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# Monopole Catalysis of Proton Decay in SO(10) Grand Unified Models

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ABSTRACT

We construct the monopole spectrum in an SO(10) grand unified theory which has an arbitrary pattern of symmetry breakdown to $SU(3) \times U(1)_{em}$. We show that if the fermion fields are in a (16) of SO(10) then it is impossible to find an SO(10) theory where monopoles do not catalyze proton decay. Furthermore, the branching ratios for monopole catalyzed proton decay are identical in SO(10) and SU(5) grand unified theories. We also present a criterion for constructing grand unified theories which do not have monopole catalysis of proton decay.

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# I. Introduction

Stable magnetic monopole solutions to the field equations of a Yang-Mills theory which is coupled to scalar bosons exist in most grand unified theories, (GUTs).[1] In such theories, the color, weak, and electromagnetic groups, SU(3) x SU(2) x U(1), ($=G_1$), are embedded in a semi-simple group $G_n$ and monopoles are typically produced at the scale where $G_n$ is broken down to $G_1$. In a theory with a hierarchy of mass scales,

$$G_n \to G_{n-1} \to \ . \ . \ . \to G_1 \to SU(3) \times U(1)_{em}, \qquad (1.1)$$

monopoles will appear when there is a non-trivial topology.[2] (That is, whenever the mapping $\Pi_1(G_n) \to \Pi_1(G_{n-1}) \to \ldots \to \Pi_1(G_1)$ has a kernel which is eventually mapped into the identity). Recently, Rubakov[3] and Callan[4] have noticed that fermion number violating processes can occur in the field of a monopole. The amplitudes for such processes are suppressed by neither instanton- like exponential factors nor by inverse powers of large vacuum expectation values of Higgs particles. Instead, Rubakov and Callan claim that these fermion number violating processes have typical strong interaction cross-sections. In particular, protons can decay in the field of a monopole with a cross-section on the order of $(1 \text{ GeV})^{-2}$.

Rubakov and Callan considered the eg=1/2 monopole occuring in the Georgi-Glashow SU(5) GUT.[5] An SU(2) doublet, $\vec{T}$, is embedded in the $(\bar{5})$ as $(\bar{5}) = (0, 0, \vec{T}, 0)$. (The fermion basis is $(\bar{5}) = (\bar{d}_1, \bar{d}_2, \bar{d}_3, e^-, \nu))$. They found a non-zero value for the matrix element between vacuum states of a $\Delta B=1$ four-fermion operator in the field of a monopole,

$$< e^- uud >^{\text{Monopole}} \neq 0, \tag{1.2}$$

which was interpreted as evidence for proton decay in a monopole field.

There are some technical problems associated with the calculations of both Rubakov and Callan. These analyses are performed in the limit in which all of the fermions are massless and in which there is only one generation of fermions--it is not clear what the effects of finite fermion masses will be. Also, all QCD effects have been ignored. Our results, however, do not depend directly on these technical problems.

The question we would like to answer is if proton decay near a monopole is a general feature of grand unified theories or if it is a specific feature of the SU(5) model. In particular, is it possible to construct GUTs in which monopoles do not catalyze proton decay? Is it possible to place restrictions upon the GUT from the features of monopole catalyzed baryon number violation, (if it is observed)? To approach these questions, we have examined the monopole spectrum in SO(10) GUTs in which there exists the possibility of intermediate stages of symmetry breakdown between $M_X$ and $M_W$. The effects of these intermediate mass scales on monopole catalysis of proton decay are considered and we present a group theoretical procedure for analyzing the proton decay occuring near monopoles in a general GUT.

In Section II, we briefly discuss SO(10) GUTs and the predictions for baryon number violating processes due to heavy gauge boson exchanges in these models. In Section III, we present our method for analyzing the monopole spectrum (and the ensuing proton decay) in a GUT model and illustrate it in the SO(10) model. In Section IV, the monopoles and the resulting proton decay for all possible symmetry breakdowns of SO(10) to SU(3) x $U(1)_{em}$ are examined and in Section V there is a discussion of our results.

## II. SO(10) Grand Unified Models

We have considered a class of SO(10) grand unified models in which the SO(10) can break directly to the low energy group $G_0$ = SU(3) x $U(1)_{em}$ or the symmetry breakdown can occur via any of the following chains:

1. $SO(10) \to SU(5) \to G_1 \to G_0$ (2.1)

2. $SO(10) \to SU(5) \times U(1) \to G_1 \to G_0$

3. $SO(10) \to SU(4) \times SU(2) \times SU(2) \to G_1 \to G_0$

4. $SO(10) \to SU(4) \times SU(2) \times U(1) \to G_1 \to G_0$

5. $SO(10) \to SU(3) \times SU(2) \times U(1) \times U(1) \to G_1 \to G_0$

The phenomenology associated with each of these patterns of symmetry breaking has been extensively discussed in the literature.[6,7] For our purposes, we note only that the mass scales which correspond to each symmetry breakdown are restricted by the requirement that the theory reproduce the experimental value for the Weinberg angle, $\sin^2\theta_W(M_W)$. The restriction that $\sin^2\theta_W(M_W) \simeq .22$ in general restricts $M_X > 10^{15}$ GeV and $M_I > 10^9$ GeV, where $M_X$ is the scale at which the SO(10) symmetry is broken and $M_I$ is the scale at which the SU(3) x SU(2) x U(1) symmetry is produced. (See Ref. 7 for explicit details on mass restrictions in these models).

The fermion fields are embedded in a left-handed sixteen dimensional spinor representation of SO(10): $\Psi = (\nu, u_1, u_2, u_3, e^-, d_1, d_2, d_3, -\bar{d}_3, \bar{d}_2, \bar{d}_1, -e^+, \bar{u}_3, -\bar{u}_2, -\bar{u}_1, \bar{\nu})_L$. The subscripts 1, 2, and 3 are color indices and $^-$ denotes charge conjugation. (The second and third generations are similarly embedded in a (16) of SO(10)). The phases of these fields are not important for our results and will therefore be ignored in the following. At this stage, the model contains a right-handed neutrino which becomes massive at the scale $M_I$.

All of the SO(10) models predict proton decay mediated by the gauge bosons which obtain mass $M_X$ at the first stage of the symmetry breaking. Further, these models predict that the dominant decay mode will be $p \rightarrow \Pi^0 e^+$ and that the proton will decay to $\nu_e$ (not $\bar{\nu}_e$) with a branching ratio of about 15%.[1]

In the next sections, we will investigate the question of whether there is proton decay in these SO(10) GUT models due to the existence of monopoles and the effects of the intermediate stages of symmetry breaking upon the predictions for monopole catalyzed proton decay. Since there are monopoles at two different mass scales in some of the models of Eq.(2.1), we are particularly interested in the question of which monopoles can catalyze proton decay and which cannot.

## III. Monopole - Fermion Interactions in GUTs

### (a) Long Range Properties of GUT Monopoles

The existence of classical monopole solutions to gauge theories which are spontaneously broken by the vacuum expectation values of scalar fields was first shown by 't Hooft and Polyakov[8] for an SU(2) gauge theory with a triplet of Higgs scalars. Using spherical symmetry ansätze, Dokos and Tomaras[9] found monopole solutions for the eg=1/2, 1, 3/2, and 2 embeddings of SU(2) in the Georgi-Glashow SU(5) GUT. Horvath and Palla[10] have extended this work and present a method for constructing static, finite energy monopole solutions in a grand unified theory. Armed with the knowledge that it is possible in general to construct spherically symmetric monopole solutions , we only consider the embeddings of SU(2) into a fermion representation of a GUT and assume that a suitable monopole ansatz can be constructed. The problem is thus reduced to a purely group theoretical one.

We first examine the monopoles formed when a unified group, $G_n$ , is broken directly to SU(3) x $U(1)_{em}$. We assume that $G_n$ has a trivial $\Pi_1$ group and that SU(3) x $U(1)_{em}$ satisfies the charge-triality relation, which forbids unconfined fractional charge. Then the topology of the problem is defined and several features of the monopole spectrum can be discussed without reference to the hierarchy of symmetry breaking. These features include the allowed magnetic quantum numbers and the stability of the monopole under small fluctuations. More detailed questions, like the monopole mass, its embedding in $G_n$ and its stability for decay to monopoles with smaller topological charges depend on the intermediate steps of symmetry breaking.

Outside the monopole core one can define a charge matrix, $Q_M$, in terms of the Dirac string potential, $\vec{A}_D$:

$$\vec{A}_D = Q_M(1-\cos\theta)\ \frac{\hat{\phi}}{r\sin\theta} \quad , \tag{3.1}$$

where $Q_M$ is a matrix in some representation of $G_n$, which we will assume to be diagonalized. Since the only long range interactions are color and electromagnetism, $Q_M$ must be a linear combination of the generators of SU (3) and $U(1)_{em}$:

$$Q_M = aQ_{em} + Q_c. \tag{3.2}$$

($Q_{em}$ is the electromagnetic charge operator and $Q_c$ is a color generator whose normalization we have not yet specified). The coefficient a and the color generator $Q_c$ are restricted by the quantization condition and by a recently derived stability condition.[11] The latter is a criterion for the absence of negative frequency modes for non-abelian point monopoles.

For our purpose the relevant non-abelian group is SU(3). Consider a color generator $Q_c' = \mathrm{diag}(q_1, q_2, q_3)$, in the fundamental representation of SU(3). Then a necessary condition for classical stability of the corresponding point monopole is:[11]

$$q_i - q_j = 0, \pm 1/2, \qquad i,j=1,2,3. \tag{3.3}$$

Since $Q_c'$ must be traceless this allows only three different values of $Q_c'$ (apart from color rotations);

$$Q_c' = 0 , \qquad Q_c' = \pm Y/2 , \tag{3.4}$$

where $Y = \mathrm{diag}\ (1/3, 1/3, -2/3)$.

The Dirac quantization condition is in this case;

$$\exp(4\Pi i Q_M) = 1. \tag{3.5}$$

Since we only consider models in which the usual charge-triality relation is satisfied, the allowed eigenvalues of $Q_{em}$ for an SU(3) triplet representation embedded in $Q_M$ are $-1/3 + m$, where m is an integer. From Eqs. (3.2), (3.4), and (3.5) we get then the following restrictions on a;

$$Q_c'=0; \qquad a=3n/2 \tag{3.6}$$

$$Q_c'=Y/2; \qquad a=1/2 + 3n/2$$

$$Q_c'=-Y/2; \qquad a=-1/2 + 3/2n,$$

where n is an integer. (Note that $Q_c$ is a representation of $G_n$ which contains some number of color triplets, each of which has an SU(3) generator, $Q_c'=0,\pm 1/2Y$). The coefficient a is equal to eg, where e is the electromagnetic coupling constant and g the magnetic strength of the monopole. For the monopoles with lowest magnetic charge one finds therefore:

$$eg=1/2; \qquad Q_M = 1/2(Q_{em} + Y) \tag{3.7}$$

$$eg=1; \qquad Q_M = Q_{em} - Y/2$$

$$eg=3/2; \qquad Q_M = (3/2)Q_{em}$$

$$eg=2; \qquad Q_M= 2Q_{em} + Y/2.$$

This specifies the eigenvalues of $Q_M$ for any representation of any group that breaks down to SU(3) x $U(1)_{em}$ and satisfies the charge-triality relation and the non-abelian stability criteria.

(b) Spherically Symmetric Monopoles

A procedure to construct a spherically symmetric monopole potential from a string potential has been described by Wilkinson and Goldhaber.[12] They prove the following theorem:

Consider a unifying group $G_n$ broken down to a subgroup H (in our case, H = SU(3) x $U(1)_{em}$). Let $\vec{T}$ be the generator of an SU(2) subgroup of $G_n$ and $\vec{I}$ the generator of an SU(2) subgroup of H, which satisfies $[\vec{I}, Q_M] = 0$. Then the string potential can be gauge-transformed to a potential which is spherically symmetric under $\vec{L} + \vec{T}$ if and only if $Q_M = I_3 - T_3$ for some choice of $\vec{I}$. (Here $\vec{L}$ is the generator of orbital angular momentum).

Spherical symmetry and the existence of an SU(2) subgroup in which the monopole is embedded is important to us for two reasons. First, it is generally believed that the spherically symmetric monopoles are the lightest in their topological class, (unless decay to two or more other monopoles is energetically and topologically allowed). Second, it is not obvious how to generalize the arguments of Refs. 3 and 4 to monopoles which are not associated with an SU(2) subgroup.

Using the relation $Q_M = I_3 - T_3$ combined with Eq. (3.7), we can find the possible $T_3$ assignments for any spherically symmetric monopole which satisfies the non-abelian stability criteria. In Table 1, we list all SU(2) subgroups of SO(10) for which these criteria can be satisfied and the values of eg of the corresponding monopoles. The $T_3$ eigenvalues for the particles in the (16) of SO(10) are not listed, but can easily be obtained. As emphasized before, these eigenvalues are the same for all SO(10) breaking patterns. However, this does not tell us which fields are connected into multiplets.

To find the fermion multiplets, we must construct the raising and lowering operators;

$$T^{\pm} = T_1 \pm iT_2 \quad . \tag{3.8}$$

The action of $T^+$ (or $T^-$) on the fermion fields of the (16) of SO(10) then defines the fermion multiplets which correspond to the monopole embedding $\vec{T}$. We repeat this procedure at each stage of the symmetry breaking where there are monopoles formed. In most cases of interest to us the raising operator can be obtained by considering the group structure. In general,however, this is not a purely group theoretical problem. When more than one embedding is allowed, one has to compute the masses of the various monopoles to determine which one is the lightest and hence the stable one.

In general, $T^2$ does not commute with the electromagnetic charge generator,

$$[T^2, Q_{em}] \neq 0. \tag{3.9}$$

When this is the case, the eigenstates of $T_3$ and $T^2$ may not be charge eigenstates. However, when a color average over the monopole embeddings is performed, all operators which have $\Delta Q_c \neq 0$ (and hence $\Delta Q_{em} \neq 0$) will have a zero vacuum expectation value between monopole states. Thus the operators which have non-zero charge have no physical consequences.

(c) Monopoles and Proton Decay

Rubakov and Callan have shown that for any GUT with $G_n$ a compact, semi-simple group which contains two massless fermion SU(2) doublets, $\Psi^1$ and $\Psi^2$,

$$< \varepsilon_{\alpha\beta}\varepsilon_{ij}\Psi^1{}_{\alpha i}\Psi^2{}_{\beta j} >^{\text{Monopole}} \neq 0 \tag{3.10}$$

where $\alpha$ and $\beta$ are Lorentz indices and i and j are SU(2) group indices. The corresponding cross section is assumed to have a magnitude typical of the strong interactions, but its precise value is uncertain. The vacuum expectation value (Eq. (3.10)) can be calculated exactly for an eg = 1/2 monopole which couples only to SU(2) massless fermion doublets. The calculation is possible because the problem can be reduced to the two dimensional Schwinger model which is exactly solvable. The generalization to other eg values and SU(2) representations is non- trivial. Fortunately, all we have to know is the structure of the relevant operators and the relevant features of that structure can be obtained by making the following observations.

First of all, the two fermion operator appearing in Eq.(3.10) is precisely the one which interacts with an instanton field without a monopole present.[13] (The same is true for the corresponding operator with four Weyl fermions. In both cases the operators can be converted to the ones given in Ref. 13 in terms of Dirac fermions.) Only the factor multiplying the operator is radically different. It seems reasonable to assume that this property is valid in general. Additional support for this assumption is the fact that the heuristic arguments of Refs. 3 are essentially unchanged if the monopole couples to higher SU(2) fermion representations.

In particular, for all cases of interest to us we find that it is possible to obtain field configurations with non-trivial winding number and infinitesimal contribution to the action. The Dirac equation can be solved explicitly in this background field, and the correct number of zero modes is obtained. This number is dictated by the anomaly of the appropriate axial fermion number with respect to the SU(2) subgroup, and the number of zero modes is equal to $1/3N_f \Delta k$ t (2t + 1)(2t + 2) for an SU(2) representation with 'spin' t, $N_f$ flavors and change in winding number $\Delta k$. (We consider each fermion multiplet to define a flavor). Therefore, we conclude that the relevant operator must be a product of 1/3 $\Delta k\, t\, (2t + 1)\, (2t + 2)$ fermion fields of each flavor. The only other assumption which we will make is that it must be a $T_3$ singlet, since $T_3$ is an unbroken generator of SU(2). We will assume that the operator, $\theta$, constructed in this manner will have a matrix element between monopole states of order one.

## IV. Results for SO(10) GUTs

In this section, we apply our procedure to the different symmetry breaking patterns of SO(10). In each case, we obtain the monopole spectrum, construct the fermion multiplets corresponding to the monopole embedding, and analyze whether or not the monopoles catalyze proton decay.

### (a) eg=1/2 Monopoles

We begin our discussion by proving a somewhat surprising result. That is, regardless of how the SO(10) symmetry is broken to SU(3) x $U(1)_{em}$ and regardless of how the monopole is embedded SO(10), the eg=1/2 monopoles of an SO(10) GUT will always mediate baryon number violating interactions with the same selection rules.[14] This statement is independent of the Higgs structure and the symmetry breaking pattern of the theory.

In Appendix A we show that it is possible to form the fermion doublets in such a way that the baryon number, B, is conserved in certain GUTs. In that construction, we allow the $T^+$ and $T^-$ operators of the monopole SU(2) group to be any linear combination of the generators of SU(16), of which SO(10) is a subgroup. Now we will show, however, that within SO(10) alone such a construction is impossible.

This can most easily be shown for the smallest non-trivial SO(10) representation, the (10). The states in the (10) have the same SU(3) x $U(1)_{em}$ quantum numbers as d, $\bar{d}$, $e^+$, $e^-$, $\upsilon$, and $\bar{\upsilon}$. We will use this identification as a convenient way to label them. From Section III, we conclude that $e^+$ and $\bar{d}_3$ have $T_3 = -1/2$ and e and $d_3$ have $T_3 = 1/2$; the other states have $T_3 = 0$. Thus there are only two doublets, compared to four for the (16), and therefore we have only a two-fold ambiguity for pairing the states in doublets. The most general doublet assignment is: (cf. Appendix A),

$$\begin{bmatrix} \alpha e^- + \beta d_3 \\ e^+ \end{bmatrix}, \qquad \begin{bmatrix} -\beta^* e^- + \alpha^* d_3 \\ \bar{d}_3 \end{bmatrix}, \qquad (4.1)$$

$$|\alpha|^2 + |\beta|^2 = 1.$$

If $\alpha \neq 0$, the operator $T^-$ must connect the state $e^+$ to $e^-$. However, the maximal difference in charge between two states that can be connected by an SO(10) generator is $\pm$ 4/3, corresponding to the maximal charge of an SO(10) gauge boson. Therefore, the only possibility is $\alpha = 0$. Now the embedding of the monopole group in SO(10) is fixed, and one can find the four doublets in the fermion (16). This is simple, since for $\alpha = 0$ we get exactly the same SU(2) group as in the SU(5) case discussed in Refs. 3 and 4 and therefore the fermion doublets are the same. It is impossible to have eg = 1/2 monopoles in SO(10) GUTs which do not catalyze proton decay. The fermion doublets for an SO(10) eg = 1/2 monopole are fixed to be;

$$\begin{bmatrix} d_3 \\ e^+ \end{bmatrix} \quad \begin{bmatrix} e^- \\ \bar{d}_3 \end{bmatrix} \quad \begin{bmatrix} \bar{u}_2 \\ u_1 \end{bmatrix} \quad \begin{bmatrix} \bar{u}_1 \\ u_2 \end{bmatrix} \tag{4.2a}$$

Hence the flavor and $T_3$ singlet operator,

$$\theta = e^- \, d_3 \, u_1 \, u_2, \tag{4.2b}$$

will have a non-zero expectation value in the presence of a monopole and will mediate proton decay.

(b) SO(10) $\rightarrow$ SU(5) $\rightarrow G_1 \rightarrow G_0$

The monopole spectrum in this breakdown of SO(10) is given in Table 1. We will analyze only those monopoles with eg =1/2, 1, and 3/2 which are formed in the breakdown of SU(5), (these monopoles are denoted by an asterisk in Table 1). There may be monopoles formed when SO(10) is broken in the first stage but these monopoles presumably decay to the SU(5) monopoles and are of no interest to us here.

Since the number of fermions in the operator $\theta$ of Section III is equal to the number of zero modes, the eg = 1/2 monopole will have $n_0 = 4$ fermion fields in $\theta$, eg = 1 will have $n_0 = 16$ fermions, and eg = 3/2 will have $n_0 = 40$ fermions for the monopoles formed in the breakdown of SU(5). Our discussion of the eg = 1 and eg =3/2 monopoles is included mainly for pedagogical reasons. (We expect that these monopoles decay into the appropriate number of eg =1/2 monopoles.)

An eg = 1 monopole can occur for two different assignments of the $T_3 = 0$ fermions. Both of these embeddings are defined by the $T_3$ eigenvalues of the $(\bar{5})$ of SU(5) ;

$$T_3 = (-1,0,0,1,0). \tag{4.3}$$

To obtain this embedding a non-zero $I_3$ is required.[15] The four triplets which correspond to the eg = 1 monopole embedding are;

$$\begin{bmatrix} e^- \\ \nu \\ \bar{d}_1 \end{bmatrix} \qquad \begin{bmatrix} d_1 \\ u_1 \\ e^+ \end{bmatrix} \qquad \begin{bmatrix} \bar{u}_3 \\ d_2 \\ u_2 \end{bmatrix} \qquad \begin{bmatrix} \bar{u}_2 \\ d_3 \\ u_3 \end{bmatrix} \tag{4.4}$$

(The alternate $T_3 = 0$ assignment gives a similar conclusion about monopole catalyzed proton decay and we will not discuss it). The operator mediating proton decay contains sixteen fermions and so a typical operator which has $\Delta B=1$ and is a flavor and $T_3$ singlet is;

$$\theta = e^- d_1 u_2 u_3 (d_1 \bar{d}_1 u_2 \bar{u}_2)^3. \tag{4.5}$$

(We have suppressed irrelevant quantum numbers). Hence even if the eg = 1 monopoles exist and are stable, their effects on proton decay will presumably be swamped by kinematic effects due to the production of a many fermion state or by dimensional factors of the order of $(m_{fermion}/m_{proton})$ due to replacing a fermion operator by a mass term.

For the eg = 1 monopoles, there are however operators with non-zero expectation values which do not exist for the eg = 1/2 case. There are flavor and $T_3$ singlet operators which mediate $n \to \nu \Pi^0$; for example,

$$\theta = \nu u_1 d_2 d_3 (\bar{d}_1 d_1 \bar{u}_2 u_2)^3. \tag{4.6}$$

The final case which we analyze for this breakdown has eg = 3/2. The fermions transform under the SU(2) corresponding to the monopole embedding as two four-plets and a five-plet;

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ e^+ \end{bmatrix} \qquad \begin{bmatrix} e^- \\ \bar{d}_3 \\ \bar{d}_2 \\ \bar{d}_1 \end{bmatrix} \qquad \begin{bmatrix} \bar{u}_3 \\ \bar{u}_2 \\ 1/\sqrt{2}(u_1+\bar{u}_1) \\ u_2 \\ u_3 \end{bmatrix}. \tag{4.7}$$

A typical $T_3$ singlet operator mediating proton decay near this monopole is then;

$$\theta = e^- d_1 u_2 u_3 (\bar{d}_2 d_2 \bar{u}_2 u_2)^9. \tag{4.8}$$

(c) $SO(10) \to SU(4) \times SU(2)_L \times SU(2)_R \to G_1 \to G_0$

In this breakdown of SO(10) there are monopoles produced at two stages of the symmetry breaking. When $SO(10) \to SU(4) \times SU(2)_L \times SU(2)_R$, monopoles are produced with masses on the order of $M_X/\alpha_X$, (we call these "heavy" monopoles) and when $SU(4) \times SU(2)_L \times SU(2)_R$ breaks to $SU(3) \times SU(2) \times U(1)$, "light" monopoles are produced with masses $M_I/\alpha_I$, ($\alpha_I$ ($\alpha_X$) is a typical coupling constant at the scale $M_I$ ($M_X$)). These monopoles have different topological properties. The light monopoles have eg equal to an integer, while the heavy ones can have half-integer eg. (We have been careless about local and global symmetries. SO(10) is actually spin (10), the universal covering group of SO(10), and what we call $SU(4) \times SU(2)_L \times SU(2)_R$ is actually $SO(6) \times SO(4)$). These monopoles and their cosmological properties have been considered by Lazarides, Shafi, and Magg.[16]

The only interesting heavy monopoles are the ones with eg=1/2, which we have already discussed in general. The light monopoles are embedded within $SU(4) \times SU(2)_L \times SU(2)_R$.

We will only consider the case eg=1 since all other integer charge monopoles will probably decay into these.

In SO(10) there are two choices of $T_3$ which lead to stable monopoles. In both cases $T_3$ can be written as a linear combination of SU(4), $SU(2)_L$, and $SU(2)_R$ generators. This leads to the following embeddings of the monopole SU(2) group in $SU(4) \times SU(2)_L \times SU(2)_R$. (The embedding is defined by the breakdown of the fundamental representations of SU(4) $\times SU(2)_L \times SU(2)_R$ under SU(2). The notation is explained in Table 1).

$$\begin{array}{lll} & SU(4) \times SU(2)_L \times SU(2)_R & \to SU(2) \\ \text{case a:} & (4,1,1) & \to (2) + 2(1) \\ & (1,2,1) & \to (2) \\ & (1,1,2) & \to (2) \end{array} \tag{4.9a}$$

and

$$\begin{array}{lll} \text{case b:} & (4,1,1) & \to 2(2) \\ & (1,2,1) & \to (2) \\ & (1,1,2) & \to (2) \quad . \end{array} \tag{4.9b}$$

In case a each factor of the gauge group contains just one doublet and therefore $T^+$ and $T^-$ are completely determined. In case b, all possible ambiguities in choosing the doublets correspond to color rotations and are thus irrelevant. Since case b is obtained from case a by embedding an extra doublet in color space this monopole probably does not exist as a stable solution, but we will nevertheless examine both.

Because $T^+$ and $T^-$ are known it is trivial to obtain the fermion multiplets in both cases;

case a: (4.10a)

$$\begin{bmatrix} d_2 \\ u_2 \end{bmatrix} \begin{bmatrix} d_1 \\ u_1 \end{bmatrix} \begin{bmatrix} \bar{u}_1 \\ \bar{d}_1 \end{bmatrix} \begin{bmatrix} \bar{u}_2 \\ \bar{d}_2 \end{bmatrix} \begin{bmatrix} e^- \\ 1/\sqrt{2}(d_3+\nu) \\ u_3 \end{bmatrix} \begin{bmatrix} \bar{u}_3 \\ 1/\sqrt{2}(\bar{d}_3+\bar{\nu}) \\ e^+ \end{bmatrix}$$

case b: (4.10b)

$$\begin{bmatrix} e^- \\ 1/\sqrt{2}(\nu+d_3) \\ u_3 \end{bmatrix} \begin{bmatrix} d_1 \\ 1/\sqrt{2}(u_1+d_2) \\ u_2 \end{bmatrix} \begin{bmatrix} \bar{u}_3 \\ 1/\sqrt{2}(\bar{\nu}+\bar{d}_3) \\ e^+ \end{bmatrix} \begin{bmatrix} \bar{u}_2 \\ 1/\sqrt{2}(\bar{u}_1+\bar{d}_2) \\ \bar{d}_1 \end{bmatrix}$$

Notice that in the second case we obtain four triplets, just as in the SU(5) case, (see Section IVb.). The grouping of the particles into multiplets is however completely different. Even if we had chosen a different embedding for the eg=1 monopoles in SU(5), it would not be possible to obtain Eq. (4.10b).

As in Eq.(4.7), the $T_3 = 0$ states are not charge eigenstates. However, charge conservation is not violated for any physical process as explained in Section III.

From the multiplets of Eq. (4.10), it is impossible to construct an operator which has $\Delta B \neq 0$, $\Delta T_3=0$, is a flavor singlet and has $\Delta Q_c=0$, (a necessary condition for the operator to survive the color averaging). For each multiplet of Eq. (4.10a), baryon number can be written as,

$$B = -T_3 - Q_{em} + \tilde{Q}, \tag{4.11}$$

where $\tilde{Q}$ is a global U(1) charge which is equal to 1/2 for the first two doublets in Eq. (4.10a), -1/2 for the second two, and zero for the triplets. Since any operator which can get a non-zero vacuum expectation value must contain all four doublets, such an operator has $\tilde{Q} = 0$. Because $\theta$ must also be a $T_3$ singlet, we find;

$$\Delta B = -\Delta T_3 - \Delta Q_{em} + \Delta\tilde{Q} = -\Delta Q_{em}. \tag{4.12}$$

Therefore, if charge is conserved baryon number is also conserved. A similar result holds for the multiplets of Eq. (4.10b). So we conclude that the monopoles of an SU(4) x $SU(2)_L \times SU(2)_R$ theory do not cause B-violating processes, (at least not with a large cross section).

This conclusion is not surprising, since the Pati-Salam model with fractional charges is known to have no proton decay by gauge boson exchange.[17] However, baryon number is a broken symmetry in any realistic model because B has an anomaly with respect to the weak SU(2) group. A monopole can in principle enhance both sources of $\Delta B \neq 0$ processes and therefore absence of proton decay by gauge boson exchange does not necessarily imply the same for proton decay by monopoles. In fact, as we shall see shortly, even in the pure Pati-Salam model, without SO(10) unification and without any perturbative violation of baryon number, we expect that monopoles will have a small cross section for catalysis of baryon number violating processes.

The fact that baryon number is anomaly free for the multiplets of Eqs. (4.10a) and (4.10b) is a consequence of the left-right symmetry of SU(3) x $U(1)_{em}$. However, this symmetry is not exact. When the model goes through the SU(3) x SU(2) x U(1) stage of symmetry breaking the left-right symmetry is obviously broken. Therefore it is not surprising to find that the fermion multiplets to which the SU(3) x SU(2) x U(1) monopole couples have a net baryon number anomaly. Consequently, SU(3) x SU(2) x U(1) monopoles can catalyze B-violating processes. Inside a radius $\sim M_W^{-1}$, SU(3) x $U(1)_{em}$ monopoles behave like SU(3) x SU(2) x U(1) monopoles and therefore the eg=1 monopole will catalyze B-violating processes. The cross section for these processes must vanish for $M_W \to \infty$ and is thus proportional to some negative power of $M_W$. The effect, even though it may be rather small, is in any case much more important than the only other B-violating process in the model, the instanton process discussed by 't Hooft.[13]

(d) Other Embeddings

(i) SO(10) → SU(5) x U(1) → SU(3) x SU(2) x U(1) x U(1) → $G_1 \to G_0$

The extra U(1) factor in the first stage of the symmetry breakdown can be identified as B-L and so it is orthogonal to the electric charge. Thus the monopoles formed when SO(10) → SU(5) x U(1) will not exist at large distances and are uninteresting to us. The monopoles formed in the breakdown of SU(5) x U(1) are the same as those discussed in Section IVb.

(ii) $SO(10) \rightarrow SU(4) \times SU(2) \times U(1) \rightarrow G_1 \rightarrow G_0$

The monopoles formed in the breakdown of SO(10) are the same as those discussed previously. There are no monopoles formed in the intermediate stage of symmetry breaking which satisfy the non-abelian stability criteria.

## V. Conclusion

In the previous sections, we have analyzed monopole catalysis of proton decay for different breaking patterns of SO(10) to $SU(3) \times U(1)_{em}$ and presented a group theoretical analysis of the monopole spectrum in SO(10). A priori, there is no reason to expect that the monopole structure of SO(10) GUTs will be the same as that of SU(5). However, we found that the structure of the lowest energy, (eg=1/2), monopole of SO(10) is uniquely determined by the group structure and is identical to the eg=1/2 monopole for SU(5). Thus monopole catalyzed proton decay can not distinguish between SO(10) and SU(5) GUTs.

Our most interesting result was found in the case where SO(10) breaks to $G_0$ via $SU(4) \times SU(2)_L \times SU(2)_R$. In this case, the monopoles formed in the intermediate stages of the symmetry breakdown (i.e. the monopoles of the Pati-Salam model) do not catalyze proton decay with a strong interaction rate. However a small proton decay cross section is expected to exist, even though there is no proton decay due to boson exchange.

There are clearly many questions about monopole catalysis of proton decay left to be answered--the most important of these being the actual value for the rate. We hope that this paper will stimulate discussion of these questions.

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## Appendix A

### GUTs Without Monopole Catalysis of Proton Decay

In this appendix we determine the fermion doublet assignments for the eg=1/2 monopole which do not lead to violation of baryon number. Let us first formulate the problem.

Consider the 16 fermions in the first generation. (To simplify the argument,we assume the existence of a right-handed $\nu$, although the same conclusions can be reached without it). The $T_3$ eigenvalues of these fermions are known, but the raising and lowering operators depend on the details of the model. In other words, one does not know a priori which $T_3 = 1/2$ fermion belongs to which $T_3 = -1/2$ fermion. The question we will answer is: Are there choices for $T^+$ and $T^-$ and corresponding doublet assignments which do not lead to B violation? This is of course only a necessary condition for absence of monopole induced B violation. It may still be impossible to construct a model which has eg=1/2 monopoles which couple to those doublets.

To answer the question we consider the maximal symmetry group of the first generation, SU(16). The results will then apply to any gauged subgroup in SU(16). We choose a basis in which the first four states are the $T_3 = 1/2$ fermions, $e^-$, $d_3$, $\bar{u}_1, \bar{u}_2$, the next four the $T_3 = -1/2$ fermions, $e^+$, $\bar{d}_3$, $u_1$, $u_2$, and the last eight the remaining $T_3=0$ fermions. The most general $T^+$ and $T^-$ that close the

SU(2) algebra are;

$$T^{+} = \begin{bmatrix} 0 & U \\ 0 & 0 \end{bmatrix} \qquad T^{-} = \begin{bmatrix} 0 & 0 \\ U^{+} & 0 \end{bmatrix} \tag{A1}$$

where $T^{+}$ and $T^{-}$ are 8 x 8 matrices which operate on the first 8 components of the basis, and U is a 4 x 4 unitary matrix. The doublet assignment appears more clearly in a different basis, obtained by a transformation;

$$T_i \rightarrow S\, T_i\, S^{-1} \tag{A2}$$

$$S = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}, \qquad SS^{+} = 1. \tag{A3}$$

Now $T^{+}$ and $T^{-}$ are,

$$T^{+} = \begin{bmatrix} 0 & S_1\, U\, S_2^{-1} \\ 0 & 0 \end{bmatrix} \qquad T^{-} = \begin{bmatrix} 0 & 0 \\ S_2\, U^{+}\, S_1^{-1} & 0 \end{bmatrix}.$$

By choosing $S_1 = 1$ and $S_2 = U$, $T^{+}$ and $T^{-}$ are brought to a simple form and the four doublets are formed by the $i^{th}$ and $(4+i)^{th}$ element of the transformed basis. Notice that the transformation S only rotates the $T_3 = -1/2$ fermions. A more general choice would be $S_2 = S_1 U$. Then the matrix S can be written as follows;

$$S = \begin{bmatrix} S_1 & 0 \\ 0 & S_1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & U \end{bmatrix}. \tag{A4}$$

This corresponds to an additional SU(4) flavor rotation of the doublets by a matrix $S_1$, which rotates the upper and lower components in the same way. The four fermion operator which gets a non-vanishing vacuum expectation value due to the monopole is an SU(4)- flavor singlet, at least in the limit considered in Refs. 3 and 4. This can be shown explicitly by repeating the calculations of Ref. 3 for the case of four doublets. Therefore the operator does not depend on $S_1$ and we can choose it equal to the unit matrix.

One can only expect $\Delta B \neq 0$ processes to be absent if B is an exact symmetry of the four-fermion operator. Otherwise $\Delta B \neq 0$ processes may manifest themselves in a different way than monopole + p → monopole + $e^+$, but they will be present in some form. Therefore B must be a linear combination of the unbroken symmetry generators.

The unbroken symmetries are $U(1)_{em}$, SU(3), and the SU(4) flavor symmetry. (The first two symmetries are not strictly symmetries of the operator, but all charge and color changing matrix elements vanish when one averages over the color embeddings of the monopole). Therefore the criterion for absence of B violation is that for the eight fields appearing in the doublets,

$$B = \alpha Q_{em} + Q_c + \tilde{M} \tag{A5}$$

where $Q_c$ is a color generator and $\tilde{M}$ is an SU(4) flavor generator. The matrix, $\tilde{M}$ is simplest in the transformed basis, in which the doublets are manifest. In that basis:

$$\tilde{M} = \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}, \tag{A6}$$

where M can be any SU(4) generator. Transforming this matrix back to the original basis we get:

$$\tilde{M}' = S^{-1}\,\tilde{M}\,S = \begin{bmatrix} M & 0 \\ 0 & S_2^{-1}\,M\,S_2 \end{bmatrix} \tag{A7}$$

Now consider the color generator $Q_c$. Any off-diagonal matrix elements involving the third component of color must vanish if Eq. (A5) is to be satisfied. Then $Q_c$ can be diagonalized in the subspace of the first two components, so that all three matrices B, $Q_{em}$, and $Q_c$ are diagonal in this basis. Then both M and $S_2^{-1}\,M\,S_2$ must be diagonal. If all eigenvalues of M are different, $S_2$ can only be a permutation of the eigenstates; if two or more are the same then rotations of these eigenstates are allowed. In any case, M corresponds to a U(1) generator which may be different for each doublet.

Now we use the fact that the $T_3 = -1/2$ fermions are the anti-particles of the $T_3 = 1/2$ ones. Therefore $Q_{em}$, M, and $Q_c$ have opposite signs, and hence the eigenvalues of M must also be opposite. Therefore, modulo permutations, the eigenvalues of M and $S_2\ M\ S_2^{-1}$ are both identical and opposite in sign. This is only possible if for any eigenvalue of M there is one with opposite sign.

The diagonal color generator $Q_c$ can be written as a linear combination of Y and $I_3$:

$$Q_c = \beta Y + \gamma I_3 \tag{A8}$$

$$Y = \mathrm{diag}(1/3,\ 1/3,\ -2/3)$$

$$I_3 = \mathrm{diag}(1/2, -1/2, 0)$$

We now use the fact that M must be traceless, (notice that the unbroken flavor group is SU(4) and not U(4), since the U(1) factor has an anomaly with respect to the SU(2) group of the monopole). Tracelessness of M leads to the following relation between $\alpha$ and $\beta$;

$$8\alpha + 4\beta = 1. \tag{A9}$$

The matrix elements of M can be expressed in terms of $\alpha$ and $\gamma$. From Eqs. (A5), (A8), and (A9) we find,

$$M = \mathrm{diag}(-\alpha,\ \alpha - 1/2,\ -1/2\gamma + 1/4,\ 1/2\gamma + 1/4). \tag{A10}$$

We must group the eigenvalues in pairs with opposite signs. There are three ways to do this. Notice, however, that equivalent results are obtained when $u_1$ and $u_2$ are interchanged. Therefore there are only two different possibilities;

1. $-\alpha = -(\alpha - 1/2);\qquad 1/2\gamma - 1/4 = 1/2\gamma + 1/4.$

This is obviously impossible.

2. $-\alpha = -(-1/2\gamma + 1/4);\qquad \alpha - 1/2 = -1/2\gamma - 1/4.$

This second case has a solution;

$$M = \mathrm{diag}(1/2\gamma - 1/4,\ -1/2\gamma - 1/4,\ -1/2\gamma + 1/4,\ 1/2\gamma + 1/4). \tag{A11}$$

This leads to the following doublets;

$$\begin{bmatrix} e^- \\ u_1 \end{bmatrix} \qquad \begin{bmatrix} d_3 \\ u_2 \end{bmatrix} \qquad \begin{bmatrix} \bar{u}_1 \\ e^+ \end{bmatrix} \qquad \begin{bmatrix} \bar{u}_2 \\ \bar{d}_3 \end{bmatrix} \tag{A12}$$

Such a doublet assignment can occur in models in which fermions and antifermions are in separate representations, for example $SU(8)_L \times SU(8)_R$ models.

There are a few more possiblities since $\gamma$ can be chosen in such a way that two eigenvalues are degenerate. Then, as mentioned before, it is allowed to consider orthogonal linear combinations of the corresponding eigenstates. The possibilities are;

$\gamma = 0$; Degenerate eigenstates: $u_1$, $u_2$ and $e^+$, $\bar{d}_3$ ,

$\gamma = 1/2$; Degenerate eigenstates; $u_1$, $e^+$,

$\gamma = -1/2$; Degenerate eigenstates; $u_2$, $\bar{d}_3$.

Restricting ourselves to charge-eigenstates we find then the following additional possibilities;

$$
\begin{array}{lllll}
(a) & \begin{bmatrix} \bar{u}_2 \\ e^+ \end{bmatrix} & \begin{bmatrix} \bar{u}_1 \\ \bar{d}_3 \end{bmatrix} & \begin{bmatrix} e^- \\ u_1 \end{bmatrix} & \begin{bmatrix} d_3 \\ u_2 \end{bmatrix} \\
(b) & \begin{bmatrix} e^- \\ e^+ \end{bmatrix} & \begin{bmatrix} \bar{u}_2 \\ \bar{d}_3 \end{bmatrix} & \begin{bmatrix} \bar{u}_1 \\ u_1 \end{bmatrix} & \begin{bmatrix} d_3 \\ u_2 \end{bmatrix} \\
(c) & \begin{bmatrix} \bar{u}_1 \\ e^+ \end{bmatrix} & \begin{bmatrix} d_3 \\ \bar{d}_3 \end{bmatrix} & \begin{bmatrix} e^- \\ u_1 \end{bmatrix} & \begin{bmatrix} \bar{u}_2 \\ u_2 \end{bmatrix}
\end{array}
\tag{A13}
$$

Combinations obtained by interchange of the color indices 1 and 2 are ignored.

In this appendix, we have constructed the most general set of fermion multiplets for eg = 1/2 monopoles embedded in SU(16) or any of its subgroups,which do not give rise to proton decay. These multiplets are given in Eqs. (A12) and (A13). Of course this result applies only to the leading contributions discussed in Refs. 3 and 4. Other less important processes may still be present.

In the models which do not satisfy our criteria,the allowed baryon number violating processes are sometimes quite complicated and may involve all three generations of fermions. This can require flavor changing interactions like W-exchange or off-diagonal elements of mass matrices,which will lead to a suppression of the rate.

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14. We have ignored an eg=1/2 monopole which exists in SU(5) and SO(10). This is the monopole defined by the embedding 4(2) in the (10) of SO(10). (See Table 1.) We do not discuss this monopole embedding as we expect the stable monopole to be the one with the largest residual global symmetry (see Ref. 8).

15. Here we differ with Ref. 9 since we have required stability for small fluctuations.

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Table I. SU(2) embeddings in SO(10) which allow spherically symmetric monopoles with stability under fluctuations. The first column gives the SU(2)-representation which is embedded in the 10 of SO(10). Irreducible SU(2) representations are specified by their dimension. The second column gives the value of eg for the corresponding monopole. Monopoles which appear in the Georgi-Glashow SU(5) GUT are denoted by an asterisk; monopoles which appear in the Pati-Salam models are denoted by a +.

| SU(2) Representation Embedded in the 10 of SO(10) | eg |
|---|---|
| 2(2)* | 1/2 |
| 2(3)*+ | 1 |
| (7) | 3 |
| 2(2) + (3)+ | 1 |
| (3) + (5) | 2 |
| 4(2)* | 1/2 |
| 2(4)* | 3/2 |
| 2(2) + (5) | 2 |