary mesons from the target will influence the shape of the focusing horn. The muon shield thickness and composition is determined by the intensity and energy of secondary muons; and, in addition, a magnetic shield might require a well collimated muon beam for its successful operation.[1] The energy and angular distributions of charged pion and kaon secondaries from the target, the effects of the focusing device and the size of the shield, in as much as it affects the location of the detector, will determine the neutrino flux and energy distribution at the detector. It is felt, therefore, that a survey of particle production spectra from proton-nucleon interactions at 200 GeV is of the utmost importance in the neutrino beam design.

A comparison of existing models[2,3,4] of particle production in the 200 GeV range indicates that disagreements of as much as a factor of five exist in certain secondary energy regions. There is also only qualitative agreement on the angular distributions of secondaries, and indeed, all models fail at lab angles greater than twenty-five degrees.[14] This situation leads to serious uncertainties in making estimates of some important beam parameters (for example, (1) the size and shape of the neutrino horn, (2) the event rate for low energy neutrinos in the detector).

It has been our hope that a review of existing models, and of newer incomplete models, might suggest some procedure to deal with the problems of the neutrino beam. The possibility of applying different models in different secondary energy regions for greater reliability was considered. Also, an attempt was made to find reasons of physics for choosing one model over another. We regret to report that no clear-cut procedure has been found.

The survey included a study of relevant experiments at accelerator energies and in cosmic-rays, as well as a review of particle yield models. The specific models studies were those of:

(1) Cocconi, Koester and Perkins.[2]

(2) Trilling.[3]

(3) Hagedorn and Ranft[4] (Thermodynamic Model).

(4) Scaling models[5] (in which systematics found at low energies are used to extrapolate spectra to the 200 GeV region).

(5) Multi-peripheral models.[6]

(6) Cocconi model of SS-61 and SS-134, 1969.

A review of other surveys was also carried out and included:

(1) CERN Survey (CERN/ECFA 67/13, Rev. 2, Appendix 4).

(2) Walker B. 5-68-24 (1968 Summer Study).

(3) Koester EA-67-79 (1967 Summer Study).

(4) Cocconi SS-61 (1969 Summer Study).

Experimental Data:

(1) The only source of measured data at 200 GeV is from cosmic ray experiments. For information concerning these measurements, refer to Fowler and Perkins,[7] and Pal and Peters.[8] The results of cosmic ray experiments can be summarized as:

1) multiplicity follows the general law $\bar{n}_s = 2E_p^{1/4}$

2) the nucleon survives the interaction with .6 to .7 of its original energy. This average elasticity is independent of primary energy. However, insufficient data exist for the energy distribution of the nucleons after the collision.

3) the mean transverse momentum of secondaries is constant, (or at least only slowly increasing) for primary energies up to at least 300 GeV.

4) the K/π ratio seems to be independent of primary energy, although current measurements suffer from poor statistics.

Cosmic ray data do not yield direct measurements of secondary pion and kaon spectra. The general features of these distributions mentioned above are determined from muon, electron and gamma ray spectra measured in the atmosphere as well as from nuclear emulsion data on cascade events.

In lieu of direct measurements of pion and kaon energy spectra and angular distributions, one would hope to obtain from existing data sufficient insight into the nature of nucleon-nucleon interactions to be able to predict the desired spectra at 200 GeV. We have, therefore, reviewed the status of measurements at currently available accelerator energies: the reliability of a particle production model is based on its ability to obtain results that are in agreement with available data at all energies. However, we find that no agreement exists on the character of nucleon-nucleon interactions in any energy range - a variety of models exist that give reasonable fits to most of the data. Table I gives a partial list of experiments.

Some of the most important data is now becoming available from the 70 GeV machine at Serpukhov.[15] However, the pion yields have large error bars (about 50%) and are in qualitative agreement with most models. What might prove to be most useful are their K/π ratios which appear to be carefully measured: In light of cosmic ray results, which indicate little variation of the K/π ratio with energy, the Serpukhov data could be used directly to extrapolate to

200 GeV. This procedure is, however, only an extrapolation and lacks physical justification. Figure 1 shows the Serpukhov yield measurements along with predictions of various models.

Perhaps the most interesting attempts to understand secondary particle spectra come from the recent measurements of center-of-mass pion distributions at Argonne and BNL. Some parameters of these experiments are given in Table II. These experiments are based on the presumption that a simple form of the spectra might exist in the nucleon-nucleon center-of-mass system. Indeed, the results of two recent experiments seem to agree on the form of the center-of-mass distributions. From Smith et.al., we write

$$\frac{d^2N}{dp\,d\Omega} = F(p_\perp)\ G\ (p_\parallel)$$

$$F(p_\perp)\,\alpha\ \frac{a_\perp^{5/2}\ p_\perp^{3/2}\ e^{-a_\perp p_\perp}}{(3/4)\,\pi}$$

$$G(p_\parallel)\ \alpha\ a_\parallel e^{-a_\parallel p_\parallel}$$

To say that the center-of-mass is in some way fundamental to nucleon-nucleon interactions is to assume that in such reactions the nuclei react with each other in total-all of the incident nucleus reacts with all of the target nucleus. For example, at high energies, one would not expect the center-of-mass to be of any particular importance in deuteronhelium collisions, since the interaction is between nucleon and nucleon and not between nucleus and nucleus. So also in our case, the center-of-mass should not be at all meaningful if the nucleon-nucleon interaction were peripheral in character.[16] Models that require a total interaction of nuclear matter would then, perhaps, be supported by the result of these experiments. In the paper of Smith et.al. it is shown, however, that predictions of the multi-peripheral model of Chew and Pignotti are also confirmed.

The fact that measured pion distributions exist in the center-of-mass at low energies, suggests a way to scale to 200 GeV, since only a factor of four in center-of-mass energy is required. One could then use the Serpukhov K/π ratios to obtain the kaon spectra. We will discuss this model later when we consider scaling models in general.

Models:

A. The CKP model[2]

This model represents one of the earliest attempts to parameterize particle yields. It is based on the results of experimental data; it makes the following assumptions:

1) multiplicity $\alpha E^{1/4}$ (See Fig. 2 taken from Fowler and Perkins).

2) mean transverse momentum independent of primary energy (See Fig. 3 taken from Fowler and Perkins).

3) constant elasticity.

The CKP formula can be written in the following form:

$$\frac{d^2n}{dpd\Omega} = \frac{n_\pi T}{2\pi p_o{}^2} \left(\frac{p}{T}\right)^2 e^{-p/T} e^{-p\theta/p_o} \qquad \text{(a)}$$

where:

n_π = mean multiplicity

T = mean energy of secondaries: $T \propto E^{3/4}$

$2p_o$ = mean transverse momentum

p = laboratory momentum

θ = laboratory production angle

It has the obvious advantage of being in a simple closed form where the physical assumptions are easily understood. The secondary kaon spectra are obtained by multiplying (a) by the K/π ratio.

The model fails in the following way:

(1) it does not predict different spectra for π^+ and π^- as expected from experiments.

(2) it does not fit accelerator data very well at small angles.

B.I. Trilling model[3]

This model assumes that in the center-of-mass the interaction products consist of a forward and backward isobar and a fireball. The decaying forward isobars contribute the high energy pions, while the fireball gives the low energy products. The Trilling formula thus has two terms:

$$\frac{d^2n}{dp\,d\Omega} = \text{fireball} + \begin{array}{l}\text{forward}\\ \text{isobar}\end{array}$$

The pion spectrum predicted by this model thus exhibits two characteristic bumps. This model predicts higher pion yields than CKP or Hagedorn Ranft. It does not have the problems of the CKP formula in that π^+ and π^- are treated separately and the formula gives reasonable fits to low energy data.

B. II. Pal and Peters[8]

A very well known model of cosmic rays is that of Pal and Peters. Here again they assume a forward and backward isobar and a fireball in the center-of-mass. Their calculations are, however, not directly applicable to accelerator use. The primary objective of their model is to describe the propagation of cosmic rays through the atmosphere. As such, they make use of the steepness of the primary cosmic ray proton spectrum to justify leaving out the contribution from the fireball. Also, their model considers only an average isobar of average mass: the model does not take into account the mass spectrum of the decaying isobars.

Comment:

In light of the results of the experiments of Day *et. al*. and Smith *et.al*. as discussed in Experimental Data, we would perhaps not be too sanguine about the success of an isobar model. On the basis of this data, however, we cannot discount the possibility that the center-of-mass distributions will change drastically at the higher energies.

C. Hagedorn-Ranft Model or Thermodynamic Model[3]

This model had its origin in the statistical model, and is in some sense an extension of that model. The incident and target nucleon interact to form a fireball in the center-of-mass system, which then decays in a manner consistent with known conservation laws, into sub-fireballs, which then further decay until the initial total energy is completely distributed among stable or semi-stable particles. The calculations are done by computer, since analytical techniques are not sufficient to fully parameterize the problem.

This is the only model that is worked out in full detail. It yields momentum and angular distributions for a wide range of secondary particles.

The difficulties of the model are; (1) it requires the use of a computer and is therefore difficult to check, and difficult to use, (2) the model is quite complex and offers no easy insight into the physics involved, (3) in general, the model predicts multiplicities that are low by about a factor of two even at low energies, (4) the mean transverse momentum of secondaries in this model are dependent on longitudinal momentum in contradiction to cosmic ray data.

D. Multipheripheral Models[6]

The model treats the nucleon-nucleon interaction by Reggi exchange; both mesonic trajectories and the Pomeranchuk are considered. This model has had some success at accelerator energies; agreement is obtained for multiple production cross-sections with nucleon-nucleon data up to 30 GeV. However, further computation must be done before the results could be applied at 200 GeV. We are currently attempting to calculate the secondary pion spectrum from this model. At this point only the secondary baryon distribution has been calculated.

E. Scaling Models[5]

Scaling models are all models that attempt to scale to 200 GeV energies by extrapolating on the basis of systematics observed in data available at lower energies. We object to these models in general, since we feel that reliable estimates can be made only if based on physics insight: that particle spectra behave in some way at low energies in no way guarantees that they will behave similarly at 200 GeV.

F. Other Models

High energy experiments at Serpukhov and electron scattering experiments from hydrogen at SLAC have led to the development of new ideas and insights into the structure of nucleons. New models are currently being developed which treat the nucleon as a collection of loosely bound mass centers,[27,28] or alternatively as divided into regions or cells.[29,30] Some results on multiplicities are already available,[30] and this new work promises new results for the near future.

Policy Statements of Other Groups

Various groups have considered the problems of how best to use existing data and models for designing beams at

200 and 300 GeV. In this section we summarize their recommendations.

The CERN/ECFA[17] report makes the following considerations:

(1) Consider only spectra based on physics, not extrapolations.

(2) Use the results of existing models to set upper and lower limits on flux. Upper limits can then be used in shielding considerations, lower limits for conservative estimates in beam design.

(3) In using the CKP formula the following recipe is recommended:

(a) for π^+ use 2 x CKP

(b) for π^- use 1/2 x CKP

(c) for K^+ use 1/10 x CKP

(4) Measurement of particle yields should be the first experiment at the 200 GeV accelerator.

Koester's[18] paper recommends that the CKP formula be used in preference to the Thermodynamic model because of computational ease.

In the summer study paper by Walker,[19] the Hagedorn-Ranft model is recommended for feasibility studies of experiments. The paper argues that this model requires less curve fitting to inadequate data, has more physics, and gives the most conservative estimates.

In a 1969 Summer Study paper Cocconi[20] proposes the use of the Thermodynamic model for fluxes at 0°, but requires constant transverse momentum for the angular distributions. In this way the good fits of the Thermodynamic model at 0° would be combined with the result of cosmic ray data of constant mean transverse momentum.

Conclusion:

We do not wish to use a thermodynamic model modified in angular distribution. If the physics in the model is correct, then we should use all of its predictions. If the assumptions are incorrect then we see no reason to believe only a part of its results. That the model fits some of the data but not all, merely advises caution in its use.

We are not hopeful of establishing upper and lower limits to particle fluxes on the basis of existing models. Experimental measurements might show in the end that all models are conservative or all are overly optimistic. We feel that we must at least be aware of the possibility that the true spectra might lie somewhere outside the limits given by existing models.

We do not see any overwhelming reasons to choose one model over another. All models have their weaknesses, although some are more completely worked out than others. Our only recourse is to estimate the effects of the uncertainties on our beam design and to make allowances for possible corrective actions at beam turn-on.

As a working procedure we propose to use the CKP formula for the initial beam design. The K/π ratio measured at Serpukhov will be used to obtain the K spectra. After the beam parameters are fixed roughly, we will study the variations due to the Thermodynamic and Trilling models and any other models that may be developed in the interim. Fig. 4 gives a summary of pi and kaon yields predicted by the more relevant models. The spectra are generated for hydrogen targets except for the K^+-Trilling spectrum which is for beryllium.

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TABLE I

Experiment	Beam Energy GeV	θ_{Lab}	Targets
Baker et.al.[9] (1961)	10, 20 25, 29.5	4-3/4°, 9° 13°, 20°	Al, Be
Diddens et. al.[10] (1964)	19, 24	116 mrad.	H_2
Dekkers et.al.[11] (1965)	8.65, 11.8, 19, 23	0°, 5.7°	H_2, Be, Pb
Lundy et.al.[12] (1965)	12.5	2°, 16°	Be
Anderson et.al.[13] (1967)	6, 10, 15 20, 30	(t) = .04 to 5 $(GeV/c)^2$	H_2 $(p+p) \rightarrow p+N^*$
Bushnin et.al.[15] (1969)	20, 43, 70	0-15 mrad	Al
Marmer et.al.[25] (1969)	12.3	0°, 11.2°	Be, Cu

TABLE II

Experiment	Beam Energy BeV
Ratner et.al.[21] (1968)	12.5
Day et.al.[22] (1969)	12.2
Smith et.al.[23] (1969)	13 to 28.5
Elbert et.al.[24] (1968)	25 GeV $\pi^- p$

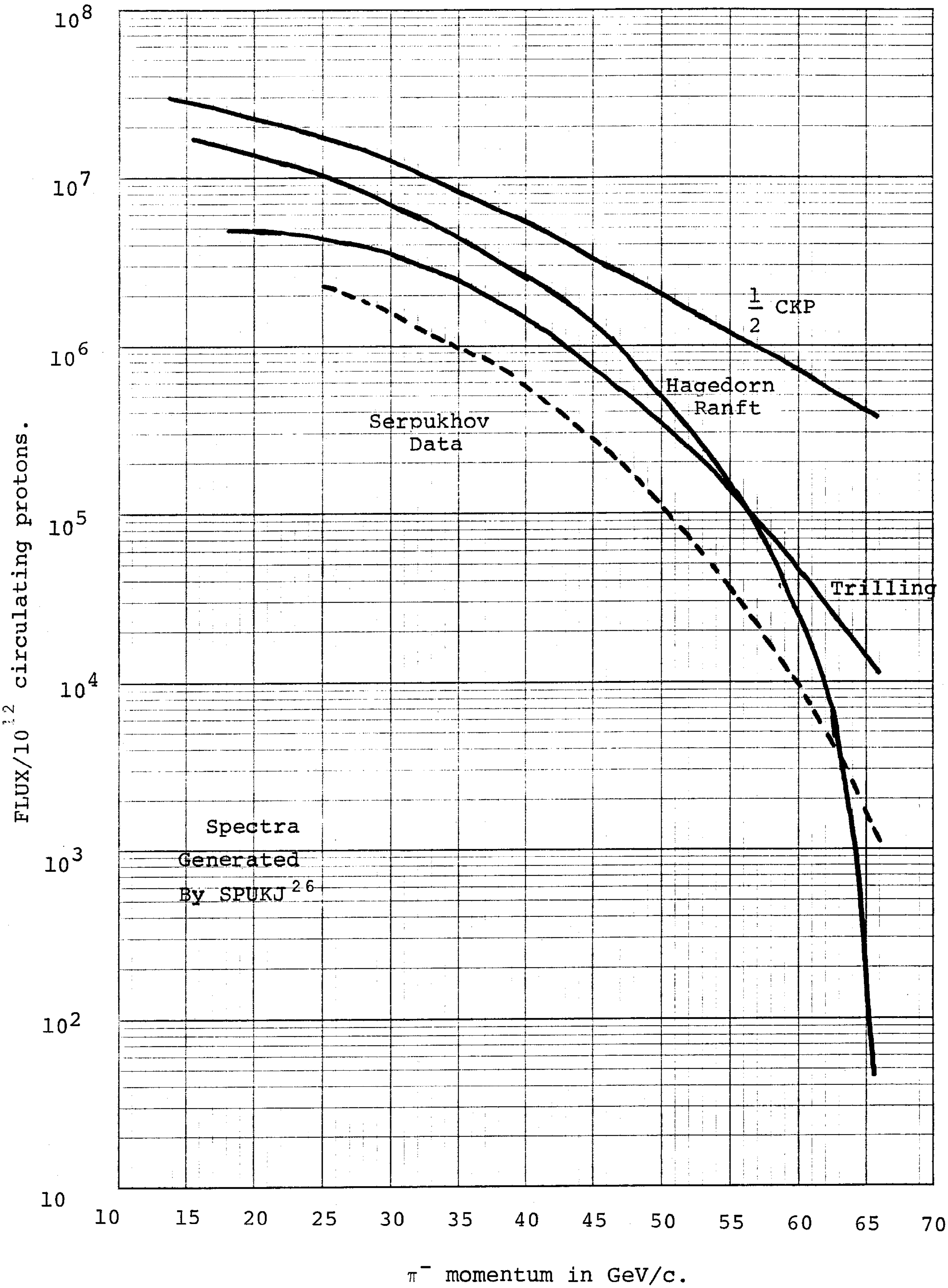
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10
FLUX/10^{12} circulating protons.
$\frac{1}{2}$ CKP
Hagedorn
Ranft
Serpukhov
Data
Trilling
Spectra
Generated
By SPUKJ[26]
10 15 20 25 30 35 40 45 50 55 60 65 70
π^- momentum in GeV/c.

Figure 1.

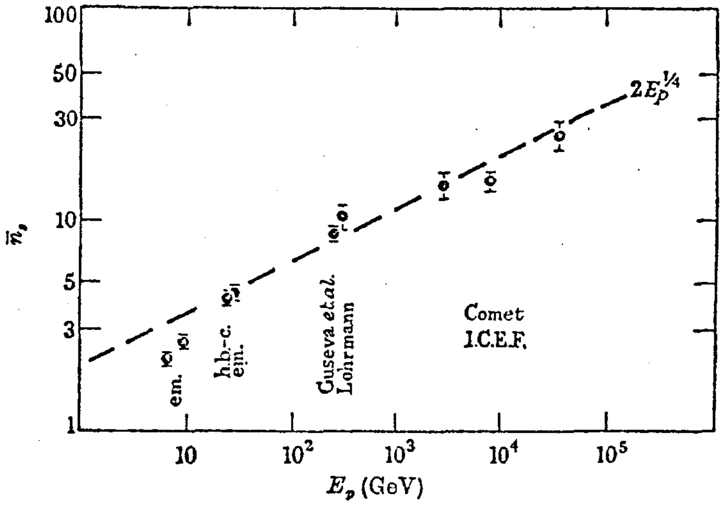

FIGURE 79. Mean shower particle multiplicity, n_s, as a function of proton energy, E_p. Below E_p = 30 GeV, the data are the result of accelerator experiments (Berkeley 6·2 GeV, Dubna 9 GeV, both in emulsion, for 'stars' in which N_h, the number of associated heavy prongs = 0: C.E.R.N. 23 GeV hydrogen bubble-chamber: 24 GeV, emulsion with N_h = 0). At E_p = 300 GeV, the data are from collisions in LiH (Guseva—see the report of Dobrotin, p.   above: and from collisions in emulsions of the fragmentation nucleons arising from the break-up of heavy primary cosmic-ray nuclei (Lohrmann). The high energy points (E_p> 10^3 GeV) are the combined results ($N_h \leqslant 5$) of the Bristol Comet stack events, and the I.C.E.F. collaboration. The primary energy in these events was taken as 10 times the energy in the resulting electromagnetic cascade (i.e. $\Sigma E_{\pi 0}$).

FIG. 2

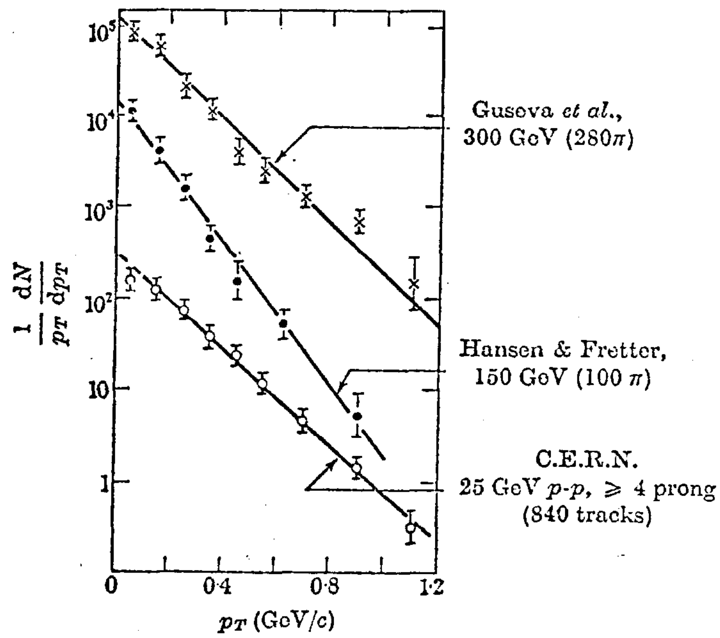

FIGURE 83. Transverse momentum distribution of charged secondaries (mostly pions). For p_T > 0·1 GeV/c, the spectrum is quite well represented by a Boltzmann distribution.

FIG. 3

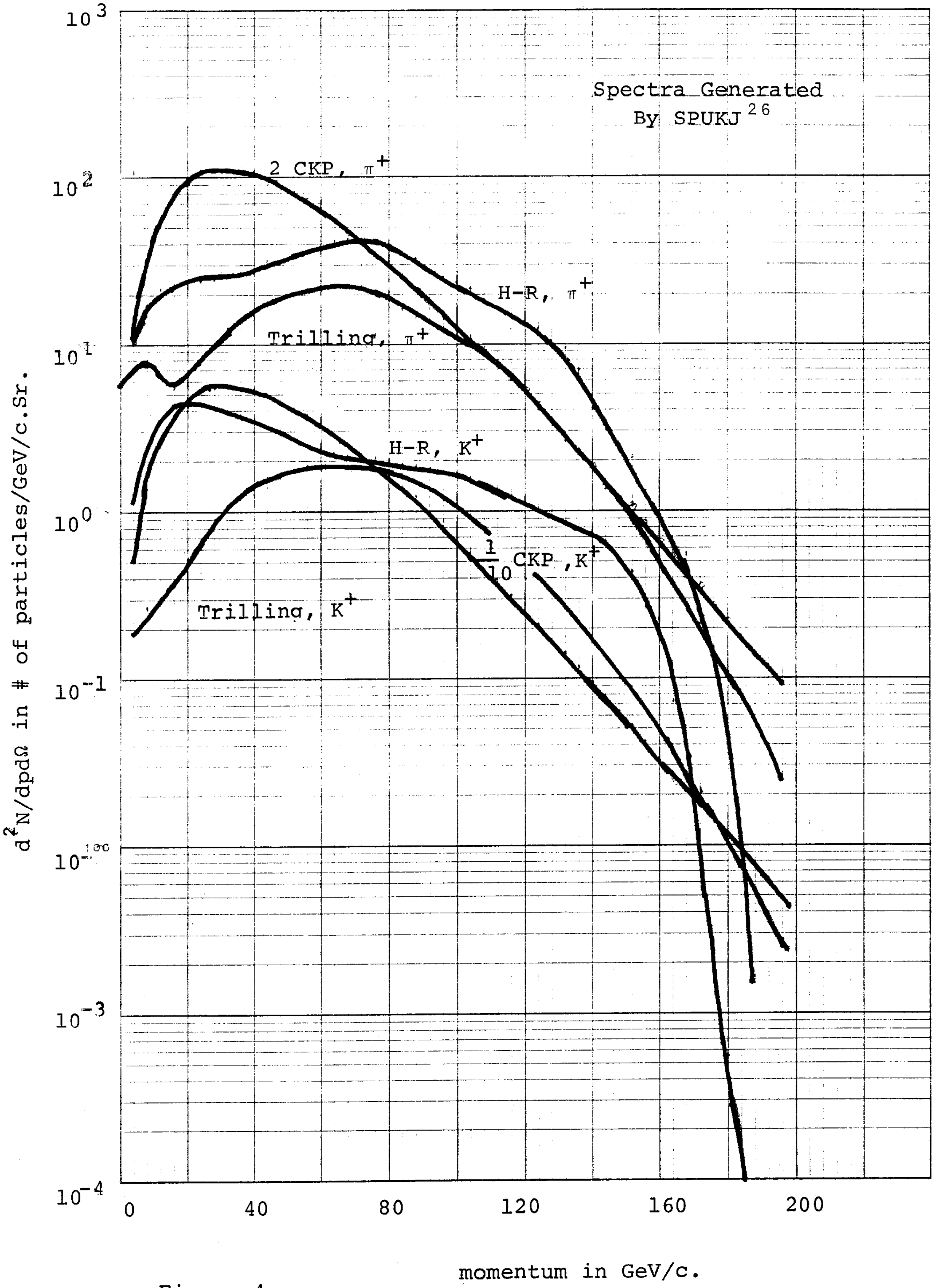
Spectra Generated
By SPUKJ 26
2 CKP, π+
H-R, π+
Trilling, π+
H-R, K+
1/10 CKP, K+
Trilling, K+
10 3
10 2
10 1
10 0
10 -1
10 -3
10 -4
0
40
80
120
160
200
d2N/dpdΩ in # of particles/GeV/c.Sr.
momentum in GeV/c.

Figure 4.