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Global spacetime topology outside global k-monopole

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Abstract. We obtain gravitational field solutions far from the core ($f \approx 1$) of nonlinear global k-monopoles in an asymptotically dS/AdS spacetime. Specifically we consider two explicit examples, $K(X) = -X - \beta^{-2}X^2$ and $K(X) = -X/(1 + X^{1/3}/\beta^{2/3})$. Using different ansatz for metric, we obtain metric spacetime for conical topology or compactification.

1. Introduction

Monopole is a non-contractible point defect, $\pi_2(\mathcal{M}) \neq \mathbb{I}$ [1, 2]. Global monopole has divergent energy due to the absence of gauge field. When coupled to gravity, it exhibits a peculiar feature: it exerts no gravitational force on the surrounding matter, save from the tiny mass at the core [3]. However the global geometry is not Euclidean; the space around monopole suffers from deficit solid angle $\Delta \equiv 8\pi G \eta^2$. The existence of deficit angle renders the existence of a critical value of $\eta_{crit} \equiv 1/\sqrt{8\pi G}$, beyond which the deficit angle consumes the entire solid angle. It is suggested in [4] that the spacetime around critical global monopole may degenerate into a cylinder; the two angular dimensions compactify into a 2-sphere. Investigation for its higher-dimensional counterpart in [5, 6] shows instead that this cigar geometry can be realized when the staticity assumption is relaxed. Thus, compactification solution can be perceived as an inflating (non-static) super-critical solution written in some particular gauge. In $4d$, it is shown numerically in [7] that regular solutions can still exist up to $\eta \lesssim \sqrt{3/8\pi G}$. The singularity above that value is interpreted as the appearance of topological inflation [8, 9].

Global k-monopole has been studied in [10, 11], while its gravitational field is investigated in [12, 13]. There, the authors showed that qualitatively the gravitational property of Barriola-Vilenkin (BV) monopole still holds. The difference is that their mass can be negative or positive (which results in whether the gravitational field is repulsive or attractive), depending on the specific model of k-term considered. Despite their numerical results, the regime outside the monopole can be studied using the vacuum approximation, where the Higgs field is approximated as $|\phi| \approx \eta$. In this approximation the analytical solutions can be found. In [14] we showed gravitational solutions for exterior of several global k-monopoles. Here in this paper we report our finding and add some more discussion on other variation of k-monopole. We focus only on two types of k-monopole models discussed in Ref. [10].

2. Black hole solutions

In this section we use metric tensor

$$ds^2 = A(r)^2 dt^2 - B(r)^2 dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right), \tag{1}$$

whose components of Ricci tensor are

$$
\begin{aligned}
R_t^t &= \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB} + \frac{2A'}{rA}\right), \\
R_r^r &= \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB} - \frac{2B'}{rB}\right), \\
R_\theta^\theta &= \frac{1}{B^2}\left(\frac{1}{r^2} + \frac{A'}{rA} - \frac{B'}{rB}\right) - \frac{1}{r^2} = R_\phi^\phi,
\end{aligned}
\tag{2}
$$

with $A' \equiv dA/dr$ and $B' \equiv dB/dr$.

2.1. Gravitational field of the first type of global k-monopole

First we begin with the Action [13, 15, 16]

$$
\mathcal{S} = \int d^4x \sqrt{-g}\left(\frac{\mathcal{R} - 2\Lambda}{16\pi G} + K(X) - \frac{\lambda}{4}(\phi^a\phi^a - \eta^2)^2\right),
\tag{3}
$$

with scalar field $\phi^a = \eta\, f(r)\, x^a/r$, ($a = 1, 2, 3$ each denotes $x^1 = r\cos\theta$, $x^2 = r\sin\theta\cos\phi$, $x^3 = r\sin\theta\sin\theta$) also

$$
X \equiv -\frac{1}{2}\partial^\mu\phi^a\partial_\mu\phi^a = \frac{\eta^2}{2}\left(\frac{f'^2}{B^2} + \frac{2f^2}{r^2}\right),
\tag{4}
$$

with $f' \equiv df/dr$.

In this subsection

$$
K(X) \equiv -X - \frac{X^2}{\beta^2}.
\tag{5}
$$

with $\beta^2 > 0$ ($\lim_{\beta^2\to\infty} K(X) = -X$ thus the weak-field limit is achieved). The components of energy-momentum tensor becomes

$$
\begin{aligned}
T_t^t &= X + \frac{X^2}{\beta^2} + \frac{\lambda\eta^4}{4}(f^2 - 1)^2 + \frac{\Lambda}{8\pi G}, \\
T_r^r &= T_t^t - \left(1 + 2\frac{X}{\beta^2}\right)\frac{\eta^2 f'^2}{B^2}, \\
T_\theta^\theta &= T_t^t - \left(1 + 2\frac{X}{\beta^2}\right)\frac{\eta^2 f^2}{r^2} = T_\phi^\phi.
\end{aligned}
\tag{6}
$$

With exterior solution approximation (r far enough from the core making $f \approx 1$ and $f' \approx 0$) make the energy-momentum tensor components becomes

$$
\begin{aligned}
T_t^t = T_r^r &= \frac{\eta^2}{r^2} + \frac{\eta^4}{\beta^2 r^4} + \frac{\Lambda}{8\pi G}, \\
T_\theta^\theta = T_\phi^\phi &= -\frac{\eta^4}{\beta^2 r^4} + \frac{\Lambda}{8\pi G}.
\end{aligned}
\tag{7}
$$

Due to $T_t^t = T_r^r$ we get $B = A^{-1}$. From the Einstein equation component (tt) we have

$$
B^{-2} = 1 - \Delta - \frac{2GM}{r} + \frac{8\pi G\eta^4}{\beta^2 r^2} - \frac{\Lambda}{3}r^2,
\tag{8}
$$

which is valid if $r > \delta$. The core is given by the same method used in Ref. [13] obtaining (31) which will get us radius of the core

$$\delta = \frac{2}{\lambda\eta^2} + \sqrt{\left(\frac{2}{\lambda\eta^2}\right)^2 + \frac{4}{\beta^2\lambda}}, \tag{9}$$

the smaller β the thicker the monopole size.

This metric can be rescaled (an example can be seen in Ref. [17]) with

$$\begin{aligned} r &\rightarrow \frac{r}{(1-\Delta)^{1/2}}, \quad t \rightarrow (1-\Delta)^{1/2}t, \\ G &\rightarrow \frac{G}{(1-\Delta)}, \quad M \rightarrow \frac{M}{(1-\Delta)^{1/2}}, \quad \beta \rightarrow (1-\Delta)^{1/2}\beta, \end{aligned} \tag{10}$$

into a Reissner-Nördstrom-like metric

$$ds^2 = \left(1 - \frac{2GM}{r} + \frac{8\pi G\eta^4}{\beta^2 r^2} - \frac{\Lambda}{3}r^2\right)dt^2 - \frac{dr^2}{\left(1 - \frac{2GM}{r} + \frac{8\pi G\eta^4}{\beta^2 r^2} - \frac{\Lambda}{3}r^2\right)} - (1-\Delta)\,r^2 d\Omega_2^2, \tag{11}$$

Considering the case $\Lambda = 0$, the metric has two roots given by

$$r_\pm = GM\left[1 \pm \sqrt{1 - \frac{8\pi\eta^4}{M^2 G\beta^2}}\right], \tag{12}$$

Real roots exist only if

$$\beta^2 > \frac{8\pi\eta^4}{GM^2} \equiv \beta_{crit}^2, \tag{13}$$

is satisfied; if not, our solution suffers from naked singularity. When η is much smaller than the Plank mass, $\beta_{crit}^2 \rightarrow 0$. It also requires $\beta \gtrsim 2/(GM)^2$ to be satisfied for black hole condition

$$GM \gg \delta. \tag{14}$$

Black hole configuration will be produced in the strongly-coupled regime ($\beta^2 < 1$) when $M \gtrsim m_P$. When $\delta < r_-$, (12) becomes Reissner-Nördstrom-like horizons and this reduces to condition (13). Inner and outer horizons coalesce when $\eta = \left(M^2 G\beta^2/8\pi\right)^{1/4}$ and the black hole becomes extremal.

Considering $\Lambda \neq 0$, a cosmological horizon will exist. This can be seen by considering a case of almost-pure global monopole-de Sitter ($\Lambda \gg M^{-2}$) which reads

$$1 + \frac{8\pi G\eta^4}{\beta^2 r^2} - \frac{\Lambda}{3}r^2 \simeq 0, \tag{15}$$

whose roots are

$$r_\pm \simeq \sqrt{\frac{3}{2\Lambda}\left(1 \pm \sqrt{1 + \frac{32\pi G\eta^4\Lambda}{3\beta^2}}\right)}. \tag{16}$$

These roots make the inner horizon vanish while the cosmological one exist when $\Lambda > 0$, and make the cosmological horizon vanish while the inner one exist when $\Lambda < 0$.

2.2. Gravitational field of the second type of global k-monopole

Now, we consider

$$K(X) = -\frac{X}{1 + X^{1/3}/\beta^{2/3}}, \tag{17}$$

with $\beta > 0$. Components of the energy-momentum tensor are

$$
\begin{aligned}
T_t^t &= \left[\frac{X}{1 + X^{1/3}/\beta^{2/3}}\right] + \frac{\lambda\eta^4}{4}(f^2 - 1)^2 + \frac{\Lambda}{8\pi G}, \\
T_r^r &= T_t^t + \left[\frac{1 + (2/3)X^{1/3}/\beta^{2/3}}{(1 + X^{1/3}/\beta^{2/3})^2}\right]\frac{\eta^2 f'^2}{B^2}, \\
T_\theta^\theta &= T_t^t + \left[\frac{1 + (2/3)X^{1/3}/\beta^{2/3}}{(1 + X^{1/3}/\beta^{2/3})^2}\right]\frac{\eta^2 f^2}{r^2} = T_\phi^\phi.
\end{aligned} \tag{18}
$$

Now as before, we focus on exterior solution which makes

$$
\begin{aligned}
T_t^t = T_r^r &= \frac{\eta^2/r^2}{1 + (\eta/r)^{2/3}/\beta^{2/3}} + \frac{\Lambda}{8\pi G}, \\
T_\theta^\theta = T_\phi^\phi &= T_t^t - \left[\frac{-1 + (2/3)(\eta/r)^{2/3}/\beta^{2/3}}{(1 + (\eta/r)^{2/3}/\beta^{2/3})^2}\right]\frac{\eta^2}{r^2}.
\end{aligned} \tag{19}
$$

Because $T_t^t = T_r^r$, we again get $A = B^{-1}$. From the Einstein equation component (tt), we get

$$(rB^{-2})' = 1 - 8\pi G r^2 \left[\frac{\eta^2/r^2}{1 + (\eta/r)^{2/3}/\beta^{2/3}}\right] - \Lambda r^2. \tag{20}$$

Integrating it, we get

$$B^{-2} = 1 - 8\pi G\eta^2 - \frac{2GM}{r} - \frac{\Lambda}{3}r^2 + 24\pi G\eta^2 \frac{\eta}{\beta r}\left[\left(\frac{\beta r}{\eta}\right)^{1/3} - \arctan\left\{\left(\frac{\beta r}{\eta}\right)^{1/3}\right\}\right], \tag{21}$$

with M is a constant of integration.

Since our solution is valid only on the asymptotic regime, we can expand $\arctan(x^{1/3})$ around $r \to \infty$ to yield

$$\arctan(x^{1/3}) \approx \frac{\pi}{2} - \frac{1}{x^{1/3}} + \frac{1}{3x} - \frac{1}{5x^{5/3}} + \frac{1}{7x^{7/3}} - \cdots. \tag{22}$$

This results in

$$B^{-2} = 1 - \Delta - \frac{2G}{r}\left(M + \frac{6\pi^2\eta^3}{\beta}\right) + \frac{24G\pi\eta^{8/3}}{\beta^{2/3}r^{2/3}} - \frac{\Lambda}{3}r^2 + O\left(\frac{1}{r^{4/3}}\right). \tag{23}$$

The metric can be rescaled with (10) into

$$
\begin{aligned}
ds^2 &= \left[1 - \frac{2G}{r}\left(M + \frac{6\pi^2\eta^3}{\beta}\right) + \frac{24G\pi\eta^{8/3}}{\beta^{2/3}r^{2/3}} - \frac{\Lambda}{3}r^2\right]dt^2 \\
&\quad - \frac{dr^2}{\left[1 - \frac{2G}{r}\left(M + \frac{6\pi^2\eta^3}{\beta}\right) + \frac{24G\pi\eta^{8/3}}{\beta^{2/3}r^{2/3}} - \frac{\Lambda}{3}r^2\right]} - (1 - \Delta)r^2 d\Omega_2^2.
\end{aligned} \tag{24}
$$

We can see once again that our solution has a deficit in the solid angle given by Δ.

For the case of $\Lambda = 0$,

$$B^{-2} \approx 1 - \frac{2G}{r}\left(M + \frac{6\pi^2\eta^3}{\beta}\right) + \frac{24G\pi\eta^{8/3}}{\beta^{2/3}r^{2/3}}, \tag{25}$$

whose real root is

$$r = \frac{\sqrt[3]{a+b}}{3\sqrt[3]{2}\beta^3} - \frac{13824\sqrt[3]{2}\pi^3\beta\eta^8G^3}{\sqrt[3]{a+b}} + \frac{2\left(6\pi^2\beta^2\eta^3G + \beta^3GM\right)}{\beta^3}, \tag{26}$$

with

$$a \equiv -4478976\pi^5\beta^6\eta^{11}G^4 - 746496\pi^3\beta^7\eta^8G^4M, \tag{27}$$

$$b \equiv \sqrt{285315214344192\pi^9\beta^{12}\eta^{24}G^9 + \left(-4478976\pi^5\beta^6\eta^{11}G^4 - 746496\pi^3\beta^7\eta^8G^4M\right)^2}.$$

Positive r exists if $\eta > 0$ and $\beta \neq 0$.

For the case of $\Lambda \neq 0$, assuming $M^{-2} \ll \Lambda$, $-(2G/r)(6\pi^2\eta^3/\beta) + (24G\pi\eta^{8/3}/\beta^{2/3}r^{2/3}) \approx (24G\pi\eta^{8/3}/\beta^{2/3}r^{2/3})$ which is reasonable for $\eta < M_p$, and $1 - \Lambda r^2/3 \approx -\Lambda r^2/3$, we obtain

$$B^{-2} \approx \frac{24\pi\eta^{8/3}G}{\beta^{2/3}r^{2/3}} - \frac{\Lambda r^2}{3}. \tag{28}$$

Its real root is

$$r = \frac{2\sqrt[8]{2}3^{3/4}\pi^{3/8}\eta G^{3/8}}{\sqrt[4]{\beta}\Lambda^{3/8}}. \tag{29}$$

Positive root exists if $\Lambda > 0$.

B^{-2} is valid only in region $r > \delta$. Here δ can be calculated using the same method as in the previous section. A rough estimate tells us that the size δ can be given as a solution of the following equation:

$$-\frac{4\pi\eta}{(\beta\delta/\eta)^{2/3}}\left[1 + \left(\frac{\beta\delta}{\eta}\right)^{2/3} + \frac{1}{1 + (\beta\delta/\eta)^{2/3}}\right] + \delta^2\eta^4\lambda \approx 0. \tag{30}$$

When $\beta \to \infty$, $\delta = \sqrt{4\pi/(\lambda\eta^3)}$, and when $\beta \to 0$, $\delta = 2\left[2\pi/(\beta^2\eta^7\lambda^3)\right]^{1/8}$; the larger β the thinner the monopole size is, the same result as the previous section.

If the blackhole exists in this theory, then it will be Schwarzschild-like rather than Reissner-Nordstrom-like. By analyzing the condition $r > \delta$ we conclude that there are two cases for the existence of the black hole as follows.

(i) For $\Lambda > 0$, the horizon vanishes when $\beta \to \infty$, but exists when $\beta \to 0$ if $\eta > \sqrt[5]{\Lambda}/(3^{2/5}\pi^{2/15}\sqrt[5]{G}\sqrt[5]{\lambda})$.

(ii) For $\Lambda = 0$, the horizon exists both when $\beta \to \infty$, if $\eta > \sqrt[3]{\pi}/(G^{2/3}\sqrt[3]{\lambda}M^{2/3})$, and when $\beta \to 0$, due to the monopole size grows slower than the horizon ($r \propto \beta^{-1}$ will be bigger than $\delta \propto \beta^{-1/4}$ when $\beta < 1$).

3. Compactification solutions

In this section, we employ

$$ds^2 = A(r)^2dt^2 - B(r)^2dr^2 - C^2\left(d\theta^2 + \sin^2\theta \, d\phi^2\right), \tag{31}$$

with C a constant, whose components of Ricci tensors are

$$R_t^t = R_r^r = \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right),$$

$$R_\theta^\theta = R_\phi^\phi = -\frac{1}{C^2}, \tag{32}$$

3.1. Compactification by the first kind of power-law global monopole
Here we consider compactification with kinetic term corresponds to (5) whose components of energy-momentum tensor are

$$T_t^t = X + \frac{X^2}{\beta^2} + \frac{\lambda\eta^4}{4}(f^2 - 1)^2 + \frac{\Lambda}{\kappa},$$
$$T_r^r = T_t^t - \left(1 + 2\frac{X}{\beta^2}\right)\frac{\eta^2 f'^2}{B^2}, \tag{33}$$
$$T_\theta^\theta = T_t^t - \left(1 + 2\frac{X}{\beta^2}\right)\frac{\eta^2 f^2}{C^2} = T_\phi^\phi.$$

The exterior condition ($f \approx 0$) reduces them into

$$T_t^t = T_r^r \;\; = \;\; \frac{\eta^2}{C^2} + \frac{\beta^{-2}\eta^4}{C^4} + \frac{\Lambda}{\kappa},$$
$$T_\theta^\theta = T_\phi^\phi \;\; = \;\; -\frac{\beta^{-2}\eta^4}{C^4} + \frac{\Lambda}{\kappa}. \tag{34}$$

Considering $\Lambda = 0$ from the Einstein equation component (tt) we obtain

$$C^2 = \frac{\kappa\eta^4/\beta^2}{1 - \kappa\eta^2}, \tag{35}$$

which puts constraint $\eta > \sqrt{1/\kappa} \equiv \eta_{crit}$ for real C. On the other hand $\Lambda \neq 0$ will give us

$$C_\pm^2 = \frac{(1 - \kappa\eta^2) \pm \sqrt{(\kappa\eta^2 - 1)^2 - 4\Lambda\kappa\eta^4\beta^{-2}}}{2\Lambda}. \tag{36}$$

We choose C_+^2 which if $\Lambda < 0$ then $\eta > 0$ and if $\Lambda > 0$ then

$$\eta \leq \frac{1}{\sqrt{\kappa + 2\sqrt{\beta^{-2}\kappa|\Lambda|}}} \equiv \eta_2. \tag{37}$$

Before calculating A and B through the Einstein equation component ($\theta\theta$)

$$\frac{1}{B^2}\left[-\frac{A''}{A} + \frac{A'B'}{AB}\right] = \kappa T_\theta^\theta, \tag{38}$$

we define $\kappa T_\theta^\theta \equiv \pm\omega^2$, which is just a constant, the plus and minus sign corresponds to $T_\theta^\theta > 0$ and $T_\theta^\theta < 0$ respectively, by keeping ω real. We also have a freedom in the metric (31) which we can fix by choosing ansatz for B.

The first ansatz we choose is $B = 1$. When $T_\theta^\theta = 0$ we get

$$ds^2 = (C_1 r + C_2)^2 dt^2 - dr^2 - C^2 d\Omega_2^2. \tag{39}$$

This solution has no singularity at $r = 0$ and $r = -C_2/C_1$ thus securing from naked singularity and both C_1 and C_2 can be set into arbitrary value, which allows us to set $C_1 = 0$ and rescale $\tilde{t} \equiv C_2 t$ that gives $ds^2 = d\tilde{t}^2 - dr^2 - C^2 d\Omega_2^2$. When $T_\theta^\theta \neq 0$, this gives us

$$ds^2 = \begin{cases} \frac{1}{\omega^2}(\sin^2\chi\, dt^2 - d\chi^2) - C^2 d\Omega_2^2, & \text{if } T_\theta^\theta > 0, \\ \frac{1}{\omega^2}(\sinh^2\chi\, dt^2 - d\chi^2) - C^2 d\Omega_2^2, & \text{if } T_\theta^\theta < 0, \end{cases} \tag{40}$$

Table 1. Conditions for the existence of T_θ^θ as a function of η in first kind of power-law monopole compactification.

	$T_\theta^\theta > 0$	$T_\theta^\theta = 0$	$T_\theta^\theta < 0$
$\Lambda > 0$	$\eta < \eta_2$	$\eta = \eta_2$	$\eta > \eta_2$
$\Lambda = 0$	does not exist	$\eta = 1/\kappa^{1/4}$	$\eta \neq 1/\kappa^{1/4}$
$\Lambda < 0$	does not exist	does not exist	$\eta > 0$

by defining $\chi \equiv \omega r$. These solutions have no singularity at $r = n\pi/\omega$ with $n = 0, 1, 2, ...$ thus there exist no naked singularities in these solutions. For $T_\theta^\theta = 0$ the solution is the Plebański-Hacyan metric ($M_2 \times S^2$) [18], while for $T_\theta^\theta \neq 0$ the solutions are Nariai ($dS_2 \times S^2$) [19] and Bertotti-Robinson ($AdS_2 \times S^2$) [20, 21] metrices.

Our second ansatz is $B = A^{-1}$. This gives us $A^2 = C_2 + C_1 r - k\omega^2 r^2$ with C_1 and C_2 constants of integration and $k = 1, 0, -1$ for $T_\theta^\theta > 0$, $T_\theta^\theta = 0$, and $T_\theta^\theta < 0$ respectively. These also free from naked singularity thus we can set $C_1 = 0$ and $C_2 = 1$,

$$ds^2 = \begin{cases} (1 - \omega^2 r^2)dt^2 - \frac{dr^2}{(1 - \omega^2 r^2)} - C^2 d\Omega_2^2, & \text{if } T_\theta^\theta > 0, \\ dt^2 - dr^2 - C^2 d\Omega_2^2, & \text{if } T_\theta^\theta = 0, \\ (1 + \omega^2 r^2)dt^2 - \frac{dr^2}{(1 + \omega^2 r^2)} - C^2 d\Omega_2^2, & \text{if } T_\theta^\theta < 0, \end{cases} \tag{41}$$

which are also the same results as the previous ansatz.

Now we continue to find the conditions for symmetry-breaking scale. We can solve the polynomial equations to obtain the range of η that allows the existence of compactification solutions by substituting the radius solutions (35) and (36) into the T_θ^θ. The results are shown in table 1. Combining with the conditions for $C^2 > 0$ to happen, we can see which compactification channels are theoretically possible. We can list the possible compactification channels in this theory as follows

$$dS_4 \longrightarrow \begin{cases} dS_2 \times S^2, \\ M_2 \times S^2, \end{cases} \tag{42}$$

$$M_4 \longrightarrow \begin{cases} M_2 \times S^2, \\ AdS_2 \times S^2, \end{cases} \tag{43}$$

$$AdS_4 \longrightarrow AdS_2 \times S^2. \tag{44}$$

The flat super-critical global monopole is possible to compactify the spacetime into an $M_2 \times S^2$ but since this is a static spacetime, we conjecture that this channel is unstable [14].

3.2. Compactification by the second kind of power-law global monopole
In general $K(X)$, the Einstein equations are

$$\frac{1}{C^2} = -\kappa K(X(f \approx 1)) + \Lambda, \tag{45}$$

$$\frac{1}{B^2}\left(\frac{A'B'}{AB} - \frac{A''}{A}\right) = \frac{1}{C^2}\left(1 - \kappa\eta^2\frac{dK}{dX}(f \approx 1)\right) \tag{46}$$

Table 2. Conditions for the existence of T_θ^θ as a function of η in the second kind of power-law monopole compactification.

	$T_\theta^\theta > 0$	$T_\theta^\theta = 0$	$T_\theta^\theta < 0$
$\Lambda > 0$	$\eta < \eta_3$	does not exist	$\eta > \eta_3$
$\Lambda = 0$	$\eta > \eta_{crit}$	$\eta = 0$ or $\eta = \eta_{crit}$	$\eta < \eta_{crit}$
$\Lambda < 0$	$\eta > \eta_3$	does not exist	$\eta < \eta_3$

with $X(f \approx 1) = \eta^2/C^2$. The right hand side of (46) is just a constant, thus the metric solutions for compactification of this flux is the same as the previous sections. Now we focus on C^2. For this section, $K(X) = -X/\left[1 + (X/\beta^2)^{1/3}\right]$ thus

$$K(X(f \approx 1)) = -\frac{(\eta/C)^2}{(1 + (\eta/C\beta)^{2/3})}, \tag{47}$$

$$\frac{dK}{dX}(f \approx 1) = -\left[\frac{1 + (2/3)(\eta/C\beta)^{2/3}}{(1 + (\eta/C\beta)^{2/3})^2}\right]. \tag{48}$$

The Einstein equations becomes

$$\frac{1}{C^2} = \left[\frac{\kappa(\eta/C)^2}{(1 + (\eta/C\beta)^{2/3})} + \Lambda\right], \tag{49}$$

$$\frac{1}{B^2}\left(\frac{A'B'}{AB} - \frac{A''}{A}\right) = \frac{1}{C^2}\left(1 + \kappa\eta^2\left[\frac{1 + (2/3)(\eta/C\beta)^{2/3}}{(1 + (\eta/C\beta)^{2/3})^2}\right]\right). \tag{50}$$

For now we focused on $\Lambda = 0$ which gives us

$$C^2 = \frac{\beta^2/\eta^2}{(\kappa\eta^2 - 1)^3}, \tag{51}$$

thus $\eta > \sqrt{1/\kappa} \equiv \eta_{crit}$. To find conditions for η at $\Lambda \neq 0$, we can consider the cosmological constant, which is

$$\Lambda = \frac{(\eta/C\beta)^{2/3} - (\kappa\eta^2 - 1)}{C^2(1 + (\eta/\beta C)^{2/3})}, \tag{52}$$

which, while assuming $C^2 > 0$, this will give results as follows. If $\Lambda > 0$ then $\eta < \eta_3$, and if $\Lambda < 0$ then $\eta > \eta_3$, with

$$\eta_3 \equiv \sqrt{\frac{1}{\kappa} + \frac{\sqrt[3]{\frac{2}{3}}\beta^2}{\sqrt[3]{9\beta^2 C^4\kappa^5 + \sqrt{3}\sqrt{27\beta^4 C^8\kappa^{10} - 4\beta^6 C^6\kappa^9}}} + \frac{\sqrt[3]{9\beta^2 C^4\kappa^5 + \sqrt{3}\sqrt{27\beta^4 C^8\kappa^{10} - 4\beta^6 C^6\kappa^9}}}{\sqrt[3]{2}3^{2/3}C^2\kappa^3}}. \tag{53}$$

The metric solutions are the same as the previous section and we can focus on the conditions of the symmetry breaking scale. The results are displayed in table 2. Combining with the conditions for C to be a real number, we get the possible solutions

$$\left.\begin{array}{c} dS_4 \\ M_4 \\ AdS_4 \end{array}\right\} \longrightarrow dS_2 \times S^2. \qquad (54)$$

From these results both from the first kind and the second kind of global k-monopole, solutions for $\mathcal{Y}_4 \to \mathcal{Z}_2 \times S^2$ (each $\mathcal{Y}$ and $\mathcal{Z}$ can be either de Sitter, Minkowski, or Anti-de Sitter spaces), which is a spontaneous compactification of $4d$ global k-monopole in to 2-dimensional spaces of constant curvatures, are produced. As suggested in [22], these are the four-dimensional analogue of the flux compactification that had been discussed, for example, in [23, 24]. The $4d$ space has a cosmological constant Λ when the $2d$ space gains the effective corresponding constant given by ω.

4. Conclusions

In this review, we presented analytical solutions of global k-monopoles which include cosmological constant for various spacetime topology. We consider two types of k-monopole with power-law types specifically. It is proposed in [10] that these kind of defects might have been formed in the very-high-energy regime in the early universe, close to the superstring scale.

For the static case, our results are both analytic and asymptotically dS/AdS versions of the otherwise numerical solutions studied in [12, 13]. They all suffer from the deficit angle $\Delta = 8\pi G\eta^2$, which does not depend on β. We then continue to study the resulting event and the cosmological horizons that are formed when a black hole swallows these global k-monopoles. They happen when $M \gtrsim m_P$.

In the case of the first kind of power-law monopole, we found that both roots are real. The monopole behaves like a Reissner-Nördstrom black hole with scalar charge. When $\Lambda \neq 0$ the cosmological horizon appears. In the almost-purely-de Sitter case, the inner horizon does not exist when $\Lambda > 0$ so the inside the cosmological horizon is exposed to naked singularity. For $\Lambda < 0$, the horizon becomes Schwarzschild-like since the cosmological horizon disappears.

For the second kind of power-law monopole, we obtain the analytic solution and expand it asymptotically to in order to better study its horizon by finding its roots. We found one real root. Thus, unlike the first kind, this monopole behaves like a Schwarzchild black hole due to having only one horizon. The horizon only appears when $\Lambda \geq 0$.

In flat spacetime, the super-critical global monopole will develop singularity and thus no static solution exists [5, 6, 7]. To cure this, the monopole is allowed to inflate [5, 6]. Our investigation shows that super-critical of both the first and second kind of global k-monopoles are able to compactify its surrounding spacetime into a product of two two-dimensional maximally symmetric spaces. This is an example of lower-dimensional spontaneous compactification, as discussed in [22, 25] for example, where in here the 2-sphere is threaded against collapsing by the flux coming from the scalar field. This singularity-free spacetime is interpreted as non-static solutions.

In the first kind of power-law monopole, we can have $M_4 \to M_2 \times S^2$ (Plebański-Hacyan compactification) and $M_4 \to AdS_2 \times S^2$ (Bertotti-Robinson compactification). Since no static solution can exist for super-critical monopole, we conjecture that the Plebański-Hacyan solution is unstable. For $\Lambda \neq 0$, we can have a set of possible compactification channels as shown in table 1. Here all solutions are all subject to the vacuum approximation ($f \approx 1$) which clearly is not an exact condition. It needs to be clarified by numerical analysis whether such compactification solutions really exist. For the second kind of power law monopole we only have Nariai compactification ($dS_2 \times S^2$) from all dS_4, M_4, and AdS_4. This confirms the appearance of topological inflation. The requirements are shown in table 2.

Acknowledgments

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