

EXPERIMENTAL TESTS OF THE VECTOR DOMINANCE MODEL IN PSEUDOSCALAR  
MESON PHOTOPRODUCTION

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Introduction: The purpose of this talk is to reexamine the experimental tests of the Vector Meson Dominance (VMD) model in photoproduction of pseudoscalar mesons<sup>1</sup>. This type of reaction has been selected since it enables clean tests of the VMD model itself rather than tests of the VMD model combined with additional model dependent assumptions. Special attention will be paid to the ambiguities associated with the model. The so-called "failures" of the model will be discussed. It will be shown that they may be due to one, or more, of the following reasons:

- (i) The application of the model in a kinematical region where it cannot be expected to be valid.
- (ii) The use of unreliable experimental data on strong production in experimental tests of the model.
- (iii) The introduction of additional model dependent assumptions, which then prevent a clean test of the VMD model itself.

Impressive evidence for the validity of the model will be presented. New possible tests of the model will be proposed. In particular clean tests of the model which should interest experimentalists will be discussed.

The VMD Model and its Ambiguities: As you may know, the VMD hypothesis for the hadron electromagnetic current<sup>2</sup> leads to relations between any photoproduction amplitude on a target T and a linear combination of the amplitudes for the corresponding strong production by transversely polarized vector mesons  $V_{tr}$

This relation can be obtained, for instance, from the current field identity<sup>3</sup>

$$eJ_\mu = g_{\rho\gamma} m_\rho^2 \rho_\mu + g_{\omega\gamma} m_\omega^2 \omega_\mu + g_{\phi\gamma} m_\phi^2 \phi_\mu , \quad (2)$$

using<sup>4</sup> standard methods of field theory. More precisely, eq. (1) is actually derived from eq. (2) for massless vector mesons, and the main assumption of the VMD model is that the amplitudes on the RHS of eq. (1) do not change markedly when we extrapolate from zero mass to the physical mass of the vector mesons. The symbols in eq. (1) and (2) have the following meaning:  $g_{V\gamma}$  are the direct vector meson-photon coupling constants (they are assumed not to show any strong dependence on the mass of the photon). They can be determined from the electromagnetic decay rates of the mesons. Indeed their values do not show any significant dependence on the mass of the photon, as it is indicated in Table I.  $J_\mu$  is the hadron electromagnetic current  $\rho_\mu$   $\omega_\mu$  and  $\phi_\mu$  are the renormalized fields for  $\omega$   $\rho$  and  $\phi$ ;  $m_\rho$   $m_\omega$  and  $m_\phi$  are their masses, respectively.

There are two fundamental difficulties associated with expression (1):

- (a) The concept of transverse polarization for massive vector mesons is not Lorentz invariant, and formula (1) is highly non-unique as long as no frame of reference is specified with respect to which transversely polarized vector mesons are to be used on the RHS of (1).
- (b) The physical regions in  $s$ ,  $t$  and  $u$  **for the strong** production and the photoproduction process in formula (1) **do not overlap** ( $s+t+u = \sum_i m_i^2$  !) We therefore cannot expect eq. (1) to be **valid everywhere**. A question then arises for what values of  $s$  and  $t$ , or  $s$  and  $u$ , etc., should eq. (1) be valid?

Obviously these two problems cannot be solved on purely kinematical grounds.

However, using quite a general class of realistic models it can be shown that:

- (i) The VMD model for photoproduction should be postulated in the s-channel helicity frame<sup>5,6</sup> ;
- (ii) The VMD relations for photoproduction should be valid only for high enough s values so that the difference between the minimum momentum transfers in the photoproduction and in the analogous strong production is small compared with  $m_\pi m_e$  , where  $m_e$  is the mass of the lightest particle that can be exchanged in these reactions<sup>6</sup> ;

$$\Delta t_{\min} = t_{\min}(\gamma) - t_{\min}(V) \ll m_\pi m_e .$$

$\gamma$  and  $V$  stand for the photoproduction reaction and for the analogous vector meson initiated reaction, respectively.

Rather than to justify these two statements at this point let me first review the VMD relations for pseudoscalar meson photoproduction and later in the Appendix come back to the justification of statements (i) and (ii).

Review of VMD Relations for Pseudoscalar Meson Photoproduction. We look for experimental tests of eq. (1). Obviously due to the short life times of the  $\rho$ ,  $\omega$  and  $\phi$  no such beams are available, and eq. (1) cannot be directly tested unless additional assumptions are introduced! The simplest assumption is the time reversal invariance. Because of it the process on the RHS on eq. (1) can be reversed so that any photoproduction amplitude is expressed as a linear combination of the corresponding three amplitudes for  $\rho$ ,  $\omega$  and  $\phi$  production.

However, experimental measurements usually give only cross sections and not the relative phases of the amplitudes. Consequently a critical check of (1) is not possible if the interference terms arising from squaring the RHS of (1) are present unless some reliable assumptions about the phases are possible.

Relation I:  $\pi^0$  Photoproduction From Isoscalar Targets. In some cases isospin conservation can be used to eliminate the contribution from the isoscalar component of the photon and from the isoscalar vector mesons to the LHS and RHS of eq. (1), respectively. One case of this kind is pion photoproduction on an isoscalar target ( $D$ ,  $He^4$ ,  $C^{12}$ , ...) where the contribution of the isoscalar component of the photon vanishes by isospin conservation. For this case we obtain<sup>7</sup>

$$\rho_{11}^H \frac{d\sigma}{dt} (\pi^\pm T_{I=0} \rightarrow \rho^\pm T_{I=0}) = \left( \frac{g_{\rho\pi\pi}}{e} \right)^2 \frac{d\sigma}{dt} (\gamma T_{I=0} \rightarrow \pi^0 T_{I=0}) , \quad (4)$$

where we used the VMD relation  $g_{\rho\gamma} = \frac{e}{g_{\rho\pi\pi}}$ .  $\rho^H$  is the density matrix in the s-channel helicity frame for  $\rho$  production. Similar relations can be obtained for linearly polarized photons. In particular for photons linearly polarized perpendicular and parallel to the production plane, the density matrix element  $\rho_{11}^H$  has to be replaced by  $\rho_\perp^H$  and  $\rho_{||}^H$ , respectively, where

$$\rho_\perp^H = \rho_{11}^H + \rho_{1-1}^H \quad (5a)$$

and

$$\rho_{||}^H = \rho_{11}^H - \rho_{1+1}^H . \quad (5b)$$

The so-called "asymmetry" is then given by

$$A = \frac{(d\sigma/dt)_{\perp} - (d\sigma/dt)_{\parallel}}{(d\sigma/dt)_{\perp} + (d\sigma/dt)_{\parallel}} = \frac{\rho_{1-1}^H}{\rho_{11}^H} , \quad (5c)$$

where  $(d\sigma/dt)_{\perp}$  and  $(d\sigma/dt)_{\parallel}$  are the differential cross sections for  $\gamma T \rightarrow \pi^0 T$ , with photons linearly polarized perpendicular and parallel, respectively, to the production plane.

The reaction  $\pi^{\pm} T_{I=0} \rightarrow \pi^{\pm} \pi^0 T_{I=0}$  has the important property that the  $\pi^{\pm}$  is produced in it without s or d wave contaminations : The  $\pi^{\pm} \pi^0$  system can be in either  $I=1$  or  $I=2$  states. Because of isospin conservation the  $I=2$  state is not produced in the above reaction, and because of Bose statistics the  $\pi^{\pm} \pi^0$  systems in an  $I=1$  state cannot be in an s-wave or in a d-wave. Note also that for a spin zero target (for instance  $H_e^4$ ) the two helicity amplitudes for the reaction  $\gamma T_{s=0} \rightarrow \pi^0 T_{s=0}$  are equal because of parity conservation. Consequently, the cross section for the reaction with photons linearly polarized parallel to the production plane vanishes, and the asymmetry (expression (5c) ) in the reaction is equal to 1.

Experimental testing of (4) would provide the most direct check on the VMD model for photoproduction, since the derivation of eq. (4) was based only on :

Assumption 1: Vector Meson Dominance

Assumption 2: Time Reversal Invariance In Strong Production

Assumption 3: Isospin Invariance In Strong Production

We therefore urge our experimentalist colleagues to perform those experiments. We stress again that comparison should be made in the s-channel helicity frame. From

Table II in the appendix we see that eq. (4) is expected to be well satisfied, say for the  $\text{He}^4$  target, for  $P_{\text{LAB}} > 2 \text{GeV}/c$ .

Relations II:  $\pi$  Photoproduction From Nucleons. Another relation which is based only on the above three assumptions is the relation<sup>7</sup>

$$\begin{aligned} \frac{d\sigma}{dt} (\gamma p \rightarrow \pi^0 p) + \frac{d\sigma}{dt} (\gamma n \rightarrow \pi^0 n) - \frac{1}{2} \frac{d\sigma}{dt} (\gamma p \rightarrow \pi^+ n) - \frac{1}{2} \frac{d\sigma}{dt} (\gamma n \rightarrow \pi^- p) \\ = g_{\rho\gamma}^2 \left[ \rho_{11}^H \frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^- p) + \rho_{11}^H \frac{d\sigma}{dt} (\pi^+ p \rightarrow \rho^+ p) - 2\rho_{11}^H \frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^0 n) \right]. \quad (6) \end{aligned}$$

Similar relations can be derived for polarized photons. In particular for photons linearly polarized perpendicular and parallel to the production plane,  $\rho_{11}^H$  in eq. (6) has to be replaced by  $\rho_{\perp}^H$  and  $\rho_{\parallel}^H$ , respectively (for their definition see eqs. (5)).

Relation (6) is expected to be valid within 10% accuracy for  $P_{\text{LAB}} \gtrsim 8 \text{ GeV}/c$ . (see the Appendix, in particular Table III there). Unfortunately no experimental data are presently available on  $\gamma n \rightarrow \pi^0 n$  at such incident energies. However, if we exclude the extremely small  $-t$  region, then relation (6) is expected to be satisfied within 10% accuracy already for  $P_{\text{LAB}} \sim 4 \text{ GeV}/c$ , where experimental data are available on all the reactions that are present in (6).

Figure 1 presents a comparison between relation (6) and experimental results at  $4 \text{ GeV}/c$ . The references from which the experimental results were taken are summarized in Table II. Figure 1 indicates that relation (6) is well satisfied by presently available data.

If in addition to assumptions 1-3 we also assume that the  $\phi$  contribution to the RHS of (1) can be neglected (Assumption 4) then eq. (1) can be tested with experimental data of much better statistics. The neglect of the  $\phi$  contribution to the RHS of eq. (1) is justified on the basis of the following relations between experimental values of cross sections<sup>8</sup> and the vector meson photon couplings of Table I:

$$g_{\rho\gamma}^2 \frac{d\sigma}{dt} (\pi p \rightarrow \rho p) \gg g_{\omega\gamma}^2 \frac{d\sigma}{dt} (\pi p \rightarrow \omega p) \gg g_{\phi\gamma}^2 \frac{d\sigma}{dt} (\pi p \rightarrow \phi p) . \quad (7)$$

Linear combinations of cross sections can be chosen, in which the interference term between isovector and isoscalar contributions is absent. If the  $\phi$  contribution is neglected, one obtains the approximate relations<sup>7</sup>:

$$\begin{aligned} g_{\rho\gamma}^2 \rho_{11}^H \frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^0 n) + g_{\omega\gamma}^2 \rho_{11}^H \frac{d\sigma}{dt} (\pi^- p \rightarrow \omega n) \\ \approx \frac{1}{2} \left[ \frac{d\sigma}{dt} (\gamma p \rightarrow \pi^+ n) + \frac{d\sigma}{dt} (\gamma n \rightarrow \pi^- p) \right] , \end{aligned} \quad (8a)$$

$$\begin{aligned} g_{\rho\gamma}^2 \rho_{11}^H \frac{d\sigma}{dt} (\pi^0 p \rightarrow \rho^0 p) + g_{\omega\gamma}^2 \rho_{11}^H \frac{d\sigma}{dt} (\pi^0 p \rightarrow \omega^0 p) \\ \approx \frac{1}{2} \left[ \frac{d\sigma}{dt} (\gamma p \rightarrow \pi^0 p) + \frac{d\sigma}{dt} (\gamma n \rightarrow \pi^0 n) \right] , \end{aligned} \quad (8b)$$

where from isospin invariance

$$\frac{d\sigma}{dt} (\pi^0 p \rightarrow \rho^0 p) = \frac{1}{2} \left[ \frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^- p) + \frac{d\sigma}{dt} (\pi^+ p \rightarrow \rho^+ p) - \frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^0 n) \right] , \quad (9a)$$

and

$$\frac{d\sigma}{dt} (\pi^0 p \rightarrow \omega^0 p) = \frac{1}{2} \frac{d\sigma}{dt} (\pi^- p \rightarrow \omega n) = \frac{1}{2} \frac{d\sigma}{dt} (\pi^+ n \rightarrow \omega p) . \quad (9b)$$

Similar relations can be derived for polarized photons. In particular for photons linearly polarized perpendicular and parallel respectively to the production plane, one has to replace  $\rho_{11}^H$  in relations (8) by  $\rho_{\perp}^H$  and  $\rho_{\parallel}^H$  respectively (see eqs. (5) for their definitions). The asymmetry ratios for the reactions are then given by<sup>9</sup>

$$A = \frac{(\overline{d\sigma/dt})_{\perp} - (\overline{d\sigma/dt})_{\parallel}}{(\overline{d\sigma/dt})_{\perp} + (\overline{d\sigma/dt})_{\parallel}} = \frac{\rho_{1-1}^H(\rho) + \alpha \rho_{1-1}^H(\omega)}{\rho_{11}^H(\rho) + \alpha \rho_{11}^H(\omega)} , \quad (8c)$$

where  $(\overline{d\sigma/dt})$  indicates cross section averaged over a proton and a neutron target, and

$$\alpha = g_{\omega\gamma}^2/g_{p\gamma}^2 \sim 1/9 .$$

Relation (8a) is expected to be satisfied within 10% accuracy for  $P_{LAB} \gtrsim 8$  GeV/c. However, if one excludes the small  $-t$  region relation (8a) is expected to be valid within 10% accuracy already at  $P_{LAB} \sim 4$  GeV/c. (see the Appendix and Table III there). Figure 2 presents a comparison between relation (8a) and experimental results at 4 and 8 GeV/c. The references from which the experimental results were taken are summarized in Table II.

Figure 2 demonstrates the well known result<sup>7,10</sup> that relation (8a) is in good agreement with experiment. Note in particular that :

- (i) although  $\frac{d\sigma}{dt} (\pi^- p \rightarrow p^0 n)$  has a narrow forward dip,<sup>11</sup> the polarized cross section  $\rho_{11}^H \frac{d\sigma}{dt} (\pi^- p \rightarrow p^0 n)$  exhibits a forward spike<sup>12</sup> analogous to the spikes observed in  $\frac{d\sigma}{dt} (\gamma p \rightarrow \pi^+ n)$ <sup>13</sup> and in  $\frac{d\sigma}{dt} (\gamma n \rightarrow \pi^- p)$ <sup>14</sup> in good agreement with the VMD hypothesis.

(ii) although the slop of  $\rho_{11}^H \frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^0 n)^{15}$  is different from the slope of  $\frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^0 n)^{15}$  it coincides with the slope of  $\frac{d\sigma}{dt} (\gamma p \rightarrow \pi^+ n) + \frac{d\sigma}{dt} (\gamma n \rightarrow \pi^- p)^{13,14,16}$ .

In view of the impressive success of relation (8a) for unpolarized photons, it is quite surprising that relation (8a) is badly violated for linearly polarized photons, as demonstrated in Figures 3 and 4: Figure 3 compares relation (8a) for photons linearly polarized perpendicular to the production plane and experimental results<sup>18</sup> at 3.4 GeV/c. Figure 4 compares the asymmetry relation (8c) and experimental results<sup>18</sup> at 3.4 GeV/c. Both relations strongly depend on the density matrix element  $\rho_{1-1}^H$ . However, recently doubts have been raised on the correct determination of  $\rho_{1-1}^H$  used in the comparisons. It was pointed out by the Notre Dame group that the matrix element  $\rho_{1-1}^H$  used in the comparisons has a large contamination from the d-wave background, as evidenced by the fact that in the  $\rho^0$  decay the quantity  $\langle \cos 2\phi \rangle$  as a function of  $\theta$  does not exhibit a  $\sin^2 \theta$  behavior as expected from a state with  $J < 2$ , where  $\phi$  and  $\theta$  are the azimuthal and polar decay angles of the  $\rho^0$  meson. In view of this and the impressive success of relation (8a) for unpolarized photons we tend to believe that the failure of (8a) for polarized photons and of relation (8c) is probably due to the poorly determined density matrix element  $\rho_{1-1}^H$ , rather than due to a failure of the VMD model. Reliable measurement of  $\rho_{1-1}^H$  for  $\pi^- p \rightarrow \rho^0 n$  is badly required.

Figure 5 presents a comparison between relation (8b) and experimental results (Table II) at  $P_{LAB} = 4$  GeV/c. The differential cross sections for  $\pi^0 p \rightarrow \rho^0_{tr} p$  and  $\pi^0 p \rightarrow \omega_{tr} p$  that were used to evaluate relation (8b) were determined from relations (9). They are presented in Figure 6. The agreement between theory and

experiment as indicated in Figure 5 is more than satisfactory. This agreement is significantly better than the one obtained in reference 7 due to new data of better quality<sup>15,20</sup> on strong production

Note also that the VMD model set both upper and lower bounds to  $\pi$  production from a single nucleon<sup>7</sup>:

$$\frac{d\sigma}{dt} (\gamma N \rightarrow \pi N) \leq \{g_{\rho\gamma} [\rho_{11}^H \frac{d\sigma}{dt} (\pi N \rightarrow \rho^0 N)]^{1/2} + g_{\omega\gamma} [\rho_{11}^H \frac{d\sigma}{dt} (\pi N \rightarrow \omega N)]^{1/2}\}^2 , \quad (8d)$$

where we have suppressed the charge indices. The upper (lower) bound is achieved when the ratio between the corresponding  $\rho$  and  $\omega$  s-channel helicity amplitudes is a real positive (negative) number independent of the helicity indices (It may however depend on  $s$  and  $t$ ).

Figures 7 and 8 present comparisons between these bounds for the neutron to proton  $\pi$ -photoproduction ratios and experimental results<sup>21</sup>. The figures demonstrate that these bounds are consistent with the experimental results.

Relations III: The Photoproduction Reactions  $\gamma N \rightarrow \pi\Delta$ . VMD relations for the reactions  $\gamma N \rightarrow \pi\Delta$  can be obtained only if we introduce an additional assumption (Assumption 5) that the relevant strong cross sections do not change under crossing from the u to the s channel.

The following relations are then obtained<sup>22</sup>:

$$\frac{1}{2} \left[ \frac{d\sigma}{dt} (\gamma p \rightarrow \pi^- \Delta) + \frac{d\sigma}{dt} (\gamma n \rightarrow \pi^+ \Delta) \right] \approx g_{\rho\gamma}^2 \rho_{11}^H \frac{d\sigma}{dt} (\pi^+ p \rightarrow \rho^0 \Delta) + g_{\omega\gamma}^2 \rho_{11}^H \frac{d\sigma}{dt} (\pi^+ p \rightarrow \omega \Delta) \quad (10a)$$

$$\frac{1}{2} \left[ \frac{d\sigma}{dt} (\gamma p \rightarrow \pi^+ \Delta) + \frac{d\sigma}{dt} (\gamma n \rightarrow \pi^- \Delta) \right] \approx g_{\rho\gamma}^2 \rho_{11}^H \frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^0 \Delta) + \frac{1}{1/3} g_{\omega\gamma}^2 \rho_{11}^H \frac{d\sigma}{dt} (\pi^+ \bar{p} \rightarrow \omega \Delta) \quad (10b)$$

$$\frac{1}{2} \left[ \frac{d\sigma}{dt} (\gamma p \rightarrow \pi^0 \Delta) + \frac{d\sigma}{dt} (\gamma n \rightarrow \pi^0 \Delta) \right] \approx g_{\rho\gamma}^2 \left[ \frac{1}{3} \rho_{11}^H \frac{d\sigma}{dt} (\pi^+ p \rightarrow \rho^0 \Delta) \right. \quad (10c)$$

$$\left. \rho_{11}^H \frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^0 \Delta) - \rho_{11}^H \frac{d\sigma}{dt} (\pi^+ n \rightarrow \rho^+ \Delta) \right] + \frac{2/3}{2} g_{\omega\gamma}^2 \rho_{11}^H \frac{d\sigma}{dt} (\pi^+ \bar{p} \rightarrow \omega \Delta)$$

The  $\phi$  contribution to the RHS of eqs. (10) was neglected due to the small production cross section for  $\pi p \rightarrow \phi \Delta$ . Note that the  $\phi$  contribution vanishes identically in relations that are obtained from relations (10) by eliminating the  $\omega$  contribution, since the  $\omega$  actually represents the whole isoscalar photon contribution to eqs. (10).

Similar relations can be written for polarized photons. In particular for photons linearly polarized perpendicular and parallel to the production plane  $\rho_{11}^H$  has to be replaced by  $\rho_{\perp}^H$  and  $\rho_{\parallel}^H$ , respectively, (see eqs. (5) for their definitions).

Figure 9 presents a comparison between relation (10a) and experimental results<sup>23</sup> at  $P_{LAB} = 8 \text{ GeV}/c$ , which is the highest energy where experimental results are available on all the reactions that appear in (10a). It is evident from this figure<sup>24</sup> that the reaction  $\pi^+ p \rightarrow \rho_{tr}^0 \Delta$  is strongly suppressed compared to the analogous photon initiated reaction. However, this is expected in view of the fact that the minimum momentum transfer in this reaction is not small compared with the mass of the lightest particle that can be exchanged in the reaction, i.e. at  $8 \text{ GeV}/c$   $t_{\min}$  does not satisfy  $t_{\min} \ll m_{\pi}^2$ . (see for details the discussion in the Appendix). Note also that the failure of relation (10a) can also be caused by a

failure of the "line reversal symmetry" (Assumption 5) for the reaction

$\pi^+ p \rightarrow \rho_{tr}^0 \Delta$ . Indeed this reaction can proceed via the exchange of particles which behave differently under u-s crossing ( $\pi$  and  $A_1$  versus  $A_2$ ). We therefore propose that the failure of relation (10a) at  $P_{LAB} \lesssim 8 \text{ GeV}/c$  is due to one or both of the following reasons:

(i) Energy too low for the relation to be valid.

(ii) Failure of "line reversal symmetry" for the reaction  $\pi^+ p \rightarrow \rho_{tr}^0 \Delta$ .

rather than due to a failure of the VMD hypothesis!

#### Relations IV: The Photoproduction Reactions $\gamma N \rightarrow \eta N$ and $\gamma N \rightarrow \chi^0 N$ .

If in addition to assumptions (1) and (2) we also assume that (i) the reactions  $\gamma N \rightarrow \eta N$  and  $\gamma N \rightarrow \chi^0 N$  are dominated by  $\rho$ ,  $\omega$  and  $B$  exchange (with or without absorption corrections), and that (ii)  $SU(6)_W$  symmetry holds for boson couplings, we then obtain the relations<sup>25,26</sup>

$$\frac{d\sigma}{dt} (\gamma p \rightarrow \eta p) + \frac{d\sigma}{dt} (\gamma n \rightarrow \eta n) \approx \frac{A^2}{3} \left[ g_{\rho\gamma}^2 \rho_{11}^H \frac{d\sigma}{dt} (\pi^0 p \rightarrow \omega p) + g_{\omega\gamma}^2 \rho_{11}^H \frac{d\sigma}{dt} (\pi^0 p \rightarrow \rho^0 p) \right], \quad (11a)$$

$$\frac{d\sigma}{dt} (\gamma p \rightarrow \chi^0 p) + \frac{d\sigma}{dt} (\gamma n \rightarrow \chi^0 n) \approx \frac{B^2}{3} \left[ g_{\rho\gamma}^2 \rho_{11}^H \frac{d\sigma}{dt} (\pi^0 p \rightarrow \omega p) + g_{\omega\gamma}^2 \rho_{11}^H \frac{d\sigma}{dt} (\pi^0 p \rightarrow \rho^0 p) \right]. \quad (11b)$$

where  $A = \cos \theta - \sqrt{2} \sin \theta$  and  $B = \sqrt{2} \cos \theta + \sin \theta$ .  $\theta$  is the  $\eta \chi^0$  mixing angle. From the quadratic mass formula one has  $\theta = -10.4^\circ$ , where the sign of  $\theta$  was determined from meson decays. The cross sections on the RHS of (11) can be determined from expressions (9). For photons linearly polarized perpendicular and parallel to the production plane  $\rho_{11}^H$  has to be replaced by  $\rho_\perp^H$  and  $\rho_\parallel^H$ , respectively.

If we make use of the same assumptions for the reactions  $\gamma N \rightarrow \pi^0 N$  then the following sum rules are obtained<sup>25</sup>

$$\frac{d\sigma}{dt} (\gamma p \rightarrow \eta p) - \frac{d\sigma}{dt} (\gamma n \rightarrow \eta n) = \frac{A^2}{3} \left[ \frac{d\sigma}{dt} (\gamma p \rightarrow \pi^0 p) - \frac{d\sigma}{dt} (\gamma n \rightarrow \pi^0 n) \right] \quad (12a)$$

$$\frac{d\sigma}{dt} (\gamma p \rightarrow \chi^0 p) - \frac{d\sigma}{dt} (\gamma n \rightarrow \chi^0 n) = \frac{B^2}{3} \left[ \frac{d\sigma}{dt} (\gamma p \rightarrow \pi^0 p) - \frac{d\sigma}{dt} (\gamma n \rightarrow \pi^0 n) \right] \quad (12b)$$

When expressions (11) and (12) are combined together they yield the relation<sup>26</sup>

$$\frac{d\sigma}{dt} (\gamma p \rightarrow \eta p) = \frac{A^2}{6} \left\{ 2 \frac{d\sigma}{dt} (\gamma p \rightarrow \pi^0 p) + (g_{\rho\gamma}^2 - g_{\omega\gamma}^2) \left[ \rho_{11}^H \frac{d\sigma}{dt} (\pi^0 p \rightarrow \omega p) - \rho_{11}^H \frac{d\sigma}{dt} (\pi^0 p \rightarrow \rho^0 p) \right] \right\}, \quad (13)$$

where the cross sections for  $\pi^0$  production of  $\rho^0$  and  $\omega^0$  can be determined from expressions (9). A similar relation is obtained for  $\chi^0$  by replacing  $A$  with  $E$ .

From Table III of the Appendix we see that relation (13) is expected to be valid within 10% accuracy already for  $P_{LAB} \gtrsim 2 \text{ GeV}/c$ . Figure 10 presents a comparison between relation (13) and experimental results<sup>27</sup> at  $4 \text{ GeV}/c$ . The agreement between theory and experiment is more than satisfactory. Note that both the photo-production data and the VMD prediction do not show significant structure around  $-t \sim .6 \text{ GeV}^2/c^2$ . In particular they do not show a pronounced dip, as expected for example in a naive Regge Pole Theory. A shallow dip however, is expected in  $\frac{d\sigma}{dt} (\gamma n \rightarrow \eta n)$  at  $-t \sim .6 \text{ GeV}^2/c^2$  from sum rule (12).

Relations V: The Photoproduction Reactions  $\gamma p \rightarrow \eta\Delta$      $\gamma p \rightarrow \eta^0\Delta$ 

Assumptions 1-3 and 5 plus the two additional assumptions (i) the reactions  $\gamma N \rightarrow \eta\Delta$  and  $\gamma N \rightarrow \eta^0\Delta$  are dominated by  $\rho$  and  $B$  exchange (with or without absorption corrections) (ii)  $SU(6)_W$  symmetry is valid for boson couplings, yield the relations:

$$\frac{d\sigma}{dt} (\gamma N \rightarrow \eta\Delta) \approx 2A^2 g_{\omega\gamma}^2 \rho_{11}^H \frac{d\sigma}{dt} (\pi^+ p \rightarrow \omega\Delta) \quad (14a)$$

$$\frac{d\sigma}{dt} (\gamma N \rightarrow \eta^0\Delta) \approx 2B^2 g_{\omega\gamma}^2 \rho_{11}^H \frac{d\sigma}{dt} (\pi^+ p \rightarrow \omega\Delta) \quad (14b)$$

For photons linearly polarized perpendicular and parallel to the production plane  $\rho_{11}^H$  has to be replaced by  $\rho_{\perp}^H$  and  $\rho_{\parallel}^H$  respectively.

From Table III of the Appendix we see that (14a) and (14b) are expected to be valid within 10% accuracy for  $P_{LAB} \gtrsim 8 \text{ GeV/c}$ . At lower energies the RHS of eqs (14) is expected to underestimate the photoproduction cross section on the LHS of these equations.

Figure 11 presents experimental results on the RHS of eq. (14a) at 4 and 8  $\text{GeV/c}$  incident  $\pi$  momentum. The RHS of eq. (14b) can be obtained by multiplying these results by the constant factor  $B^2/A^2 \sim 0.9$ .

Relation VI: K Photoproduction from Nucleons. VMD relations for photoproduction of kaons can be derived from assumption 1-3 and 5:

$$\frac{d\sigma}{dt} (\gamma p \rightarrow K^+ \Sigma^0) + \frac{d\sigma}{dt} (\gamma n \rightarrow K^0 \Sigma^0) - \frac{1}{2} \frac{d\sigma}{dt} (\gamma p \rightarrow K^0 \Sigma^+) - \frac{1}{2} \frac{d\sigma}{dt} (\gamma n \rightarrow K^+ \Sigma^-)$$

$$\approx g_{\rho\gamma}^2 \left[ 2\rho_{11}^H \frac{d\sigma}{dt} (K^- p \rightarrow \rho \Sigma^0) - \rho_{11}^H \frac{d\sigma}{dt} (\bar{K}^0 p \rightarrow \rho \Sigma^+) \right]. \quad (15)$$

Relation (15) is analogous to relation (6). The only new assumption involved in its derivation is "line reversal symmetry" (Assumption 5). This assumption is rather dangerous for the reactions  $KN \rightarrow \rho \Sigma$  in view of the fact that particles which behave differently under  $u$ - $s$  crossing can be exchanged in the same reaction (e.g.  $K$  and  $K_A^+(1^+)$  versus  $K^*(1^-)$  and  $K_N^+(2^+)$ ). Note also that the condition  $\Delta t_{\min} \ll m_\pi m_K$  requires incident  $K$  momenta that satisfy  $P_{LAB} \gtrsim 20 \text{ GeV}/c$ . At lower energy the hadron initiated reactions in relation (15) should be strongly suppressed compared to the analogous photon initiated reactions. Absence of relevant experimental data prevents any meaningful test of relation (15).

Relations analogous to relations (8) can also be derived under assumptions 1-3 and 5:

$$\frac{1}{2} \left[ \frac{d\sigma}{dt} (\gamma p \rightarrow K^+ \Lambda) + \frac{d\sigma}{dt} (\gamma n \rightarrow K^0 \Lambda) \right] = \sum_{V=\rho, \omega, \phi} g_{V\gamma}^2 \rho_{11}^H (V) \frac{d\sigma}{dt} (K^- p \rightarrow V \Lambda) + \text{Interference term } (\omega\phi) \quad (16a)$$

$$\frac{1}{2} \left[ \frac{d\sigma}{dt} (\gamma p \rightarrow K^+ \Sigma^0) + \frac{d\sigma}{dt} (\gamma n \rightarrow K^0 \Sigma^0) \right] = \sum_{V=\rho, \omega, \phi} g_{V\gamma}^2 \rho_{11}^H (V) \frac{d\sigma}{dt} (K^- p \rightarrow V \Sigma^0) + \text{Interference term } (\omega\phi) \quad (16b)$$

$$\frac{1}{2} \left[ \frac{d\sigma}{dt} (\gamma p \rightarrow K^0 \Sigma^+) + \frac{d\sigma}{dt} (\gamma n \rightarrow K^+ \Sigma^-) \right] = \sum_{V=\rho, \omega, \phi} g_{V\gamma}^2 \rho_{11}^H (V) \frac{d\sigma}{dt} (\bar{K}^0 p \rightarrow V \Sigma^+) + \text{Interference term } (\omega\phi) . \quad (16c)$$

Note that one cannot neglect the  $\phi$  contribution to eqs. (16) since all three cross sections on the RHS of these equations are comparable. Eqs. (16) can be tested experimentally only if additional assumption is made on the relative phase of the  $\omega$  and  $\phi$  contributions. The condition  $\Delta t_{\min} \ll m_\pi m_K$  requires K momenta that satisfy  $P_{LAB} \gtrsim 10, 20$  and  $20 \text{ GeV}/c$  for relations (16a), (16b) and (16c), respectively. At lower energies the hadron initiated reactions in these relations should be strongly suppressed compared with the analogous photon initiated reactions. Possible failure<sup>28</sup> of relations (16), especially at lower energies, do not provide evidence against the VMD model. It may be due to one or more of the following reasons:

- (i) energy too low for the VMD model to be valid
- (ii) failure of line reversal symmetry<sup>28</sup>
- (iii) wrong assumption about the relative phase of the  $\omega$  and  $\phi$  contributions,<sup>28</sup> rather than due to failure of the VMD hypothesis.

### CONCLUSIONS

We believe that the experimental tests of the VMD model in pseudoscalar meson photoproduction that we discussed here provide a firm basis for the following conclusions:

- (a) The VMD hypothesis if postulated in the s-channel helicity frame is qualitatively a very successful hypothesis for pseudoscalar meson photoproduction. The various features of pseudoscalar meson photoproduction are indeed similar to those observed in the analogous hadron initiated reactions. For instance:

(i) Although  $\frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^0 n)$  has a forward dip  $\rho_{11}^H \frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^0 n)$  has a forward spike analogous to spikes observed in the corresponding reactions  $\gamma N \rightarrow \pi^\pm N$

(ii) Although the slope of  $\rho_{11}^H \frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^0 n)$  is different from the slope of  $\frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^0 n)$ , it is similar to the slope of  $\frac{d\sigma}{dt} (\gamma p \rightarrow \pi^+ n) + \frac{d\sigma}{dt} (\gamma n \rightarrow \pi^- p)$ .

(iii) The forward dip and the second one at  $-t \sim .6 \text{ GeV}^2/c^2$  in  $\frac{d\sigma}{dt} (\gamma N \rightarrow \pi^0 n)$  are also present in the analogous reaction  $\pi^0 p \rightarrow \rho^0_{tr} p$ .

(iv) No pronounced dip at  $-t \sim .6 \text{ GeV}^2/c^2$  has been observed in  $\frac{d\sigma}{dt} (\gamma p \rightarrow \eta p)$ , nor in the analogous reaction  $\pi N \rightarrow \omega N$ .

(v) Although  $\frac{d\sigma}{dt} (\pi^+ p \rightarrow \rho^0 \Delta)$  has a forward spike,  $\rho_{11}^H \frac{d\sigma}{dt} (\pi^+ p \rightarrow \rho^0 \Delta)$  has a narrow forward dip analogous to the dips observed in the corresponding reactions  $\gamma N \rightarrow \pi^\pm \Delta$ .

(vi) Although the slope of  $\rho_{11}^H \frac{d\sigma}{dt} (\pi^+ p \rightarrow \rho^0 \Delta)$  is significantly different from the slope of  $\frac{d\sigma}{dt} (\pi^+ p \rightarrow \rho^0 \Delta)$  it is similar to the slope of  $\frac{d\sigma}{dt} (\gamma p \rightarrow \pi^- \Delta) + \frac{d\sigma}{dt} (\gamma n \rightarrow \pi^+ \Delta)$ .

(vii) The neutron to proton photoproduction ratios are consistent with the lower and upper bounds set by the VMD model.

(b) The VMD model is quantitatively successful for photoproduction of  $\pi^\pm$ ,  $\pi^0$  and  $\eta$  from nucleons with unpolarized photon beams.

view of the unreliable experimental results on the density matrix element  $\rho_{1-1}^H$  in the reaction  $\pi^- p \rightarrow \rho^0 n$

(d) The "failure" of the VMD model for  $\pi\Delta$  photoproduction<sup>23</sup> and K photo-production from nucleons<sup>28</sup> may be due to one or more of the following reasons :

(i) Energy too low for the model to be valid

(ii) Failure of the "line reversal symmetry",

rather than due to failures of the VMD model itself. Indeed there are known cases where line reversal symmetry is violated by a factor of 2 or more, e.g. the cross sections for  $\pi^+ p \rightarrow K^+ \Sigma^+$  and  $K^- p \rightarrow \pi^- \Sigma^+$  which should be the same if "line reversal symmetry" holds, are different<sup>29</sup> by about a factor 3 in the region  $P_{LAB} \sim 2 \text{ GeV}/c$  to  $P_{LAB} \sim 10 \text{ GeV}/c$ .

(e) More accurate data on the hadron reactions that are involved in the VMD relations are needed for accurate tests of the VMD model. In particular reliable data on  $\rho_{1-1}^H$  in  $\pi^- p \rightarrow \rho^0 n$  are badly needed.

(f) Experimental test of relation (4) would provide the most clean test of the VMD model

(g) "Primakoff Peaks" should be found in the differential cross sections  $\rho_{11}^H \frac{d\sigma}{dt} (\pi^\pm p \rightarrow \rho^\pm p)$  around  $t \sim 0$  analogous to the Primakoff Peaks in  $\gamma N \rightarrow \pi^0 N$  and  $\gamma N \rightarrow \pi^0 N$ .

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## APPENDIX

In this appendix I would like to justify the statements that (i) the VMD relations for photoproduction should be postulated for vector mesons transversely polarized in the s-channel helicity frame, and (ii) the VMD relations should be valid for  $P_{LAB}$  values that satisfy  $\Delta t_{min} \ll m_\pi m_e$ .

The Choice of the Helicity Frame

Obviously the question of a privileged frame for postulating the VMD model cannot be settled by purely kinematical arguments. Consequently many authors have tried to study the VMD relations within the framework of theoretical models for high energy reactions<sup>5,6</sup>. All the authors arrived at the conclusion that the VMD model has to be postulated for vector mesons transversely polarized in the helicity frame. In particular Cho and Sakurai<sup>5</sup> have shown that the simplest model that one can think of, i.e. the gauge invariant one pion exchange model, does not only support this conclusion, but also correctly describes charged pion photoproduction at small  $-t$  values. However, although this model and the above conclusion can be generalized for any particle exchange<sup>30</sup>, the model fails for  $t$  values larger than  $m_\pi^2$  and for other photoproduction reactions such as  $K$  photoproduction. Its success therefore may be considered as an accident. More realistic models should be examined before any final conclusions are drawn. Such a model is discussed below:

The "realistic" model<sup>31</sup> that we propose to study is based on two general assumptions:

- (i) Exchange Reactions are "Surface Reactions": the s channel partial waves which give the dominant contribution to an exchange reaction are the peripheral partial waves corresponding to s channel resonances

lying on an effective Regge trajectory  $j + \frac{1}{2} \sim kR$ , where  $R$  is the "hadronic radius" and it increases only logarithmically with  $s$ ; and  $k$  is the c.m. momentum in the  $s$  channel. This follows<sup>32</sup> from Regge behavior and duality. This picture is also consistent with that underlying the absorption models<sup>33</sup>, where the contribution of partial waves with  $j + \frac{1}{2} < kR$  is strongly suppressed due to competition of many open channels.

(ii) The peripheral partial waves of an exchange amplitude are approximately described by the corresponding partial waves of the pole closest to the physical region (the corresponding partial waves of the Born Approximation for an exchange of the lightest particle lying on the exchanged Regge trajectory). This follows from the dispersion relations that an exchange amplitude satisfies).

What are the implications of (i) and (ii) for high energy exchange reactions?

To answer this question let us examine the impact parameter expansion of an  $s$  channel helicity amplitude for a reaction  $a+b \rightarrow c+d$  with the helicity situation  $[\lambda]$ ;  $[\lambda] = [\lambda_a, \lambda_b, \lambda_c, \lambda_d]$ . It is given by

$$F_{[\lambda]} = k^2 \int_0^\infty b db J_{\Delta\lambda} (b \sqrt{-t'}) f_{[\lambda]}(b) \quad (1)$$

$f_{[\lambda]}(b)$  is the contribution at impact parameter  $b$  to the scattering amplitude  $F_{[\lambda]}$ .  $J_{\Delta\lambda} (b \sqrt{-t'})$ , the cylindrical Bessel function of order  $\Delta\lambda$ , is a small angle and large  $j$  approximation for the rotation functions  $d_{\mu\nu}^j (\cos \theta)$ ,  $j$  being related to the impact parameter  $b$  through the classical relation  $j + \frac{1}{2} \sim kb$ .  $\mu = \lambda_a - \lambda_b$ ,  $\nu = \lambda_c - \lambda_d$  and  $\Delta\lambda = \mu - \nu$  is the total helicity change in the

reaction,  $t' = t - t_{\min}$ , where  $t_{\min}$  is the minimum momentum transfer allowed by the kinematics of the reaction. According to (i)  $f_{[\lambda]}^{(b)}$  is appreciable only for  $b \gtrsim R$ , i.e.

$$\begin{aligned} f_{[\lambda]}^{(b)} &\sim 0 \\ b &\gtrsim R \end{aligned} \tag{2}$$

According to (ii) :

$$\begin{aligned} f_{[\lambda]}^{(b)} &\sim B_{[\lambda]}^{(b)}, \\ b &\gtrsim R \end{aligned} \tag{3}$$

where  $B_{[\lambda]}$  is the Born approximation expression for the contribution of impact parameter  $b$  to the  $s$  channel helicity amplitude with the helicity situation  $[\lambda]$ .  $B_{[\lambda]}^{(b)}$  for the exchange of a particle with mass  $m_e$  and spin  $J_e$  is given by<sup>33</sup>

$$B_{[\lambda]}^{(b)} = C_{[\lambda]} \left( \frac{s}{s_0} \right)^{J_e - 1} K_{\Delta\lambda}(\mu_e b), \quad kb \gg 1 \tag{4}$$

where  $\mu_e^2 = m_e^2 - t_{\min}$ .  $K_{\Delta\lambda}$  is the cylindrical Bessel function of second kind and of the order  $\Delta\lambda$ . It approaches rather quickly its asymptotic behavior:

$$K_{\Delta\lambda}(x) \sim \sqrt{\frac{\pi}{2x}} e^{-x} \quad x \gtrsim \Delta\lambda \tag{5}$$

$C_{[\lambda]}$  in the high energy limit depends only on the helicity situation, but not on  $b$  or on  $s$ .

When expressions (2), (3) and (4) are inserted in expression (1) they yield<sup>33</sup>

$$\begin{aligned}
 F[\lambda] &\sim c[\lambda] \left(\frac{s}{s_0}\right)^{J_e-1} k^2 \int_{R(s)}^{\infty} bdb J_{\Delta\lambda} (b\sqrt{-t'}) K_{\Delta\lambda} (b\mu_e) \\
 &\sim c[\lambda] \left(\frac{s}{s_0}\right)^{J_e-1} \sqrt{\mu_e R(s)} e^{-\mu_e R(s)} \frac{J_{\Delta\lambda} (R\sqrt{-t'})}{\mu_e^2 - t'} ,
 \end{aligned} \tag{6}$$

For large  $s$  values  $t_{\min} \rightarrow 0$  so that  $t' \rightarrow t$  and  $\mu_e \rightarrow m_e$ . From eq. (6) we then conclude that in the high  $s$  limit the helicity amplitude  $F[\lambda]$  does not depend on the mass of the external vector meson provided the coefficient  $c[\lambda]$  does not depend on that mass. Let us therefore examine the coefficients  $c[\lambda]$  for high energy exchange reactions.

Since all the VMD relations that we are interested in here involve reactions of the type



where  $M$ ,  $N$ ,  $V$  and  $B$  stand for pseudoscalar meson, nucleon, vector meson and baryon, respectively, we shall limit our examination of the coefficients  $c[\lambda]$  for these reactions. Let us first consider  $\pi$  exchange. The coefficients  $c[\lambda]$  can be determined in the following way: Let us write the Born approximation for pion exchange in the form (all our calculations are equally valid for any pseudoscalar meson exchange)

$$B[\lambda]^{(s,t)} = \frac{v_{\lambda_V \lambda_M}^{(s,t)} v_{\lambda_B \lambda_N}^{(s,t)}}{m_\pi^2 - t} \tag{8}$$

where the  $V$ 's are the vertex function for the corresponding Feynman diagram. It is then easy to show that the coefficients  $C_{[\lambda]}$  are given by

$$C_{[\lambda]} = \frac{v_{\lambda_V \lambda_M}(s, m_\pi^2) v_{\lambda_B \lambda_N}(s, m_\pi^2)}{m_\pi^2 - t} \quad (9)$$

From eq. (9) we see that  $C_{[\lambda]}$  can depend on  $m_V$ , the mass of the external vector meson, only through the vertex function  $v_{\lambda_V \lambda_M}(s, m_\pi^2)$ . This vertex function is given by:

$$v_{\lambda_V \lambda_M}(s, m_\pi^2) = 2 s e_\mu^* (p(V), \lambda_V) p^\mu(M) \quad (10)$$

where  $e_\mu$  is the spin one polarization vector of  $V$ , and  $g$  is a coupling constant.

In the limit  $s \rightarrow \infty$ , in the  $s$  channel helicity frame,

$$v_{10}(s, m_\pi^2) = -\sqrt{2} g m_\pi \quad (11a)$$

$$v_{00}(s, m_\pi^2) = -g \frac{m_V^2 - m_\pi^2 - m_M^2}{m_V} \quad (11b)$$

Eqs. (9) and (11a) tell us that for vector mesons transversely polarized in the  $s$ -channel helicity frame ( $\lambda_V = \pm 1$ ) the coefficients  $C_{[\lambda]}$  do not depend on the mass of the external vector meson. Since in (6) we demonstrated that in the high  $s$  limit  $F_{[\lambda]}$  can depend on  $m_V$  only through  $C_{[\lambda]}$  we conclude that for vector mesons transversely polarized in the  $s$ -channel helicity frame the amplitudes  $F_{[\lambda]}$  do not depend in the high  $s$  limit, on the mass of the vector meson. Note however that for longitudinally polarized vector mesons ( $\lambda_V = 0$ ) the coefficients  $C_{[\lambda]}$ , and consequently the amplitudes  $F_{[\lambda]}$  depend strongly on  $m_V$  through the vertex function (11b).

Consider now the amplitudes for production of vector mesons transversely polarized in another frame. If the transformation from the s-channel helicity frame to this frame mixes s-channel helicity amplitudes with  $|\lambda_V| = 1$  and  $\lambda_V = 0$ , and/or depends on  $m_V$ , the mass of the external vector meson, then it will introduce strong dependence on  $m_V$  in the new production amplitudes. In particular this happens when transforming to the Gottfried-Jackson frame and to the Donohue - Hoggrasen frame. We thus conclude that our model indicates that VMD hypothesis has to be postulated in the s-channel helicity frame. Although the proof presented here is applicable only to pseudoscalar meson exchange reactions it can be generalized for the case of exchange of a particle with an arbitrary spin and parity. Since the proof involves lengthy algebra it will be given elsewhere<sup>30</sup>.

The Condition  $\Delta t_{\min} \ll m_V m_e$

Since experiments are performed at finite energies the effect of  $m_V \neq 0$  on the various VMD relations at finite s values is worth studying:

Eq. (6) tells us that the  $t'$  dependence of a photoproduction amplitude and the analogous strong production amplitude will be similar provided the effective masses  $\mu_e$ , are about the same for the two reactions, i.e. that

$$\Delta t_{\min} \ll m_e^2 \quad (12)$$

(Recall the relations:  $t' = t - t_{\min}$ ,  $m_e^2 = m_e^2 - t_{\min}^2$  and  $t_{\min} = \mu_e^2(\gamma) - \mu_e^2(v) = t_{\min}(\gamma) - t_{\min}(v)$ , where  $t_{\min}$  is the minimum momentum transfer allowed by the actual masses measured in the reaction, and  $\gamma$  and  $v$

stand for the photoproduction reaction and the analogous hadron initiated reaction, respectively). Condition (12) is well satisfied already at medium energies (say  $P_{LAB} \sim 4 \text{ GeV}/c$ ), except for  $\pi$  exchange. We therefore expect that already at relatively low energies the photoproduction reactions that can not proceed via  $\pi$  exchange and the analogous hadron initiated reactions will exhibit the same  $t'$  dependence! What about their relative magnitude? Eq. (6) tells us that the cross sections for the hadron initiated reactions are suppressed compared with the cross sections for the analogous photon initiated reactions. Under condition (12) the suppression factor is approximately given by

$$(1 - \Delta\mu_e R) \exp(-2\Delta\mu_e R) \sim \exp(-\Delta t_{\min} R/2m_e), \quad (13)$$

where  $\Delta\mu_e = \mu_e(v) - \mu_e(\gamma)$ . For an "hadronic radius"  $R \sim m_\pi^{-1}$ , approximate equality of the cross sections for photoproduction and the analogous strong production is obtained therefore if, and only if,

$$\Delta t_{\min} \ll m_\pi m_e \quad (14)$$

Condition (14) coincides with condition (12) for  $\pi$  exchange reactions. However for exchange of other particles condition (14) is stronger than condition (12). It implies that the shapes of the differential cross sections for a photoproduction reaction that cannot proceed via  $\pi$  exchange and those of the analogous strong production match at relatively lower energies than the energies where their magnitudes start matching.

Now consider  $\pi$  exchange reactions. The condition  $\Delta t_{\min} \ll m_\pi^2$  requires energies relatively high, especially for reactions where mass changes take place

at both vertices of the reaction:

$$-t_{\min} \sim \frac{(m_V^2 - m_M^2)(m_B^2 - m_N^2)}{s} \quad \text{if } m_V \neq m_M \text{ and } m_B \neq m_N$$

$$-t_{\min} \sim \frac{m_N^2 (m_V^2 - m_M^2)^2}{s^2} \quad \text{if } m_B = m_N$$

If  $-t_{\min} \lesssim m_\pi^2$ , i.e. if  $\mu_\pi^2 = m_\pi^2 - t_{\min} \sim m_\pi^2$  where  $\mu_\pi^2$  is the distance from the physical  $t$  region to the pion pole, then the strong variation of the pion propagator in (6) introduces strong variation in the cross section near  $t' = 0$ . This strong variation of the pion propagator near  $t' = 0$  when  $\mu_\pi^2 \sim m_\pi^2$  is responsible for the narrow forward structure observed in high energy  $\pi$  exchange reactions. (For a detailed discussion of narrow forward structure in high energy  $\pi$  exchange reactions see my second talk at this meeting<sup>31</sup>). In particular such structures have been observed in the photoproduction reactions  $\gamma N \rightarrow \pi^\pm N^*$  and  $\gamma N \rightarrow \pi^\pm \Delta$ , where the condition  $-t_{\min} \lesssim m_\pi^2$  is easily satisfied. The analogous narrow structures in hadron initiated reactions should also be found at high enough energies such that  $-t_{\min} \lesssim m_\pi^2$ .

When the energy decreases  $\mu_e^2$  increases, the narrow forward structure becomes broader, and the strong production is strongly suppressed at small  $-t'$  values compared with the analogous photoproduction, mainly due to the pion propagator  $(\mu_e^2 - t')^{-1}$  in (6). (The physical  $t$  region moves away from the pion pole when the energy decreases, much faster in the strong production than in the analogous photoproduction).

Table II summarizes the lowest incident energies where we may expect the VMD relations to be valid within 10% accuracy both in shape and magnitude. Note in particular the extremely high energies required in order that relations (10a) and (10b) be valid within at least 10% accuracy.

TABLE I

Determination of vector meson coupling constants<sup>34</sup>

Experimental Results	$f_V^2/4\pi$	$g_{V\gamma}^2 = e^2/f_V^2$
	$V = \rho$	
$\Gamma(\rho \rightarrow 2\pi) = 110 \pm 10 \text{ MeV}$	$2.2 \pm 0.2$	
$\Gamma(\rho \rightarrow 2e) = 7.5 \pm 1 \text{ KeV}$	$2.0 \pm 0.3$	
$\Gamma(\rho \rightarrow 2\mu) = 7.5 \pm 1 \text{ KeV}$	$2.0 \pm 0.3$	
$\Gamma(\omega \rightarrow \pi^0 \gamma) = 1.2 \pm 2 \text{ MeV}$	$2.8 \pm 0.4$	
$\Gamma(\omega \rightarrow 3\pi) = 12.7 \pm 1.7 \text{ MeV}$		
	$(f_\rho^2/4\pi)_{\text{average}} = 2.2$	$g_{\rho\gamma}^2 = 3.5 \times 10^{-3}$
	$V = \omega$	
$\Gamma(\omega \rightarrow 2e) = 0.94 \pm 0.18 \text{ KeV}$	$15 \pm 3$	
$\Gamma(\omega \rightarrow \pi^0 \gamma) = 1.2 \pm 0.2 \text{ MeV}$	$24 \pm 4$	
$\Gamma(\pi^0 \rightarrow 2\gamma) = 11.7 \pm 1.7 \text{ eV}$		
	$(f_\omega^2/4\pi)_{\text{average}} = 19.5$	$g_{\omega\gamma}^2 = 3.7 \times 10^{-4}$
	$V = \phi$	
$\Gamma(\phi \rightarrow 2e) = 1.5 \pm 0.3 \text{ KeV}$	11.5	
$\Gamma(\phi \rightarrow 2K) = 3.9 \pm 0.4 \text{ MeV}$	12.1	
	$(f_\phi^2/4\pi)_{\text{average}} = 11.8$	$g_{\phi\gamma}^2 = 6.1 \times 10^{-4}$

TABLE II

Reaction	$P_{LAB}$ in GeV/c	References	Remarks
$\gamma P \rightarrow \pi^+ n$			
$d\sigma/dt$	3.7 5 - 16	CEA (13) DESY (13) SLAC (13)	We interpolated all these data to 4 GeV and 8 GeV assuming $(S-M^2)^2 d\sigma/dt$ to be constant.
$d\sigma_{\perp}/dt$	3.4 ; 5	DESY (18)	
$d\sigma_{  }/dt$	3	CEA (18)	
$\gamma n \rightarrow \pi^- p$			
$d\sigma/dt$	3.4 ; 3 3.4 ; 5 8	CEA (16) DESY (14) SLAC (14)	We used these data to interpolate to GeV, assuming $(S-M^2)^2 (d\sigma/dt)$ to be constant.
$d\sigma_{\perp}/dt$	3.4	DESY (18)	
$d\sigma_{  }/dt$	3	CEA (18)	
$\gamma P \rightarrow \pi^0 p$	4 4	CEA (19) DESY (19)	
$\gamma n \rightarrow \pi^0 n$	4	CEA (21)	
$\gamma P \rightarrow \eta^0 p$	4 4	CEA (27) BONN-DESY (27)	
$\gamma P \rightarrow \pi^- \Delta^{++}$	8	SLAC (23)	
$\gamma d \rightarrow \pi^+ \Delta^- p$	16	SLAC (23)	We used this ratio to calculate $\gamma n \rightarrow \pi^+ \Delta^-$ at 8 GeV
$\gamma d \rightarrow \pi^- \Delta^{++} n$			

Reaction	$P_{LAB}$ in GeV/c	References	Remarks
$\pi^- p \rightarrow \rho^0 n$	4	Johnson et al. (15) Scharenguivel et al. (12)	
	8	Poirier et al. (15)	
$\pi^- p \rightarrow \rho^- p$	4	Johnson et al. (15)	
$\pi^+ p \rightarrow \rho^+ p$	4	Aderholz et al. (35)	
$\pi^+ n \rightarrow \omega p$	3.6	Benson (20)	
	4.19	Abrams et al. (20)	
$\pi^+ p \rightarrow \rho^0 \Delta^{++}$	4	Brown (35)	
	8	Aderholz et al. (23) Morrison (23)	
$\pi^+ p \rightarrow \omega \Delta^{++}$	4	Brown (35)	
	8	Aderholz et al. (23) Morrison (23)	

TABLE III

Minimum Incident Momentum For VMD Relations

Relation	$t' \lesssim m_{\pi}^2, P_{LAB} \gtrsim$	$t' \gtrsim 5m_{\pi}^2, P_{LAB} \gtrsim$
(4) $T=H_e 4$	2 GeV/c	2 GeV/c
(6)	8 "	4 "
(8a)	8 "	4 "
(8b)	2 "	2 "
(10a)	200 "	30 "
(10b)	200 "	30 "
(10c)	8 "	5 "
(11a)	2 "	2 "
(11b)	2 "	2 "
(13)	2 "	2 "
(14a)	8 "	5 "
(14b)	8 "	5 "
(15)	20 "	20 "
(16a)	10 "	10 "
(16b)	20 "	20 "
(16c)	20 "	20 "

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FIGURE CAPTIONS

Fig. 1 Comparison between the VMD relation (6) for photoproduction from nucleons and experimental results at 4 GeV, quoted in Table II.

Fig. 2 Comparison between the VMD relation (8a) for photoproduction of charged pions from nucleons and experimental results at 4 GeV quoted in Table II.

Fig. 3 Comparison between the VMD relation (8a) for photons linearly polarized perpendicular to the production plane and experimental results at 3.4 GeV quoted in Table II.

Fig. 4 Comparison between the VMD relation (8c) for the asymmetry ratio in photoproduction of charged pions and experimental results at 4 GeV quoted in Table II.

Fig. 5. Comparison between the VMD relation (8b) for  $\pi^0$  photoproduction and experimental results at 4 GeV, quoted in Table II.

Fig. 6 The differential cross sections  $\frac{d\sigma}{dt} (\pi^0 p \rightarrow \rho^0_{tr} p)$  and  $\frac{d\sigma}{dt} (\pi^0 p \rightarrow \omega_{tr} p)$  as calculated from relations (9) and experimental results at  $P_{LAB} = 4 \text{ GeV}/c$ , quoted in Table II.

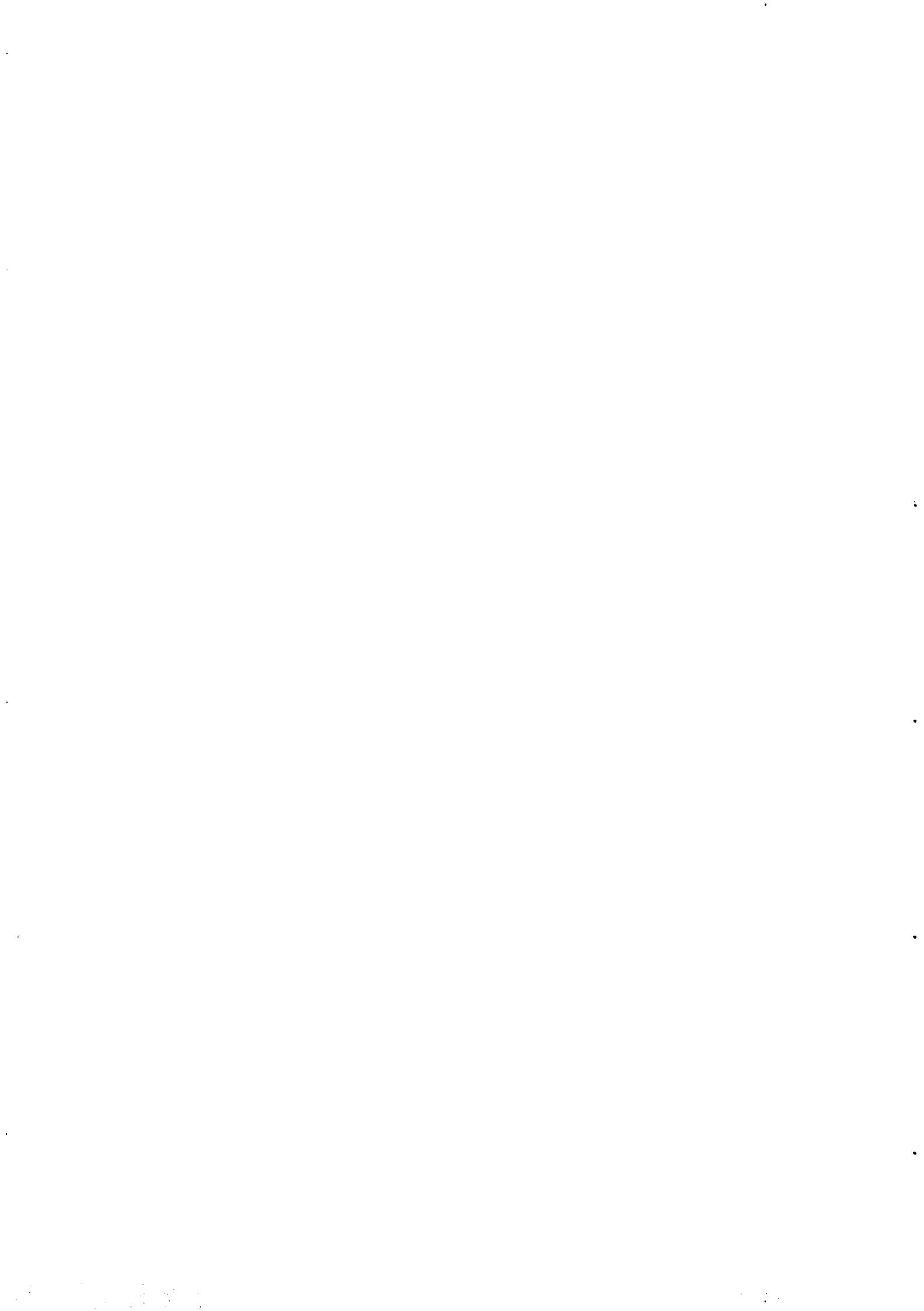
Fig. 7 Comparison between the VMD lower bound to the photoproduction ratio  $\frac{d\sigma}{dt}(\gamma n \rightarrow \pi^- p)/\frac{d\sigma}{dt}(\gamma p \rightarrow \pi^+ n)$  calculated from relations (8d) and experimental results at 3.4 GeV quoted in Table II.

**Fig. 8** Comparison between the VMD lower bound to the photoproduction ratio  $\frac{d\sigma}{dt}(\gamma n \rightarrow \pi^0 n) / \frac{d\sigma}{dt}(\gamma p \rightarrow \pi^0 p)$  calculated from relation (8d) and experimental results at 3.4 GeV quoted in Table II.

**Fig. 9** Comparison between the VMD relation (10a) for photoproduction of  $\pi\Delta$  from nucleons and experimental results at 8 GeV quoted in Table II.

**Fig. 10** Comparison between the VMD relation (13) for photoproduction of  $\eta$  from protons and experimental results at 4 GeV, quoted in Table II.

**Fig. 11** VMD predictions for the differential cross section  $\frac{d\sigma}{dt}(\gamma p \rightarrow \eta\Delta)$  at 4 and 8 GeV calculated from relation (14a) and the experimental data of reference (23).



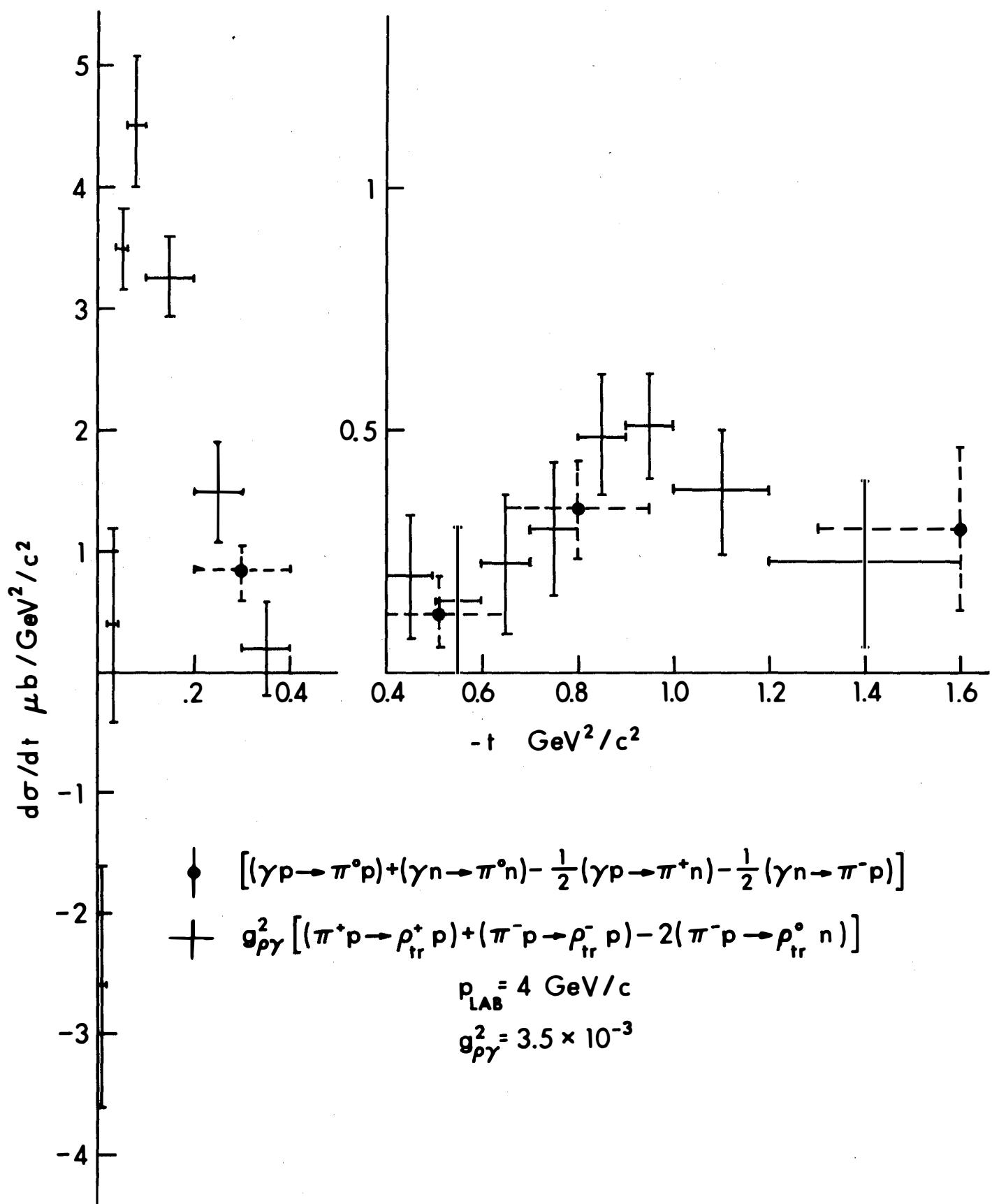


Fig. 1.

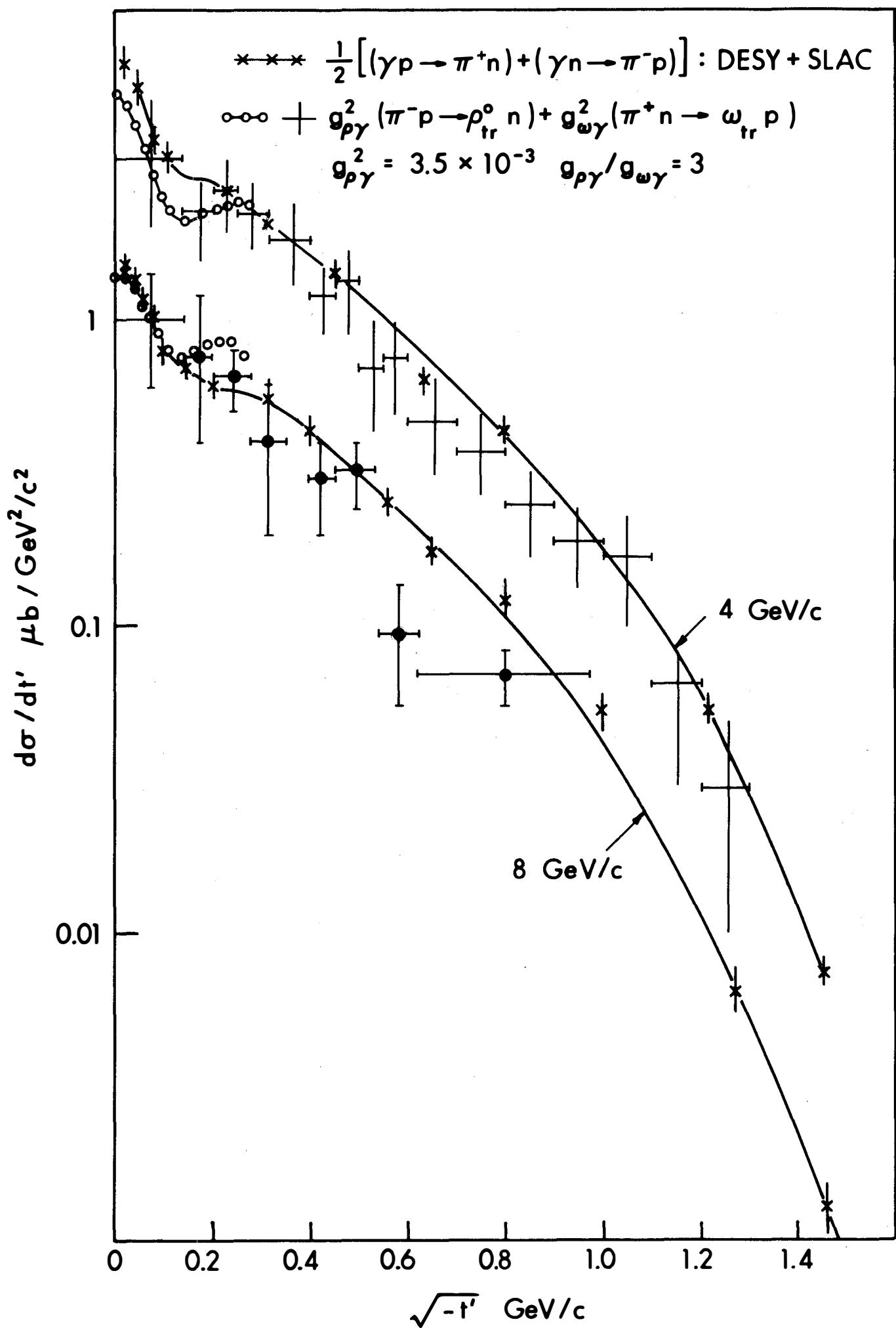


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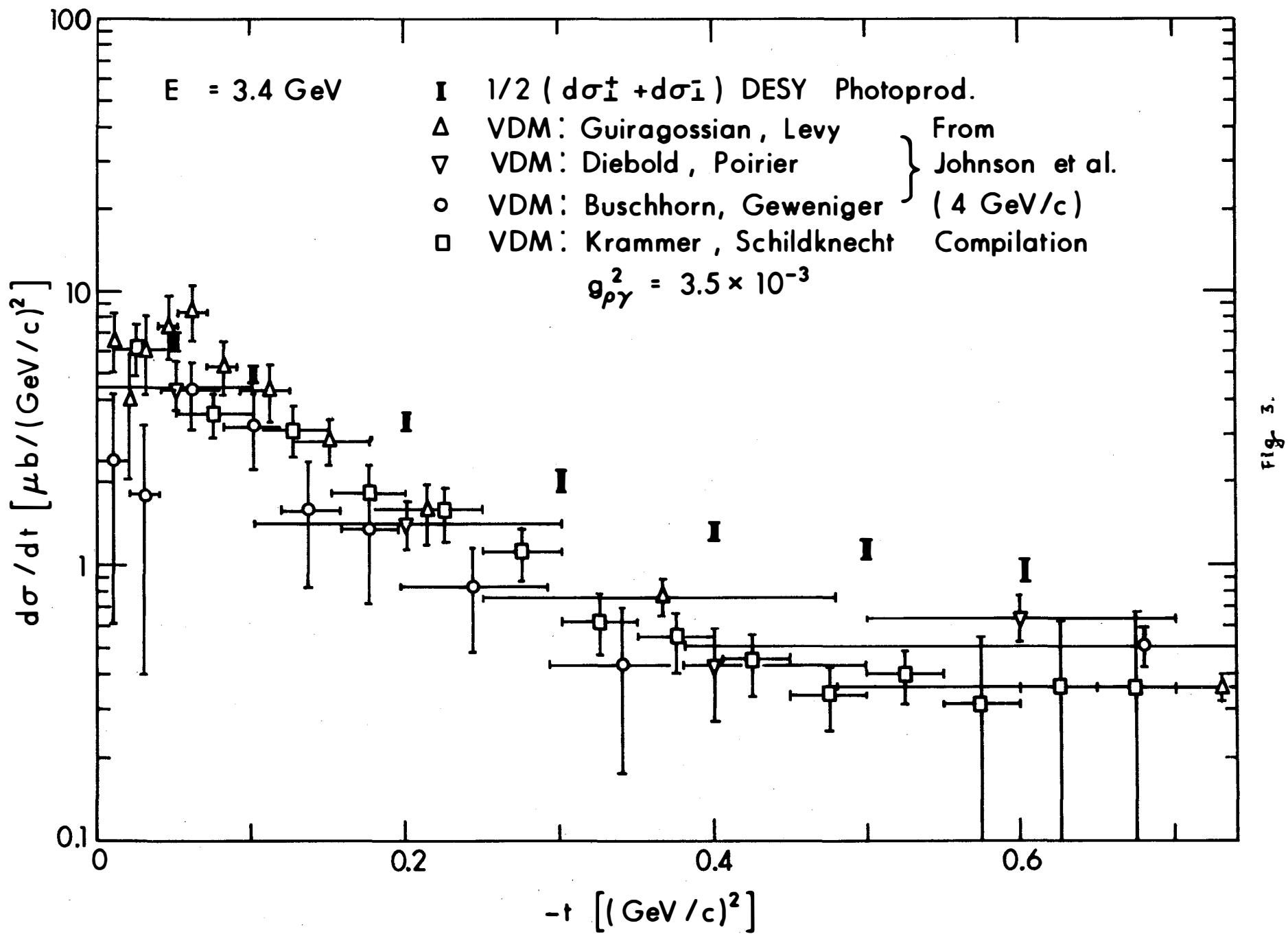


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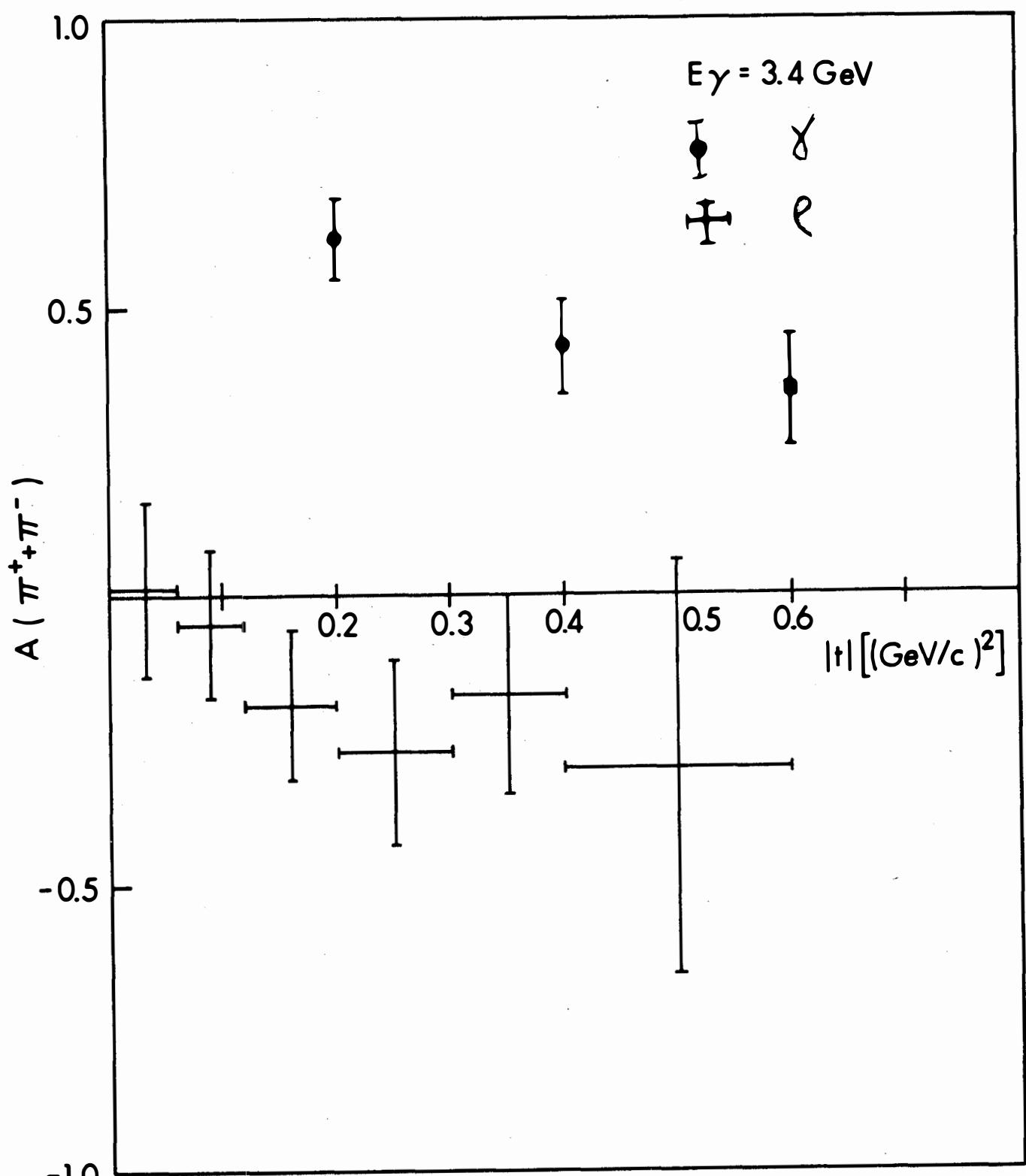


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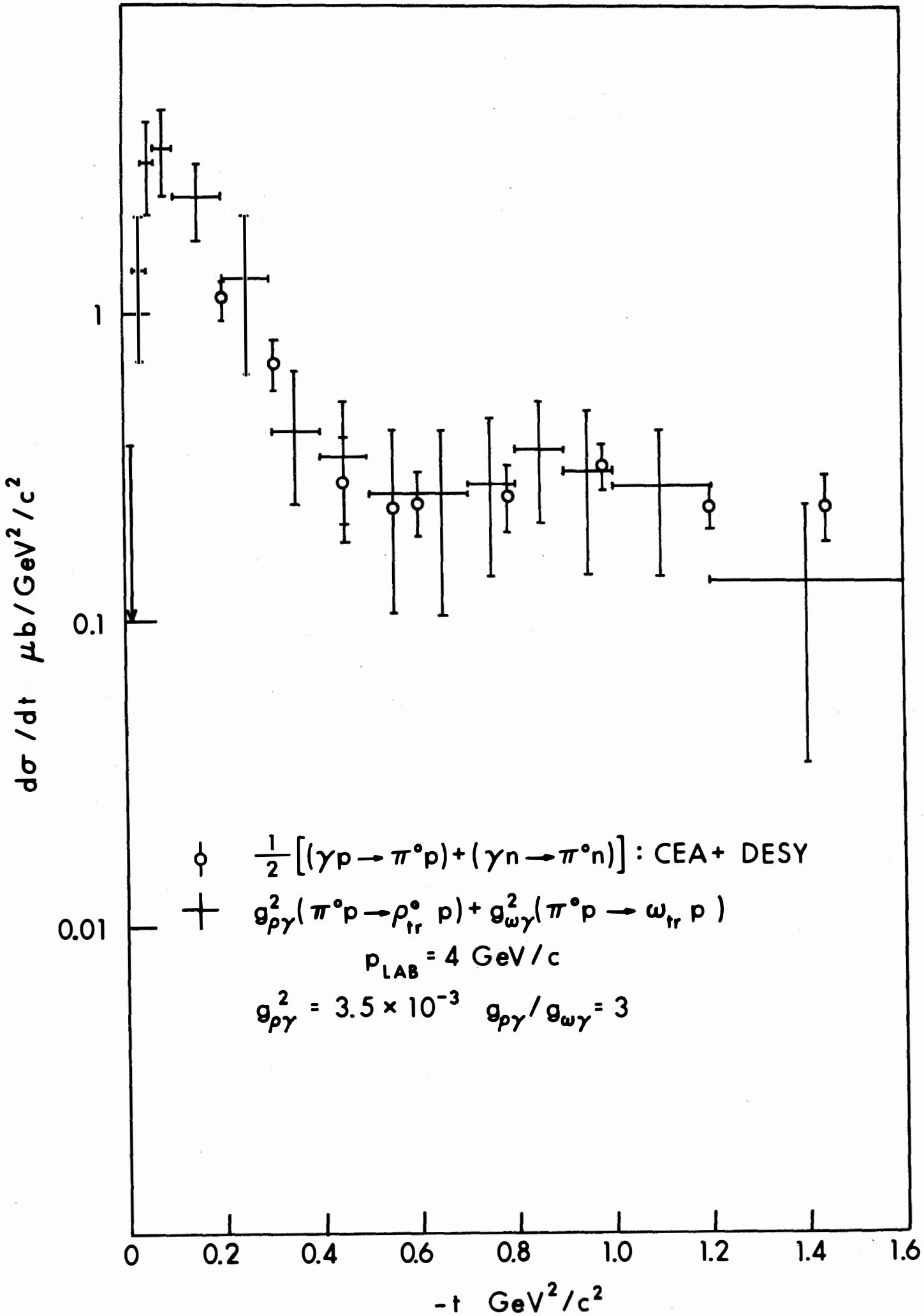


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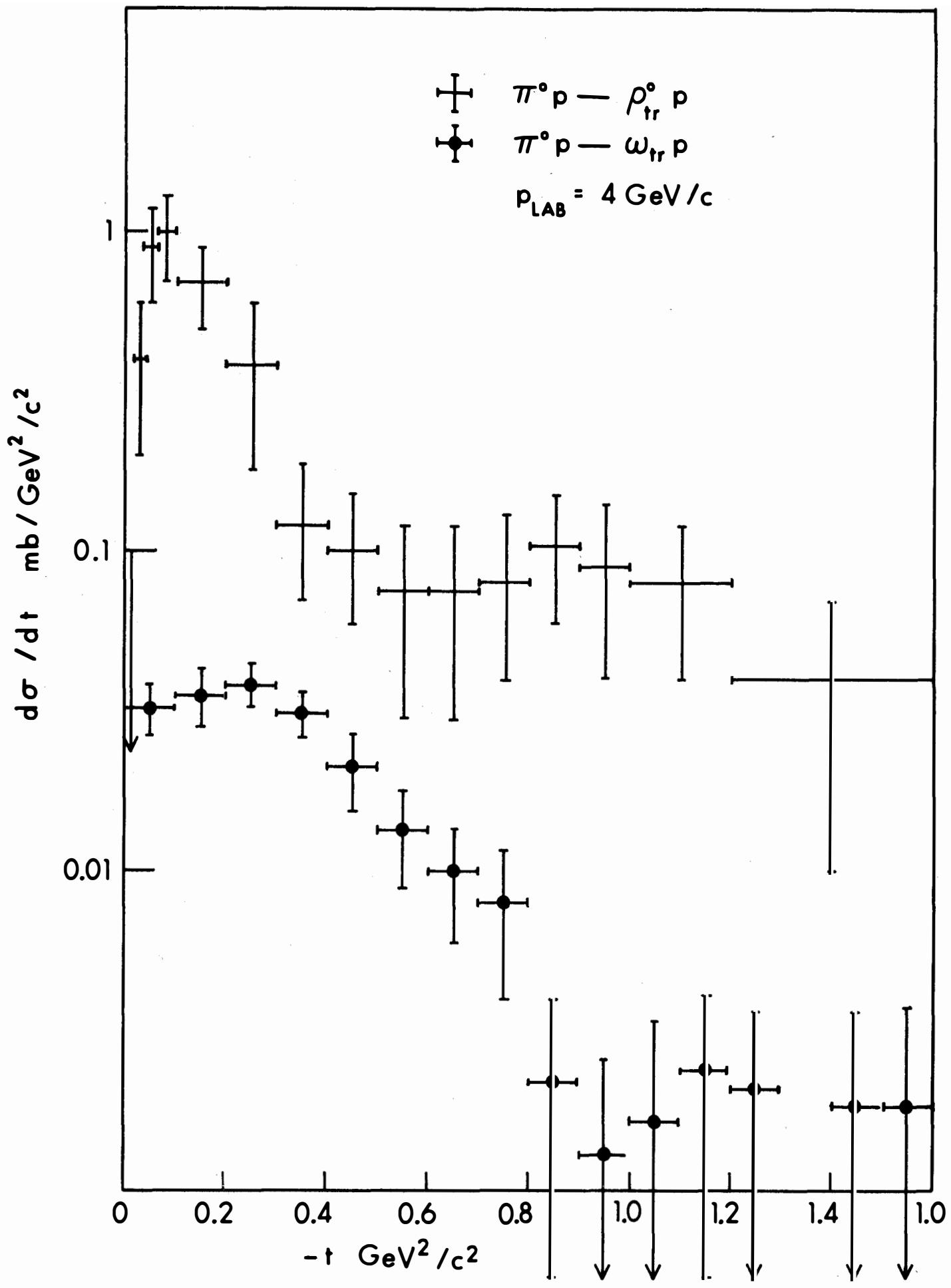


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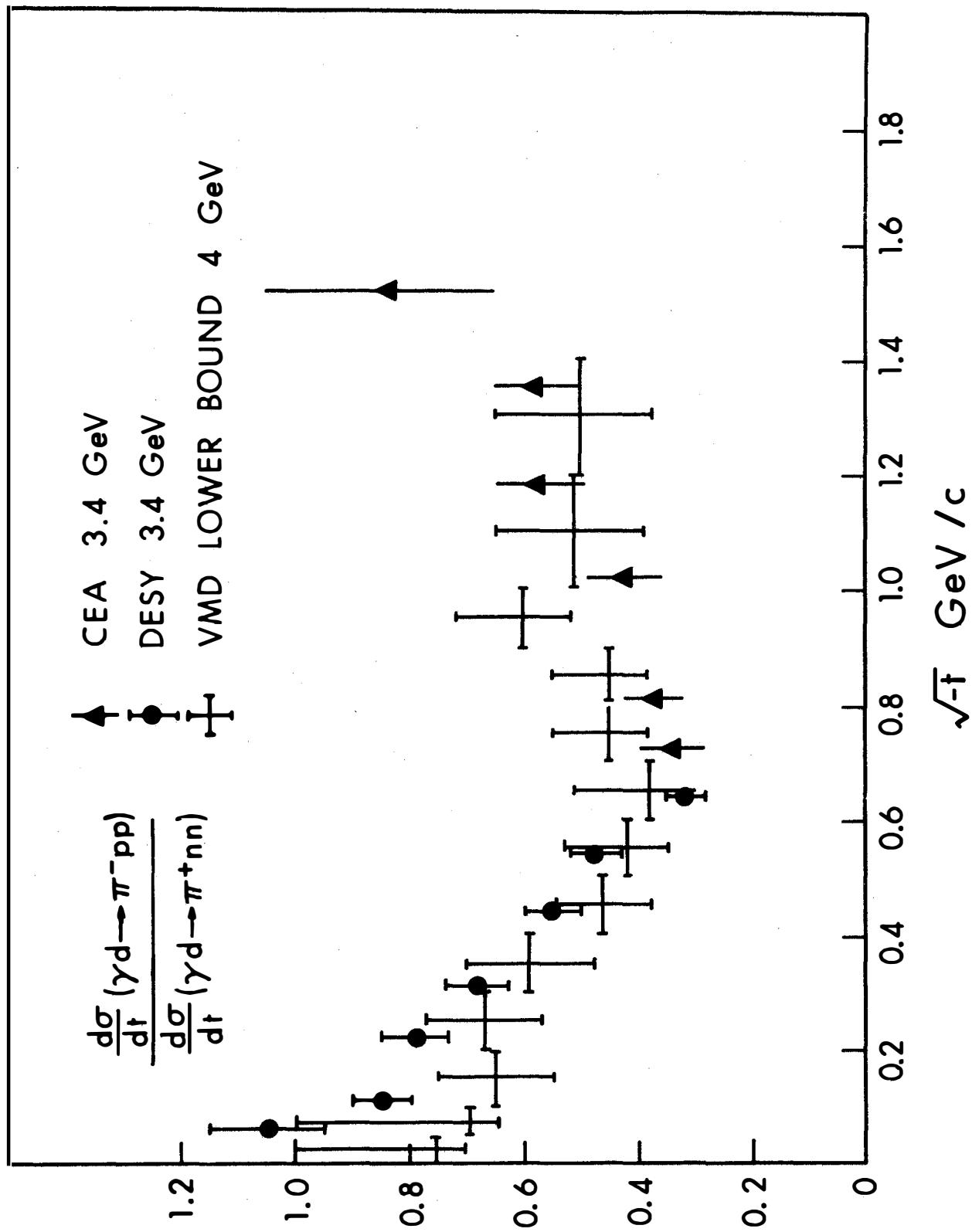
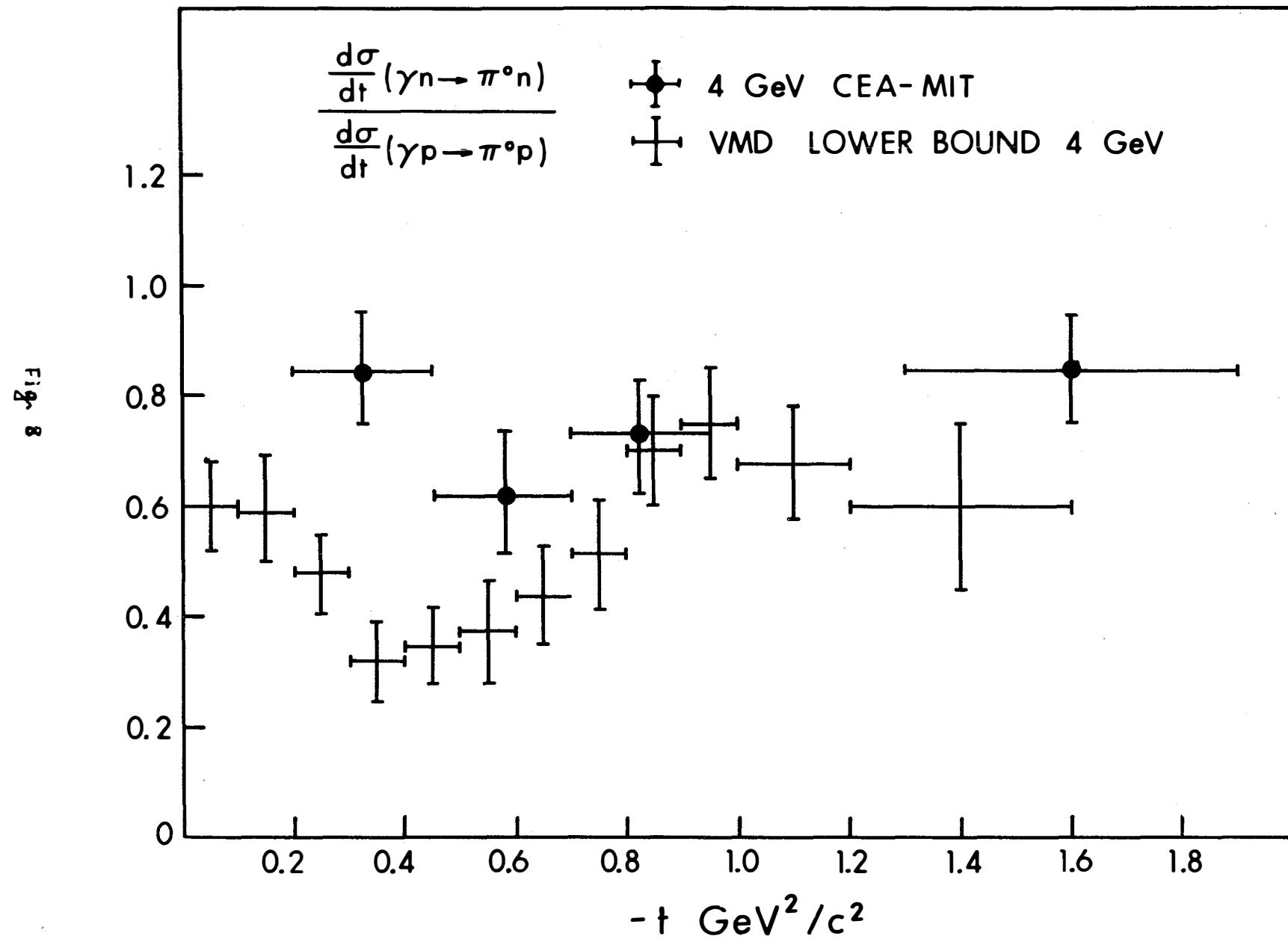


Fig. 7.



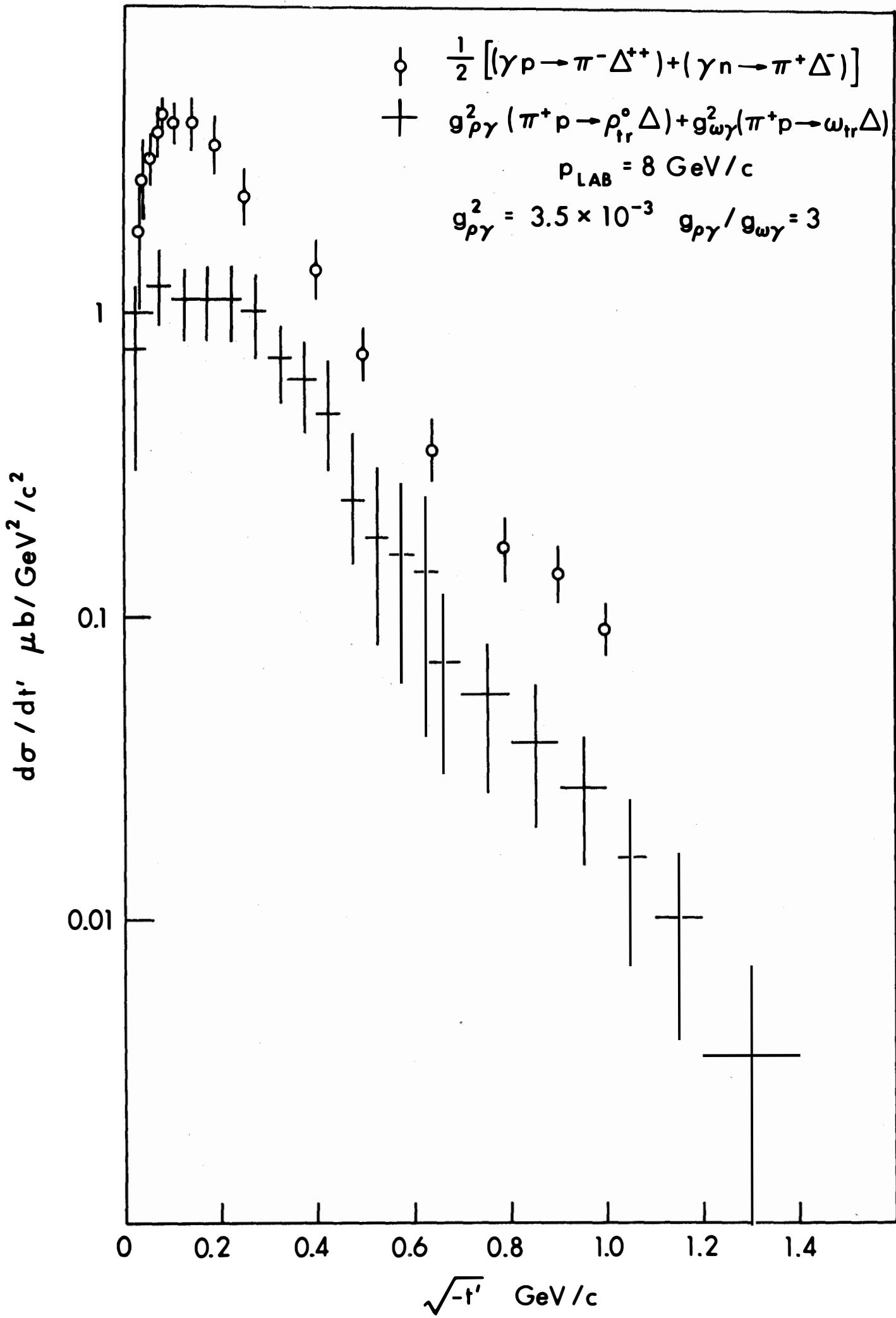


Fig. 9.

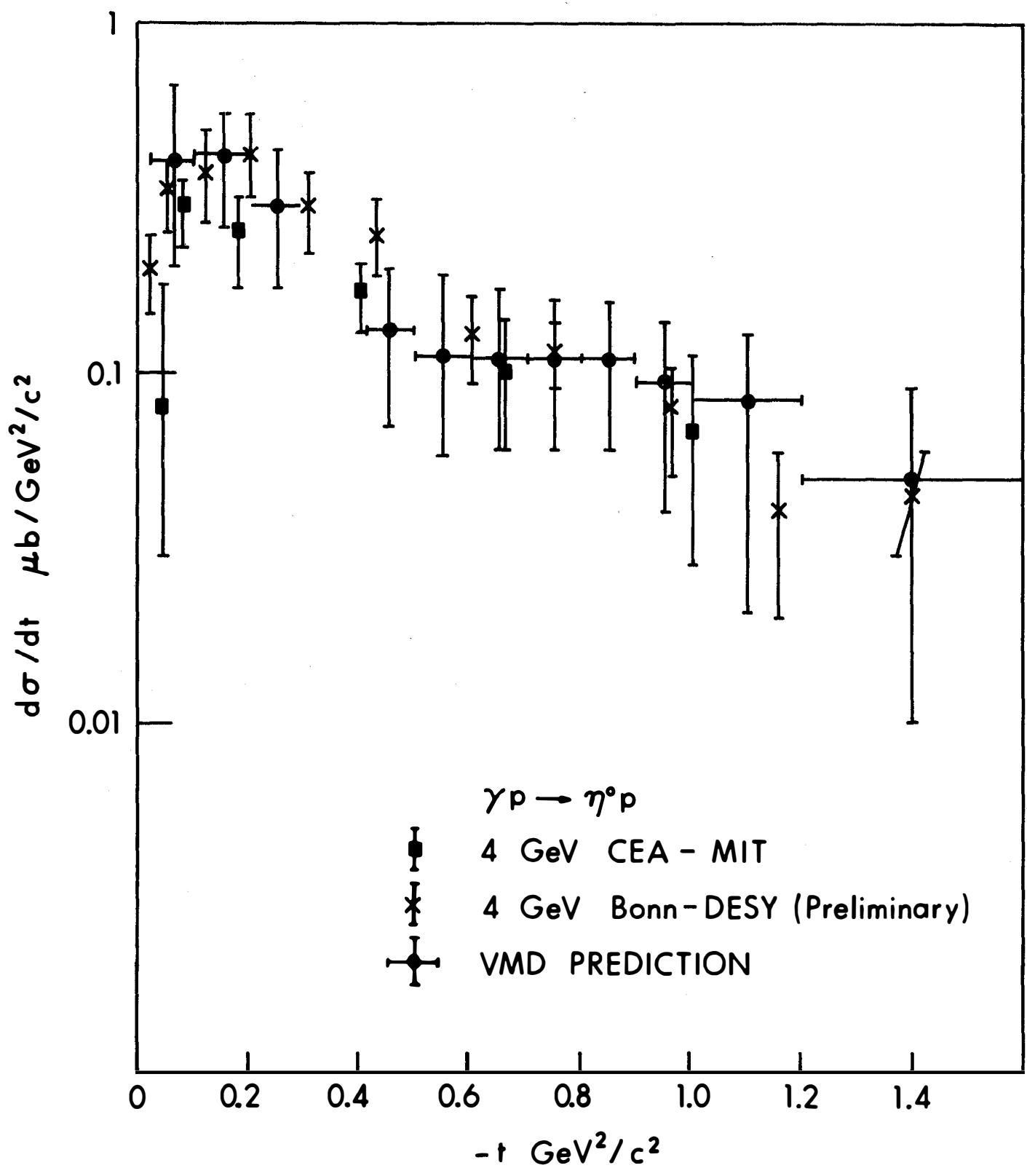


Fig. 10.

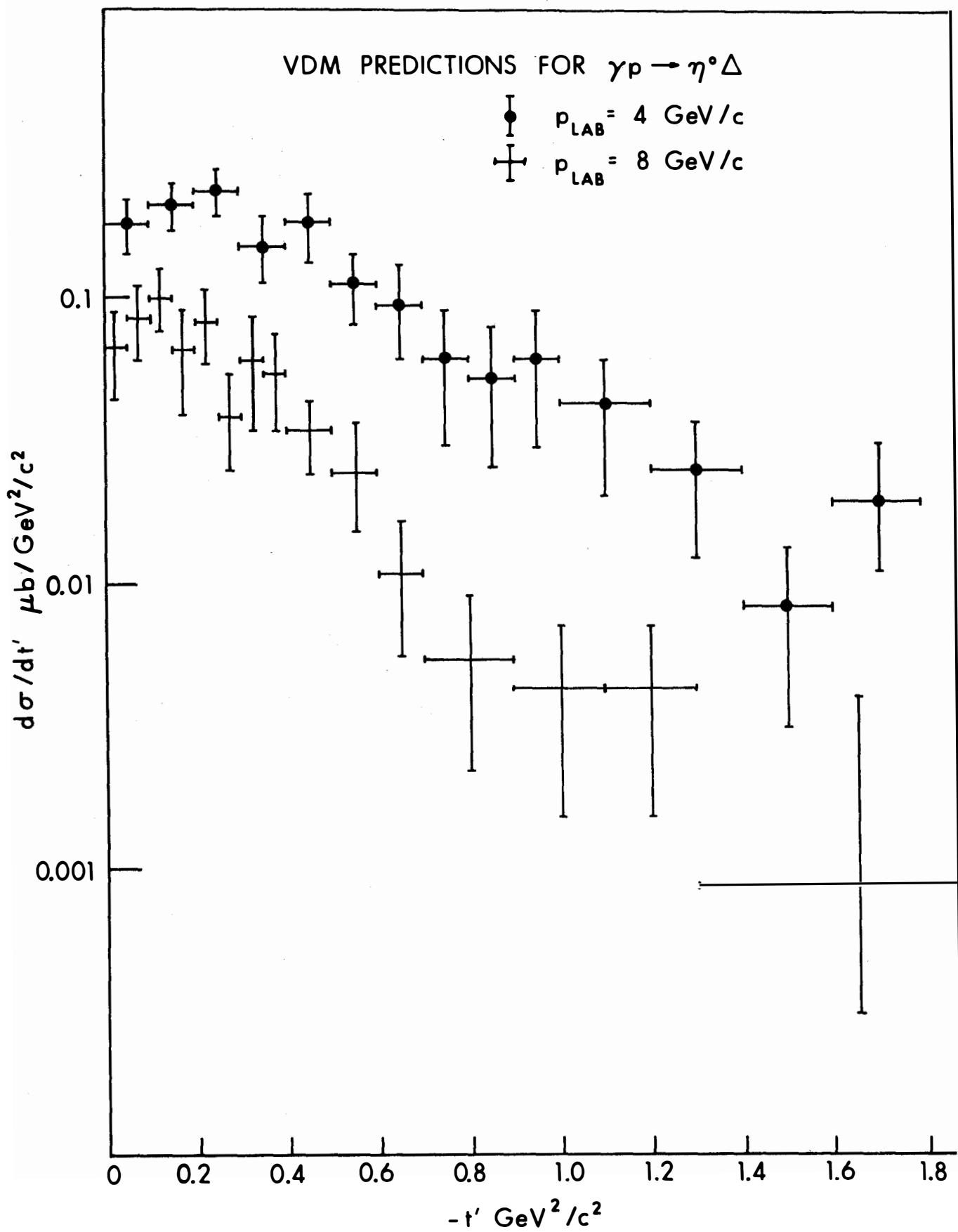


Fig. 11.