

# The Black Hole Paradoxes and Possible Solutions

**Xuer Dai**

Faculty of science, Zhejiang Sci-Tech University, Hangzhou, Zhejiang Province

Corresponding author's address: [angela@cas-harbour.org](mailto:angela@cas-harbour.org)

**Abstract.** Quantum mechanics and general relativity have some irreconcilable contradictions, causing the black hole information and firewall paradoxes. This paper investigates the cause of these two paradoxes and provides some possible solutions. The information paradox wants to explore if the information falling into the black hole really gets lost. The article briefly introduces the principle of Maldacena's Duality, the black hole complementarity principle and other models to solve it. The firewall paradox tries to find if things passing the black hole horizon would be destroyed by a firewall. The introduction of computational complexity and the model of ER=EPR may resolve this paradox. Moreover, if the firewall really exists, the rebound of the gravitational wave when hitting the firewall may help detect it. In general, the solutions of the black hole paradoxes may provide us with a possible way of unifying quantum mechanics and general relativity.

## 1. Introduction

Black hole is a peculiar object where quantum mechanics and general relativity meet, and those two theories have some uncoordinated conflicts which become obvious considering the black hole and lead to two famous paradoxes—the black hole information paradox and the firewall paradox. This article investigates those two famous paradoxes. For the information paradox, it is necessary to either modify quantum mechanics to allow the loss of information or modify the general relativity for allowing the information to escape from the black hole. For the firewall paradox, when taking quanta into consideration, one assumption of equivalence principle, Hawking radiation and quantum entanglement must be eliminated to make the theory self-consistent, and it means that we can either admit the existence of a firewall or revise the quantum theory. This article aims to find some reasonable explanations and solutions to those two paradoxes. Moreover, if the information does not get lost, the author wants to find a calculation method to restore the previous information, and if the black hole firewall really exists how to checkout its existence by experiments. There are plenty of other promising theories of solving the black hole paradox these days, like the fuzzball model and the black hole soft hair model. But they all have some problems that need to be improved. Solving the black hole paradoxes may help unify quantum mechanics and general relativity by eliminating the conflicts between those two theoretical backbones of modern physics. In the future, string theory may be an ideal candidate for solving the black hole paradoxes.

## 2. The black hole information paradox

Hawking believes when things fall into the black hole, its information will get lost. The loss of information means there is no possible way to predict what happens inside the black hole through Hawking radiation because the interior of a black hole is cut off from the external universe. Nothing



beneath the event horizon can influence the exterior universe since no signal can escape the event horizon to carry that influence.

The loss of information violates the law of energy conservation as well as quantum mechanics by the principle of micro-reversibility which means all things are microscopically reversible, in other words, one can always reconstruct the initial configurations from the final products. The violation of microscopic reversibility destroys energy conservation.

This is how the black hole information paradox forms, and if the information gets lost, it is necessary to modify quantum mechanics. If the information does not get lost, it is essential to modify general relativity to allow the escape of information through Hawking radiation.

## 2.1. Solutions

### 2.1.1. The Maldacena Duality.

Researchers have proposed some solutions to this paradox. Maldacena came out with a solution, which is called the Maldacena Duality (ADS/CFT)[1]. Duality is the odd equivalence between two things that seem very different from each other. Maldacena's theory of Duality shows a quantum gravity theory, based on string theory and combining quantum mechanics with gravity, that is mathematically equivalent to a general quantum theory under certain conditions. Characteristically, the quantum physics of a black hole is equivalent to that of a mass of highly heated primordial gas.

Through the Maldacena Duality, physicists found another way to describe the theory of quantum mechanics of black holes. If the hypothesis of Maldacena is true, then the law of the quantum theory is also applicable to gravity, for this information is not lost. The evidence also suggests that black hole radiation leaves no residue, because all information must be reemitted with the Hawking radiation. Therefore, the information is not actually lost.

However, it is impossible for this theory to explain clearly how the information comes back from the inside of the black hole to the outside and it does not describe how the information is stored.

### 2.1.2. The complementarity principle.

Susskind and t'Hooft pointed another possible solution known as the complementarity principle[2]. They believe as objects approach horizon, there exists two different scenarios which are not contradictory but complementary. From an outside view the object slows down and seems to freeze in the event horizon while from a falling perspective, everything looks the same and nothing special about the horizon according to general relativity will be noticed.

Therefore, for the observer entering the black hole horizon, he will find the information inside the black hole. While for the observer standing outside the black hole, he will not see the information falling into the black hole, the speed of things will be slower as it gets nearer to the horizon and finally gets rived. The debris gets frozen in the event horizon and finally he will see the information of debris get out through Hawking radiation.

There is no contradiction between the two, because those two groups of observers cannot communicate with each other. Therefore, you cannot say that the information gets lost.

### 2.1.3. Other solution.

There are also other possible ways of solving the information paradox. Maybe the black hole will not be completely evaporated out; rather, it will eventually take the form of a very small remnant containing the information of the original black hole. However, such a solution contains some problems. For example, to store such a large amount of information of a heavenly scale in a small body will contradict the Bekenstein-Hawking's theory of black hole entropy.

In 2002, Mathur proposed the fuzzball model that the black hole horizon of a classical black hole is clearly bounded, but the black hole in this model is fuzzy on the corresponding boundary, possibly a fuzzball wrapped by a bunch of superstrings. In this model, the various semi-classical properties of the black hole, such as the event horizon of the black hole, the entropy of the black hole, the Hawking

temperature should be equal to the statistical average of all the possible quantum states of the string inside the black hole. Since all the strings that fall into the black hole are components of the fuzzball, the information they carry are preserved, so there is no loss of information in the fuzzball model.

In 2016, Hawking, Perry and Strominger proposed the black hole soft-haired model. When Hawking first proposed the black hole radiation, he assumed that black holes still obey the hairless law, which states that all the properties of black holes are determined by only three parameters: mass, charge and angular momentum. However, they recently discovered that when new charges fall into the event horizon of a black hole, extremely low-energy soft photons, or what they call "soft hairs", would be generated. Calculations show that with each Hawking radiation, a soft hair which records information about particles falling into the black hole will grow on the black hole's event horizon, and information in the soft hair is then re-radiated as the black hole evaporates. This theory has the potential to solve the problem of information loss, but it only considers the electromagnetic field, not the gravitational field, and the soft hairs may not be able to store all the material information falling into the event horizon, so this model needs further research and generalization.

## 2.2. Discussion

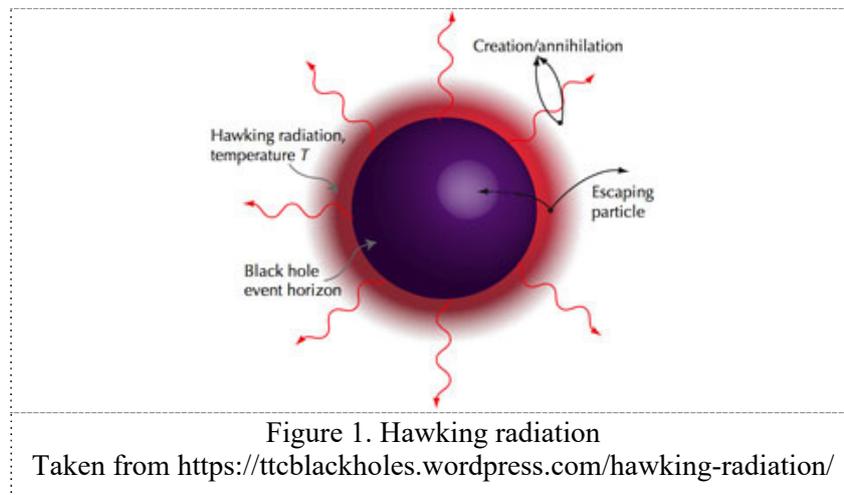
Now the question is, how to restore the previous information through Hawking radiation? The solution is the holographic principle and ADS/CFT Duality. Holographic principle shows that time and space are fundamentally different from common knowledge, and what we think of as a three-dimensional universe is actually a projection of information written on a two-dimensional plane[3]. Therefore, the information that goes into a black hole is stored on the surface of it in two dimensions instead of being destroyed, which means the observer outside the black hole knows what happens to the information by investigating the surface of the black hole using the ADS/CFT Duality.

In summary, according to the holographic principle, the information is just encoded in another way, and the Hawking radiation is able to learn about the information encoded in the event horizon of the black hole and carry them away, so we can completely revive the previous information through Hawking radiation, in other words, the information does not get lost.

It seems that the principle of Maldacena's Duality and the black hole complementarity principle are able to eliminate all the paradoxes by the supplement of some details. However, Ahmed Almheiri, Donald Marolf, Joseph Polchinski and James Sully (AMPS) met a lot of problems when trying to build a model that would combine the Maldacena Duality with the black hole complementarity. They realized that the problem lied not in their lack of mathematical ability, but in fact there were still some conflicts existing[4]. The conflicts become apparent when the quanta is taken into consideration.

## 3. The black hole firewall paradox

According to quantum theory, Hawking pointed out that a vacuum fluctuation generating a pair of positive and negative particles would annihilate quickly when it happens near the event horizon. The gravitational field of a black hole can tear these virtual particles apart in the vacuum, and as one particle falls into the black hole, the other would run away from the black hole with some mass. Finally, all the mass of the black hole may be consumed through this process called Hawking radiation, as can be seen in figure 1. Because the pair of particles is generated from the vacuum at the same time. According to quantum mechanics, they must be in an entangled state no matter how far the particles run out.



Now we assume that there is “no drama” in the event horizon and a Hawking photon B is radiated. According to Hawking radiation, A—the companion particle of B, will fall into the black hole and A is entangled with B. If the information is not lost, we are supposed to know what has fallen into the black hole before by investigating the particle of Hawking radiation. However, because B is entangled with A, some of the information of B gets lost in the black hole. Therefore, according to the conservation of information, B must be entangled with a particle C which is radiated later, so that the entire information through Hawking radiation can be got. And that causes a contradiction, B is entangled with A and C; however, according to a “Monogamy of entanglement” in the quantum theory, one particle can only be fully entangled with one other particle.

It seems that some combinations of the equivalence principle (EP), Hawking radiation (HR) and quantum entanglement can work, but not all three. One assumption must be eliminated to make the theory self-consistent. If the entanglement is eliminated, it means that Hawking radiation (HR) will no longer be linked to particles that have passed the Event Horizon (EH), and we cannot predict the information which fell into the horizon through Hawking radiation; that is to say, the information will be lost, which violates the conservation of information.

To save the conservation of information, which means to reserve Hawking radiation (HR) and quantum entanglement so that the information does not get lost, we need to keep the entanglement between B and C, and destroy the entanglement between A and B. But doing so requires energy that needs to be carried by particles. Therefore, a firewall composed of a pile of high-energy particles will emerge near the horizon of the original black hole and the information that entered the black hole is stored in this firewall. Once you pass through the event horizon (EH), the firewall will kill you, which violates the equivalence principle (EP) predicted by general relativity, showing that you would not feel anything when passing through the event horizon.

And if Hawking radiation (HR) is abandoned, the conservation of energy will be destroyed. In conclusion, we can either admit the existence of the firewall or revise the quantum theory.

### 3.1. Solutions

#### 3.1.1. Same information inside and outside the black hole.

There is one point trying to get past the firewall that the information inside the black hole is just a copy of certain things outside the universe[6]. Although they are in different positions, the photons behind the event horizon may be equivalent to the information encoded in early Hawking radiation in a certain way. But this would violate a critical principle in quantum mechanics, that means the no-cloning theory which shows any process that replicates quantum information perfectly is impossible[7].

Fortunately, this problem can be got around nicely. It is something similar to the complementarity principle that an observer outside the event horizon of a black hole cannot communicate with an observer

inside the event horizon, so no one can observe both inside and outside the black hole simultaneously. Therefore, this hypothesis may provide us with a possible way of solving the black hole firewall paradox.

The complementarity principle indeed prevents us from violating the no-cloning theory. However, another special circumstance should be considered: what if the observer can decode the radiated information first, and then jump into the black hole? During the fall process, the decoded information can be compared with the information inside the black hole.

### 3.1.2. Computational complexity analysis.

AMPS reasoned that nature would prevent the situation above from happening by creating a hot wall of fire close to the event horizon of the black hole. Because of the wall of fire, space would come to an abruptly end at the event horizon, which contradict Einstein's general relativity that predicts space must be perfectly continuous at the event horizon.

To solve this contradiction, the observer's ability to decode the information from the Hawking radiation was introduced. P. Hayden and D. Harlow brought computational complexity into the debate for the first time[8]. Computational complexity analysis shows that as the number of radiating and information-carrying particles increases, the number of logical steps that need to decode the information increases exponentially, so that no calculations can be done before the black hole is exhausted. When the black hole disappears, the internal information disappears. Therefore, there is no need for the black hole fire wall to exist and the contradictions are solved naturally.

### 3.1.3. $ER=EPR$ .

Another possible solution pointed out by Susskind and Maldacena is called  $ER=EPR$ [9]. The wormhole, a direct result of general relativity, can also be the result of quantum mechanics. According to general relativity, two black holes several light years apart can still be connected through space-time channels called wormholes, as shown in figure 2. But according to quantum theory, two black holes that are far apart can be linked by "entanglement" of states, which means they can share information about quantum states to some degree.

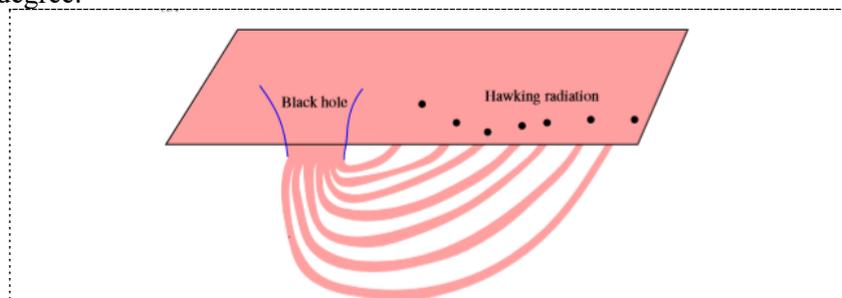


Figure 2. Black hole and Hawking radiation is connected through wormhole, Taken from "Cool horizons for entangled black holes"[8]

Susskind and Maldacena claimed that any two entangled particles were connected by those tiny space-time wormholes. Therefore, the black hole could connect to all the particles of Hawking radiation through a virtual wormhole, and each Hawking radiation particle would act as the entrance to the wormhole as well as a large space-time region like the inside of a black hole can be composed of a large number of entangled wormholes. The information can get out of the black hole in this way without causing contradiction of quantum entanglement or opening the firewall.

However, some people questioned the work because theory predicted that the wormhole was too small for qubits to pass through, equaling to for the information to get out of the black hole. What is more, there is not much calculation can be done about this theory at this moment. In general relativity, wormholes are the results of extremely distorted space-time, and it takes enough energy or matters to bend space-time this way, so the necessary conditions of whether wormholes can be created between

arbitrary entangled particles or there needs to be enough particles in system to create wormholes are yet to be further studied.

### 3.2. Discussion

Besides, if the firewall really exists, what is it? One view is that the firewall is the end of space. Maybe the conditions inside a black hole are not able for space-time to form. Therefore, a falling object that touches the event horizon will be decomposed into qubits that stay in this boundary.

Moreover, there may be evidence to prove the existence of the firewall. If the firewall is real, then the gravitational waves generated by the merger of black holes once hit the horizon, it will rebound again, producing an echo similar to a bell. This echo can be detected in the signal of the wave. Looking through the Earth to check the LIGO data, the team led by Jahed Abedi and Hannah Dykaar found that there was an echo[5]. But unfortunately, their findings lacked statistical significance and could not be used as the basis of this result.

## 4. Conclusion

The Maldacena Duality, the complementarity principle and other models such as the fuzzball model and the soft hair model may solve the information paradox. Moreover, the holographic principle and ADS/CFT Duality can be used to restore the previous information. For the firewall paradox, the paper points out some solutions including computational complexity analysis and ER=EPR. Additionally, we can detect if the firewall really exists with the help of gravitational waves. However, the theory of the Maldacena Duality does not describe how the information is stored, and for the model of ER=EPR, the necessary conditions for the wormholes to form and if the information can pass through such tiny wormholes need to be further studied. Besides, the grey hole theory raised by Hawking provides us with a whole new idea, and we can investigate more about this rough theory in the future. Moreover, string theory may bring the black hole paradoxes a bright future.

Finding paradoxes in physics offers a splendid chance of finding something new, something that would expand our theory to a boarder sense. The black hole paradoxes derive from the contradictions between quantum mechanics and general relativity. Therefore, studying the black hole paradoxes may help us find a possible way to unify these two theoretical pillars of modern physics.

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