

**EXPERIMENTAL SUPPORT FOR THE GEOMETRICAL PICTURE OF  
MULTIPARTICLE EMISSION IN  $e^+e^-$  COLLISIONS**

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**ABSTRACT**

Main predictions and their experimental support are reviewed for the geometrical picture of multiparticle emission for two-jet events in  $e^+e^-$  annihilation.

The geometrical model of multiparticle production in hadron-hadron collisions has provided a good picture<sup>1,2</sup> for the description of some important features in such collisions. These features are the broad distribution of total multiplicity, the distinctly narrow fluctuation of multiplicities in the forward and backward hemispheres, and the single-particle momentum distribution. The main ideas of the geometrical picture for inelastic hadron-hadron collisions are summarized below:

(a) The phenomenon of inelastic hadron-hadron collision at high energies is essentially an incoherent superposition of many collisions at different impact parameters. For a fixed angular momentum of the incoming system, the multiplicity fluctuates very little and follows a narrow distribution.<sup>3,4</sup> But the superposition of a multitude of such narrow distributions due to a wide range of values of the total angular momentum in hadron-hadron collisions gives rise to the observed broad fluctuation of the total multiplicity which is characterized by the approximate KNO-scaling or negative binomial distribution.

(b) The exact form of the narrow multiplicity distribution for fixed impact parameter  $b$  is postulated<sup>5</sup> to be a product of two Poisson distributions, one in  $n_F$  and one in  $n_B$ , both with

average  $n_0(b)/2$ , i.e.,

$$f(n_0/4, n_F/2) f(n_0/4, n_B/2) \quad (1)$$

where  $f(a, m) = \exp(-a) a^m/m!$ . We tacitly assumed in (1) that particles are emitted in the forward and backward hemispheres in pairs of opposite charges, hence the factors of  $1/2$ .

Expression (1) can be written alternatively as

$$2^{-n/2} f(n_0/2, n/2) C_{n_F/2}^{n/2} \quad (1')$$

which yields the narrow Poisson multiplicity distribution  $f(n_0/2, n/2)$  for fixed impact parameter and the experimentally observed binomial distribution<sup>4</sup> for the forward-backward charge asymmetry for fixed  $n$ .

(c) Similar to the stochastic partition of particles in the two hemispheres, the partition of energy among outgoing particles for a fixed  $n$  in each hemisphere is also stochastic in nature. Considerations along this line led to a simple and concise expression<sup>1</sup> for the single-particle spectrum

$$dn = (d^3p/E) g(p_T) \exp(-E/T_p) \quad (2)$$

where  $g(p_T)$  is a transverse momentum cutoff factor, and  $T_p$  is a parameter which we named the partition temperature. For each total multiplicity  $n$  or a narrow range of  $n$ , there is a partition temperature  $T_p$  which can fit<sup>1</sup> experimental momentum distribution very well.

The physical picture which emerged for inelastic hadron-hadron collisions was applied<sup>5</sup> to hadron productions in two-jet events in high-energy  $e^+e^-$  annihilation experiments. The following conjectures have been inferred.

(i) In  $e^+e^-$  annihilation the system goes through an intermediate state of a virtual photon with angular momentum 0 or 1, and the total angular momentum in this case does not extend over a wide range as in hadron-hadron collisions. Since in our view for each angular momentum state of the incoming system, the multiplicity distribution is narrow and follows a Poisson law, the multiplicity for  $e^+e^-$  collisions, being the superposition of two almost identical Poissons, must be an essentially Poisson-type of distribution.

(ii) The distribution of the forward, backward multiplicities ( $n_F, n_B$ ) is of a product form with the separate distribution in each hemisphere being Poisson with a cluster size  $k$ :

$$P(n_F, n_B) = f(n/2k, n_F/k) f(n/2k, n_B/k). \quad (3)$$

In analogy with the distribution (1) for  $p\bar{p}$  collisions, we surmised further that  $k=2$  for  $e^+e^-$  two-jet events. An immediate consequence of our product distribution prediction (3) is that for fixed  $n_F$  in  $e^+e^-$  collisions, the average of  $n_B$  is a fixed number independent of  $n_F$ . In other words, the forward-backward multiplicity correlation parameter must vanish.<sup>6</sup>

(iii) The same argument that we have given for the single-particle spectrum in hadron-hadron collisions also applies to  $e^+e^-$  annihilation. The only difference<sup>5</sup> is that for  $e^+e^-$  collisions there is only one partition temperature because there is only one impact parameter (viz.,  $b \approx 0$ ) for a given incident energy. Hence the charged-particle spectrum for  $e^+e^-$  collisions takes the same form (2) as for hadron-hadron collisions, but  $p_T$  is now the transverse momentum with respect to the jet or sphericity axis instead of the collision axis.

Are our predictions for high-energy electron-positron annihilations in agreement with experiments? We summarize in the following

available experimental evidence in support of our conjectures.

(I) The best experimental data available for multiplicity distributions in  $e^+e^-$  collisions which covers an extended energy region has been supplied by the TASSO group<sup>7,8</sup> ( $W = 14$  to  $34$  GeV). Because the range of values of the average multiplicity is not large enough, a definitive conclusion cannot be reached about whether KNO scaling or Poisson distribution obtains for  $e^+e^-$  collisions. However, if one compares the multiplicity plots for  $e^+e^-$  and hadron-hadron experiments (Fig. 1), some distinctive characteristics<sup>8</sup> of these curves stand out which in our view favor Poisson distributions: Each  $e^+e^-$  curve exhibits a region of multiplicity to the left of the peak, where the curve has a positive curvature. The presence of this region makes the peak almost left-right symmetrical. Furthermore as  $\bar{n}$  increases, this region widens toward the right. All of these qualitative features are characteristic of Poisson distributions but not of the  $p\bar{p}$  curves.

The  $e^+e^-$  multiplicity distributions have also been studied by other experimental groups. The 29-GeV data gathered by the HRS group<sup>9</sup> at PEP is remarkably close to the Poisson curve, and the more recent results of the AMY Collaboration<sup>10</sup> at TRISTAN also reported a narrow multiplicity distribution which may be fitted with a Poisson distribution. To provide a quantitative comparison we list in Table I values of  $(\Delta n)^2/\bar{n}$  for multiplicity distribution obtained at different  $e^+e^-$  collision energies.<sup>8-10</sup>

(II) Our conjecture about the  $e^+e^-$  multiplicity distribution being narrow and Poissonian with a cluster size  $k=2$  implies that  $(\Delta n)^2/\bar{n} = k = 2$ .<sup>11</sup> The values exhibited in Table I are all very small in the present energy range of 14 to 60 GeV, indicating the narrowness of the  $e^+e^-$  distribution; nevertheless they are not equal to 2 as we had predicted. However, the cluster size  $k$  does appear to be increasing steadily with energy, and it is quite possible that  $k$  will eventually approach our expected value of 2 at high energies. But why does

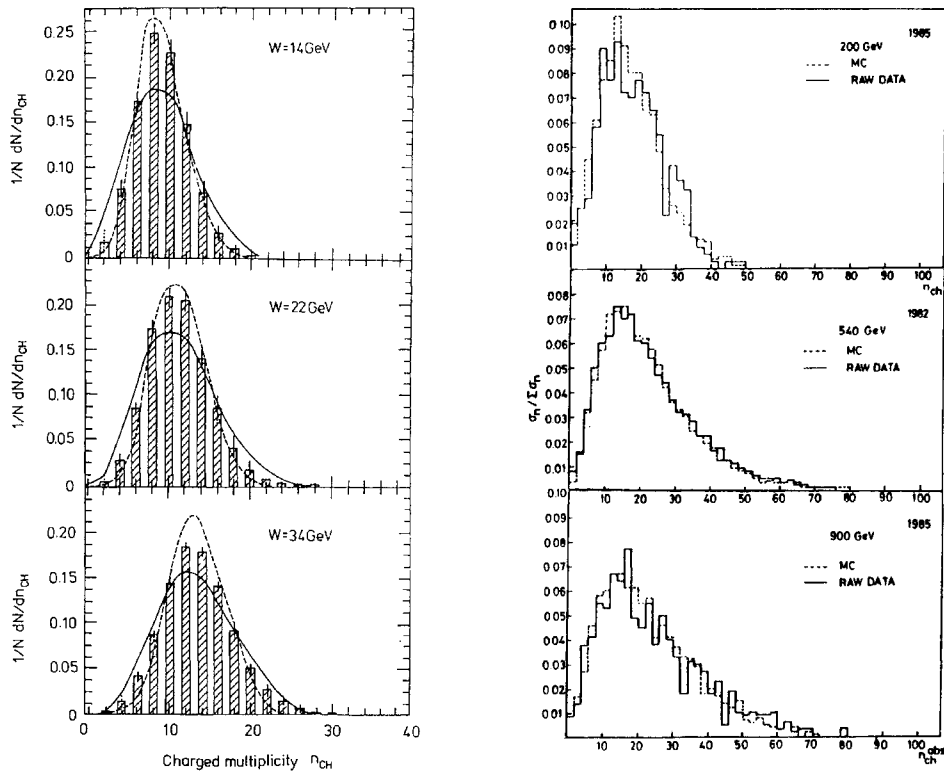


Fig. 1. Comparison of charged-particle multiplicity distribution for  $e^+e^-$  annihilation and pp collisions. The  $e^+e^-$  curves (left column) are reproduced from Ref. 7. The pp curves (right column) are reproduced from a UA5 report by J. G. Rushbrooke, in *Multiparticle Dynamics 1985* (World Scientific, Singapore, 1985).

TABLE I Properties of  $e^+e^-$  charged-multiplicity distributions

W(GeV)	$\bar{n}$	$(\Delta n)^2/\bar{n}$
14.0	$9.30 \pm 0.06 \pm 0.41$	$1.01 \pm 0.19$
22.0	$11.30 \pm 0.08 \pm 0.46$	$1.12 \pm 0.02$
29.0	$12.87 \pm 0.03 \pm 0.3$	$1.05 \pm 0.11$
34.8	$13.59 \pm 0.02 \pm 0.46$	$1.26 \pm 0.24$
43.6	$15.08 \pm 0.06 \pm 0.47$	$1.40 \pm 0.23$
57.0	$17.19 \pm 0.07$	$1.47 \pm 0.04$

the cluster size increase with energy? We do not have a good explanation for it except to offer the following observation: For  $\bar{p}p$  collisions at lower energies, the cluster size inferred<sup>12</sup> from a forward-backward multiplicity correlation analysis also showed a gradual increase with energy. This trend, if indeed true, strongly suggests the similarity of the hadronization processes in  $e^+e^-$  and  $\bar{p}p$  collisions.

(III) According to our view for  $e^+e^-$  annihilation, the  $n_F-n_B$  distribution is a product of

two Poisson distributions with respect to  $n_F$  and  $n_B$ . For such a product distribution one expects the forward-backward multiplicity correlation parameter  $b_c=0$ . The HRS Collaboration has reported<sup>9</sup> a value of  $b_c$  consistent with zero at  $W = 29$  GeV. A new analysis<sup>8</sup> of the TASSO data for  $W = 14$  to 44 GeV yields a weak positive correlation. These values are tabulated in Table II. The slight difference between the results of the HRS and the TASSO Collaborations could be due to the different event selection procedures used in the two experiments as noted in Ref.8. But indisputably,

TABLE II Forward-backward multiplicity correlation parameter  $b_c$

Experiment	W(GeV)	$b_c$
TASSO	14.0	$0.085 \pm 0.014$
TASSO	22.0	$0.084 \pm 0.016$
HRS	29.0	$-0.001 \pm 0.015$
TASSO	34.8	$0.089 \pm 0.003$
TASSO	43.6	$0.111 \pm 0.009$

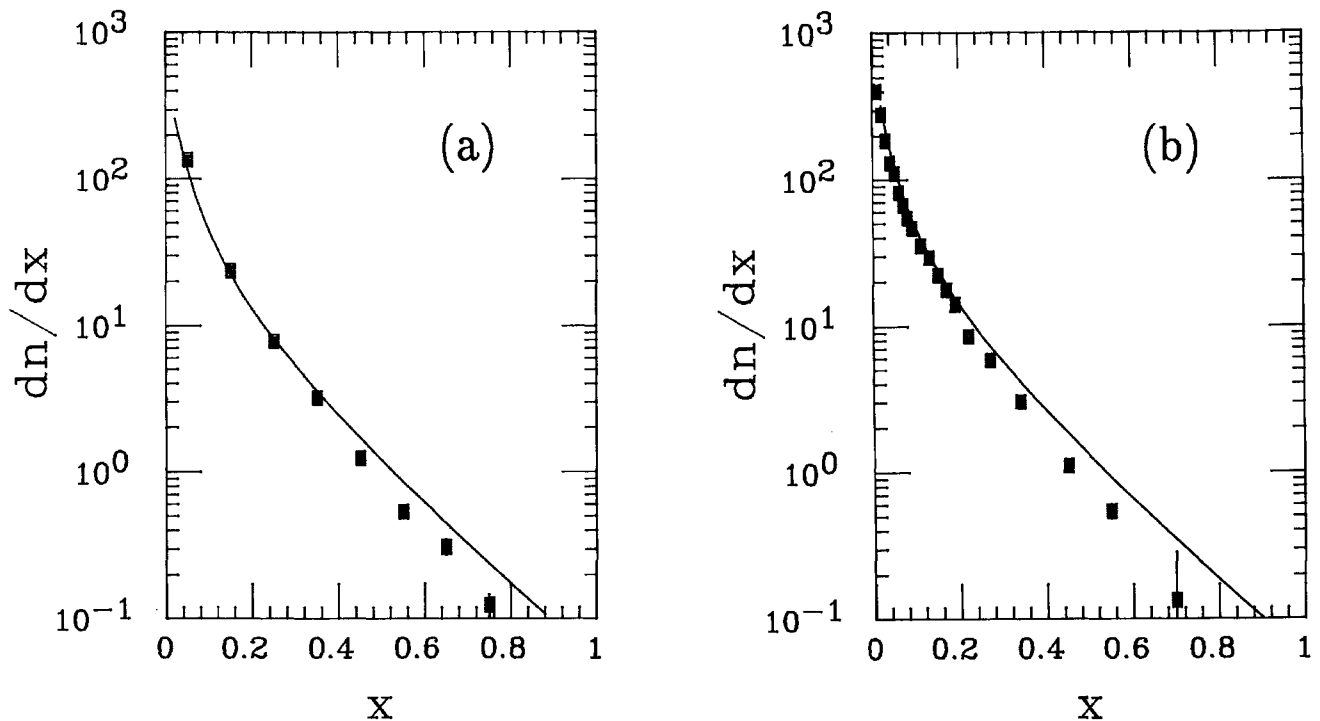


Fig. 2. The charged-particle momentum spectra  $dn/dx$  ( $x = 2p/W$ ) for  $e^+e^-$  annihilation at (a)  $W = 57$  GeV, and (b)  $W = 91$  GeV. The curves are calculated from theory and the experimental data points are taken from Refs. 14 and 15.

the  $b_c$  values for  $e^+e^-$  collisions are very much smaller than those measured in hadron-hadron collisions, in agreement with the expectation of the geometrical model.

(IV) The single-particle momentum distributions at PETRA energies of  $e^+e^-$  collisions have been fitted very well with eq.(2). The only adjustable parameter is the partition temperature  $T_p$ , which was found<sup>13</sup> to have the values 1.4 GeV and 3.5 GeV respectively for  $W = 14$  GeV and 34 GeV. We have extrapolated the momentum spectra to high values of  $W$  using a procedure described in Ref.13. The computed single-particle momentum distributions at  $W = 57$  GeV (TRISTAN) and 91 GeV (LEP) are exhibited in Fig. 2 together with experimental data points taken by the AMY<sup>14</sup> and the ALEPH<sup>15</sup> Collaborations. The agreement appears quite satisfactory.

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## DISCUSSION

**Q. A. K. Wroblewski (Warsaw University):** For  $e^+e^-$  there seems to be KNO scaling up to 91 GeV (see latest DELPHI paper). The dispersion  $D$  is proportional to  $\langle n \rangle$  experimentally which is a necessary condition for KNO scaling. Also the trend of experimental data points of  $\langle n \rangle/D$  does not seem to approach the Poisson behaviour which you predict.

There seems to be a difference in semantics. You said that the multiplicity distribution for  $p\bar{p}$  is wide and KNO, while for  $e^+e^-$  is narrow and cannot obey KNO. But KNO scaled distribution does not need to be wide, it can also be narrow, which is the case for  $e^+e^-$  interventions.

**A. C.N. Yang:** To answer Prof. Wroblewski, may I say something to highlight Prof Chou's report?

There is a fundamental difference between our prediction and KNO (or approximate KNO) behavior for  $e^+e^-$  collisions. The difference between our prediction and negative binomial behavior is even larger. Our prediction gives Poisson-like distributions while the others do not.

Of course, we must accept existing experimental data. But we do not believe existing data is accurate enough, and covers a wide enough range of average multiplicity, to differentiate definitively about the difference predictions.

I would like to emphasize that it is very important to differentiate between the different predictions because on this matter hangs *a priori* conceptual understanding of the approximate KNO behavior for  $p\bar{p}$  collisions, and of the whole multiparticle production mechanism: If our prediction of Poisson-like behavior is correct, it means in  $e^+e^-$  collisions the multiparticle production events form a pure simple, or non-mixed sample. In contrast, for proton-antiproton collision it is a mixed sample (consisting of different angular momenta states). To appreciate the important difference between non-mixed and mixed cases, we remark that if one plots in a racially homogeneous country the frequency distribution of the heights of boys of age 10, one gets a non-mixed case. But if one plots the same for both boys and girls of age 10, one gets a mixed case.

**A. Giovannini (Univ. Torino):** It seems to me that one should be careful before drawing conclusions on the occurrence of KNO scaling in  $e^+e^-$  annihilation (full phase space) in the energy range covered by TASSO and DELPHI. The present situation if confirmed by three experiments at LEP—reminds me of what happened in  $p\bar{p}$  collisions in the ISR energy range. At the time people were convinced of the occurrence of KNO scaling just because  $1/\bar{n}$  and  $1/k$ , ( $\frac{D^2}{\bar{n}^2} = \frac{1}{\bar{n}} + \frac{1}{k}$ ), were balanced in the mentioned energy domain to produce constant  $\frac{D}{\bar{n}}$ . As soon as a new energy domain became available with the  $p\bar{p}$  collider the UA5 collaboration found that  $D/\bar{n}$  was widening breaks to the simultaneous increasing of  $\bar{n}$  and decreasing of  $k$  with energy.