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RESEARCH ARTICLE

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Time evolution of $T_{\mu\nu}$ and the cosmological constant problem

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Abstract We study the cosmic time evolution of an effective quantum field theory energy-momentum tensor $T_{\mu\nu}$ and show that, as a consequence of the effective nature of the theory, $T_{\mu\nu}$ is such that the vacuum energy decreases with time. We find that the zero point energy at present time is washed out by the cosmological evolution. The implications of this finding for the cosmological constant problem are investigated.

Keywords Effective quantum field theories, Vacuum energy, Cosmological constant problem

1 Introduction

A generic feature of systems with an infinite (very large) number of degrees of freedom is that fluctuations at arbitrarily close points are independent. When computing physical quantities, this results in the appearance of divergent terms. This is the case of quantum field theories, where such terms are generated as soon as the quantum fluctuations are taken into account. In particular, the calculation of zero point energies leads to divergences whose leading term, when using an ultraviolet (UV) momentum cutoff Λ, goes as Λ^4. According to standard analysis, these terms contribute to the cosmological constant.

One sometimes takes the point of view that the divergences have no physical meaning and that the definition of the theory has to be completed by some appropriate renormalisation procedure that allows to remove them. In this perspective, the regularization is just a mathematical step in the calculation of observable quantities.

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From a deeper physical point of view, however, it is more satisfactory to consider a quantum field theory as an effective theory valid up to a certain scale Λ, which takes the meaning of "scale of new physics", and consider a hierarchy of theories each having higher and higher energy range of validity [1]. This hierarchical structure is usually believed to end at the Planck scale M_P where a different theory, most probably string theory, is supposed to replace ordinary quantum field theories and should account for the unification of gravity with the other interactions. This naturally leads to the idea that, whatever theory describes physics before the Planck time t_P, for $t > t_P$ physics is appropriately described by one or a small number of effective quantum field theories with physical cutoff $\Lambda = M_P$.

Going back to the problem of divergences, we note that, for the zero point energies, the physical meaning of the divergences is deeply rooted in the underlying harmonic oscillator structure of a quantum field theory; this is automatically lost if we cancel out those terms with the help of a formal procedure such as normal ordering [2].

Another important ingredient in the formulation of a relativistic quantum field theory is the selection of the ground state, which is done by referring to the Lorentz symmetry. According to [3], a Lorentz invariant vacuum $|0>$ is characterised by the requirement that $\hat{P}_\mu|0>=0$, where $\hat{P}_\mu$ is the field four-momentum operator. As clearly explained in [4] and [5], however, this statement is too restrictive. This is easily seen if we consider the energy-momentum tensor of a perfect fluid: $T_{\mu\nu} = (\rho+p)\,u_\mu u_\nu - \rho g_{\mu\nu}$ (where u_μ is the fluid four-velocity, p the pressure and ρ the energy density). In order to have a Lorentz invariant vacuum, all we need is the vacuum expectation value of the energy-momentum tensor operator $\hat{T}_{\mu\nu}$ to be of the form:

$$< 0|\hat{T}_{\mu\nu}|0 >= -\rho g_{\mu\nu}. \qquad (1)$$

Equation (1) contains $\hat{P}_\mu|0>=0$ as a special case. However, it is more general and allows for the presence of vacuum condensates.

On the cosmological side (for some reviews on the cosmological constant problem see [5; 6; 7; 8]), the importance of the quantum field theoretic contribution to the energy momentum tensor that appears in the Einstein equations was firstly recognised in [9] and [4]. In accordance with the idea that the divergences are unphysical and have to be discarded, the divergent terms which do not respect the constraint imposed by Eq. (1) were removed with the help of a renormalization procedure (more precisely, Pauli-Villars regulators were used in [4]). Such a formal approach is thoroughly analysed and criticised in [2]. Still, a popular prescription (often used nowadays) for the automatic (yet formal) cancellation of these divergences is the dimensional regularization scheme. In this respect, see [10] (and also [11] and [12]).

In the present work, we would like to pursue a different point of view. We consider an effective field theory at a fixed time t with momentum cut-off Λ, where t and Λ, as discussed below, are taken to coincide respectively with the Planck time t_P and the Planck scale M_P, and then compute the thermal average $\ll \hat{T}_{\mu\nu} \gg$ of the energy momentum tensor operator. $\ll \hat{T}_{\mu\nu} \gg$ contains two additive contributions:

$$\ll \hat{T}_{\mu\nu} \gg = T^m_{\mu\nu} + T^v_{\mu\nu}, \tag{2}$$

where $T^v_{\mu\nu}$ is the vacuum expectation value of $\hat{T}_{\mu\nu}$ (the superscript v stands for "vacuum"), while $T^m_{\mu\nu}$ corresponds to the equilibrium thermal average of the field excitations above the vacuum at temperature T (the superscript m stands for "matter"). For weakly interacting fields, $T^m_{\mu\nu}$ can be regarded as the thermal average of the energy momentum tensor of a gas of non interacting particles. At $T = 0$, one clearly has $T^m_{\mu\nu} = 0$.

The vacuum contribution $T^v_{\mu\nu}$ is of special interest for our analysis. In fact, due to the well known form of the Planck (or Fermi-Dirac) distribution, $T^m_{\mu\nu}$ is finite and does not contain any reference to the physical cut-off M_P. On the contrary, $T^v_{\mu\nu}$ contains terms proportional to $M_P^4, m^2 M_P^2$ and $m^4 \ln M_P$, where m is the particle mass [see Eqs. (10) and (11) below].

According to our effective field theory point of view, in the r.h.s. of the Einstein equation,

$$G_{\mu\nu} - \lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{3}$$

we consider for $T_{\mu\nu}$ the full contribution coming from Eq. (2), i.e. we take

$$T_{\mu\nu} \equiv \ll \hat{T}_{\mu\nu} \gg = T^m_{\mu\nu} + T^v_{\mu\nu}, \tag{4}$$

without discarding any of the terms that appear in this equation. Finally, starting at the Planck time $t = t_P$, we follow the cosmic evolution of $\ll \hat{T}_{\mu\nu} \gg$, in particular of $T^v_{\mu\nu}$, with the help of the corresponding Friedman equations.

Let us call ρ^v the vacuum energy density and p^v the vacuum pressure. Had we considered a renormalization scheme such as dimensional or Pauli-Villars regularization, the coefficient w in the equation of state (EOS) $p^v = w\rho^v$ (after discarding the divergent terms) would have been $w = -1$ [4; 10; 11; 12]. Accordingly, ρ^v would not evolve with time and could be interpreted as the vacuum energy contribution to the cosmological constant. This is the standard view.

Conversely, within our effective field theory approach, where we keep the large but finite terms proportional to M_P^4 and M_P^2, we get a different EOS for p^v and ρ^v, namely $w = 1/3$, as for the relativistic matter case. The immediate implication of this result is that the zero point energy density of a quantum field red-shifts with time. Another interesting approach which also leads to a red-shifting vacuum energy density can be found in [13].

The most natural framework to study the evolution of the effective quantum field is the Friedmann–Robertson–Walker (FRW) metric rather than the flat Minkowski space–time. However, this would introduce technical complications due to the formulation of quantum field theories on a non-flat space–time. Since the essential results we are interested in can be recovered in both approaches, we adopt the latter description for the sake of simplicity. In the last part of Sect. 3 we shall briefly come back to the FRW metric to show more in detail that our results hold even in this case.

Within the framework of our effective field theory approach, a crucial question to pose is that of setting an initial time at which the effective theory begins to provide a reliable description of the physical phenomena. A natural choice for that is the Planck time t_P as it indicates the typical scale at which the gravitational strength is comparable with the other forces, while at earlier times a non-field theoretical description of physics is presumably required. At later times (and lower energy scales), instead, gravity is weaker and it is generally believed that quantum field theory is a reliable tool to describe the relevant interactions.

The above considerations are at the basis of our fundamental assumption: we introduce one (or few) fundamental field (fields) at t_P with an UV cut-off at M_P and investigate the fate of its (their) zero point energy density. Clearly, this general picture could be more specifically implemented by taking particular models. However, as our aim is that of illustrating our general ideas, we limit ourselves to the simplest set-up.

In this respect, it is also very important to note that, in the spirit of effective fields, the effective theories that become relevant below some particular scale, say $\Lambda \ll M_P$, do not provide any novel contribution to the zero point energy density because their degrees of freedom (dof) have to be regarded as effective dof, derived from those pertaining to the more fundamental theories valid above Λ. The inclusion of zero point energies of the low energy effective fields would produce an erroneous result due to a multiple counting of dof.

One last very important point, is that in the higher order computations of any physical quantity within a specific effective theory [Standard Model (SM), etc.], the UV cut-off to be used to regularize the loop divergences is the momentum scale Λ which is the UV limit of validity of the effective theory considered. In other words, the UV cut-off scale of any low energy theory should not be confused with the red-shifted energy density of the fundamental field which is decreasing with the cosmic time. We shall come back to this point at the end of Sect. 3.

The outline of the paper is the following. Section 2 is devoted to the computation of the pressure and density of an effective field theory in order to determine its EOS, while the cosmic time evolution of the density will be analyzed in Sect. 3. Some considerations on the zero point energy of effective field theories are presented in Sect. 4. The conclusions are contained in Sect. 5.

2 Effective field energy-momentum tensor

Let us begin by considering a free real single component scalar field theory. The energy-momentum operator is:

$$\hat{T}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\mathscr{L} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2\right), \qquad (5)$$

and $\mathscr{L}$ is the corresponding Lagrangian density.

After considering the standard Fourier decomposition of ϕ in creation and annihilation operators $a^\dagger_{\vec{k}}$ and $a_{\vec{k}}$, the energy-momentum tensor $T_{\mu\nu}$ of Eq. (4) (i.e. the energy-momentum tensor that appears in the r.h.s. of the Einstein equation (3)) is obtained by taking the thermal average of (5) for a statistical equilibrium distribution at temperature T. The non-diagonal terms vanish, while the diagonal ones take the form:

$$T_{00} = \ll \hat{T}_{00} \gg = \frac{1}{V}\sum_{\vec{k}}\sum_{n} < n|\rho_T|n > n_{\vec{k}}\,\omega_{\vec{k}} + \frac{1}{V}\sum_{\vec{k}}\frac{\omega_{\vec{k}}}{2} \tag{6}$$

$$T_{ii} = \ll \hat{T}_{ii} \gg = \frac{1}{V}\sum_{\vec{k}}\sum_{n} < n|\rho_T|n > n_{\vec{k}}\,\frac{(k^i)^2}{\omega_{\vec{k}}} + \frac{1}{V}\sum_{\vec{k}}\frac{(k^i)^2}{2\omega_{\vec{k}}}, \tag{7}$$

where $\ll \cdots \gg$ indicates the quantum-statistical average, $|n>$ is a compact notation for the generic element of the Fock space basis, ρ_T is the density operator at temperature T, $n_{\vec{k}} = < n|a^{\dagger}_{\vec{k}}a_{\vec{k}}|n >, \omega_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$ and V is the quantization volume. By performing the sum over n in Eqs. (6) and (7), we get the matter and the vacuum contributions to the energy density $\rho = \ll \hat{T}_{00} \gg$ and pressure $p = \ll \hat{T}_{ii} \gg$ (due to rotational invariance, $\ll \hat{T}_{11} \gg = \ll \hat{T}_{22} \gg = \ll \hat{T}_{33} \gg$):

$$\rho = \frac{1}{V}\sum_{\vec{k}} n_{BE}\,\omega_{\vec{k}} + \frac{1}{V}\sum_{\vec{k}}\frac{\omega_{\vec{k}}}{2} \equiv \rho^m + \rho^v \tag{8}$$

$$p = \frac{1}{3V}\sum_{\vec{k}} n_{BE}\,\frac{\vec{k}^2}{\omega_{\vec{k}}} + \frac{1}{3V}\sum_{\vec{k}}\frac{\vec{k}^2}{2\omega_{\vec{k}}} \equiv p^m + p^v, \tag{9}$$

where $n_{BE} = n_{BE}(\vec{k}^2, T)$ is the Bose–Einstein distribution at temperature T. Again, the superscripts "m" and "v" are for "*matter*" and "*vacuum*", respectively.

The first terms in the r.h.s. of Eqs. (8) and (9), ρ^m and p^m, come from the thermal average of the number operators $a^{\dagger}_{\vec{k}}a_{\vec{k}}$ and are the matter contribution to $T_{\mu\nu}$. It is worth to note that this is the only contribution usually considered in Eq. (3): the energy momentum tensor of the relativistic gas of particles. On the other hand, ρ^v and p^v come from the thermal average of the commutators $[a^{\dagger}_{\vec{k}}, a_{\vec{k}}]$, i.e. from c-numbers, and coincide with the vacuum expectation values of the components of $\hat{T}_{\mu\nu}$. Note also that ρ^v is nothing but the term which is usually recognised as the zero point energy contribution to the cosmological constant. Equations (8) and (9) provide an explicit example of the general relation shown in Eq. (2).

This elementary computation shows that the matter and the vacuum contributions to $T_{\mu\nu}$ do not come as separate entities. They are the result of a unique operation, namely the thermal average of the operator $\hat{T}_{\mu\nu}$ with respect to the Bose–Einstein distribution. Both ρ and p contain on the same footing contributions from the matter and from the vacuum content of the theory. However, while the first terms in the r.h.s. of Eqs. (8) and (9) are convergent (due to the cutoff role played by the Bose–Einstein distribution), the second ones, i.e. the vacuum contributions, diverge. By explicitly performing the computation with the help of an UV momentum cutoff we get:

$$\rho^v = \frac{1}{16\pi^2}\left[\Lambda(\Lambda^2+m^2)^{\frac{3}{2}} - \frac{\Lambda m^2(\Lambda^2+m^2)^{\frac{1}{2}}}{2} - \frac{m^4}{4}\ln\left(\frac{(\Lambda+(\Lambda^2+m^2)^{\frac{1}{2}})^2}{m^2}\right)\right], \tag{10}$$

$$p^v = \frac{1}{16\pi^2}\left[\frac{\Lambda^3(\Lambda^2+m^2)^{\frac{1}{2}}}{3} - \frac{\Lambda m^2(\Lambda^2+m^2)^{\frac{1}{2}}}{2} + \frac{m^4}{4}\ln\left(\frac{(\Lambda+(\Lambda^2+m^2)^{\frac{1}{2}})^2}{m^2}\right)\right]. \tag{11}$$

As we have said before, we are considering the theory defined at the Planck time t_P with the momentum cutoff taken at the Planck scale, i.e. $\Lambda = M_p \gg m$. The ratio between p^v and ρ^v is then essentially $1/3$:

$$p^v \sim \frac{\rho^v}{3}. \tag{12}$$

Moreover, when the matter content is relativistic, this is also the ratio between p^m and ρ^m and the EOS for the field ϕ is:

$$p = p^v + p^m \sim \frac{\rho^v+\rho^m}{3} = \frac{\rho}{3}. \tag{13}$$

These results are totally different from the usual ones, where for the vacuum component one has $p^v = -\rho^v$, i.e. a value of w which is different from the matter one. As we have already noted, if we manage to get rid of the quartic and quadratic divergences with the help of some formal regularization procedure, the remaining terms in p^v and ρ^v would obey the usual vacuum equation of state with $w = -1$. We also note that, as in Eq. (13) w turns out to be $\sim 1/3$, the above finding does not change the well known scaling of ρ^m.

So far we have considered the simple example of a free theory (see Eq. (5)). However, these same steps can be repeated for any, even interacting, field theory. Of course the presence of interaction terms such as $g\phi^4$ induces corrections to the Lagrangian parameters. In the case of mass, for instance, these corrections are proportional to $g\Lambda^2 + O(g^2)$. As long as g is perturbative, we expect these terms not to spoil the above analysis.

3 Time evolution of the vacuum energy density

Up to now we have considered the theory at the Planck time. From now on we consider the time evolution and, for the sake of simplicity, in the following we shall consider $m^2 = 0$. As from Eqs. (10) and (11) we see that $w = 1/3$, we have for $\rho = \rho^v + \rho^m$ the well known time evolution of relativistic matter, which is governed by the continuity and Friedman equations:

$$\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)(\rho + p) = 0 \tag{14}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho, \tag{15}$$

where $a(t)$ is the cosmic scale factor (consistently with the present observations, we have considered a flat space, $k = 0$). Note also that in Eq. (15) we have neglected the "classical" (i.e. not originated from quantum vacuum fluctuations) λ term in the Einstein equation (3). As is well known, the solution of Eq. (14) is:

$$\rho(t) \propto a(t)^{-4}. \tag{16}$$

Although Eq. (14) and the corresponding solution (16) are obtained for $\rho = \rho^v + \rho^m$, we expect them to hold also for ρ^v and ρ^m separately. In fact, when no matter is present, ρ reduces to ρ^v so that Eq. (14) is valid for ρ^v alone. Then, if no substantial change in the behaviour of ρ^v is induced by the presence of matter, ρ^m satisfies Eq. (14) too. Such a time evolution of ρ^m is nothing but the well known evolution of relativistic matter: in the usual treatment, it is obtained by neglecting ρ^v in the continuity equation (14).

At early cosmological times (and therefore at high temperatures T) one has $T \gg m$ (we have taken the Boltzmann constant $k_B = 1$) and this corresponds to the radiation, i.e. relativistic matter, dominated era:

$$\rho^m(t) = \frac{\pi^2}{30}T^4 \propto a^{-4}. \tag{17}$$

As we noticed above, as long as matter is relativistic, ρ^m and ρ^v have the same scaling ($\rho^{m,v} \propto a^{-4}$) so that we can write

$$\rho^v(t) = \frac{\rho^v(t_P)}{\rho^m(t_P)}\rho^m(t), \tag{18}$$

where we have chosen as initial time $t = t_P$, with $t_P = (M_P)^{-1}$, the Planck time. Moreover, from Eq. (17) we have that $\dot{a}/a = -\dot{T}/T$ and Eq. (15) can be written as:

$$\left(\frac{\dot{T}}{T}\right)^2 = \frac{8\pi G}{3}\left(1 + \frac{\rho^v(t_P)}{\rho^m(t_P)}\right)\rho^m(t) = \frac{4\pi^3 G}{45}\left(1 + \frac{\rho^v(t_P)}{\rho^m(t_P)}\right)T^4, \tag{19}$$

By integrating the above equation we get:

$$T = \left(\frac{45}{16\pi^3 K G}\right)^{\frac{1}{4}} t^{-\frac{1}{2}}, \tag{20}$$

with $K = 1 + \rho^v(t_P)/\rho^m(t_P)$. Note that in the standard approach, where $\rho^v(t)$ is not taken into account in the Friedman equation (15), $K = 1$.

Let us consider now the theory defined at the Planck time, t_P. If the cutoff is taken to be at the Planck scale, $\Lambda = M_P = 1.22\times 10^{19}GeV$, the leading contribution to the vacuum energy density at t_P is:

$$\rho^v(t_P) = \frac{M_P^4}{16\pi^2}. \tag{21}$$

From Eqs. (17) and (20) we then find:

$$\frac{\rho^m(t_P)}{\rho^v(t_P)} = \frac{3\pi}{2} - 1 \sim 3.71, \tag{22}$$

where we have used $G = M_P^{-2} = t_P^2$. In passing, we note that from Eq. (22) we have that $K \sim 1.27$. When this value of K is inserted in Eq. (20), we get a slight correction to the result obtained in the standard approach, where $K = 1$.

The relevance of the result contained in Eq. (22), however, lies elsewhere. In fact, Eq. (18) predicts that, as long as matter is relativistic, the ratio $\rho^m(t)/\rho^v(t)$ is constant and given by Eq. (22). In particular, if we consider a massless field which is relativistic at any time, this ratio keeps such a value up to the present time t_0. Therefore, $\rho^m(t_0)$ is about four times $\rho^v(t_0)$. As the background photon density $\rho_\gamma(t)$ follows precisely this scaling, we find that: $\rho_\gamma(t_0) \sim 4\rho^v(t_0)$. Therefore, since we know that at present time $t = t_0$ the contribution of $\rho_\gamma(t_0)$ to the total energy density is negligible, the same must hold true for $\rho^v(t_0)$.

Few comments are in order. As we have already said in the Introduction, our fundamental assumption is to consider an effective field theory defined at $t = t_P$ with UV cut-off $\Lambda = M_P$. This is why we have Eq. (21), i.e. $\rho^v(t_P) \sim M_P^4$, which in turn gives $\rho^m(t)/\rho^v(t) \sim 4$. It is possible that, within other frameworks and with other assumptions, one could get a different boundary value $\overline{\rho}^v(t_P)$, although it would be rather questionable to start with an energy density larger than our value in Eq. (21), i.e. $\overline{\rho}^v(t_P) \gg M_P^4$. Then, considering a boundary $\overline{\rho}^v(t_P) < M_P^4$ would simply have the effect of magnifying the washing out of the vacuum energy density according to the mechanism discussed above. In other words, the choice $\rho^v(t_P) \sim M_P^4$ in Eq. (21) gives the highest possible value of $\rho^v(t_0)$ at present time t_0 which, as we have seen above, is $\rho^v(t_0) \sim 0.25\rho_\gamma(t_0)$.[1]

As T decreases, matter evolves towards the non-relativistic regime (opposite limit, $T \ll m$) where $\rho^m \propto a^{-3}(t)$, while ρ^v continues to follow its previous scaling, $\rho^v \propto a^{-4}(t)$. During this epoch, the expansion of the universe, i.e. its scale factor $a(t)$, is controlled by non-relativistic matter so that, starting from $t = t_{\rm eq}$, when $\rho_{\rm rel}(t_{\rm eq}) = \rho_{\rm nrel}(t_{\rm eq})$, the scaling of ρ^v with t changes.

It is not difficult to estimate the value of ρ^v at the present time t_0. The computation goes as follows. By integrating Eq. (14) for ρ^v from t_P down to $t_{\rm eq}$, i.e.

[1] These considerations can be rephrased as follows. If (see the Introduction) we compute $\rho^v(t)$ by considering our field theory in a FRW background, for $\rho^v(t)$ we get $\rho^v(t) \sim \Lambda^4 a(t)^{-4} \sim M_P^4 a(t)^{-4}$. As $\rho^m(t) \sim T(t)^4$ and $a(t) \propto T(t)^{-1}$, which means that $a(t)T(t)$ defines a constant temperature, $a(t)T(t) = const. = T_1$, we immediately have $\rho^v(t_P) \sim (T_P/T_1)^4 M_P^4$. With our choice of the boundary $\rho^v(t_P) \sim M_P^4$ one has $T_1 = T_P$, while other choices of $\rho^v(t_P)$ lead to a difference between the values of T_P and T_1. From the above considerations, the highest vacuum energy density at present time corresponds to $T_1 = T_P$.

during the radiation era, as $a(t) \sim t^{1/2}$ we get:

$$\rho^{v}(t_{\rm eq}) = \rho^{v}(t_P)\left(\frac{t_P}{t_{\rm eq}}\right)^2 . \qquad (23)$$

During the successive period, the matter dominated era, it is still $\rho^v \propto a^{-4}$, but now $a(t) \sim t^{2/3}$. Therefore, by integrating Eq. (14) for ρ^v from $t_{\rm eq}$ down to t_0 we have:

$$\rho^{v}(t_0) = \rho^{v}(t_{\rm eq})\left(\frac{t_{\rm eq}}{t_0}\right)^{\frac{8}{3}} , \qquad (24)$$

so that, at the present time, $\rho^v(t_0)$ is:

$$\rho^{v}(t_0) = \rho^{v}(t_P)\left(\frac{t_P}{t_0}\right)^2 \cdot \left(\frac{t_{\rm eq}}{t_0}\right)^{\frac{2}{3}} = \rho^{v}(t_P)\left(\frac{t_P}{t_0}\right)^2 \cdot \frac{a_{\rm eq}}{a_0} . \qquad (25)$$

By inserting now in Eq. (25) $\rho^v(t_P)$ given in Eq. (21), $t_P \sim 5\times10^{-44}\,$s, $t_0 \sim 2/(3H_0)$, with $(H_0)^{-1} \sim 13.7$ Gy and $a_{\rm eq}/a_0 \sim 1/3048$ [14], we finally find:

$$\rho^{v}(t_0) \sim \left(1.93\times10^{-4}\,{\rm eV}\right)^4 . \qquad (26)$$

We would like to compare now this result for $\rho^v(t_0)$ with the determination of ρ_γ at present time [14],

$$\rho_{\gamma}(t_0) \sim \left(2.11\times10^{-4}\,{\rm eV}\right)^4 . \qquad (27)$$

As can be easily checked, compatibly with the numerical uncertainties of the various quantities involved, the ratio between $\rho_\gamma(t_0)$ and $\rho^v(t_0)$ is in substantial agreement with the prediction of Eq. (22). As photons are always relativistic, this is precisely what should be expected from our previous analysis. In fact, as the measure of ρ_γ is an experimental input totally independent from our analysis, we can consider this finding as a check on our ideas. Moreover, Eq. (26) shows that, as is the case for photons, the contribution of ρ^v is nowadays negligible.

To summarize, we suggest that the cosmological evolution itself provides the mechanism that dilutes the zero point energy contribution to the total energy density of the universe down to a value which is negligible if compared to the current matter and cosmological constant determinations.

Another interesting outcome of our analysis is the following. As already noted, when the energy momentum tensor of the vacuum is not of the form $T_{\mu\nu} \propto g_{\mu\nu}$, the Lorentz invariance of the theory is lacking. Our $<\hat{T}_{\mu\nu}>$ at Planck time has not a Lorentz invariant form, but the cosmic evolution allows to recover Lorentz invariance at our time. We think that the connection between our findings and the whole subject of Lorentz violation at Plank scale is worth of further investigations.

Before ending this section, we would like to add some comments which should help in making more transparent the entire setup of our proposal. First of all, we think it is worth to spend some words on the underlying field theoretical framework of our work. As is clear from the previous section, up to now we have considered a Fock space in a flat Minkowski space–time. Clearly, a more rigorous treatment of the problem would have required the use of quantum field theory in an expanding universe, as is the case (of interest for us) of a FRW background.

As we shall show in a moment, however, it is not difficult to convince ourselves that such a refinement is irrelevant for the issue under investigation. Actually, we have deliberately chosen to work on a flat space–time since our goal is to present the mechanism of the washing out of the zero point energies of the effective field in the simplest possible framework, avoiding any unnecessary technical detail.

In fact, let us consider a scalar quantum field in a FRW background, a problem largely investigated in the literature [15; 16; 17; 10]. Regularization procedures based on "point splitting" or on "adiabatic regularization" both give the same result for the leading divergences in the vacuum pressure and density, namely $p^v = \rho^v/3$.

Clearly, from our effective field theory point of view, the "adiabatic basis" approach [15; 16], which allows for a mode decomposition, is the most appropriate. In fact, this property allows for the definition of a Fock space at each time, similarly to what happens in the flat case. Moreover, it is easy to see that the leading "divergent" term of the vacuum energy density scales as $\rho^v \sim a(t)^{-4}\Lambda^4$, where $a(t)$ is the scale factor in the FRW metric and Λ is the UV cut-off. This is nothing but our result.

In this respect, it is important to stress once again that our results are derived in the framework of an Effective Field Theory approach. This is completely different from a renormalized theory, which is the point of view considered in the above mentioned literature, where the divergent terms are treated as unphysical and are accordingly cancelled out. In our Effective Theory approach the physical cut-off is part of the definition of the theory itself and plays an important role in establishing the physical results. In such a framework, the cut-off dependence of ρ^v and p^v is an essential physical aspect of our analysis. It is worth to spend some additional words on this point.

The very notion of Effective Field Theory is related to the presence of a physical cut-off in the definition of the theory, which plays the role of "scale of new physics". Only when the UV completion of this low energy theory is known, the cut-off can be naturally related to some physical parameter (such as a renormalized mass) of the higher energy theory. In the low energy effective theory, it is just a (physical) momentum cut-off.

For instance, when in the SM the impact of dimension 5 or 6 operators is studied, the whole point is the following. The SM is considered as an effective theory valid up a scale Λ, the physical cut-off, and the ratios which appear in the expressions for the physical quantities (the so called "suppression factors") are not ratios of SM renormalized masses but rather ratios between the physical scale μ involved in the computation of the quantity under consideration and the cut-off. The contributions of these dimension 5 or 6 operators are typically suppressed by μ/Λ and μ^2/Λ^2 respectively but, if the cut-off Λ (the scale of new physics) is not too high, these ratios are not too small and can give sizable contributions to physical quantities.

Another example of this kind is chiral perturbation theory, where again we have a low energy effective theory, intrinsically defined with a cut-off, and the (would be) irrelevant operators coming from the higher terms of the gradient expansion can give sizable contributions to physical quantities. Again, the ratios that appear in this case are not ratios between renormalized masses of a renormalized theory but rather powers of p^2/Λ^2, where Λ is the physical cut-off and p the scale of the physical process. The cut-off, far from being just a theoretical tool for computation, is physical: it plays the role of "scale of new physics" and enters the theoretical predictions of physical quantities.

Therefore, our momentum cut-off, rather than being an alternative way of doing computations, is physically and deeply motivated by the effective field theory point of view.

Let us remind now that in the standard approach, when we compute a generic physical quantity, the large and potentially dangerous terms which appear because of the use of such a non-Lorentz invariant cut-off are actually cancelled by suitable (Lorentz violating) counter-terms, so that eventually the low energy theory is fully Lorentz covariant. This clearly applies to all phenomena that do not involve the absolute value of the vacuum energy, namely those observable phenomena which deal with energy differences and, as is well known, do not show any Lorentz violating effect up to very high precision, as is for instance the case of atomic spectra.

In this respect, our predictions totally coincide with the usual ones. Our point is that the only modification to this standard picture concerns the vacuum energy density which is unrelated to the various interactions and is typically (and correctly) cancelled with no consequences, for instance, in the description of scattering processes or of bound states. Our proposal concerns only the zero point energy contribution to the energy density. Instead of being cancelled with the help of an "ad hoc" counter-term (a fine tuning problem known as the 120 orders of magnitude problem), we have shown that it is possible to interpret this energy density as a "physical" contribution at early cosmological times and that the corresponding density at present time is washed out by the evolution and is comparable with the present energy density of radiation.

4 The counting of the degrees of freedom

Up to now we have considered the cosmological evolution of the (thermal average of the) energy-momentum tensor of a quantum scalar field starting at the Planck time t_P, with the assumption that at $t \sim t_P$ and $E \sim M_P$ physics is entirely described by one quantum field (or a small number of fields) and that the known lower energy theories were born during the cosmic time evolution.[2] This assumption appears natural in view of our ideas on the effective nature of particle physics theories and fits our current views on the cosmological evolution. In this respect, the lower energy new fields, new dof, are nothing but a convenient manner to parametrise the theory at a lower scale. Therefore, when computing the vacuum contribution to the cosmological constant, one should not include the zero point energies of the effective low energy theories as this would result in a multiple counting of dof. The zero point energies coming from the dof of the original quantum field already account for the whole contribution to the vacuum energy.

Before we can conclude that our findings can be of some relevance for the cosmological constant problem, we still have to address another issue. As is well

[2] As we have already said, a different (probably string) theory is supposed to describe the physics at times earlier than t_P.

known, some of our low energy theories, for instance the Higgs sector of the SM, are characterised by the presence of condensates. In the standard approach, these terms are considered to give very large contributions to the cosmological constant as they enter the energy momentum tensor as $\rho_c g_{\mu\nu}$, where ρ_c is the vacuum energy density associated with the condensate. However, according to our previous discussion, there are no such additional terms as the whole contribution is already contained in the zero point energies of the original theory. Again, taking into account these terms would result in a double counting of dof. A similar point of view has already been expressed within a different approach to the cosmological constant problem [18].

Below we try to elucidate the arguments of the previous two paragraphs with the
help of an example inspired to the work on the top quark condensates of Bardeen et al. [19].

Following [19], let us consider a Nambu Jona-Lasinio theory defined at the high energy scale Λ by:

$$Z=\int D\bar{\psi}D\psi \exp\left[i\int d^4x\left(\bar{\psi}(i\gamma^\mu\partial_\mu-M)\psi+\frac{g^2}{2m_0^2}\bar{\psi}\psi\bar{\psi}\psi\right)\right]. \qquad (28)$$

An Hubbard-Stratonovic transformation introduces a new scalar field ϕ so that Eq. (28) can be rewritten as:

$$Z=\frac{1}{\mathscr{N}}\int D\bar{\psi}D\psi D\phi \exp\left[i\int d^4x\left(\bar{\psi}(i\gamma^\mu\partial_\mu-M)\psi-\frac{m_0^2}{2}\phi^2+g\bar{\psi}\psi\phi\right)\right], \qquad (29)$$

where the normalisation factor $\mathscr{N}$ ensures the equality of Eqs. (28) and (29). Obviously, any quartic divergent term which apparently comes from the zero point energies of ϕ cannot induce any change in the quartic divergences of Eq. (28) as they are cancelled by $\mathscr{N}$.

The next step in [19] consists in the integration of the high frequency modes of the fermion and scalar fields from Λ to the lower energy scale μ:

$$Z=\frac{\mathscr{Q}}{\mathscr{N}}\int D\bar{\psi}_l D\psi_l D\phi_l \exp\left[i\int d^4x\left(\bar{\psi}_l(i\gamma^\mu\partial_\mu-M-\delta M)\psi_l\right.\right.$$
$$\left.\left.+g\bar{\psi}_l\psi_l\phi_l+\frac{1}{2}Z_\phi\partial^\mu\phi_l\partial_\mu\phi_l-\frac{m_0^2+\delta m_0^2}{2}\phi_l^2-\frac{\lambda}{24}\phi_l^4\right)\right] \qquad (30)$$

where ϕ_l and ψ_l are the scalar and fermion fields with Fourier components up to μ. This integration generates new dynamical dof [20] in the Lagrangian of Eq. (30).

This example is relevant to our problem for the following reason. When one deals with the effective Lagrangian of Eq. (30), the normalisation factor $\mathscr{Q}/\mathscr{N}$ is not
considered as one has no knowledge of the higher energy theory. Clearly, this has no effect in the evaluation of the low energy Green's functions, i.e. for typical scattering processes. However, if we compute the vacuum energy from the quartic divergences of this effective Lagrangian, we end up with a result which differs from the one obtained from the "fundamental" theory of Eq. (28) because of an

erroneous counting of the dof. Only if we take into account the normalisation factor $\mathscr{Q}/\mathscr{N}$ we recover the original result. Clearly, the same argument applies when additional contributions to the vacuum energy come from the appearance of condensates such as, for instance, a vacuum expectation value for ϕ_l.

We can also consider an alternative, but equivalent, argument which allows to understand the suppression of the Λ^4 and the condensate terms. Let us consider the appearance of a condensate below some temperature T_{SB} through a symmetry breaking mechanism. The cutoff of the low energy theory which describes the broken symmetry phase is nothing but the temperature T_{SB} at which the transition takes place. Moreover, the cutoff and the condensate contributions to ρ^v and p^v come in the same combination as in Eqs. (10) and (11), where the m^4 terms are now accompanied by the additional v^4 condensate contribution (v is the value of the condensate). As is always the $\Lambda^4 = T_{SB}^4$ term which dominates, we obtain for ρ^v the same scaling as before, regardless of the Lorentz invariant nature of the condensate contribution. Being T_{SB} the cutoff, again we find that these contributions at present time are suppressed.

5 Summary and conclusions

We have found that if we consider that at the Planck time t_P physics is described by an effective field theory with UV cutoff M_P, the corresponding vacuum energy density undergoes a cosmic scaling that makes it negligible at present time t_0 when compared to non-relativistic matter and cosmological constant densities, much in the same way as the cosmological scaling makes the photon density negligible nowadays. The reason for this behaviour is that for an effective field theory $< \hat{T}_{\mu\nu} >$ is such that $p^v \sim \rho^v/3$.

Moreover, our analysis predicts a constant ratio, Eq. (22), between the vacuum and the radiation densities. When the theoretical determination of the vacuum energy

density at present time, given in Eq. (26) and obtained by a proper rescaling of the Planck time vacuum density of Eq. (21), is compared with the experimentally determined photon energy density in Eq. (27), we find substantial agreement with our prediction. In case one takes the energy density scale in Eq. (21) just as the maximum acceptable value for this quantity, then the corresponding derived value at present time, $\rho^v(t_0)$, should be regarded as the upper limit of the present vacuum energy density.

We believe that this supports the central idea put forward in the present work, namely that zero point energy and condensate contributions to the universe energy density are washed out by the cosmological evolution. Moreover, these terms, being $w \sim 1/3$, cannot contribute to the cosmological constant, for which we know that the measured value of w is $w \sim -1$. In our opinion, this result points towards a gravitational origin of the (measured) cosmological constant.

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