zero quark mass, the ground state and a multi--quark state at a lattice site can be degenerated in energy. When all lattice sites are combined this becomes an infinite degeneracy.

The third problem is formulating the free quark theory on the lattice. The question is what to write as the finite difference form of $\gamma \cdot \nabla$ or $\gamma_\mu \nabla_\mu$. The simplest finite difference form of ∇_μ, unfortunately, gives zero energy to eight different momentum states (the states with $P_i = 0$ or $\frac{\pi}{a}$ for each i). This means one has eight quark species. All known methods of avoiding the multiplication destroy chiral symmetry, unless one uses the very nonlocal approach of Drell et al. Each group has their own favorite method for dealing with the problem. These methods are always designed to restore chiral symmetry in the continuum limit.

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POWER COUNTING THEOREM FOR INFRARED LOGARITHMS IN NON-ABELIAN GAUGE THEORIES

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In recent years the theory of quarks coupled to non-Abelian gauge bosons has emerged as a promising field theoretical model for understanding the spectroscopy of hadrons and their reactions. In spite of its qualitative successes, however, the basic premise that quarks and gluons themselves are permanently bound within hadrons and never appear as asymptotic particles is yet to be verified.

It has been speculated /1/ that the severe infrared divergence of non-Abelian gauge theories, as is indicated by the growth of the effective gauge coupling constant at large distances, may provide a mechanism for such confinement. The viability of this idea has been demonstrated in 2-dimensional gauge theories /2/ in which free quarks are indeed eliminated from the particle spectrum because of infrared divergence.

(However, this may be due to the low dimensionality of the space-time, having nothing to do with the non-Abelian nature of the gauge group. Thus, there may be no easy generalization of these results to the 4-dimensional case).

On the other hand, the information on the 4-dimensional case is rather limited at present: Conrwall and Tiktopoulos /3/ have attempted to sum up the leading infrared logarithms. Other recent works are concerned with the study of relatively low orders or directed toward an alternate verification or extension of the cancellation theorem /4,5,6,7/ . These works, however, do not answer the questions concerning the nature of infrared singularities of non-Abelian gauge theories such as 1. What are the differences of non-Abelian gauge theories and QED? 2. Does the Bloch-Nordsieck scheme work in such a theory? 3. What is the large distance behaviour of quark-antiquark potential? 4. Do their theories intrinsically require non-perturbative approach?

As the first step for analyzing their problem, we have developed within the Feynman diagram framework a general method for precisely determining the power of infrared logarithms for arbitrary diagrams [8]. It is applied specifically to the quark electromagnetic form factor shown in Fig.1 in which quarks have <u>finite</u> mass m and the momentum transfer $q = p' - p''$ is also <u>finite</u>.

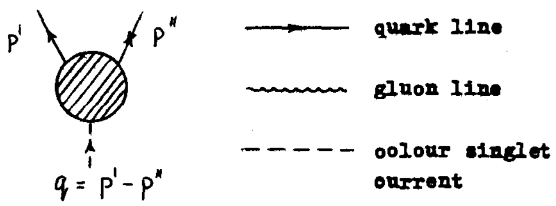


Fig.1

The result of our analysis may be stated (with some over-simplification for the sake of clarity) as follows:

<u>Power counting theorem.</u> Let G be an arbitrary 1PI colour singlet current-fermion--fermion vertex diagram and let F_G be the corresponding renormalized Feynman amplitude for which all renormalizations (except for the fermion mass which is renormalized on the mass shell) are performed at an off-mass-shell point μ. Suppose (the "electric" form factor of) F_G behaves as

$$F_G = a(\ln\lambda)^{m_G} + \dots$$

in the limit in which an appropriately chosen infrared cut-off tends to zero, where a depends on q^2, m^2, μ^2 and the gauge coupling constant g only. Then we have

m_G = the number of nontrivial infrared singular subdiagrams of G.

To explain the content of this theorem, we must define some words.

i. Infrared singular subdiagram. It can be shown that the only subdiagrams of G (including G itself) which are infrared singular are the following:

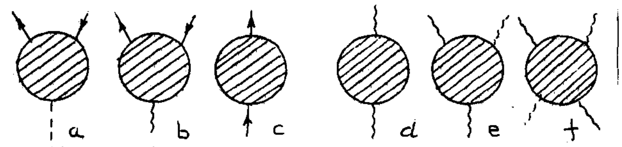


Fig.2

ii. All digrams of Fig.2a are <u>nontrivial</u> i.e. contribute one unit to m_G.

iii. Diagrams of Fig.2b,c are nontrivial if and only if they contain no triple-gluon vertex like

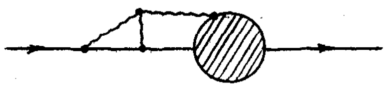


Fig.3

iv. Diagrams of Fig.2d,e,f count as L nontrivial infrared singular subdiagram where L is the maximum number of nonoverlapping[9] ultraviolet divergent subdiagrams within such a subdiagram.

To illustrate this theorem, the values of m_G for some simple diagrams are shown in Fig.4.

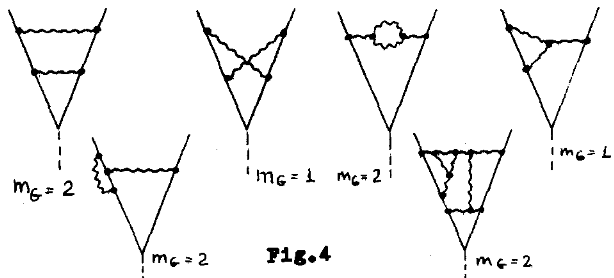
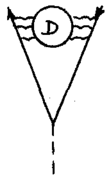


Fig.4

<u>Outline of the proof.</u> The Feynman amplitude F_G corresponding to the diagram G of Fig.1 can be expressed in the form

$$F_G = \int_0^\infty \delta(1 - \sum_{i\in G} z_i) \prod_{i\in G} dz_i\, J_G(z) , \qquad (1)$$

where $z_1, z_2, \dots$ are Feynman parameters. The diagrams with quark loops are found to be less infrared-singular than those of the same order without quark loops. Thus, we shall consider only the diagrams shown in Fig.5:



$G = D \oplus P$ where P consists of quark lines only and D consists of gluon lines only.

Fig.5

For such diagrams all infrared divergences arise from the region of parametric space where $z_i \to 0$ for all $i \in P$.

Thus, the infrared divergent part of F_G is contained in the integral

$$\tilde{F}_G = \int \theta(\delta_0 - \sum_{i\in P} z_i) \prod_{i\in P} dz_i \int \delta(1 - \sum_{j\in D} z_j) \prod_{j \in D} dz_j \, J_G(z), \qquad (2)$$

where δ_0 is a small fixed positive constant. If we introduce the function

$$J_D(z) = \int \delta(1 - \sum_{i\in D} z_i) \prod_{i\in D} dz_i \, J_G(z), \qquad (3)$$

which is a function of z_j, belonging to the quark path P only, $\tilde{F}_G$ be written as

$$\tilde{F}_G = \int \theta(\delta_0 - \sum_{i\in P} z_i) \prod_{i\in P} dz_i \, J_D(z). \qquad (4)$$

Now, to deal first with the infrared divergences arising from quark self-energy and quark vertex parts, let us consider the scale transformation

$$z_{i_1} = \rho z'_{i_1}, \; z_{i_2} = \rho z'_{i_2}, \; \dots, \; z_{i_\ell} = \rho z'_{i_\ell}, \qquad (5)$$

where $\{i_1, \dots, i_\ell\}$ is an arbitrary subset of P. Then, we find for $\rho \to 0$

$$J_D \to \rho^{-d} \times (\text{power of } \ln\rho \text{ if } d \le 0). \qquad (6)$$

We obtain the following relation between ℓ and d:

<u>Lemma</u>. (i) $d \le \ell$ (ii) $d = \ell$ holds if and only if $i_1, \dots, i_\ell$ make up a continuous quark path of an infrared singular subdiagram (see Fig.2).

To illustrate this lemma, consider the following diagram:

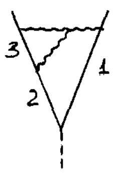


Fig.6

For the continuous quark paths $\{1,2,3\}$, $\{1,2\}$ and $\{3\}$ we find

$$J_D(z_1, z_2, z_3) \longrightarrow \frac{1}{(z_1+z_2+z_3)^3}, \; d = \ell = 3, \qquad (7a)$$

$$\longrightarrow \frac{1}{z_1+z_2}, \; d = 1, \; \ell = 2, \qquad (7b)$$

$$\longrightarrow \frac{1}{z_3}, \; d = \ell = 1. \qquad (7c)$$

Thus, (7a) gives a logarithmic divergence on integration over z_1, z_2, z_3, (7b) gives a finite result, and (7c) is logarithmic but actually becomes finite because of the infrared suppression of the three-gluon vertex (see Fig.3). The integrations over the remaining quark paths $\{1\}$, $\{2\}$, $\{2,3\}$ are all infrared finite.

This lemma leads to the restriction of infrared divergent subdiagrams involving quark lines to those of Fig.2a,b,c.

From this analysis it follows that the infrared singular part of J_D factorises into a product of contribution of all infrared divergent subdiagrams. This property enables us to prove the theorem with the help of Ward's identity and mathematical induction.

The infrared singularities arising from the gluon diagrams of Fig.2d,e,f can be included in the theorem making use of the singularity family introduced by Speer /10/.

<u>Remarks.</u> 1. This theorem is stated for the Feynman gauge. If other gauge are used, it must be modified accordingly. 2. For simplicity it is stated for the "electric" form factor only. For the "magnetic" form factor, the power of $\ln\lambda$ must be reduced by one. 3. It is formulated for the case of <u>massive</u> quarks and <u>fixed</u> q^2. Other interesting cases such as <u>massless</u> quarks and the leading behaviour for very large q^2 can be analyzed by a slight modification of our technique. 4. In the case where G contains only subdiagrams of Fig.2a,b the theorem shows that the leading infrared singular diagrams are restricted to those which are planar and have nests of these subdiagrams. Such a structure of

leading diagrams is found to persist even after subdiagrams of Fig. 2c,d,e,f are included. Thus, all leading diagrams can be generated by a set of Dyson-Schwinger integral equations. This may be useful for the purpose of summing up the leading infrared singularities. It may also serve as a starting point of search for a renormalization group like equations for infrared divergence.

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MACROSCOPIC INFLUENCE ON THE SPONTANEOUS SYMMETRY BREAKING IN QUANTUM FIELD THEORY

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I. Introduction

The aim of this report is to give the brief review of the results concerning macroscopic influence (heating, compression, external field and current) on elementary particles systems with spontaneous symmetry breaking.*) The study of this problem has been stimulated by recent progress in unified renormalizable theory of elementary particles. Typically it appears that at some values of external parameters (temperature, field, density, current) a phase transition with symmetry restoration takes place. There exists a profound and far going analogy with phase transition in many body physics especially with superconductivity phenomenon (SC). Some applications to Cosmology are also considered in this report.

2. The model

The Higgs model is used for description of broken symmetry

$$L = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + |D_\mu \varphi|^2 + \frac{\mu^2}{2}|\varphi|^2 - \frac{\lambda}{4}|\varphi|^4 + L_{fermion}, \qquad (1)$$

where $D_\mu = \partial_\mu + igA_\mu$, $\lambda \ll 1$, $g \ll 1$.

This is a relativistic analogue of Ginzburg-Landau theory of SC. From eq. (1) one can obtain the effective potential $V(\sigma)$ (energy or free energy) in the "external field" $\sigma = \langle\varphi\rangle$

The equilibrium condition is

$$\frac{\partial V(\sigma)}{\partial \sigma} = \lambda\sigma^3 - \mu^2\sigma + \Gamma = 0, \qquad (2)$$

where Γ is diagrams of the current of the field φ .

At small g the Higgs mechanism gives $\sigma_0 \neq 0$ because of Bose-Einstein condensation of φ -quanta (see fig.1) and $m_\varphi^2 \sim \mu^2 > 0$, $m_A^2 \sim g^2\sigma_0^2$, $\sigma_0^2 \sim \frac{\mu^2}{\lambda}$.

*) For more details and References see LG = Lebedev Group= A.D.Linde + repporteur, Annals of Physics, N.Y. (in press)