HOW TO LEARN ABOUT HADRON DYNAMICS FROM AN UNDERLYING QUARK-GLUON FIELD THEORY

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Abstract: A solution is proposed to the problem of how physics can be abstracted from a fundamental quark-gluon field theory for hadrons, without having solved the problem of confinement.

Résumé: On propose une solution au problème qui consiste à extraire certaines propriétés physiques des hadrons d'une théorie des champs fondamentale des quarks et des gluons sans avoir à résoudre le problème du confinement.

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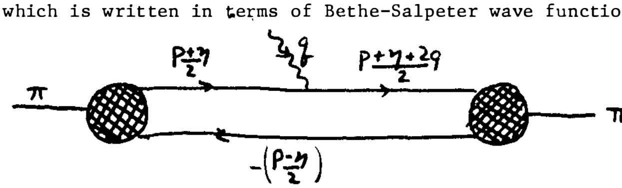
There is ample evidence that the quark model has something to do with nature, as shown by the success of its predictions for current algebra, hadron spectroscopy and the large t behavior of form factors. However, the problem of actually constructing a theory which has fundamental quark and gluon fields, and yet insures that only hadrons are observed, remains unsolved. Evidently, promiscuous use of low-order perturbation theory to learn about hadron dynamics is a mistake, since that neglects the complicated coherent processes that necessarily are present and responsible for confinement. A question thus presents itself: Can we formulate a prescription as to when perturbation theory may be legitimately used? That is, is there a class of phenomena in which the physics responsible for confinement does not modify beyond recognition the results of low-order perturbation theory?

I wish to present a possible answer, or partial answer, to that question. However, it must be stressed that without having solved the problem of confinement,the validity of my proposal cannot be proved theoretically. From a theoretical standpoint, the important issues therefore are its self-consistency, reasonableness, and attractiveness. Experiment is the best test of its validity.

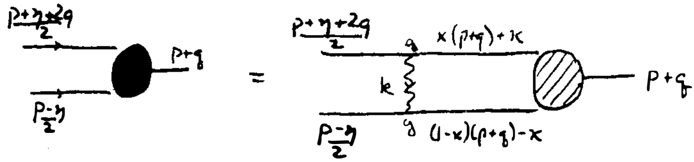
In order to be specific I will choose what seems to me to be the most likely candidate for a correct field theory of the hadrons - a non-abelian Yang-Mills theory of quarks and vector gluons with an exact SU(3) color symmetry. This theory possesses the desirable feature that the effective quark-gluon coupling vanishes logarithmically as the momenta-squared of the quarks and gluon become large and spacelike. Furthermore, the interaction of very soft gluons is so singular that it is conceivable that an infrared catastrophe insures confinement. I will go further and imagine that the quark-gluon coupling is small even for momentum transfers (q^2) of the order of 1 GeV^2, and is only effectively large if very soft processes are occurring[1).

Thus we should expect that perturbation theory would be a useful tool for studying processes that selectively probe regimes of small coupling, for instance large momentum transfer scattering (inclusive or exclusive), or reactions necessarily involving quarks of very dissimilar momenta, such as massive lepton pair production (e.g., $pp \to \ell^+\ell^- + X$) or the propagation of fast secondaries through nuclear material. In what follows I will discuss each of these in turn, facing the issue of when binding effects "factorize" and do not significantly modify the perturbation theory results, and when they are crucial. An application of perturbation theory which I will not discuss is to the dynamics of bound states of very heavy quarks. For instance it has been conjectured that, assuming the ψ is a $c\bar{c}$ bound state, its relative narrowness is due to a decrease in the quark gluon coupling as the quark mass increases[2)]. Although similar in spirit, that is not the same as the assumption of importance to us.

Let us begin, as a simple illustration, with the pion form factor at large q^2 which is written in terms of Bethe-Salpeter wave functions as:

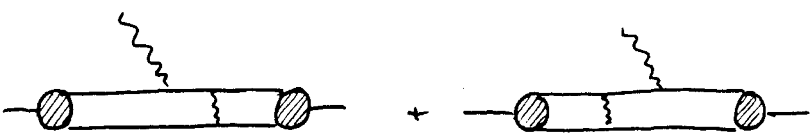


Since $p^2 = (p+q)^2 = m_\pi^2$, at least one of $(\frac{p+\eta}{2})^2$, $(\frac{p-\eta}{2})^2$ or $(\frac{p+\eta+2q}{2})^2$ is of order q^2. In general the Bethe-Salpeter wave functions are not computable, nor will they be until we have solved the problem of confinement. They must therefore be taken to be unknown functions. However, the wave function for those very improbable configurations in which one of the quarks has an invariant mass much larger than any natural length scale is computable from perturbation theory. Let us introduce some terminology: the "normal" part of the wave function is that part in which the quarks have finite momenta-squared, of the order of some characteristic hadronic mass scale such as m_ρ^2; the "exceptional" part of the wave function involves at least one quark whose momentum-squared is very large on that scale. In terms of infinite momentum wave functions, the normal part has both constituents carrying a finite fraction x, $0 < x < 1$, of the total momentum and having limited transverse momentum; the exceptional part involves either large transverse momenta or x not in the range $0 < x < 1$. Given the normal wave function ψ_N, [] the exceptional part ψ_E [] can be computed from perturbation theory:



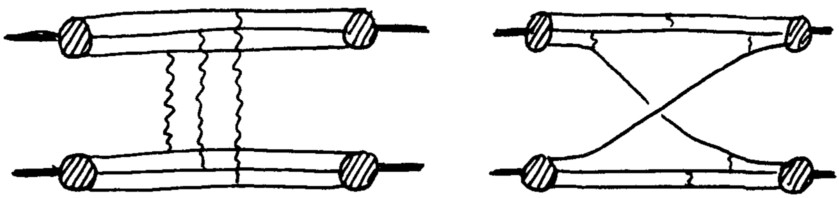
If $\eta \sim xp$ so that $(\frac{p+\eta+2q}{2})^2 \sim q^2$, then $k^2 \sim q^2$ so that our ansatz of g being small applies. As mentioned above, this procedure can only be proved to be legitimate when detailed information on the properties of ψ_N at large distances is known. The necessary conditions on ψ_N are discussed in detail in Ref. 3.

Thus the calculation of the large q^2 pion form factor amounts to evaluating:



The q^2 dependence is completely determined, independent of details of the normal wave functions. One finds modulo logarithms[4)] $1/q^2$ for the pion, and for the proton $G_E(q^2) \sim G_M(q^2) \sim \frac{1}{q^4}$, consistent with experiment as discussed in Ref. 3. In fact, this prediction of the power with which the leading form factor decreases is not specific to the choice of a non-abelian gauge theory; it is the same in any renormalizable field theory with small coupling constant for $q^2 > 1$ GeV^2. However the result that G_E/G_M scales, as appears to be the case experimentally, follows only in a theory with vector gluons[3)].

In the form factor examples, binding simply generates the normal wave function and makes no serious modification to the power law, assuming the wave function is reasonably well behaved. However, if we wish to discuss hadron-hadron scattering the problem is somewhat more complex. Shown below are two possible lowest order diagrams for $pp \to pp$ scattering:



Only gluons carrying large q^2 are shown, and all wave functions shown are "normal." As pointed out by Landshoff[5], the three-gluon exchange diagram is dominated by the configuration in which each gluon carries almost exactly one third of the total momentum transfer, and gives an asymptotic (large s, θ_{cm} fixed) behavior $\frac{d\sigma}{dt}(pp \to pp) \sim \frac{g^{12}}{s^8} f_1(\theta)$. All diagrams involving at least one $q\bar{q}$ pair in the t channel[6], such as the one on the right above, give $\frac{d\sigma}{dt}(pp \to pp) \sim \frac{g^{20}}{s^{10}} f_2(\theta)$. If the effects of binding can be ignored here, so that perturbation theory can be used, we would expect that the three gluon exchange diagram would dominate. Experimentally that is clearly not the case. The ratio of theory to experiment with three gluon exchange varies by nearly three orders of magnitude between θ_{cm} of 30° and 90°[7]. Furthermore a simple fit to the energy dependence suggests $s^{-9.7 \pm 0.5}$[8].

What is the explanation for this? I propose that it is simple. Binding modifies our field theory diagrams in precisely one way - it replaces free quarks, antiquarks, and gluons by hadrons in each physical channel, as required by dispersion theory. If that can be accomplished without additional large momentum transfers, then the field theory result for that diagram is unmodified. If not, then I argue that the diagram as written does not contribute and the additional large momentum transfer interactions necessary to give physical particles in the s,t and u channels must be explicitly included. We have no evidence of the existence of hadrons which consist exclusively of "hard" glue (epoxyons), at least for masses less than about 2 GeV^2. In fact, the narrowness of the ψ, if it is a $c\bar{c}$ state, can be "explained" as due to the absence of any vector epoxyon through which it could mix with a $u\bar{u}$, $d\bar{d}$ or $s\bar{s}$ state and hence to ordinary hadrons. The reason that such states appear not to bind or at least couple very weakly to ordinary hadrons is in this approach an unexplained mystery of the confinement mechanism, which while nice to understand, need not be understood in order to analyze large $p_\perp$ scattering [9].

I propose then that the expected confinement mechanism eliminates Landshoff's diagram, at least at present values of t, leaving those with at least $q\bar{q}$ or qqq in each channel[10]. It is easily seen that all remaining allowed diagrams give the asymptotic behavior[3] (modulo logarithms)

$$\frac{d\sigma}{dt} \sim s^{2-n} f(\theta) \ , \qquad (1)$$

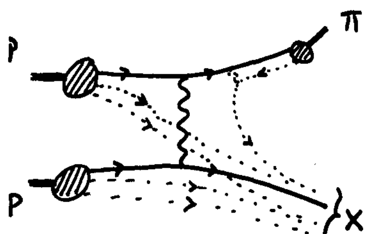
where n is the total number of elementary fields in the initial and final states, including photons, leptons and quarks (e.g., for pp scattering n = 12). This is the same result obtained by dimensional analysis if $s^{-1/2}$ is the only length scale in the problem[3,11]. Equation (1) can be generalized to 2 → N scattering such as $\pi N \to \pi\pi N$ and scattering of hadrons with non-zero orbital angular momentum. When applied to eh → eh or $e^+e^- \to h\bar{h}$ one obtains for the spin averaged form factor

$$F_h(t) \sim \frac{1}{t^{n_h - 1}} \ , \qquad (2)$$

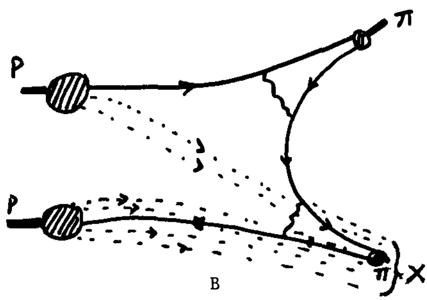
where n_h is the minimum number of elementary fields in hadron h.

These dimensional scaling laws are in good agreement with available data[3]. Nonetheless it is very difficult over the measured ranges of s and t to experimentally rule out modified exponentials, etc. A more stringent test of the validity of this use of perturbation theory, which also specifically checks whether the non-abelian color gauge theory is the correct underlying field theory, is the angular dependence of large s and t elastic scattering. While the scaling behavior, eq. (1), is independent of the details of the normal wave function, the angular dependence in general is not. However, it is often the case that the quark scattering amplitude is sharply peaked at the configuration in which the momenta of the hadrons are evenly shared by their constituents, e.g., in which the q and $\bar{q}$ of a pion each carry half the pion momentum[12]. If that is generally true, then the angular dependence of high energy wide angle elastic scattering will be insensitive to details of the normal wave functions and thus computable[13].

The analysis of large $p_\perp$ inclusive scattering is more difficult than for elastic scattering because the minimal quark scattering diagrams are not so easily identified. Some of the possible diagrams for $pp \to \pi + X$ are shown below.



A



B

Quark, antiquark and gluon lines which are involved in the large momentum transfer are solid; "spectator" lines are dotted. Call M the amplitude for the minimal high $p_\perp$ scattering; then $Ed\sigma/d^3p \sim \frac{1}{s^2}|M|^2$. Direct computation[3] shows that $M \sim \sqrt{s}^{\,4-n}$, where n is the number of quarks and antiquarks in the minimal large $p_\perp$ scattering. In diagram (A) $qq \to qq$ is the minimal large $p_\perp$ scattering so that n = 4 and $M \sim g^2$, giving $Ed\sigma/d^3p \sim g^4/p_\perp^4\, f(\theta,x)$[14]; in diagram (B) which has $q\bar{q} \to \pi\pi$ as the minimal process, $M \sim g^4/s$ so that $Ed\sigma/d^3p \sim g^8/p_\perp^8\, f(\theta,x)$. Naive application of perturbation theory without considering the effects of binding indicates that the $qq \to qq$ subprocess (diagram (A)) should dominate. It is certainly not dominant experimentally[15].

This can be qualitatively accounted for by requiring physical hadrons in each channel. In particular the t channel in case (A) can be shown to require a gluon carrying a large fraction of the t channel momentum. While it is well known that roughly half a proton's momentum is carried by glue, there is no evidence that any individual gluon carries a substantial fraction of the momentum. In fact, the absence of excited baryons with the additional degrees of freedom implied by "valence glue" is evidence that known hadrons do not contain fast gluons. (This is not surprising in view of the absence of other exotic states such as $q\bar{q}q\bar{q}$. The more natural expectation is for the gluon distribution to resemble that of antiquarks, which is only non-negligible at very small momenta. If we assume that intermediate states in the s,t and u channels which require a gluon to carry an asymptotically finite fraction of the momentum are not physically allowed, then $qq \to qq$ is not a possible minimal subprocess and $E d\sigma/d^3p$ falls faster than $p_\perp^{-4}$. Whether $p_\perp^{-8}$ is the leading allowed power is not yet known. A detailed analysis of the energy, angle, and particle species dependence of inclusive scattering data is underway[13) which will provide additional tests of these ideas.

We have assumed above that the elementary quark-gluon coupling is inherently small unless very soft processes are occurring, and have seen that we thereby obtain a reasonable picture of large $p_\perp$ scattering. Presumably low $p_\perp$ strong interactions are due to the multiple soft interactions of quark and antiquark constituents of hadrons which have small relative momenta and large effective couplings. Quarks which have large relative momenta may be assumed to interact very weakly. Thus any hadronic process which selectively involves fast quarks should be amenable to analysis. Two examples are the nuclear size (A) dependence of large $p_\perp$ or $p_\parallel$ inclusive scattering and massive lepton pair production in high energy hadron-hadron collisions.

From our analysis of large $p_\perp$ scattering we have learned that it is well described as resulting from a few hard scatterings of constituents, rather than multiple soft scatterings. Thus to obtain a hadron of large $|P_{cm}|$

requires a q or $\bar{q}$ from each initial hadron which has cm momentum $\geq |P_{cm}|$. Similarly to create a lepton pair of large invariant mass $\sqrt{Q^2}$ at rest in the cm, requires a q and $\bar{q}$ of cm momentum $\sqrt{Q^2}/2$. Thus if $|P_{cm}|$ and $\sqrt{Q^2}/2$ are large, the quarks and antiquarks involved will interact only very weakly with the rest of the hadronic material. Thus in each case we expect an A dependence which is A^1 (no shadowing) rather than the $A^{2/3}$ (shadowing) which is seen at low $|P_{cm}|$ or low $\sqrt{Q^2}/2$. The value of $|P_{cm}|$ or $\sqrt{Q^2}/2$ at which the A^1 dependence takes over can be estimated as follows[16]: If the incident particle is a proton of cm momentum P, consider its three fast valence quarks. These three bare quarks may be written as a superposition of baryons, e.g., p and N^* (plus higher states which we will truncate) by cleverly arranging their phases. As they propagate their relative phases change, since they have different masses. After a distance z, the relative phase is

$$\phi_{N^*} - \phi_p = \left[\frac{m_{N^*}^2}{P} - \frac{m_p^2}{P}\right] z \quad .$$

When the relative phase becomes large, say > 1 radian, the three fast quarks have evolved into a state which no longer looks like three bare quarks but instead looks like hadrons. That is, the three fast quarks have grown a "tail" of quarks, antiquarks and glue with which they can interact hadronically. Thus if P is such that in a distance z, $\phi_{N^*} - \phi_p < 1$, the fast quarks will interact very weakly (usually not at all) within that distance z. The diameter in the cm of a nucleus of atomic number A is $D \approx A^{1/3}/m_\pi P$. Hence requiring $|\Delta\phi| < 1$ for $z < D$ gives

$$[P/1.3 > A^{1/3}/m_\pi P] \quad . \tag{3}$$

For typical nuclei $A^{1/3} \sim 4$ so $P \approx 6$ GeV/c. Since the valence quarks in a proton typically carry 1/6 - 1/3 of the momentum, we may deduce that quarks having $P^q_{cm} > 1$-2 GeV/c will not be shadowed. Thus processes involving lepton pairs with $\sqrt{Q^2}/2 > 1$-2 GeV or pions of $|P_{cm}| > 1$-3 GeV/c will have no shadowing. The latter has been observed experimentally two ways: Cronin et al.[17], measure the A dependence, parameterized by $A^{n(p_\perp)}$ of the inclusive cross section for π's produced at 90° in the cm and $\sqrt{s} \approx 23$ GeV. Their results for $n(p_\perp)$ are shown in Fig. 1.

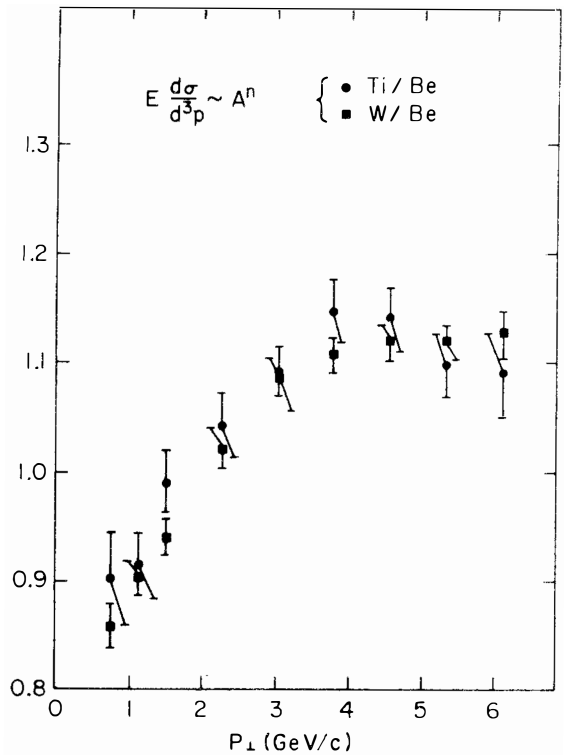
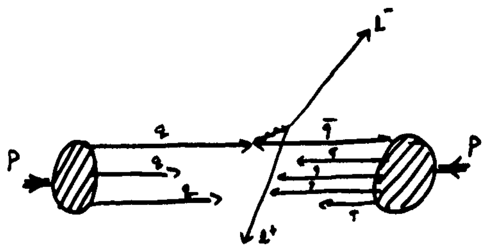


Fig. 1: The A dependence of high $p_\perp$ pion production at p_{lab} = 300 GeV/c and θ_{cm} = 90° taken from Ref. 17.

The qualitative agreement with our result is remarkable. However our discussion does not explain why $n(p_\perp)$ should become larger than 1, as is observed. Possible reasons for this are discussed in Ref. 16. The second observation is that the multiplicity of particles produced forward in the cm in p-nucleus collisions is independent of A[18], as expected from our argument but far from obvious in conventional Glauber models[19]. No measurement has been made of the A dependence of $pp \to \mu^+\mu^- + X$.

We can use our analysis of quark propagation through nuclear matter to decide when the Drell-Yan model[20] for massive lepton pair production is applicable. In that model the virtual photon is produced by the annihilation of a q from one initial hadron with a $\bar{q}$ from the other:



The cross section is:

$$\frac{d}{dQ^2 d\xi} = \frac{4\pi\alpha^2}{3Q^4}\left(\frac{1}{x_A+x_B}\right)\left\{\frac{4}{9}x_A u_A(x_A)x_B\bar{u}_B(x_B) + \frac{1}{9}(u \leftrightarrow d) + \underbrace{\frac{1}{9}(u \leftrightarrow s)}_{\text{negligible}} + \text{c.c.}\right\}$$

where $\xi = \frac{2Q_{\parallel}}{\sqrt{s}}$ and $x_A = \frac{1}{2}\left[\xi + \sqrt{\xi^2 + 4Q^2/s}\right]$, $x_B = x_A - \xi$. The $u_A(x)$ is the probability of finding an up quark in hadron A with fraction x of the momentum, etc. Taking eq. (3) with $A^{1/3} = 1$, gives $P^q_{min} \approx .5$-1 GeV. Thus if both $x_A\sqrt{s}/2$ and $x_B\sqrt{s}/2$ are > 1/2 - 1 GeV, the distributions u(x), $\bar{u}(x)$, d(x) and $\bar{d}(x)$ will be just those which are measured in electron and neutrino deep inelastic scattering. Consequently the theory makes a definite prediction if the q and $\bar{q}$ distributions are known. Fig. 2 shows such a prediction (solid line) for $pp \to \mu^+\mu^- + X$ based on a particular (reasonable) guess for the antiquark distributions[21], assuming there are three colors of quarks. Also shown (circles) is the data of Christenson et al.[22], which they extracted from $p + U \to \mu^+\mu^- + X$, assuming essentially that the cross section scales as $A^{2/3}$. The triangles show that data "renormalized" by using the $A^{n(\sqrt{Q^2})}$ dependence from Cronin et al.[17], (Fig. 1) for $\sqrt{Q^2} = 2p_\perp < 4$ GeV and n = 1 for $\sqrt{Q^2} > 4$ GeV. In addition the dotted line shows their data with the new resonances removed[23] and renormalized by the correct A dependence. The prediction of the theory is within about a factor of three of the experiment, which is within the range of the theoretical and experimental uncertainties.

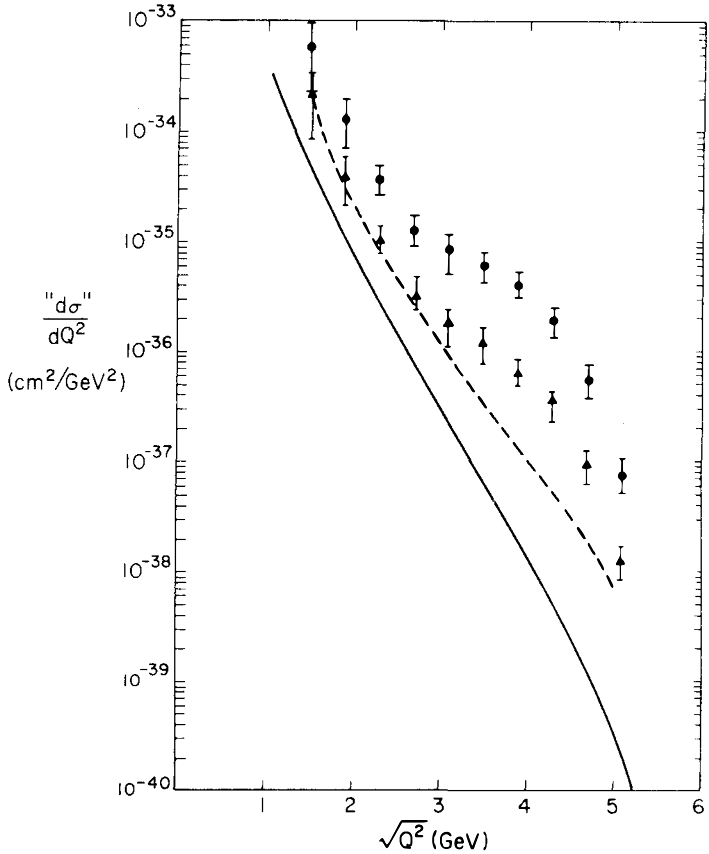


Fig. 2. The cross section (folded with experimental cuts) for $\bar{p}p \to \mu^+\mu^- + X$ as a function of the $\mu^+\mu^-$ invariant mass, $\sqrt{Q^2}$, at P_{lab} = 28.5 GeV/c.

To summarize, I have attempted to present here a unified description of several hadronic phenomena. The basic assumption is that only confinement and other soft interactions involve a large effective quark gluon coupling constant. I have proposed that in large $p_\perp$ inclusive and exclusive scattering low order perturbation theory should give a correct description of the energy and angle dependence of cross sections, with the effect of confinement being to eliminate the contribution of all diagrams which have fast gluons in any physical channel. Theoretical predictions for angular dependences, except in ep $\rightarrow$ ep, are not yet available for comparison with the data. Exclusive scattering is in good agreement with the predicted energy dependence. A complete analysis of inclusive scattering $p_\perp$ dependence is not yet available, however absence of a $p_\perp^{-4}$ falloff is evidence in favor of these ideas on the role of confinement. Further evidence of the fundamentally small strength of quark-gluon coupling is the absence of shadowing in large $p_\perp$ hadron production from nuclei and the A-independence of the multiplicity of forward produced pions. The qualitative agreement of theory and experiment for $pp \rightarrow \mu^+\mu^- + X$, when account is taken of the expected A dependence, is an encouraging indication of the consistency of all of these ideas. The most crucial missing tests of the validity of using perturbation theory as we propose are: 1) angular distributions for exclusive scattering must be predicted and compared with data; 2) data on the pion form factor at larger t and $\pi p \rightarrow \pi p$ at larger s and t is needed to check the energy scaling behavior of eq. (1); 3) $\bar{p}p \rightarrow \ell^+\ell^- + X$ and the A dependence of $pA \rightarrow \ell^+\ell^- + X$ should be measured. In addition, a complete theoretical analysis of inclusive high $p_\perp$ scattering and further experimental exploration of it are of great importance.

These ideas presented here have evolved over the past two years as a result of collaborations and conversations with a number of people to whom I am indebted. They include S. J. Brodsky, R. P. Feynman, M. Gell-Mann, A. Schwimmer and C. C. Wu.

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$$g^2(q^2)/4\pi = \left[\frac{g^2(M^2)/4\pi}{1 + g^2/16\pi^2\, b\, \ell n\,(q^2/M^2)}\right].$$

M^2 is a mass at which $g^2(M^2)/4\pi$ is already small (we are assuming $M^2 \approx 1\ \text{GeV}^2$). In addition there is a factor $[\ell n\,(q^2/m^2)]^{n-1}$, where n is the number of constituents, which comes from the integrations over the mass of the n-1 far-off shell quarks.
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