Study of Gauge Theory in the Light of Constrained Dynamics

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Certified that this thesis entitled 'Study of gauge theory in the light of constrained dynamics' submitted by Safia Yasmin, who has got her name registered on 14.12.2011 with Reg. No. Phy/Sc/225 for the award of Ph. D. (Science) degree of the University of Burdwan is absolutely based on her own work done under my supervision and that neither this thesis nor any part of it has been submitted for any degree/diploma or any other academic award anywhere before.

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DECLARATION

I hereby declare that the matter embodied in this Thesis entitled **"Study of Gauge Theory in the Light of Constrained Dynamics"** is based on my own work done under the supervision of Dr. Anisur Rahaman, Associate Professor, Hooghly Mohsin College, Chinsurah, Hooghly. This Thesis or any part of it has not been submitted and will not be submitted for the award of any other Degree or Diploma elsewhere.

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Contents

List of Symbols

A_1, A_0	Gauge Fields
η, θ	Auxiliary Field in the Extended Phase Space
ψ	Fermion Fields
$\pi_1, \pi_0, \pi_\eta, \pi_\theta,$	Momentum corresponding to the Fields A_1, A_0, η, θ
u, v	Lagrange Multipliers
G	Gauss Law Constraints
C_{ij}	Square Matrix
m	Mass
e	Charge
$Z(A)$	Generating Functional
w, Ω	Constraints
λ	Lagrange multiplier
α, β, γ	Arbitrary Parameters
$g_{\mu\nu}$	Metric in Two Dimensional Space $= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\epsilon_{\mu\nu}$	Levichevite Symbol , where $\epsilon_{01} = 1$
H_R	Reduced Hamiltonian
P_R	Momentum Operator
M_R	Lorentz Boost Generator
H_{BRST}	BRST invariant Hamiltonian
Q	BRST Charge
X	Gauge fixing Condition
$[D\mu]$	Livolle Measure
$C, \bar{P}, P, \bar{C}$	Ghost, Anti Ghost fields and their Momentum
S	Action
B, N	Multiplier Fields
h, B, X, F	Fock Space Fields
Θ	Finite Field Dependent Parameter
w_{ij}	Hessian Matrix
y^{a_i}	Lagrangian Primary Constraint
γ	Dirac Matrix

where $\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}$ $\sigma' s$ are the Pauli matrices.

Chapter 1

1 Introduction

Symmetry plays an important role in the development of the various aspects of theoretical physics. Gauge theory in this context is of particular interest. A theory which is symmetric under a local gauge transformation is known as gauge theory. This gauge symmetry provides the essential principle to describe fundamental interactions of nature except one namely gravitational interaction.

In modern language a gauge theory is characterized by the first class constraint [1, 2, 3]. The terminology first class constraint was initially used by Dirac. To study the gauge theory, therefore, concept of constraints along with the formalism to handle the constrained system (known as constraints dynamics) enters automatically into the study related to gauge theory. We often get confronted to the constrained theory to study theoretical physics. From Maxwell's theory of electromagnetism to the latest developed string theory, everywhere presence of constraint in the phase space is found in an essential way. So the study of physics related to the gauge theory in the light of constraint dynamics has wide applications in the arena of theoretical physics.

Constraint means velocity independent relation between coordinate and momentum [1, 2]. So all the velocities of the dynamical variables of a theory can not be determined in terms of momenta and as a result the precise canonical quantization gets threatened when a system contains constraints in its phase space. So quantization of this type of system is interesting in its own right. The constraints imposes restriction on the degrees of freedom too. The physical degrees of freedom are manifested through the Hamiltonian only when all the constraints are imposed in it.

Sometimes it may be the case that a theory does not show any symmetry in its usual phase space. However these theories may have symmetry in the extended phase space. These theories are also considered as gauge theory and constrained dynamics developed by Dirac is equally useful to these systems. The main purpose of our investigation is the study of some field theoretical models which has gauge symmetry in the usual phase space or which can be made gauge symmetric in the extended phase space taking the help of some auxiliary fields [4]. It is interesting to mention that these auxiliary fields render their incredible services towards restoration of the gauge symmetry without disturbing the physical sector at all.

Our investigations not only limited to gauge symmetry. It has been extended to the two other important symmetry like BRST symmetry [5, 6, 7] and Poincare symmetry. Unlike the BRST symmetry Poincar'e symmetry has no direct link with gauge symmetry. However presence of constraints sometimes found to have considerable influence on the Poincar'e symmetry. In this context, we mention that even a theory which is not Lorentz covariant to start with through it has been found to satisfy the correct Poincar'e algebra [8, 9].

BRST symmetry [5, 6, 7] has direct link with gauge symmetry. In fact, it is an improvement over gauge symmetry. It is a symmetry of the gauge fixed action after all gauge fixed theory is the ultimate description of a gauged theory since what we need is the physical sector of a theory. For covariant quantization of a theory BRST formulation is instrumental. The BRST symmetry also ensures the unitarity and renormalization of a theory [5, 6, 7]. Since it has direct link with gauge symmetry, constraint structure of a theory has the crucial link to the BRST formulation. So special emphasis towards BRST invariant reformulation is given in our investigation [10, 11, 12].

FFBRST is an important extension over BRST [13]. Here BRST transformation parameter become field dependent and anti-commutating in nature. It is also a symmetry of the gauge fixed action like the BRST. It indeed protect nilpotency. Few recent interesting investigations related to FFBRST are available in [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24]. So investigation related to FFBRST is also included in our studies [12].

Quantization in the usual phase space as well as in the extended phase space have also acquired a considerable part of our investigation [10, 11, 12]. Like the previous studies [25, 26, 27, 28, 29] quantization in the extended phase space has been termed as alternative quantization in our study. When the phase space of a system is enlarged in order to bring back the gauge symmetry of the theory introducing Wess-Zumino fields [4] the composite system contains usual as well as Wess-Zumino fields. So it is a matter of study whether these extra fields has influence on the physical subspace or not. To investigate it our study [12, 26] is extended with Falck and Kramer formalism [30] which enables us to conclude that the Wess-Zumino fields helps to bring back the symmetry of the theory without disturbing the physical sector. Proper gauge fixing maps the theory of the extended phase space onto the usual phase space.

Study of constrained system is not limited to Hamiltonian formulation only. Lagrangian formulation of constrained system is also possible [31]. One of the drawback of Hamiltonian formulation is its inability of presenting a theory in a covariant manner, which can be overcome in the Lagrangian formulation. In spite of that, it is fair to say that this formalism it is not so popular like Hamiltonian formulation [32, 33, 34, 35, 36, 37]. However because of the available advantage of covariant description it can not be ignored. In [31], a formulation is developed which not only enables to identify the constraint but also general gauge transformation generator can be constructed like the Hamiltonian formulation. It is capable of giving well judgement whether a theory has gauge symmetry or not. This formulation is equally applicable both in the usual and extended phase space. Our investigation, therefore, include the application of this formulation on some field theoretical models both in the usual and the extended phase space [38]. This thesis has been organized in the following way.

Chapter 2, contains general description of constraint dynamics with an example of gauged chiral boson with Faddeevian anomaly [8, 9]. In Chapter 3, a gauge invariant reformulation of the chiral Schwinger model with Faddeevian anomaly has been carried out [10] with the help of Mitra-Rajaraman prescription [39, 40]. A comparison is also made between the result obtained for the chiral Schwinger model with Faddeevian anomaly with the gauge invariant version of the usual chiral Schwinger model for a = 2. In Chapter 4, we have studied the Poincar'e invariance of a model where both vector and axial vector interaction get mixed up with different weight [41, 42, 43]. Therefore, investigation through the Poincare algebra has been carried out to find the appropriate masslike term which is capable of describing a physically sensible theory, using a very generalized mass like term for the gauge field [26]. An attempt is also made to single out the real physical canonical pairs embedded within the phase space of the allowed physically sensible theory. In Chapter 5, Batalin, Fradkin, Vilkovisky (BFV) formalism [44, 45, 46, 47, 48] is used to obtain BRST invariant reformulation of the generalized version of quantum electro dynamics (GVQED) and chiral Schwinger model with Faddevian anomaly and Gauged Floreanini-Jackiw type chiral Boson. In Chapter 7, an alternative quantization of the gauge invariant version of the GVQED and the Gauged Floreanini -Jackiw type chiral Boson are made. Using the method developed by Falck and Kramer [30] one can see that an appropriate gauge fixing can correlate between the gauge invariant theory of the extended phase space and the gauge non-invariant theory in the usual phase. In Chapter 7, a brief introduction of FFBRST and anti-FFBRST formulation is given. This formulation is applied to the BRST invariant effective action of GVQED to get back the original gauge non-invariant form of the action through the field dependent parameter of FFBRST and anti-FFBRST [12]. In Chapter 8, we study the constrained systems with the lagrangian formulation. With a brief description of Shirzad's formalism [31] the symmetry property of few lower dimensional models has been investigated.

Chapter 2

2 General Description of Constrained Dynamics

2.1 Definition and Classifications of Constraints

A constraint in general is a velocity independent relations between coordinate and momentum in the phase space of a theory. The presence of which make some velocities inexpressible in terms of momentum and as a result naive Poisson's brackets become inadequate to quantize the system. How to deal with a system endowed with constraints let us consider a system which is described by the lagrangian

$$L = L(q_i, \dot{q}_i), \tag{1}$$

where $q_i, \dot{q}_i$ represent coordinate and velocity. This lagrangian is said to be singular when

$$\det[\frac{\partial^2 L}{\partial \dot{q}_i \partial \dot{q}_j}] = 0. \tag{2}$$

The above equation (2) signifies that the system posses some primary constraints [1, 2, 3]. In order to extract out the constraint of a given system described by the lagrangian (1) the momenta corresponding to the variables q_i are required to find out. The momentum variables P^i is defined by

$$P^i = \frac{\partial L}{\partial \dot{q}^i}, \tag{3}$$

with $i = 1............n$, where n is the number of canonical coordinates. When some momenta are not expressible in terms of the velocities then there exist certain relation among the momenta and coordinate variables:

$$\phi_m(q, p) \approx 0, \tag{4}$$

where m=1,..........M. The above relations are known as primary constraints of the theory. The constraints are all weak condition and to indicate weak condition the symbol $\approx$ is used in place of =. The canonical Hamiltonian of the system is given by

$$H_c(q_i, p_i) = p_i\dot{q}_i - L(q_i, \dot{q}_i, t). \tag{5}$$

The corresponding Hamiltonian equations of motion are

$$\frac{dq_i}{dt} = \frac{\partial H_i}{\partial p^i}, \tag{6}$$

$$\frac{dp_i}{dt} = -\frac{\partial H_i}{\partial q_i}. \tag{7}$$

Hamiltonian in equation (5) is not unique. We may replace it by effective Hamiltonian

$$H_e = H_i + u_m\phi_m(q, p) \approx H_e. \tag{8}$$

Here u_m are known as Lagrange multipliers. H_e gives new equations of motion as follows

$$\frac{dq_i}{dt} = \frac{\partial H_i}{\partial p^i} + u_m\frac{\partial \phi_m}{\partial p^i}, \tag{9}$$

$$\frac{dp_i}{dt} = \frac{\partial H_i}{\partial q_i} + u_m\frac{\partial \phi_m}{\partial q_i}. \tag{10}$$

Physical consistency demands that the time derivative of the primary constraints are to be zero for all time and the necessary condition for that is

$$\dot{\phi_m} = [\phi_m, H_{eff}] \approx 0. \tag{11}$$

This preservation may lead to two possibilities [1, 2, 3]: it may gives new constraints or it may fix the velocities u_m. The new constraints that evolve out are termed as secondary constraints. The process is to be repeated until all independent constraints and conditions on u_m have been explicitly found out. In this way all the constraints and velocities of a theory are determined. If we get K number of additional secondary constraints,

$$\phi_n(q, p) \approx 0, \tag{12}$$

where n=1,2,.............K, we find the complete set of constraints

$$\phi_a(q,p) \approx 0, \tag{13}$$

where a=1,2,.............K+M=T. T is the total number of constraints embedded in the system. Consistency of all constraints with Lagrange equations of motion require that there exist solutions of u_m as a function of q and p:

$$u_m = u_m(q,p). \tag{14}$$

According to Dirac [1], the constraints are classified into two categories, e.g. first class constraints and second class constraints. If Poission brackets of a particular constraint with itself and with the other constraints of the theory get vanished then that constraint is called first class constraints else it is called second class constraints. A function of coordinate and momenta, R(q,p) is said to be first class if it has zero Poission bracket with all the constraints, i.e.,

$$[R, \phi_a] \approx 0, \tag{15}$$

where a=1.............A. R(q,p) is said to be second class if $[R, \phi_a] \neq 0$. Sometimes a linear combination of second class constraints form first class constraints which is of interest for gauge invariant reformulation of a theory. All constraints may not be independent of each other. Constraints are divided into two sets. One set consists of all linearly independent first class constraints, i.e.,

$$\psi_i(q,p) \approx 0, \tag{16}$$

where i=1,.......P, and other set consists of remaining T-P number of second class constraints

$$\phi_\alpha(q,p) \approx 0, \tag{17}$$

where $\alpha = 1...........T - P$. For instance the second class constraints give a nonsingular matrix constructed with the Poisson bracket among themselves,

$$C_{\alpha,\beta} = [\phi_\alpha, \phi_\beta]. \tag{18}$$

Since it is known that anti-symmetric matrices can be inverted if and only if they have an even number of rows and columns. So we assume that the system consists of an even number of second class constraints, i.e. $C_{\alpha,\beta}$ is a nonsingular matrix and its inverse $C^{-1}_{\alpha,\beta}$ exist:

$$C_{\alpha,\beta} C^{-1}_{\alpha,\beta} = 1. \tag{19}$$

2.2 Dirac Bracket and its importance

The presence of second class constraints make the ordinary Poission bracket in adequate for its analysis. So we need to modify the naive Poisson bracket. A new dynamical variable which has zero Poission bracket with all second class constraints can be defined by

$$A^{'} = A - [A, \phi_{\alpha}]C^{-1}_{\alpha,\beta}\phi_{\beta}. \tag{20}$$

We see that

$$[A^{'}, \phi_{\gamma}] = [A, \phi_{\gamma}] - [A, \phi_{\gamma}]C_{\alpha,\beta}C^{-1}_{\alpha,\beta} = 0. \tag{21}$$

With the construction (20), the variables give vanishing Poission brackets with the constraints. Let us now change the Poission bracket of two variable A and B by their primed variables [2],

$$[A, B] \to [A^{'}, B^{'}]. \tag{22}$$

Though $A \approx A^{'}$, $B \approx B^{'}$ the Poission bracket between A, B is not weakly equal to Poisson bracket of $A^{'}, B^{'}$. So to overcome the above problem all Poission bracket have to be replaced by Dirac bracket which is defined by

$$[A, B]^{*} = [A, B] - [A, \phi_{\alpha}]C^{-1}_{\alpha,\beta}[\phi_{\beta}, B]. \tag{23}$$

Now we observe that $[A, B]^{*} \approx [A^{'}, B^{'}]^{*} \approx [A^{'}, B]^{*} \approx [A, B^{'}]^{*}$.

The presence of second class constraints imply that the system contains some non dynamical degrees of freedom. Thus the naive Poission bracket needs to be modified, if we impose the second class constraints in the theory as a strong condition to get the theory involving only the dynamical degrees of freedom. The Dirac bracket of a constraint with an arbitrary phase space variable vanishes by construction. This is the essential condition to set all second class constraints strongly to zero.

$$[A^{'}, \phi_{\gamma}]^{*} \approx [A, \phi_{\gamma}] - [A, \phi_{\gamma}]C_{\alpha,\beta}C^{-1}_{\alpha,\beta} = 0. \tag{24}$$

A very useful property of Dirac bracket is its iterative property. If there are a large number of constraints, it is not always convenient to invert the large matrix. So one can choose a smaller subset of second class constraints and find the intermediate Dirac bracket and so on. The process is to be repeated

until all the second class constraints are exhausted. Identical result can be achieved calculating the final Dirac bracket in a single step.
So first class replacement of the Hamiltonian is obtained by imposing the constraints as strong condition in (8) redefining the Hamiltonian by

$$H_R = H_e - [H_e, \phi_\alpha] C^{-1}_{\alpha\beta} \phi_\beta, \tag{25}$$

and then we find

$$u_\alpha = -[H_c, \phi_\alpha] C^{-1}_{\alpha\beta}. \tag{26}$$

With this choice too, Hamiltonian is not completely determined. However H_t defined by

$$H_t = H_R + v_i \psi_i(q, p), \tag{27}$$

keeps the equations of motion and the constraints unaltered. Where $\psi_i(q,p)$ stands for the first class constraints and $v_i(\tau)$ are the arbitrary velocity function. This Hamiltonian (27) is found consistent with the Dirac bracket. Now it is possible to find out the new equations of motion since $\psi_i(q,p)$ do not have vanishing Poission bracket with the canonical variables. Now the time derivative of q and p are given by

$$\dot{q}_i = [q_i, H_t] + v_i [q_i, \psi_j], \tag{28}$$

$$\dot{p}_i = [p_i, H_t] + v_i [p_i, \psi_j]. \tag{29}$$

$\psi_i(q,p)$ in equation (28) and (29) generates infinitesimal contact transformation of the q's and p's, under which the physical content of the theory remains the same. This is known as gauge transformation.

Appearance of arbitrary functions v_i in H_t occurs when the original lagrangian contains of gauge degrees of freedom associated with the first class constraints. The velocities v_i can be fixed by the gauge fixing condition

$$\gamma_i(q, p, \tau) \approx 0, \tag{30}$$

where i=1.................J. Condition (30) looks like a constraint which however does not follow from the lagrangian. The gauge fixing conditions are to be chosen in such a way that the constriants ψ_i and gauge fixing conditions all together form a second class set. So the matrix $[\psi_i, \gamma_i]$ becomes nonsingular.

One needs to replace all the Poission bracket by Dirac bracket which is consistent with the ψ_i and γ_i, then arbiratariness due to v_i will automatically disappear.
If at this stage one likes to express the systems in terms of true independent canonical variables only, then it is needed to impose some invariant relations on the system [2]. It may be the situation that the gauge choices do not completely reduce the phase space available for particle momentum down to the size implied by the Eular Lagrange equations. Therefore additional constraints are needed to define the physical system completely. These are termed as invariant relations. The function $\zeta_i(q,p)$ be an invariant relation of the form

$$\zeta_i(q,p) \approx 0. \tag{31}$$

at $\tau = 0$, and

$$\frac{d\zeta_i}{d\tau} \approx M_{ij}\zeta_j. \tag{32}$$

In(31) ($\approx$) sign indicates that the all the constraints including the ζ_i are set to zero. ζ will remain weakly zero for all τ if equation (31) is satisfied. Invariant relations are different from the equations of motion and these are the conditions which should be satisfied in order for a solution to be considered as a physical one. If one can choose the gauge constraints and invariant relations properly, then all the constraints turned into second class. In the second-class theory, the variables which have vanishing Poission bracket with the constraints, can be considered as independent degrees of freedom.
As it is discussed earlier for a theory with second class constraints, Poission brackets have to be replaced by Dirac brackets. The system then can be quantized by converting the Dirac brackets into equaltime commutators. We give an example which is usually helpful.

2.3 Application of Constrained Dynamics on chiral Schwinger Model with Faddeevian Anomaly

We consider the lagrangian of chiral Schwinger model with Faddeevian anomaly [8, 9]. The model when described in terms of chiral Boson [49, 50, 51] looks

$$\mathcal{L}_{\mathcal{CH}} = \dot{\phi}\phi' - \phi'^2 + 2e\phi'(A_0 - A_1) - 2e^2A_1^2. \tag{33}$$

From the standard definition, the momentum corresponding to the fields A_0, A_1 and ϕ are:

$$\frac{\partial \mathcal{L}_{\mathcal{CH}}}{\partial \dot{A}_0} = \pi_0 = 0, \tag{34}$$

$$\frac{\partial \mathcal{L}_{\mathcal{CH}}}{\partial \dot{A}_1} = \pi_1 = \dot{A}_1 - A_0', \tag{35}$$

$$\frac{\partial \mathcal{L}_{\mathcal{CH}}}{\partial \dot{\phi}} = \pi_\phi = \phi'. \tag{36}$$

$\pi_0 \approx 0$ and $\pi_\phi = \phi' \approx 0$ are identified as the primary constraints of the theory. The effective Hamiltonian follows from the equation (34), (35) and (36)

$$H_p = \int d\,x[H_C + u\pi_0 + \nu(\pi_\phi - \phi')], \tag{37}$$

where the canonical Hamiltonian is

$$H_C = \int dx[\frac{1}{2}\pi_1^2 + \pi_1 A_0' + \phi'^2 - 2e(A_0 - A_1)\phi' + 2e^2 A_1^2]. \tag{38}$$

Here u and v are two required lagrange multipliers. The gauss law constraints of the theory is

$$G = \pi_1' + 2e\phi' \approx 0. \tag{39}$$

The preservation of constraint $\pi_\phi = \phi' \approx 0$, with respect to the Hamiltonian gives a new constraint

$$(A_1 + A_0) \approx 0. \tag{40}$$

The Lagrange multipliers u and v take the following expressions

$$u = -(\pi_1 + A_0'), \tag{41}$$

$$v = \phi - e(A_0 - A_1). \tag{42}$$

The reduced Hamiltonian for this system is

$$H_r = \int dx[\frac{1}{2}\pi_1^2 + \pi_1 A_1' + \frac{1}{4e^2}\pi'^2 + 4e^2 A_1^2]. \tag{43}$$

The Dirac bracket between the two variables A and B is defined by

$$[A(x), B(y)]^* = [A(x), B(y)] - \int [A(x), \omega_i(\eta)] C_{ij}^{-1} [\omega_i(\eta), B(x)] d\eta dz, \quad (44)$$

where C_{ij}^{-1} is given by

$$\int C_{ij}^{-1}(x, z) [\omega_i(z), \omega_j(y)] dz = 1. \quad (45)$$

Four constraints of the system under consideration are

$$\omega_1 = \pi_0 \approx 0, \quad (46)$$

$$\omega_2 = \pi_\phi - \phi' \approx 0, \quad (47)$$

$$\omega_3 = \pi_1 + 2e\phi \approx 0, \quad (48)$$

$$\omega_4 = (A_1 + A_0) \approx 0. \quad (49)$$

For this system

$$C_{ij} = \begin{pmatrix} 0 & 0 & 0 & -\delta(x-y) \\ 0 & -2\delta'(x-y) & -2e\delta(x-y) & 0 \\ 0 & 2e\delta(x-y) & 0 & -\delta(x-y) \\ \delta(x-y) & 0 & \delta(x-y) & 0 \end{pmatrix} \quad (50)$$

The matrix C_{ij}^{-1} exists since C_{ij} is nonsingular

$$C_{ij}^{-1} = \begin{pmatrix} -\frac{\partial}{2e^2} & \frac{1}{2e} & \frac{\partial}{2e^2} & 1 \\ -\frac{1}{2e} & 0 & \frac{1}{2e} & 0 \\ \frac{\partial}{2e^2} & \frac{1}{2e} & -\frac{\partial}{2e^2} & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \delta(x-y). \quad (51)$$

The Dirac brackets between the fields describing the reduced Hamiltonian are

$$[A_1, \pi_1]^* = \delta(x-y), \quad (52)$$

$$[A_1, A_1]^* = -\frac{1}{2e^2}\delta'(x-y). \quad (53)$$

Using Dirac bracket (52), (53) and the reduced Hamiltonian (43), we obtain the following first order equations of motion

$$\dot{\pi}_1 = \pi_1' - 4e^2 A_1, \quad (54)$$

$$\dot{A}_1 = \pi_1 - A_1'. \tag{55}$$

The second order differential follows from (54) and (55) is

$$[\Box + 4e^2]A_1 = 0. \tag{56}$$

So A_1 describes a massive boson with square of the mass $m^2 = 4e^2$. So the theoretical spectrum contains a massive boson with mass $2e$ [8, 9]. This example shows an application of constraint dynamics towards the exact solution of this model.

Chapter 3

3 Gauge Invariant Reformulation in the Usual Phase Space

3.1 Introduction

Symmetry plays a crucial role in the understanding of theoretical physics and a gauge symmetric theory always scores over the theory where this symmetry is lacking. So the study related to restoration of gauge symmetry is of interest. Gauge symmetry can be restored in two different ways. Extension of phase space by auxiliary fields to bring back the gauge symmetry was known from long past. Another interesting way of restoration of gauge symmetry is available from the work of Mitra and Rajaraman [39, 40]. Here extension of Phase space is not needed. Restoration of symmetry takes place in the usual phase space here. Therefore, applications of this technique on any gauge non symmetric model would be instructive. In this content, we consider the chiral Schwinger model with Faddeevian anomaly [8, 9]. Here gauge symmetry breaks down at the quantum mechanical level. The ancestor of the model known as Jackiw-Rajaraman version of chiral Schwinger model too did not have gauge symmetry which was restored by Mitra and Rajaraman in [39, 40]. In [10] we have made the restoration of gauge symmetry of the chiral Shwinger model with Faddeevian anomaly which we are going to describe here.

Before restoration of gauges symmety has been carried out on the model, the bosonization of the fermionic version of chiral Schwinger model has been done with Faddeevian type regularization and imposing a chiral constraint in the phase space the model has been expressed in terms of chiral boson [49, 50, 51].

3.2 Bosonization of Fermionic Model and Imposition of Chiral Constraint

Chiral Schwinger model is described by the following generating functional

$$Z[A] = \int d\psi d\bar{\psi} e^{\int d^2x\mathcal{L}}, \tag{57}$$

with

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\gamma^\mu[i\partial_\mu + e\sqrt{\pi}A_\mu(1-\gamma_5)]\psi \\ &= \bar{\psi}_R\gamma^\mu i\partial_\mu\psi_R + \bar{\psi}_L\gamma^\mu(i\partial_\mu + 2e\sqrt{\pi}A_\mu)\psi_L.\end{aligned} \tag{58}$$

The right handed fermion remains uncoupled in this type of chiral interaction. So integration over this right handed part leads to field independent counter part which can be absorbed within the normalization. Integration over left handed fermion leads to

$$Z[A] = \exp\left[\frac{ie^2}{2}\int d^2x A_\mu\left[M^{\mu\nu} - (\partial^\mu + \bar{\partial}^\mu)\frac{1}{\Box}(\partial^\nu + \bar{\partial}^\nu)\right]A_\nu\right]. \tag{59}$$

$M_{\mu\nu} = ag_{\mu\nu}$, for Jackiw-Rajaraman regularization [32] where the parameter a represents the regularization ambiguity and

$$M_{\mu\nu} = \begin{pmatrix} 1 & -1 \\ -1 & -3 \end{pmatrix}\delta(x-y),$$

for an alternative version proposed by Mitra in [8, 9]. Writing down the generating functional in terms of the auxiliary field $\phi(x)$, it turns out to the following

$$Z[A] = \int d\phi e^{i\int d^2x\mathcal{L}_\mathcal{B}}, \tag{60}$$

with

$$\begin{aligned}\mathcal{L}_\mathcal{B} &= \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + e(g^{\mu\nu} - \epsilon^{\mu\nu})\partial_\mu\phi A_\nu + \frac{1}{2}e^2A_\mu M^{\mu\nu}A_\nu \\ &= \frac{1}{2}(\dot{\phi}^2 - \phi'^2) + e(\dot{\phi} + \phi')(A_0 - A_1) \\ &+ \frac{1}{2}e^2(A_0^2 - 2A_0A_1 - 3A_1^2).\end{aligned} \tag{61}$$

Here $\epsilon^{01} = -\epsilon_{01} = 1$ and the Minkowski metric $g^{\mu\nu} = diag(1, -1)$. Equation (61) was initially found in [8] where Mitra termed it as chiral Schwinger model with Faddeevian regularization. In [8], we find that the Gauss law constraint of this theory is

$$G = \pi_1' + e(\pi_\phi + \phi'). \tag{62}$$

It is found that the Poisson bracket between $G(x)$ and $G(y)$ is

$$[G(x), G(y)] = 2\delta'(x - y). \tag{63}$$

This Poission bracket of the Gauss law constraint (63) was found to gave the vanishing contribution for the usual chiral Schwinger model [32]. Faddeev initially noticed that anomaly made Poission bracket between G(x) and G(y) nonzero [52, 53]. The constraint became second class itself and gauge invariance was lost. He, however, argued that it would be possible to quantize the theory but in this situation system may posses more degrees of freedom. From the standard definition, the momentum corresponding to the field ϕ is found out to be

$$\frac{\partial \mathcal{L}_{\mathcal{B}}}{\partial \dot{\phi}} = \pi_\phi = \dot{\phi} + e(A_0 - A_1). \tag{64}$$

The following Legendre transformation

$$H_B = \int d^2x [\pi_\phi \dot{\phi} - \mathcal{L}_{\mathcal{B}}], \tag{65}$$

leads to the Hamiltonian density

$$\begin{aligned} \mathcal{H}_{\mathcal{B}} &= \frac{1}{2}[\pi_\phi - e(A_0 - A_1)]^2 + \frac{1}{2}\phi'^2 \\ &- 2e\phi'(A_0 - A_1) - \frac{1}{2}e^2(A_0^2 - 2A_0A_1 - 3A_1^2). \end{aligned} \tag{66}$$

In order to suppress one chirality at this stage we impose the chiral constraint

$$\omega(x) = \pi_\phi(x) - \phi'(x) \approx 0. \tag{67}$$

It is a second class constraint itself since

$$[\omega(x), \omega(y)] = -2\delta'(x - y). \tag{68}$$

After imposing the constraint $\omega(x) \approx 0$, into the generating functional we arrived at the following

$$
\begin{aligned}
Z_{CH} &= \int d\phi d\pi_\phi \delta(\pi_\phi - \phi')\sqrt{det[\omega,\omega]}e^{i\int d^2x[\pi_\phi\dot{\phi}-\mathcal{H}_{\mathcal{B}}]} \\
&= \int d\phi e^{i\int d^2x L_{CH}}. \qquad (69)
\end{aligned}
$$

with

$$
L_{CH} = \dot{\phi}\phi' - \phi'^2 + 2e\phi'(A_0 - A_1) - 2e^2A_1^2. \qquad (70)
$$

We obtained the gauged lagrangian density for chiral boson from the bosonized lagrangian with Faddeevian regularization [8] just by imposing the chiral constraint in its phase space. Harada in [35], obtained the same type of result for the usual chiral Schwinger model with one parameter class of regularization proposed by Jackiw and Rajaraman [32]. The lagrangian (70) can be thought of as the gauged version of chiral boson [49, 50] described by Floreanini and Jackiw [51]. A discussion related to the theoretical spectrum has given in Chapter 1. One can find that theoretical spectra contains a massive boson with mass $m = 2e$. The equation of massive boson was

$$
[\Box + 4e^2]A_1 = 0. \qquad (71)
$$

Equation (71) was interpreted there as the photon acquired mass via a dynamical symmetry breaking and the fermion got confined.

3.3 Role of Constraint in the Gauge Invariant Reformulation

The formalism of making a theory gauge invariant by the reduction of the number of second class constraint was first developed by Mitra and Rajaraman [39, 40]. The formalism strictly depends on the constraint structure of the theory. Depending on the constraint structure of the theory different gauge invariant version is possible for a particular theory. No extension of phase space is needed in this formalism. So the physical contents of all the gauge invariant actions remain the same. In [39, 40], the authors gave a reasonably general theory relating to a large class of systems with second class constraints to corresponding class of gauge invariant systems having the same dynamical content. A gauge theory in a generalized sense means a theory with some first class constraints. To covert it into an equivalent second class system is well known. One generally fix the gauge, i.e., impose a suitable number of gauge fixing conditions. These gauge fixing conditions together with the original first class set of constraint form a second class set and the theory gets converted into an equivalent second class system. An inverse procedure is suggested in [39, 40] where a formalism is developed for construction of a gauge invariant system equivalent to a given second class theory. The authors argued there as follows. If a dynamical system possess 2n constraints and the constraints all together form a second class set and if n of these constraints are found to have mutually vanishing Poission brackets then these n constraints can be used as gauge generator of the gauge invariant reformulation. The remaining n constraints may be thought of as the gauge fixing condition. The Hamiltonian needs the required modification accordingly. So in [39, 40] the authors suggested to reduce half of the constraint from a second class set of constraint retaining the first class set only in order to get the gauge invariant reformulation. The obtained gauge invariant theory can be treated in the similar way as any standard gauge invariant theory is treated. What follows next is the application of the formalism in the presently considered mode. To apply this formalism in a model it is essential to know the constraint structure of that theory. In our case which is already given in Chapter 2.

Now we are going to describe the gauge invariant reformulation of chiral Schwinger model with Faddeevian anomaly using Mitra Rajaraman's formalism, the investigation in that respect was carried out in [10]. The lagrangian of chiral Schwinger model with Faddeevian anomaly in terms of chiral boson gets the following shape

$$\mathcal{L}_{CH} = \dot{\phi}\phi' - \phi'^2 + 2e\phi'(A_0 - A_1) - 2e^2 A_1^2. \tag{72}$$

In Chapter 2, we found that the theory under consideration contains four constraints in its phase space. Precisely, the constraints were

$$\omega_1 = \pi_\phi - \phi' \approx 0, \tag{73}$$

$$\omega_2 = \pi_0 \approx 0, \tag{74}$$

$$\omega_3 = \pi_1 + 2e\phi \approx 0, \tag{75}$$

$$\omega_4 = -(A_1 + A_0) \approx 0. \tag{76}$$

The combination $\omega_2 \approx 0$ and $\omega_3 \approx 0$ form a first class set. If we retain only these two constraints as stated above, following the suggestion available in [39, 40], we require a modification of the Hamiltonian density of the second class system (38) in the following manner in order to get a first class system.

$$\begin{aligned} \mathcal{H} &= \frac{1}{2}\pi_1^2 + \pi_1 A_0' - e(A_0 - A_1)\phi' + 2e^2 A_1^2 + \pi_\phi \phi' - e\pi_\phi(A_0 - A_1) \\ &\quad + e(\pi_\phi - \phi')(A_0 + A_1) + \frac{1}{2}(\pi_\phi - \phi')^2 + u\pi_0. \end{aligned} \tag{77}$$

The modification certainly keeps the physical contents of the theory intact. This modified Hamiltonian density (77) contains only the two first class constraints $\omega_2 \approx 0$ and $\omega_3 \approx 0$. The equation of motion with respect to the Hamiltonian (77) are found out as follows

$$\dot\phi = [\phi,\ H] = \pi_\phi + 2eA_1, \tag{78}$$

$$\dot A_0 = [A_0,\ H] = -u, \tag{79}$$

$$\dot A_1 = [A_1,\ H] = \pi_1. \tag{80}$$

We have kept the first class constraints only modifying the system accordingly and we have got desired first class lagrangian from the modified Hamiltonian (77)

$$\begin{aligned} L_1 &= \int dx[\pi_\phi \dot\phi + \pi_1 \dot A_1 + \pi_0 \dot A_0 - [\frac{\pi_1^2}{2} + \pi_1 A_0' + 2eA_1\pi_\phi + \pi_\phi\phi' - 2eA_0\phi' \\ &+ \frac{1}{2}(\pi_\phi - \phi')^2 + u\pi_0 + 2e^2A_1^2]. \end{aligned} \tag{81}$$

Using equation (78),(79),(80) the lagrangian density (81) can be converted to a very simplified form

$$L_2 = \frac{1}{2}(\dot\phi^2 - \phi'^2) - 2e(A_1\dot\phi - A_0\phi') + \frac{1}{2}(\dot A_1 - A_0')^2. \tag{82}$$

The lagrangian density (82), is consistent with the Hamiltonian density (77), and the equations of motion (78), (79) and (80). To see whether the lagrangian density (82) stems out from the modified Hamiltonian density (77) contains only the two first class constraints (74) and (75) in its phase space let us calculate the momenta corresponding to the field A_0

$$\pi_0 = \frac{\partial L_2}{\partial \dot{A}_0} = 0. \tag{83}$$

It gives back the primary constraint (74) and the preservation of this once again gives the Gauss law constraint

$$G = \pi_1' + 2e\phi' \approx 0. \tag{84}$$

No other constraints come out from the preservation of (84). These two first class constraints help us to construct the gauge transformation generator. The generator is given by

$$G = \int dx(\lambda_1\omega_1 + \lambda_2\omega_2). \tag{85}$$

Here λ_1 and λ_2 are two arbitrary parameters. The transformations evolved out of the generator (85) for the fields ϕ, A_1 and A_0 respectively are

$$\delta\phi = 0, \qquad \delta A_1 = -\lambda_1', \qquad \delta A_0 = -\lambda_2. \tag{86}$$

The lagrangian (82) is found to remain unchanged under the transformation (86) if the parameter satisfy the following relations .

$$\lambda_2 = \dot{\lambda_1}. \tag{87}$$

A note worthy thing is that this transformation is equivalent to the transformation $A_\mu \rightarrow A_\mu + \frac{1}{2e}\partial_\mu\lambda$. There is some thing interesting that we must mention here. The first class lagrangian that comes out from our investigation is the bosonized lagrangian of the well known vector Schwinger model [54, 55]. Here coupling strength is 2e. It does not come as a great surprise because the theoretical spectrum of the model under consideration is identical to the vector Schwinger model. To be precise, both the models contain the massive boson with mass m = 2e. We have mentioned earlier that the gauge invariant reformulation follows from this prescription depends crucially on the constraint structure of the model. There are other possibilities to get first class set of constraints from the set of constraints (73), (74), (75) and (76). However that possibilities fail to give consistent first class theories.

3.4 Comparison of the Result Obtained in Section 3.3 with the Gauge Invariant Chiral Schwinger Model for $a = 2$

Let us compare our result with the work of the Shatashvili [36] because seeing their apparent similarities at a first glance one may think that these two results are identical. But a careful look revels that this is not so. In his work Shatashvili considered the non-Abelian gauge invariant version of the chiral Schwinger model and showed that the interacting degrees of freedom gets reduced if the choice a $= 2$ is made. For a $= 2$, the mass term of Shatashvili's model become identical to our model but there lies a basic difference which we would like to address. Here we consider the gauge invariant Abelian bosonized version of that model [36] because this version would be compatible for comparison with our work. Unlike the non- Abelian version the Abelian version of it is exactly solvable.
It is described by the lagrangian density

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + e(g^{\mu\nu} - \epsilon^{\mu\nu})\partial_\mu\phi A_\nu + \frac{1}{2}ae^2 A_\mu A^\mu \\ &- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + L_{WESS} \end{aligned} \tag{88}$$

where L_{WESS} is given as follows

$$L_{WESS} = \frac{1}{2}(a-1)(\partial_\mu\eta)(\partial^\mu\eta) + e[(a-1)g^{\mu\nu} + \epsilon^{\mu\nu}]\partial_\mu\eta A_\nu. \tag{89}$$

The lagrangian is invariant under the gauge transformation $A_\mu \to A_\mu + \frac{1}{e}\partial_\mu\Lambda$, $\phi \to \phi + \Lambda, \quad \eta \to \eta - \Lambda$. The momenta corresponding to the fields A_0, A_1, ϕ and η are

$$\frac{\partial\mathcal{L}}{\partial\dot{A}_0} = \pi_0 = 0, \tag{90}$$

$$\frac{\partial\mathcal{L}}{\partial\dot{A}_1} = \pi_1 = \dot{A}_1 - A_0', \tag{91}$$

$$\frac{\partial\mathcal{L}}{\partial\dot{\phi}} = \pi_\phi = \dot{\phi} + e(A_0 - A_1), \tag{92}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\eta}} = \pi_\eta \;\; = \;\; (a-1)\dot{\eta} + e[(a-1)A_0 + A_1]. \tag{93}$$

Equation (90), (91) and (92) are independent of the parameter a. The choice a = 2 brings change only in (93) and with that choice that turns into

$$\pi_\eta = \dot{\eta} + e(A_0 + A_1). \tag{94}$$

The canonical Hamiltonian density for the model with $a = 2$ is

$$\begin{aligned} \mathcal{H} \;\; = \;\; & \frac{1}{2}[\pi_1^2 + \pi_\phi^2 + \phi'^2] - eA_1(\pi_\phi - \phi') + 2e^2 A_1^2 + \frac{1}{2}[\pi_\eta^2 + \eta^2] - eA_1(\pi_\eta \\ + \;\; & \eta') - A_0\pi_1' + e(\pi_{\phi'} - \phi') - e(\pi_\eta + \eta'). \end{aligned} \tag{95}$$

The phase space of the model contains the following two constraints [27]

$$\Omega_1 = \pi_0 \approx 0, \tag{96}$$

$$\Omega_2 = \pi_1' + e(\pi_\phi - \phi') - e(\pi_\eta + \eta') \approx 0. \tag{97}$$

The constraint (97) appears as a secondary constraint in order to preserve the constraint (96). The two constraints are first class. The first class constraints shows a clear indication of reduction of degrees of freedom because to quantize the theory two gauge fixing conditions are to be needed. Bosonized version of vector Schwinger model (82), appeared out as the gauge invariant version of chiral Schwinger model with Faddeevian anomaly in previous Section, contains the following two constraint

$$\omega_{VS1} = \pi_0 \approx 0, \tag{98}$$

$$\omega_{VS2} = \pi_1 + 2e\phi \approx 0. \tag{99}$$

The Hamiltonian density of this bosonized version of vector Schwinger model (82) comes out to be

$$H_{VS} = \frac{1}{2}(\pi_1^2 + \pi_\phi^2 + \phi'^2) + \pi_1 A_0' + 2e(A_1\pi_\phi - A_0\phi'). \tag{100}$$

It is true that both the models are gauge invariant and the massive fields which comes out form (100) and (95) looks almost identical. Square of the mass of the boson in each case is $m^2 = 4e^2$. However the Hamiltonian (95) cannot be made free from Wess-Zumino field η using the constraints (96) and (97) and the constraints (98) and (99) also do not map on to the constraints of the vector Schwinger model. On the contrary the Gauge invariant version as obtained in (82), using Mitra-Rajaraman prescription, does not contain this type of field. Here gauge invariance is resulted in the usual phase space.

3.5 Discussion

Gauge invariant reformulation of chiral Schwinger model with Faddeevian anomaly has been carried out using Mitra-Rajaraman's prescription [39, 40]. Here gauge invariance takes place in the usual phase space and that is a special feature of Mitra-Rajaraman's prescription. However, in this situation we have to be satisfied with the gauge invariant reformulation only because the formalism developed till now is not adequate to obtain BRST invariant action of the gauge fixed version of the gauge invariant reformulation obtained through this formalism. In spite of the existence of more than one possibilities, only a particular possibility leads to a gauge invariant action in this situation. Surprisingly, the other possibilities fail to do so. Only that possibility has explored to obtain gauge invariant reformulation which renders a very interesting result. The gauge invariant model that comes out is found to be identical to the lagrangian of the well known vector Schwinger model and gauge invariance of which is obvious. It is explicitly shown here too. It is true that the gauge non invariant version of this model under consideration too contains a massive boson like vector Schwinger model [54, 55]. We have already mentioned it. But the exact mapping of this model onto the vector Schwinger model is an interesting and novel findings. The counting of degrees of freedom also found to be consistent. It would be interesting to investigate how a particular Faddeevian regularized version of the chiral Schwinger model maps onto the vector Schwinger model in its gauge invariant version. We compare the gauge invariant lagrangian of chiral Schwinger model contains Faddeevian type of anomaly with the gauge invariant version of the Abelian chiral Schwinger model setting a = 2. Both the model is gauge invariant and contains a massive field with the same mass. But for the former one gauge invariance has occurred in its usual phase space whereas for the later it does occur in the extended phase space.

Chapter 4

4 Study of a Constrained Field Theoretical Model where Vector and Axial Vector Interaction Get Mixed up with Different Weight

4.1 Introduction

In terms of fundamental interaction, Quantum Electrodynamics (QED) in (1+1) dimension can be categorized into two different classes. The first way of description was originated from vector type of interaction between matter and gauge fields. The models which belong to this class are well known vector Schwinger model [54, 55] and Thiring-Wess model [56]. The other way of description originated from chiral interaction between matter and gauge fields. Chiral Schwinger model [32] along with its different variants [9, 35, 57, 58] and chiral Thiring-Wess model [59, 60] are the example of this class. In the chiral Schwinger model [32] and in its different variants [9, 35, 57, 58] we find that vector and axial vector interaction get mixed up with equal weight. Few years ago, the authors in [41] presented a model where unlike chiral Schwinger model, vector and axial vector interactions did not mix up with equal weight. Few extensions over this model are also found in [42, 43]. The mixing of interaction with different weight may be regarded as a generalized version of QED (GVQED) which covers all the fundamentally different interaction and their mixing [8, 35, 54, 56]. The beauty of this model is that it is capable of interpolating both the QED and chiral QED. Both the Schwinger model [54] and the chiral Schwinger model [32] can be achieved through the different choices of its mixing weight factor of interaction. For unit weight factor it describes the chiral Schwinger model [32] and for vanishing weight it describes the vector Schwinger model [54].

Standard quantization scheme furnishes that these two models are fundamentally different so far theoretical spectrum and confinement aspect of fermion are concerned [33, 34, 55]. Needless to mention that Schwinger model [54], and its chiral generation, e.g., chiral Schwinger model [32, 61] and the GVQED as presented in [41, 42, 43] which covers the both into its own, are of considerable interest because of their ability to describe different physical aspects which are found to exist even in (3+1) dimension. Schwinger model acquired popularity not only for its ability of describing mass generation via dynamical symmetry breaking [54, 55] but also it can describe the confinement aspect of fermion in lower dimension [54, 55] which is a real (3+1) dimensional phenomena of QCD. On the other hand, chiral Schwinger model is capable of describing mass generation as well like vector Schwinger model [54, 55], however fermions are found to get liberated here which can be considered as lower dimensional de-confining state of fermion [8, 32, 35]. Since the GVQED presented in [41, 42, 43] interpolates both the Schwinger model and chiral generation of that, it is natural that all the surprises involved within the Schwinger model and chiral Schwinger model lies significantly in this GVQED. All these models along with the GVQED are so rich in describing, different surprises like dynamical mass generation, confinement and de-confinement aspects of fermion, that till now investigation over these models has been carried out [57, 58, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84] and these models still remains as a fertile field to carry out further investigations. Our objective in this work is to carry out few investigations over the GVQED coined in [41] concerning the Poincar'e and gauge symmetry. An attempt is also made here to single out the real physical canonical pairs embedded within the phase space of the system. It is true that a systematic quantization of this model is available in [41], however the definite identification of real physical canonical pairs lying within the phase space is found to be absent. In order to make it a compliment to the quantization part of the work [41], again quantization of this model has been pursued using Dirac's scheme of quantization of constrained system.

4.2 Brief Review of the Model

A Model where we find both vector and axial vector interaction get mixed up with different weight is given by the following generating functional [41]

$$Z(A) = \int d\psi d\bar{\psi} exp[i \int d^2x \mathcal{L_F}]. \tag{101}$$

with $\mathcal{L_F} = \bar{\psi}\gamma^\mu[i\partial_\mu + e\sqrt{\pi}A_\mu(1 - r\gamma_5)]\psi$. The integration over the fermionic degrees of freedom ψ leads to a determinant which is singular in nature [33, 34]. In order to remove the singularity we need to regularize the theory. After proper regularization if we express the fermionic determinant in terms of auxiliary scalar field ϕ, we get

$$Z(A) = \int d\phi exp[i \int d^2x \mathcal{L_B}]. \tag{102}$$

with

$$\begin{aligned} \mathcal{L_B} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + e\epsilon_{\mu\nu}\partial^\nu\phi A^\mu + erg_{\mu\nu}\partial^\nu\phi A^\mu \\ &+ \frac{e^2}{2}(\alpha A_0^2 + 2\beta A_0 A_1 + \gamma A_1^2). \end{aligned} \tag{103}$$

where $\tilde{\partial} = \epsilon_{\mu\nu}\partial^\nu$ and $\epsilon^{01} = 1$. A generalized masslike term has been included here as counter term in place of standard $\frac{1}{2}ae^2 A_\mu A^\mu$ term since we are intended to study whether any other alternative masslike term can serve as a physically sensible counter term for regularization like the chiral Schwinger model [8, 9, 57, 58]. The parameters α, β and γ, therefore, stand as the regularization ambiguity parameter. Needless to mention that in this situation ambiguity emerged out during the process of regularization in order to remove the divergence of the fermionic determinant. If we now take into account the kinetic term of the back ground electromagnetic field the lagrange density then turns into

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + e\epsilon_{\mu\nu}\partial^\nu\phi A^\mu + erg_{\mu\nu}\partial^\nu\phi A^\mu \\ &+ \frac{e^2}{2}(\alpha A_0^2 + 2\beta A_0 A_1 + \gamma A_1^2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \end{aligned} \tag{104}$$

What follows next is the invariant property of the theory described by the lagrangian (104).

4.3 Lorentz Transformation of the Fields and the Requirement to be the Physically Sensible

Investigation has been carried out over the GVQED [41] to study the Poincar'e symmetry in [26]. An attempt is also made to single out the real physical canonical pairs embedded within the phase space of the theory. Starting with the generalized masslike term we now proceed to investigate which type of term leads to a physically sensible theory. The word physically sensible implies a structure that not only maintains physical Lorentz invariance but also leads to an exactly solvable nature at the same time. To this end, we would like to study the Lorentz transformation property of the fields and the Poincar'e algebra of the theory in an explicit manner following the guideline available in [8]. In this context we need to calculate the momenta of the fields describing the theory. From the standard definition the momenta corresponding to the fields ϕ , A_0 and A_1 are found out:

$$\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \pi_\phi = \dot{\phi} - eA_1 + erA_0, \tag{105}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{A}_0} = \pi_0 = 0, \tag{106}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{A}_1} = \pi_1 = \dot{A}_1 - A_0'. \tag{107}$$

For this theory $\omega_1 = \pi_0 \approx 0$, is the primary constraint. A Legendre transformation leads to the the following canonical Hamiltonian density.

$$\begin{aligned} \mathcal{H}_\mathcal{C} &= \frac{1}{2}(\pi_\phi^2 + \phi'^2 + \pi_1^2) + \pi_1 A_0' + \frac{1}{2}e^2(A_1 - rA_0)^2 \\ &+ e\pi_\phi(A_1 - rA_0) - e(A_0\phi' - rA_1\phi') \\ &- \frac{e^2}{2}(\alpha A_0^2 + 2\beta A_0 A_1 + \gamma A_1^2). \end{aligned} \tag{108}$$

Time evolution of primary constraint with respect to the Hamiltonian gives a secondary constraint,

$$\omega_2 = \pi_1' + e^2(\alpha - r^2)A_0 + e^2(r + \beta)A_1 + er\pi_\phi + e\phi' \approx 0. \tag{109}$$

The constraints are all weak conditions at this stage. To impose it as a strong condition into the system we need to have the expression of A_0 . Equation (109), gives

$$A_0 = -\frac{1}{e^2(\alpha - r^2)}(\pi_1' + er\pi_\phi + e\phi' + e^2(r+\beta)A_1). \tag{110}$$

Inserting the expression of A_0 in equation (108) we get the following reduced Hamiltonian.

$$\begin{aligned} H_R &= \int dx[\frac{\pi_1^2}{2} + \frac{\pi_1'^2}{2e^2(\alpha - r^2)} + \frac{1}{2}\frac{\alpha\pi_\phi^2}{(\alpha - r^2)} + \frac{e^2}{2}[(1-\gamma) + \frac{(\beta + r)^2}{(\alpha - r^2)}]A_1^2 \\ &+ \frac{(1+\alpha - r^2)}{(\alpha - r^2)}\frac{\phi'^2}{2} \\ &+ \frac{(\beta + r + \alpha r - r^3)}{(\alpha - r^2)}eA_1\phi' + \frac{(\alpha + \beta r)}{(\alpha - r^2)}eA_1\pi_\phi \\ &+ \frac{(\beta + r)}{(\alpha - r^2)}\pi_1' A_1 + \frac{r}{(\alpha - r^2)}\phi'\pi_\phi \\ &+ \frac{\phi'\pi_1'}{e(\alpha - r^2)} + \frac{r}{e(\alpha - r^2)}\pi_\phi\pi_1']. \end{aligned} \tag{111}$$

For this reduced Hamiltonian the ordinary Poission brackets become inadequate [2]. So it becomes essential to calculate the Dirac brackets between the fields describing the Hamiltonian to proceed further. The Dirac bracket [1] between the two variables A and B is defined by

$$[A(x), B(y)]^* = [A(x), B(y)] - \int [A(x), \omega_i(\eta)]C_{ij}^{-1}(\eta, z)[\omega_i(\eta), B(y)]d\eta dz, \tag{112}$$

where C_{ij}^{-1} is given by

$$\int C_{ij}^{-1}(x, z)[\omega_i(z), \omega_j(y)]dz = 1. \tag{113}$$

Here $\omega_i's$ represents the second class constraints that remains embedded within the phase space of the theory. The matrix C_{ij}^{-1} for the theory under consideration is

$$C_{ij}^{-1} = \frac{1}{e^2(\alpha - r^2)}\begin{pmatrix} 0 & \delta(x-y) \\ -\delta(x-y) & 0 \end{pmatrix}. \tag{114}$$

Our task becomes little easier since it is found that the Dirac brackets between the fields remains canonical.

$$[A_1(x), \pi_1(y)]^* = \delta(x-y), \quad (115)$$

$$[\phi(x), \pi_\phi(y)]^* = \delta(x-y), \quad (116)$$

$$[A_1(x), \phi(y)]^* = 0. \quad (117)$$

The reduced Hamiltonian can be expressed in the following form

$$\begin{aligned} H_R &= \int dx[\frac{1}{2}(\pi_\phi^2 + \phi'^2 + \pi_1^2) + \frac{e^2}{2}(1-\gamma)A_1^2 \\ &+ \frac{1}{2e^2(\alpha - r^2)}(\xi^2 + 2\partial_1(\xi\pi_1)) + eA_1(\pi_\phi + r\phi')]. \end{aligned} \quad (118)$$

The total momentum and the boost generator in (1+1) dimension are defined by

$$P = \int dx[\pi_\phi\phi' + \pi_1 A_1' + \pi_0 A_0']. \quad (119)$$

$$M = t(\pi_\phi\phi' + \pi_1 A_1' + \pi_0 A_0') + \int dx[xH_R + \pi_1 A_0 + \pi_0 A_1]. \quad (120)$$

In the reduced phase space that is in the constrained subspace the equations (119) and (120) reads

$$P_R = \int dx[\pi_\phi\phi' + \pi_1 A_1']. \quad (121)$$

$$M_R = t(\pi_\phi\phi' + \pi_1 A_1') + \int dx[xH_R - \frac{\xi}{e^2(\alpha - r^2)}\pi_1], \quad (122)$$

where

$$\xi = \pi_1' + er\pi_\phi + e\phi' + e^2(r+\beta)A_1. \quad (123)$$

and the total Hamiltonian H_R and the Hamiltonian density $\mathcal{H_R}$ are related by $H_R = \int dx \mathcal{H_R}$. The momentum operator P_R transform the fields within the constrained subspace. Similarly, the Hamiltonian operator H_R generate the time translation of the same. The time translation of the fields are given by

$$\dot{\phi} = \pi_\phi + eA_1 + \frac{r}{e(\alpha - r^2)}\xi, \quad (124)$$

$$\dot{A}_1 = \pi_1 - \frac{1}{e^2(\alpha - r^2)}\xi'. \tag{125}$$

However, the most interesting one is the action of the Lorentz-boost generator M_R, on the fields in the constrained subspace. We now turn to observe that. Let us now see how the fields get transformed under the Loentz-boost. Calculating the Poission brackets of the fields ϕ and A_1 with the Lorentz-boost and expressing these in terms of $\dot{\phi}$ and $\dot{A}_1$ using equation (124) and (125), we find the expected transformation of the fields ϕ and A_1 under the Lorentz-boost.

$$[\phi, M_R] = t\phi' + x\dot{\phi}, \tag{126}$$

$$[A_1, M_R] = tA_1' + x\dot{A}_1 + A_0. \tag{127}$$

With the use of the above transformation rules (126) and (127), and the Dirac brackets (115), (116) and (117), it is straight forward to see that the following Poincare algebra

$$[P_R, H_R]^* = 0, \tag{128}$$

$$[P_R, M_R]^* = H_R, \tag{129}$$

$$\begin{aligned}[H_R, M_R]^* &= \pi_\phi\phi' + \frac{2e^2\beta\xi^2}{e^2(\alpha - r^2)} + \frac{2e^2\beta\xi\pi_1}{e^4(\alpha - r^2)} \\ &+ [\frac{(1-\gamma)}{(\alpha - r^2)} - \frac{(r^2+1)}{(\alpha - r^2)}]\pi_1 A_1' = P_R. \end{aligned} \tag{130}$$

is satisfied if and only if $\beta = 0$ and $\alpha = -\gamma$ [26]. We should mention here that it is valid only for the very structure of the constraints which are given in equation (106) and (109). If we set $\alpha = r^2$, the constraint structure will get altered and in that case total scenario will be different. In fact, the number of constraint will be greater than two in this situation like the Faddeevian [52, 53] class of regularization of chiral Schwinger model [8, 9, 57, 58]. To study the aforesaid situation let us set $\alpha = r^2$ and carry out the Poincar'e algebra for this special case. The constraint ω_2 now takes the form

$$\tilde{\omega_2} = \pi_1' + e^2(r + \beta)A_1 + er\pi_\phi + e\phi'. \tag{131}$$

The effective Hamiltonian of this theory in the present situation can be written down as

$$H_{eff} = H + v\tilde{\omega}_2 + u\tilde{\omega}_1. \tag{132}$$

The consistency of $\tilde{\omega_2}$ with time requires $\dot{\tilde{\omega_2}} = 0$, which fixes the velocity v. The velocity v is found out to be

$$v = A_0 + \frac{\gamma + r^2}{2\beta} A_1. \tag{133}$$

With this velocity v the $[\tilde{\omega_2}, H(y)]$ gives birth of a new constraint

$$\tilde{\omega_3} = (r + \beta)\pi_1 + 2\beta A_0' + (\gamma + r^2) A_1'. \tag{134}$$

So in the present situation, three constraints are embedded in the phase space of the theory and the constraints are

$$\tilde{\omega_1} = \pi_0, \tag{135}$$

$$\tilde{\omega_2} = \pi_1' + e^2(r + \beta)A_1 + er\pi_\phi + e\phi', \tag{136}$$

$$\tilde{\omega_3} = (r + \beta)\pi_1 + 2\beta A_0' + (\gamma + r^2) A_1'. \tag{137}$$

The matrix constructed out of the Poission brackets within the constraints is $C_{ij} =$

$$\begin{pmatrix} 0 & 0 & 2\beta\partial_1 \\ 0 & -2e^2\beta\partial_1 & (r^2+\gamma)\partial_1^2 + e^2(r+\beta)^2 \\ 2\beta\partial_1 & -(r^2+\gamma)\partial_1^2 - e^2(r+\beta)^2 & 2(r+\beta)(r^2+\gamma)\partial_1 \end{pmatrix} \delta(x-y) \tag{138}$$

The Hamiltonian in the reduced phase space in this situation reads

$$\begin{aligned} H_r &= \int dx [\frac{(1+r^2)}{r^2}\frac{\phi'^2}{2} + \frac{\pi_1^2}{2} + \frac{1}{2e^2r^2}\pi_1'^2 + \frac{e^2}{2}(\frac{\beta^2}{r^2} - \gamma)A_1^2 \\ &+ e(r + \frac{\beta}{r^2})A_1\phi' + \frac{1}{er^2}\phi'\pi_1' + \frac{\beta}{r^2}\pi_1' A_1]. \end{aligned} \tag{139}$$

The Dirac brackets of the fields with which the reduced Hamiltonian is constituted with are computed as follows.

$$[A_1(x), A_1(y)]^* = \frac{1}{2e^2\beta}\delta'(x-y), \tag{140}$$

$$[A_1(x), \pi_1(y)]^* = \frac{(\beta + r)}{2\beta}\delta(x - y), \tag{141}$$

$$[\pi_1(x), \pi_1(y)]^* = \frac{e^2(r + \beta)^2}{4\beta}\epsilon(x - y), \tag{142}$$

$$[\phi(x), \pi_1(y)]^* = -\frac{er}{4\beta}(r + \beta)\epsilon(x - y), \tag{143}$$

$$[\phi(x), \phi(y)]^* = \frac{r^2}{4\beta}\epsilon(x - y). \tag{144}$$

Let us now proceed to calculate the Poincar'e algebra for this special situation. There are three elements in this algebra like the previous situation. One of the elements of course, is H_r, which is given in equation (139), and the rest of the two are two are the total momentum and the boost generator. These two respectively are

$$\bar{P} = \int dx[\pi_\phi \phi' + \pi_1 A_1' + \pi_0 A_0']. \tag{145}$$

and

$$\bar{M} = t(\pi_\phi \phi' + \pi_1 A_1' + \pi_0 A_0') + \int [xH_R + \pi_1 A_0 + \pi_0 A_1]dx, \tag{146}$$

In the constrained subspace these two reduce to

$$\bar{P}_r = \pi_\phi \phi' + \pi_1 A_1', \tag{147}$$

and

$$\bar{M}_r = t(\pi_\phi \phi' + \pi_1 A_1') \int xH_R dx - \pi_1[\frac{1}{2\beta}(r + \beta)\partial^{-1}\pi_1 + \frac{(\gamma + r^2)}{2\beta}A_1], \tag{148}$$

respectively. The the action of the Lorentz-boost generator M_r on the fields in the constrained subspace for this case are

$$[\phi, M_r] = t\phi' + x\dot{\phi}, \tag{149}$$

$$[A_1, M_r] = tA_1' + x\dot{A}_1 + A_0, \tag{150}$$

$$[A_0, M_r] = tA_0' + x\dot{A}_1 + A_1. \tag{151}$$

With the use of the above transformation rules (149) (150) and (151) and the Dirac brackets (140), (141), (142), (143) and (144) to see that the following Poincare algebra

$$[P_R, H_R]^* = 0, \tag{152}$$

$$[P_R, M_R]^* = H_R. \tag{153}$$

$$[H_R, M_R]^* \tag{154}$$

$$\begin{aligned}
&= -\frac{\pi'\phi'}{er} - e^2\frac{(r+\beta)}{er}A_1\phi' - \frac{e}{er}\phi'\phi' \\
&+ [\frac{(\gamma+r^2)}{2\beta}((\gamma+r^2) + \frac{2\beta}{r}) + \frac{(\beta^2-\gamma r^2)}{r^4} \\
&+ \frac{(\gamma+r^2)}{r^2} - \frac{\beta^2}{r^2}]\pi_1 A_1' \\
&= \pi_\phi\phi' + \pi_1 A_1' = P_R.
\end{aligned} \tag{155}$$

holds if the conditions $r^2 = 1$ and $2\beta + r(1+\gamma) = 0$ are satisfied simultaneously. This result agrees with result available in [8, 9, 57, 58] for weight factor $r = -1$ with the choice of parameters $\beta = -1$ and $\gamma = -3$. The result also reminds the result obtained in [59]. At this point we would like to end up our the investigation through Poincar'e algebra on this model and would like to proceed with the Lorentz covariant mass like term for the gauge field (which of course is a result obtained from the Poincar'e algebra) and carry out investigation to shed light on some of the important facts those which would be of orth unraveling for this model.

4.4 Identification of the Real Physical Canonical Pair Using Dirac Quantization Scheme

Putting $\beta = 0$ and $\alpha = -\gamma = a$ (a condition for maintenance of Lorentz invariance) we get a Lorentz covariant masslike term for gauge field and the reduced Hamiltonian with this setting reads

$$H_R = \frac{\pi_1^2}{2} + \frac{\pi_1'^2}{2e^2(a-r^2)} + \frac{a\pi_\phi^2}{2(a-r^2)} + \frac{e^2}{2}\frac{a(1+a-r^2)}{(a-r^2)}A_1^2$$

$$
\begin{aligned}
&+ \quad \frac{(1+a-r^2)}{(a-r^2)}\frac{\phi'^2}{2} + \frac{r\phi'\pi_\phi}{(a-r^2)} + \frac{\phi'\pi_1'}{e(a-r^2)} \\
&+ \quad \frac{r\pi_\phi\pi_1'}{e(a-r^2)} + er\frac{(1+a-r^2)}{(a-r^2)}A_1\phi' + e\frac{a}{(a-r^2)}A_1\pi_\phi \\
&+ \quad \frac{r\pi_1' A_1}{(a-r^2)}. \qquad (156)
\end{aligned}
$$

Using Dirac bracket (115),(116) and (117) we get the following first order differential equations of motion for the fields describing the theory in the constrained subspace.

$$\dot{A}_1 = \pi_1 - \frac{1}{e^2(a-r^2)}\pi_1'' - \frac{r}{(a-r^2)}A_1' - \frac{\phi''}{e(a-r^2)} - \frac{r\pi_\phi'}{e(a-r^2)}, \qquad (157)$$

$$\dot{\pi}_1 = -e^2a\frac{(1+a-r^2)}{(a-r^2)}A_1 - er\frac{(1+a-r^2)}{(a-r^2)}\phi' - e\frac{a}{(a-r^2)}\pi_\phi - \frac{r}{(a-r^2)}\pi_1', \qquad (158)$$

$$\dot{\pi}_\phi = \frac{(1+a-r^2)}{(a-r^2)}\phi'' + e\frac{r(1+a-r^2)}{a-r^2}A_1' + \frac{r}{a-r^2}\pi_\phi' + \frac{\pi_1''}{e(a-r^2)}, \qquad (159)$$

$$\dot{\phi} = \frac{a}{(a-r^2)}\pi_\phi + \frac{ea}{(a-r^2)}A_1 + \frac{r}{e(a-r^2)}\pi_1' + \frac{r}{(a-r^2)}\phi'. \qquad (160)$$

Using equation (157),(158),(159) and (160) we have obtained second order differential equations

$$[\Box + e^2\frac{a(1+a-r^2)}{(a-r^2)}]\pi_1 = 0, \qquad (161)$$

$$\Box[\phi + e\frac{1}{(1+a-r^2)}\pi_1] = 0, \qquad (162)$$

$$\Box[\Box + e^2\frac{a(1+a-r^2)}{(a-r^2)}](A_1 + \frac{r}{ea}\phi') = 0, \qquad (163)$$

$$\Box(\frac{r}{ea}\pi_1' + \pi_\phi) = 0. \qquad (164)$$

Now a careful look reveals that within the above four equations (161), (162), (163) and (164) the theoretical spectra are hidden in a significant manner.

Note that the equation (161) describes a massive boson with square of the mass

$$m^2 = \frac{e^2 a(1 + a - r^2)}{(a - r^2)} \tag{165}$$

and equation (162) describes a massless boson which is equivalent to a free fermion in (1 + 1) dimension. So unlike the Schwinger model, fermions gets deconfined here. We have noticed that equation (163) and (164) describe the Klein-Gordon type equations for a massive and a massless excitation respectively. The fields describing equations (163) and (164), can be considered as the momenta corresponding to the fields satisfying equation (161) and (162). Note that the fields satisfying equation (161) and (163), satisfy canonical Poission brackets between themselves. Similarly, the fields satisfying equation (162) and (164), satisfy the same canonical condition. So our description gives a transparent picture not only for the theoretical spectrum but also for the physical canonical pairs of the phase space. Therefore this section, will certainly complement the quantization part of the work reported in [43]. We end up the discussion related to the theoretical spectra and identification of the real canonical pair of the gauge non-invariant version of the GVQED here.

4.5 Discussion

We have considered the GVQED coined in [41], with a generalized masslike term for gauge fields. It is added as a counter term to remove the divergence of the fermionic determinant. In this context, we should mention that all possible masslike term are not admissible as it gets restricted in order to be physically sensible, however massslike term may take some generic shape. It may even take a structure which looks Lorentz non-covariant however it does not stand as a hindrance in the way of the theory to be exactly physical Lorentz invariant [9, 35, 57, 58]. In this respect, an investigation through the Poincar'e algebra has been carried out using a generalized masslike term for the gauge field. The algebra has imposed some condition on the parameters used in the generalized masslike term and on the weight factor of mixing. In fact, we have found two possibilities. In the first case it does not put any restriction on the weight factor of mixing, however, it suggests a restriction that admits the Lorentz covariant structure of the masslike term. No other masslike term is admissible for this theory as long as its phase space contains two constraints. In the second case, i.e., when $\alpha = r^2$, it imposes restriction on both the weight factor and the parameters within the masslike term simultaneously. We have found that the number of constraint in this situation is more than two and the masslike term is of Lorentz noncovariant in nature. It is worth mentioning here that the mixing weight $r \neq 1$ fails to provide any physically sensible theory having Lorentz non-covariant masslike term. With the admissible masslike term obtained from the first possibility of the Poincar'e algebra we quantize the theory using the Dirac's scheme of quantization of constrained system. The result though was known from the work available in [41] that the theoretical spectrum contains a massive and massless boson, nevertheless a more transparent calculation has been presented here with the identification of real canonical pairs of the phase space. Massive boson as usual can be considered as photon acquires mass via a dynamical symmetry breaking. On the other hand, the massless boson of the theoretical spectrum may be considered as free fermion. So fermion gets liberated here which can be thought of as de-confinement in lower dimension. So the model may be useful to study the lower dimensional QGP phenomena. The quantization of the theory with masslike term as obtained in the second possibility may get a ready idea form the work of one of us [60], with few redefinitions of the parameter used there.

Chapter 5

5 Study of BRST Symmetry of Few Field Theoretical Models

5.1 Introduction

Quite often dynamical equations of physical system cannot be described in terms of observable physical degrees of freedom. As a result the physical interpretation of evaluation equation cannot be done in a straightforward manner. In some cases, certain solution needs to be excluded since they do not describe the real physical situation or it may be the case that certain class of apparently different solutions are physically indistinguishable. BRST formalism has been developed specifically to deal with such system. BRST is a technique to enlarge the phase space of a gauge theory and to restore the symmetry of the gauge fixed action in the extended phase space keeping the physical contents of the theory intact. The unphysical ghost field acquires prominent status bringing back the symmetry of the gauge fixed action preserving unitarity in a significant manner. Since this symmetry mixes all the fields (physical and ghost) in such a way that ghost field along with the other fields needs to be treated on the same footing and that forces to regard the ghost field along with all the other field as a different component of a single geometrical object.

BRST formalism provide a natural framework of covariant quantization of field theoretical models and is interesting in its own right since it ensures unitarity and renormalizability of the theory [5, 6, 7]. So BRST invariant reformulation of any field theoretical model would be interesting and add new contribution to the field theoretical regime. We, therefore, carry out BRST quantization of three interesting field theoretical models. The models are 1. A generalized (1+1) dimensional quantum electrodynamical model where axial and vector interaction get mixed up with different weight [41, 42, 43] 2. Chiral Schwinger model with Fadeevian anomaly [8, 9] and 3. Gauged model of Floreanini-Jackiw type chiral Boson [51]. BRST invariant reformulation of these models have been done with the help of Batalin, Fradkin and Vilkovisky (BFV) formalism [44, 45, 46, 47, 48] .

The scheme developed by Batalin, Fradkin and Vilkovisky [44, 45, 46, 47, 48] towards the conversion of a set second class constraint into first class set helps to get this symmetric transmuted form. It is known that for the above transmutation some extra fields are needed. These fields are known as auxiliary fields [44, 45, 46, 47, 48, 82, 83]. These auxiliary fields turn into Wess-Zumino scalar [4] with appropriate choice of gauge fixing conditions for some favorable situations. In fact, we have used the improved version presented by Fujiwara and Igarishi and Kubo (FIK) [46], since it is known that it generally helps to obtain the Wess-Zumino [4] action associated with the model in most of the cases. What follows next are systematic description of BRST quantization of the said three models which we have done in [10, 11, 12]. We are going to give the detailed description of these one by one with a brief introduction of BFV formalism.

5.2 BRST Invariant Reformulation using BFV Formalism

Let us start with the brief introduction of the BFV formalism. BFV formalism consists of two steps. First step consists of converting the second class system to a first class system. Auxiliary fields are needed for this conversion. In the second step the ghost and anti-ghost fields are needed to be introduced and along with that few gauge fixing function are needed to be chosen. This allows one to define BRST charge and obtain BRST transformation of the fields.

We consider a canonical Hamiltonian described by the canonical pairs $q^i, p_i (i = 1, 2................n)$. The pairs are subjected to a set of constraints $\Omega_i(q^i, p_i) \approx 0$, $a = 1, 2, ..., n$, and it is assumed that the constraints satisfy the following algebra [48, 82].

$$[\Omega_a, \Omega_b] = i\Omega_c U^c_{ab}, \tag{166}$$

$$[H_c, \Omega_a] = i\Omega_b V^b_c. \tag{167}$$

where U^c_{ab} and V^b_c are structure coefficients. Then n number of additional condition $\phi^a \approx 0$ with $det[\phi_a, \Omega_b] \neq 0$ have to be imposed in order to single out the physical degrees of freedom. The constraints $\Omega_a \approx 0$ and $\phi_a \approx 0$ together with Hamiltonian equation of motion is obtained from the action

$$S = \int [p_i \dot{q^i} - H_c(p_i, q^i) - \lambda^a \Omega_a + \pi_a \phi^a] dt, \tag{168}$$

where λ^a, π_a are the Lagrange multiplier fields canonically conjugate to each other having the relation $[\lambda^a, \pi_a] = i\delta^a_b$ between themselves.
Now introducing a pair of canonical ghost field $(C^a \bar{P}_a)$ and a pair of canonical anti-ghost field $(P^a, \bar{C}_a)$ for each pair of constraints an equivalence can be made to the initial theory with constraints in the reduced phase space. So the quantum theory can be described by the partition function where the action [44, 45, 46, 47, 82, 83] in its exponent will be

$$S = \int dt [p_i \dot{q^i} + \pi_a \dot{\lambda^a} + \bar{P}^a \dot{C}_a + \bar{C}^a \dot{P}_a - H_m + i[Q, \psi]]. \tag{169}$$

H_m is the minimal Hamiltonian [44, 45], as termed by Batalin, Fradkin is defined by

$$H_m = H_c + \bar{P}_a V^a_b C^b. \tag{170}$$

The $BRST$ charge Q and the fermionic gauge fixing function ψ are respectively given by [48, 82, 83]

$$Q = C^a \omega_a - \frac{1}{2} C^b C_c U^c_{ab} \bar{P}^a + P^a \pi_a, \tag{171}$$

$$\psi = \bar{C}_c \chi_a + \bar{P}^a \lambda^a, \tag{172}$$

BRST invariant Hamiltonian is

$$H_{BRST} = H_m + \int dx [Q, \psi]. \tag{173}$$

where χ_a's are expressed through the gauge fixing condition

$$\Phi_a = \dot{\lambda_a} + \chi_a. \tag{174}$$

In order to show the equivalence between the BFV and the reduced phase space quantization, one has to consider the quantum effects associated with the ghosts and the pure degrees of freedom they mutually cancel each other.

5.3 Study of BRST Quantization of GVQED

Before describing BRST quantization let us start with the brief introduction of the model GVQED. It is a (1+1) dimensional model where vector and axial vector mixes with different weight [41, 42, 43]. The most interesting feature of this model is its ability to interpolates the vector schwinger model [54] and chiral Schwinger model [32] through its mixing weight factor. Schwinger model started a glorious journey for its potential of describing the mass generation along with its ability to describe the confinement aspect of fermion in lower dimension. Chiral generalization of this model too has been studied with great interest after the removal of its unitarity problem by Jackiw and Rajaraman [32, 63, 64, 67, 68, 69, 70].

Recently, an attempts has been made by us in [12] to quantize both the gauge invariant and gauge noninvariant version of GVQED [41]. This model in its bosonized version does not posses the local gauge symmetry, since it becomes essential to take into account the anomaly to protect the unitarity of this model. Here mass generation takes place indeed, via a kind of dynamical symmetry breaking. However, unlike Schwinger model [54], here the fermions are found to get liberated which may be considered as de-confinement phase of fermions. We should mention here that the fermion are found to remain confined when the model turns into Schwinger model in absence of its axial interaction part. So naturally, the extension of the model coined in [41], which has the ability of combining these two models into a single structure would be of worth investigations. Besides, in order to protect unitarity, inclusion of anomaly become essential and it adds further interest in another direction, because one loop correction enters there holding the hand of anomaly. But it certainly breaks the local gauge symmetry. So the study related to the restoration of symmetry would be instructive.

Let us now proceed to describe the BRST invariant reformulation of this model which we have done in [12]. It has been done here by the use of Batalin, Fradkin and Vilkovisky (BFV) formalism. The bosonized lagrangian density for this theory is

$$\mathcal{L_B} \;=\; \frac{1}{2}(\dot{\phi}^2 - {\phi'}^2) + e(A_0\phi' - A_1\dot{\phi}) + er(A_0\dot{\phi} - A_1\phi')$$

$$+ \quad \frac{e^2}{2}a(A_0^2 - A_1^2) + \frac{1}{2}(\dot{A}_1 - A_0')^2. \tag{175}$$

We are now in a state to proceed towards the BRST invariant reformulation of the lagrangian given in (175). In order to proceed to that end, we need to know the constraint structure of the theory described by the lagrangian (175). The momentum corresponding to the fields ϕ, A_0 and A_1 respectively are

$$\frac{\partial \mathcal{L_B}}{\partial \dot{\phi}} = \pi_\phi = \dot{\phi} - eA_1 + erA_0, \tag{176}$$

$$\frac{\partial \mathcal{L_B}}{\partial \dot{A}_0} = \pi_0 = 0, \tag{177}$$

$$\frac{\partial \mathcal{L_B}}{\partial \dot{A}_1} = \pi_1 = \dot{A}_1 - A_0'. \tag{178}$$

The equation $\Omega_1 = \pi_0 \approx 0$, is identified as the primary constraint of the theory.
The canonical Hamiltonian is $H_c = \pi_1 \dot{A}_1 + \pi_\phi \dot{\phi} - L$.
By the Legendre transformation we obtain the following canonical Hamiltonian:

$$\begin{aligned} H_c &= \int dx[\pi_\phi \dot{\phi} + \pi_1 \dot{A}_1 - (\frac{1}{2}(\dot{\phi}^2 - \phi'^2) + e(A_0\phi' - A_1\dot{\phi}) \\ &+ er(A_0\dot{\phi} - A_1\phi') + \frac{e^2}{2}a(A_0^2 - A_1^2) + \frac{1}{2}(\dot{A}_1 - A_0')^2]. \end{aligned} \tag{179}$$

Putting the expression of $\dot{\phi}$ and $\dot{A}_1$ from equations (176) and (178) we obtain the simplified form of Hamiltonian

$$\begin{aligned} H_c &= \int dx[\frac{1}{2}(\pi_\phi^2 + \pi_1^2 + \phi'^2) + \pi_1 A_0' + e\pi_\phi(A_1 - rA_0) \\ &- \frac{e^2}{2}a(A_0^2 - A_1^2) + \frac{1}{2}e^2(A_1 - rA_0)^2 + e\phi'(rA_1 - A_0)]. \end{aligned} \tag{180}$$

The consistency of the primary constraint with respect to the time evolution leads to the secondary constraint

$$\Omega_2 = [\pi_0, H_c] \tag{181}$$

$$= \pi_1' + e^2(a - r^2)A_0 + e^2 rA_1 + e\phi' + er\pi_\phi \approx 0. \tag{182}$$

So the constraints that are embedded within the phase space of the theory are

$$\Omega_1 = \pi_0 \approx 0, \tag{183}$$

$$\Omega_2 = \pi_1' + e^2(a - r^2)A_0 + e^2 r A_1 + e(\phi' + r\pi_\phi) \approx 0. \tag{184}$$

Therefore, the effective Hamiltonian in this situation reads

$$H_{eff} = H_c + u\Omega_1, \tag{185}$$

where u is the Lagrange multipliers. The preservation of Ω_2 with respect to the Hamiltonian determines the velocity u as follows.

$$\dot{\Omega}_2 = [\Omega_2, H_{eff}] = e^2(a - r^2)u - e^2(a - r^2)A_1' + e^2 r\pi_1 = 0. \tag{186}$$

Equation (186) gives

$$u = A_1' - \frac{r}{(a - r^2)}\pi_1. \tag{187}$$

Therefore, substituting u in (185) we get

$$\begin{aligned} H_{eff} &= \frac{1}{2}(\pi_1^2 + \pi_\phi^2 + \phi'^2) - \frac{e^2}{2}a(A_0^2 - A_1^2) \\ &+ \frac{1}{2}e^2(A_1 - rA_0)^2 + e\phi'(rA_1 - A_0) \\ &+ e\pi_\phi(A_1 - rA_0) + \pi_0(A_1' - \frac{r}{(a - r^2)}\pi_1). \end{aligned} \tag{188}$$

These two constraints form a second class set as they gives nonvanishing Poission bracket between themselves

$$[\Omega_1, \Omega_2] = -e^2(a - r^2)\delta(x - y). \tag{189}$$

The closures of the constraints with respect to the Hamiltonian are

$$\dot{\Omega}_1 = [\Omega_1, H_{eff}] = \Omega_2, \tag{190}$$

$$\dot{\Omega}_2 = [\omega_2, H_{eff}] = \Omega_1'' - \frac{e^2 r^2}{(a - r^2)}\Omega_1. \tag{191}$$

For BRST invariant reformulation the system with second class constraints (190) and (191) are needed to be convert these into a first class set. In this respect, we introduce the auxiliary field θ and π_θ. This set of auxiliary fields satisfy the following canonical relation.

$$[\theta(x), \pi_\theta(y)] = i\delta(x-y). \tag{192}$$

The auxiliary fields are known as BF fields. With some suitable linear combinations of BF fields the second class constraints get convert into first class constraints in the following way.

$$\bar{\Omega_1} = \Omega_1 + e(a-r^2)\theta, \tag{193}$$

$$\bar{\Omega_2} = \Omega_2 + e\pi_\theta. \tag{194}$$

In general, the first class Hamiltonian consistent with the constraints will be the original Hamiltonian added with a polynomial of BF field. And the polynomial will be determined by the condition that the new first class constraints will satisfy the same time involution like the old second class set of constraint (183) and (184). The first class Hamiltonian in the extended phase space is found out as

$$\bar{H} = H_{eff} + H_{BF}. \tag{195}$$

where H_{BF} for this theory is found out to be

$$H_{BF} = \int dx[\frac{1}{2(a-r^2)}\pi_\theta^2 + \frac{1}{2}(a-r^2)\theta'^2 + \frac{1}{2}e^2r^2\theta^2]. \tag{196}$$

The constraints (193) and (194), $\bar{\Omega_1}$ and $\bar{\Omega_2}$ need to satisfy the same closures as satisfied by Ω_1 and Ω_2 for consistency:

$$[\bar{H}, \bar{\Omega_1}] = \bar{\Omega_2}, \tag{197}$$

$$[\bar{H}, \bar{\Omega_2}] = \bar{\Omega_1}'' - \frac{e^2r^2}{a-r^2}\bar{\Omega_1}. \tag{198}$$

We are now in a position to introduce the two pairs of ghost and anti-ghost fields $(C^i, \bar{P}_i)$ and $(P^i, \bar{C}_i)$. We also need a pair of multiplier fields (N_i, B_i). The fields satisfy the following canonical Poission bracket

$$[C^i, \bar{P}_i], [P^i, \bar{C}_i], [N^i, B_j] = i\delta^i_j\delta(x-y). \tag{199}$$

From the definition (170), we can write the BRST invariant Hamiltonian for the theory under the present situation:

$$H_{BRST} = H_{eff} + H_{BF} + \int [Q, \psi]dx + \bar{P}_a V_b^a C^b. \tag{200}$$

The BRST charge Q is a nilpotent operator that satisfies the equation

$$Q^2 = [Q, Q] = 0. \tag{201}$$

Here BRST charge Q and the fermionic gauge fixing function ψ are defined by

$$Q = \int dx(C^1\bar{\Omega}_1 + C^2\bar{\Omega}_2 + P^1 B_1 + P^2 B_2), \tag{202}$$

$$\psi = \int dx(\bar{C}_1 X^1 + \bar{C}_2 X^2 + \bar{P}_1 N^1 + \bar{P}_2 N^2). \tag{203}$$

Right now we have to fix up the gauge condition which is very crucial for getting appropriate Wess-Zumino term. It is found that these two very conditions only meet our need successfully.

$$X_1 = A_0, \tag{204}$$

$$X_2 = A_1' + \frac{\alpha}{2} B_2. \tag{205}$$

Let us now calculate the commutation relation in between BRST charge and gauge fixing function:

$$\begin{aligned} [Q, \psi] &= [B_i P^i + C^i \bar{w}_i, \bar{C}_j X^j + \bar{P}_j N^j] \\ &= B_1 X^1 + B_2 X^2 \\ &- P^1 \bar{P}_1 - P^2 \bar{P}_2 - C^1 \bar{C}_1 + C^2 \bar{C}_2'' \\ &+ \bar{\Omega}_1 N^1 + \bar{\Omega}_2 N^2. \end{aligned} \tag{206}$$

Using equation (200), BRST invariant Hamiltonian is obtained which is given by

$$H_{BRST} = H_{eff} + H_{BF} + \int [Q, \psi]dx + \bar{P}_2 C_1 + \bar{P}_1{}'' C_2 - \frac{e^2 r^2}{a - r^2} \bar{P}_1 C_2. \tag{207}$$

The generating functional for this system can now be written down as

$$Z = \int [D\mu] exp^{iS}. \tag{208}$$

where $[D\mu]$ is the Liouville measure in the extended phase space

$$[D\mu] = [d\phi][d\pi_\phi]\sum_{i=0}^{1}[dA_i][d\pi_i][d\eta][d\pi_\eta][d\theta][d\pi_\theta] \times \sum_{k=1}^{2}[dN^k][dB_k][dC^k],[d\bar{C}_k][dP^k],[d\bar{P}_k]. \quad (209)$$

There exists a simplification

$$\int d^2x(B_1N^1 + \bar{C}_1\dot{P}_1) = -i[Q\int d^2x\bar{C}_1\dot{N}^1]. \quad (210)$$

with the Legendre transformation $B^i = B^i + N^i$.
The action S in equation (208)reads

$$\begin{aligned} S &= \int d^2x[\pi_\phi\dot{\phi} + \pi_0\dot{A}_0 + \pi_1\dot{A}_1 + \dot{\theta}\pi_\theta + \dot{N}^1B_1 \\ &+ \dot{N}^2B_2 - H_{BRST}]. \end{aligned} \quad (211)$$

The explicit form of H_{BRST} lying in equation (211) is

$$\begin{aligned} H_{BRST} &= H_{eff} + H_{BF} - P^1\bar{P}_1 - P^2\bar{P}_2 - C^1\bar{C}_1 + C^2\bar{C}_2'' \\ &+ \bar{\Omega}_1N^1 + \bar{\Omega}_2N^2 + \bar{P}_2C_1 + \bar{P}_1{}''C_2 \\ &- \frac{e^2r^2}{a-r^2}\bar{P}_1C_2 + B_1A^0 + B_2(A_1' + \frac{\alpha}{2}B_2). \end{aligned} \quad (212)$$

To get the action in the desired form it is necessary to integrate out the fields $B_1, N^1, \pi_0, \pi_1, \pi_\phi, \bar{P}_1, C_1$ and $\bar{C}_1$. After integrating out of these fields we obtain a simplified form of the generating functional with the following action:

$$\begin{aligned} S &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + e\epsilon_{\mu\nu}A^\mu\partial^\nu\phi + erg_{\mu\nu}A^\mu\partial^\nu\phi \\ &+ \frac{1}{2}ae^2A_\mu A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(a-r^2)\partial_\mu\theta\partial^\mu\theta \\ &+ e(a-r^2)g_{\mu\nu}A^\mu\partial^\nu\theta - er\epsilon_{\mu\nu}A^\mu\partial^\nu\theta \\ &+ \partial_\mu C\partial^\mu\bar{C} + \alpha\frac{B^2}{2} + B\partial_\mu A^\mu. \end{aligned} \quad (213)$$

We have to choose $C^2 = C, N^2 = A_0, B_2 = B$ to reach equation (213) from equation (208). It is interesting to see that the action (213) is invariant under the transformation

$$\delta\phi = er\lambda C, \delta N_0 = -\lambda\dot{C}, \delta A_1 = -\lambda C', \tag{214}$$

$$\delta\theta = -\lambda eC, \delta C = 0, \delta\bar{C} = -\lambda B. \tag{215}$$

These are the very BRST transformations corresponding to the fields that describe the system under consideration. It would be of worth to reiterate that the choice of gauge fixing is very crucial here. The choice of gauge fixing which we have considered here renders a great service to obtain the appropriate Wess-Zumino term. The Wess-Zumino term L_{wz} can easily be identified as

$$L_{wz} = \frac{1}{2}(a - r^2)\partial_\mu\theta\partial^\mu\theta + e(a - r^2)g_{\mu\nu}A^\mu\partial^\nu\theta - er\epsilon_{\mu\nu}A^\mu\partial^\nu\theta. \tag{216}$$

Wess Zumino terms appears automatically during the process of quantization. In this formalism the fields needed for the extension of the phase space keep themselves allocated in the unphysical sector of the theory. With this we end up the BRST quantization part of the GVQED and proceed towards BRST quantization of gauged model of chiral boson with the siegel type kinetic term in the following two Sections. Among these two first one contains the description of the model itself.

5.4 A Gauged Model of Chiral Boson with the Siegel Type Kinetic Term

Free chiral boson is interesting because it is considered as the basic ingredient of heterotic string theory [85, 86, 87, 88]. The obvious generalization of free chiral boson is to take into account of the interaction of gauge field with that and this interacting field theoretical model is known as gauged chiral boson. The interacting theory of chiral boson was first described by Bellucci, Golterman and Petcher [50] with Seigel like kinetic term for chiral boson. So, naturally, the theory of interacting chiral boson with FJ type kinetic was wanted for as free FJ type chiral boson became available in [51] and that was

successfully met up by Harada [35]. After the work of Harada [35], interacting model of chiral boson based on FJ type kinetic term attracted considerable attention [84] in spite of the fact that this theory of interacting chiral boson was not derived from any fundamental principle. Harada obtained it from Jackiw-Rajaraman (JR) version of chiral Schwinger model [32] imposing a chiral constraint into it by hand. An attempt towards search for a link is therefore a natural extension which we would like to explore. In fact, we want to show whether the gauged FJ type chiral boson is contained within the gauged chiral boson of Seigel type chiral boson which is available in [50]. The study of the model may be beneficial from another point of view indeed; where anomaly is the central issue of investigation [8, 9, 32, 35, 65], since it is known from Ref. [35] that the model took birth from the JR version of chiral Schwinger model and this very chiral Schwinger model viz., chiral generation of Schwinger model [54] gets secured from unitarity problem when anomaly was taken into consideration [32]. In this respect, the recent chiral generation of Thirring model is of worth mentioning [59, 60]. So when the issue related to the search of desired link gets settled down a natural extension that comes automatically in mind is to study the symmetry underlying in the model and perform the quantization of the model. BRST quantization in this context scores over other.
The gauged chiral boson with the Siegel type of kinetic term is described by the lagrangian density

$$\begin{aligned}\mathcal{L_B} &= \frac{1}{2}(\dot\phi^2 - \phi'^2) + e(\phi' + \dot\phi)(A_0 - A_1) \\ &+ \frac{\lambda}{2}((\dot\phi - \phi')^2 + e^2(A_0 - A_1)^2 + 2e(A_0 - A_1)(\dot\phi - \phi')) \\ &+ \frac{1}{2}(\dot A_1 - A_0')^2 + \frac{1}{2}ae^2(A_0^2 - A_1^2). \qquad (217)\end{aligned}$$

Here over dot and over prime represent the time and space derivative respectively. The momenta corresponding to the field A_0, A_1, λ and ϕ respectively are

$$\frac{\partial\mathcal{L_B}}{\partial\dot A_0} = \pi_0 = 0, \qquad (218)$$

$$\frac{\partial\mathcal{L_B}}{\partial\dot A_1} = \pi_1 = \dot A_1 - A_0'. \qquad (219)$$

$$\frac{\partial \mathcal{L}_{\mathcal{B}}}{\partial \dot{\lambda}} = \pi_\lambda = 0, \tag{220}$$

$$\frac{\partial \mathcal{L}_{\mathcal{B}}}{\partial \dot{\phi}} = \pi_\phi = (1+\lambda)\dot{\phi} - \lambda\phi' + e(1+\lambda)(A_0 - A_1), \tag{221}$$

The canonical Hamiltonian density of the system is obtained through a Legendre transformation:

$$\mathcal{H} = \pi_\phi \dot{\phi} + \pi_1 \dot{A}_1 - \mathcal{L}. \tag{222}$$

Using equations (218), (219), (220) and (221), we find that H_c takes the following form

$$\begin{aligned} H_c &= \int dx [\frac{\pi_1^2}{2} + \pi_1 A_0' + \pi_\phi \phi' + \frac{1}{2} e^2 (A_1 - A_0)^2 \\ &- e(\pi_\phi + \phi')(A_0 - A_1) - \frac{ae^2}{2}(A_0^2 - A_1^2) \\ &+ \frac{1}{2(1+\lambda)}(\pi_\phi - \phi')^2 + u\pi_0 + v\pi_\lambda]. \end{aligned} \tag{223}$$

In equation (223) u and v are the two Lagrange multipliers. The primary constraints of this system are identified as

$$\Omega_1 = \pi_0 \approx 0, \tag{224}$$

$$\Omega_2 = \pi_\lambda \approx 0. \tag{225}$$

since these two expression do not contain the time derivative of the fields. The preservation of the constraints (224) and (225) leads to the following two constraints:

$$\Omega_3 = \pi_1' + e(\pi_\phi + \phi') + e^2[(a-1)A_0 + A_1] \approx 0, \tag{226}$$

$$\Omega_4 = \pi_\phi - \phi' \approx 0. \tag{227}$$

In order to single out the physical degrees of freedom we proceed to quantize the theory with the following gauge fixing condition

$$\Omega_5 = \lambda - f \approx 0. \tag{228}$$

Generating functional for this system can be written down as

$$\begin{aligned} Z = \int dA_0 dA_1 d\pi_1 d\phi d\pi_\phi d\lambda d\pi_\lambda \exp^{i\int d^2x[\pi_\phi\dot{\phi} + \pi_1\dot{A}_1 - H]} \times \\ \delta(\Omega_1)\delta(\Omega_2)\delta(\Omega_3)\delta(\Omega_4)\delta(\Omega_5) \end{aligned} \tag{229}$$

After integrating out of the momenta of the fields we get the generating functional Z in the following form

$$Z = \int dA_0 dA_1 d\phi d\lambda \exp^{i\int d^2x L_{GCB}}. \tag{230}$$

where

$$\begin{aligned} L_{GCB} &= \dot{\phi}\phi' - \phi'^2 + 2e\phi'(A_0 - A_1) - \frac{1}{2}e^2(A_0 - A_1)^2 \\ &+ \frac{1}{2}ae^2(A_0^2 - A_1^2) + \frac{1}{2}(\dot{A}_1 - A_0')^2. \end{aligned} \tag{231}$$

This is the gauged model of chiral boson with FJ type kinetic term. Note that L_{GCB} is an action generated from $\mathcal{L}_{\mathcal{B}}$ and it agrees with the lagrangian found in [35]. So we find that the gauged model of chiral boson with FJ type kinetic term is contained within the gauged version of Siegel like chiral boson. Imposition of chiral constraint by hand like the work [35] is not needed here. Here $\Omega's$ stands for the standing second class constraints embedded in the phase space of the theory. The constraints of the theory explicitly are

$$\Omega_1 = \pi_0 \approx 0, \tag{232}$$

$$\Omega_2 = \pi_\lambda \approx 0, \tag{233}$$

$$\Omega_3 = \pi_\phi - \phi' \approx 0, \tag{234}$$

$$\Omega_4 = \pi_1' + e(\pi_\phi + \phi') + e^2[(a-1)A_0 + A_1] \approx 0, \tag{235}$$

$$\Omega_5 = \lambda - f \approx 0. \tag{236}$$

Therefore, to compute Dirac brackets we need to construct the matrix constituted with the Poission brackets between the constraints (232), (233), (234) ,(235) and (236). The required matrix is

$$C_{ij} = \begin{pmatrix} 0 & 0 & 0 & -e^2(a-1) & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & -2\partial & 0 & 0 \\ e^2(a-1) & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \delta(x-y). \tag{237}$$

The matrix C^1_{ij} is nonsingular. So inverse of it exists which is found out to be

$$C^{-1}_{ij} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{e^2(a-1)} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{2\partial_x} & 0 & 0 \\ -\frac{1}{e^2(a-1)} & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{pmatrix} \delta(x-y). \tag{238}$$

The Dirac brackets between the field variables are found to be

$$[A_0(x), A_1(y)]^* = \frac{1}{e^2(a-1)} \partial_x \delta(x-y), \tag{239}$$

$$[\phi(x), \phi(y)]^* = -\frac{1}{2\partial_x} \delta(x-y), \tag{240}$$

$$[A_0(x), \phi(y)]^* = \frac{1}{e(a-1)} \delta(x-y), \tag{241}$$

$$[A_0(x), \pi_1(y)]^* = -\frac{1}{(a-1)} \delta(x-y), \tag{242}$$

$$[A_0(x), \pi_\phi(y)]^* = -\frac{1}{e(a-1)} \partial_x \delta(x-y), \tag{243}$$

$$[A_1(x), \pi_1(y)]^* = \delta(x-y), \tag{244}$$

$$[\phi(x), \pi_\phi(y)]^* = \delta(x-y), \tag{245}$$

Here (*) indicate the Dirac bracket. It is the rightpoint to end up the description of this Section. In the following section we will proceed towards BRST quantization of this model.

5.5 Study of BRST Quantization of Gauged Floreanini-Jackiw Type Chiral Boson

We have carried out the BRST quantization of the gauged chiral boson with FJ type kinetic term using BVF formalism in [11] which we are going to describe here. The lagrangian density of gauged FJ type chiral boson is given by

$$\begin{aligned}\mathcal{L} &= \dot{\phi}\phi' - \phi'^2 + 2e\phi'(A_0 - A_1) \\ &- \frac{1}{2}e^2(A_0 - A_1)^2 + \frac{1}{2}ae^2(A_0^2 - A_1^2) + \frac{1}{2}(\dot{A}_1 - A_0')^2. \end{aligned} \tag{246}$$

For this lagrangian density (246) the canonical momenta corresponding to the field ϕ, A_0 and A_1 respectively are

$$\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \pi_\phi = \phi', \tag{247}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{A}_0} = \pi_0 = 0, \tag{248}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{A}_1} = \pi_1 = \dot{A}_1 - A_0'. \tag{249}$$

Equation (247) and (248) do not contain any time derivative of the fields. So these two are the primary constraint of the theory.

$$\omega_1 = \pi_\phi - \phi' \approx 0, \tag{250}$$

$$\omega_2 = \pi_0 \approx 0. \tag{251}$$

The canonical Hamiltonian density of the system is obtained through a Legendre transformation:

$$H_c = \pi_\phi \dot{\phi} + \pi_1 \dot{A}_1 - L \tag{252}$$

The canonical Hamiltonian can be calculated using equations (247),(248) and (249) through a Legendre transformation as done earlier:

$$\begin{aligned}H_c &= \int dx[\frac{1}{2}\pi_1^2 + \pi_1 A_0' + \phi'^2 - 2e\phi'(A_0 - A_1) \\ &+ \frac{1}{2}e^2(A_0 - A_1)^2 - \frac{1}{2}ae^2(A_0^2 - A_1^2)]\end{aligned} \tag{253}$$

Therefore, the effective Hamiltonian reads

$$\begin{aligned}H_P &= \int dx[\frac{1}{2}\pi_1^2 + \pi_1 A_0' + \phi'^2 - 2e\phi'(A_0 - A_1) \\ &+ \frac{1}{2}e^2(A_0 - A_1)^2 - \frac{1}{2}ae^2(A_0^2 - A_1^2) + u(\pi_\phi - \phi') + v\pi_0],\end{aligned} \tag{254}$$

where u and v are Lagrange multipliers. The preservation of ω_2 renders the following new constraint

$$\omega_3 = \pi_1' + 2e\phi' + e^2(a-1)A_0 + e^2 A_1 \tag{255}$$

$$\approx \pi_1' + e\phi' + e\pi_\phi + e^2(a-1)A_0 + e^2 A_1 \approx 0. \tag{256}$$

The preservations of ω_1 and ω_3 however do not give rise to any new constraint. These two conditions fix the velocities u and v respectively:

$$u = A_1' - \frac{1}{(a-1)}\pi_1. \tag{257}$$

and

$$v = \phi' - e(A_0 - A_1). \tag{258}$$

Therefore, the theory under consideration contains three constraints in its phase space. Precisely, the constraints are

$$\omega_1 = \pi_\phi - \phi' \approx 0, \tag{259}$$

$$\omega_2 = \pi_0 \approx 0, \tag{260}$$

$$\omega_3 = \pi_1' + 2e\phi' + e^2(a-1)A_0 + e^2 A_1 \approx 0. \tag{261}$$

Imposing the expression of u and v in (254) the Hamiltonian turns into

$$\begin{aligned} H_P &= \int dx[\frac{1}{2}\pi_1^2 + \pi_1 A_0' + \pi_\phi(\phi' - e(A_0 - A_1)) \\ &- e\phi'(A_0 - A_1) + \frac{1}{2}e^2(A_0 - A_1)^2 \\ &- \frac{1}{2}ae^2(A_0^2 - A_1^2) + \pi_0(A_1' - \frac{1}{(a-1)}\pi_1)] \end{aligned} \tag{262}$$

The constraints of the theory satisfy the following Poission brackets among themselves

$$[\omega_1, \omega_1] = -2i\delta(x-y), \tag{263}$$

$$[\omega_1, \omega_3] = 0, \tag{264}$$

$$[\omega_2, \omega_2] = 0, \tag{265}$$

$$[\omega_2, \omega_3] = -ie^2(a-1)\delta(x-y). \tag{266}$$

The involution relation between the Hamiltonian (262) and the constraints ω_1 , ω_2 and ω_3 are

$$-i[\omega_1, H_P] = \omega_1', \tag{267}$$

$$-i[\omega_2, H_P] = \omega_3, \tag{268}$$

$$-i[\omega_3, H_P] = \omega_2'' - \frac{e^2}{(a-1)}\omega_2. \tag{269}$$

The set of second class constraints ω_1 , ω_2 and ω_3 can be converted into a first a class set with the help of two auxiliary canonical pairs (θ, π_θ) and (η, π_η).

$$[\eta, \pi_\eta] = \delta(x-y), \tag{270}$$

$$[\theta, \pi_\theta] = \delta(x-y). \tag{271}$$

The first class set of constraints that are constructed from the said second class set of constraints using these auxiliary fields are the following

$$\bar{\omega_1} = \omega_1 + \pi_\theta + \theta', \tag{272}$$

$$\bar{\omega_2} = \omega_2 - \pi_\eta, \tag{273}$$

$$\bar{\omega_3} = \omega_3 + e^2(a-1)\eta. \tag{274}$$

The Hamiltonian consistent with the first class set of constraint (272), (273) and (274) is

$$\bar{H} = H_P + H_{BF}. \tag{275}$$

where H_{BF} would certainly be constituted with the auxiliary fields which is found out to be

$$H_{BF} = \int dx[\frac{1}{4}(\pi_\theta + \theta')^2 + \frac{1}{2}e^2(a-1)\eta^2 + \frac{1}{2e^2(a-1)}\pi_\eta'^2 + \frac{1}{2(a-1)^2}\pi_\eta^2]. \tag{276}$$

For consistency, the time evaluation of these first class set (272), (273) and (274) must be identical to the (267), (268) and (269). Precisely these are the following.

$$-i[\bar{\omega_1}, \bar{H}] = \bar{\omega_1'}, \tag{277}$$

$$-i[\bar{\omega_2}, \bar{H}] = \bar{\omega_3}, \tag{278}$$

$$-i[\bar{\omega_3}, \bar{H}] = \bar{\omega_2}'' - \frac{e^2}{(a-1)}\bar{\omega_2}. \tag{279}$$

The stage is now set to introduce the two pairs of ghost and anti-ghost fields $(C^i, \bar{P}_i)$ and $(P^i, \bar{C}_i)$. We also need to introduce a pair of multiplier fields (N_i, B_i). The multipliers and the ghost anti-ghost pairs satisfy the following canonical Poission brackets: $[P^i, \bar{C}_i] = [C^i, \bar{P}_i] = [N^i, B_j] = i\delta^i_j\delta(x-y)$, where i = 1, 2, 3. According to the definition

$$H_{BRST} = H_P + H_{BF} + \bar{P}_a V^a_b C^b + \int [Q, \psi] dx. \tag{280}$$

In this situation BRST charge Q and the fermonic gauge fixing function ψ can be written down as

$$Q = \int dx (C^i \bar{\omega}_i + P^i B_i), \tag{281}$$

$$\psi = \int dx (\bar{C}_i \chi^i + \bar{P}_i N^i). \tag{282}$$

We are now in a position to fix up the gauge condition which is very crucial for getting appropiate Weiss Zumino term. It is found that the following condition help to reach our goal successfully.

$$\chi_1 = \pi_\phi - \phi', \tag{283}$$

$$\chi_2 = \dot{N}^2 + A_0, \tag{284}$$

$$\chi_3 = -A_1' + \frac{\alpha}{2} B_3. \tag{285}$$

Let us now calculate the commutation relation in between the BRST charge, and gauge fixing function:

$$[Q, \psi] = B_i \chi^i + P_i P^i - C^3 \bar{C}_3'' - C^2 \bar{C}_2 - 2C^1 \bar{C}_1 + \bar{\omega}_i N^i. \tag{286}$$

Generating functional for this system can be written down as

$$Z = \int [D\mu] \exp^{iS}. \tag{287}$$

where $[D\mu]$ is the Liouville measure in the extended phase space.

$$\begin{aligned}[D\mu] \quad &= \quad [d\phi][d\pi_\phi] \sum_{i=0}^{1} [dA_i][d\pi_i][d\eta][d\pi_\eta][d\theta][d\pi_\theta] \times \\ &\sum_{k=1}^{3} [dN^k][dB_k][dC^k], [d\bar{C}_k][dP^k], [d\bar{P}_k]. \end{aligned} \tag{288}$$

and the action S is explicitly given by

$$\begin{aligned} S &= \int d^2x[\dot{\phi}\pi_\phi + \dot{A}_1\pi_1 + \dot{A}_0\pi_0 + \dot{\eta}\pi_\eta + \dot{\theta}\pi_\theta \\ &+ B_i\dot{N}^i + \bar{P}_i\dot{C}^i + \bar{C}_i\dot{P}^i - H_{BRST}]. \end{aligned} \tag{289}$$

The above formulation allows the following simplification:

$$\int dx(B_iN^i + \bar{C}_i\dot{P}^i) = i[Q, \int dx\bar{C}_i\dot{N}^i]. \tag{290}$$

Exploiting the above simplification (290), we obtain the effective action in the following form

$$\begin{aligned} S_{eff} &= \int dx[\dot{\phi}\pi_\phi + \dot{A}_1\pi_1 + \dot{A}_0\pi_0 + \dot{\eta}\pi_\eta + \dot{\theta}\pi_\theta + \dot{N}^2B_2 \\ &+ \dot{N}^3B_3 + \bar{P}_1\dot{C}^1 + \bar{P}_2\dot{C}^2 + \bar{P}_3\dot{C}^3 + \bar{C}_2\dot{P}^2 + \bar{C}_3\dot{P}^3 \\ &- P_1\bar{P}^1 - P_2\bar{P}^2 - P_3\bar{P}^3 - [\pi_\phi(\phi' - e(A_0 - A_1)) \\ &+ \frac{1}{2}\pi_1^2 + \pi_1A_0' - e\phi'(A_0 - A_1) + \frac{1}{2}e^2(A_0 - A_1)^2 \\ &- \frac{1}{2}ae^2(A_0^2 - A_1^2) + \pi_0(A_1' - \frac{1}{(a-1)}\pi_1) + \frac{1}{4}(\pi_\theta + \theta')^2 \\ &+ \frac{1}{2}e^2(a-1)\eta^2 + \frac{1}{2e^2(a-1)}\pi_\eta'^2 + \frac{1}{2(a-1)^2}\pi_\eta^2] \\ &+ (\pi_\phi - \phi' + \pi_\theta + \theta')N^1 + (\pi_0 - \pi_\eta)N^2 \\ &+ (\pi_1' + e\phi' + e\pi_\phi + e^2(a-1)A_0 + e^2A_1 + e^2(a-1)\eta)N^3 \\ &- B_1\chi^1 - B_2\chi^2 - B_3\chi^3 - \bar{P}_1C_1' + \bar{P}_3C_2 - \bar{P}_2''C_3 \\ &- \frac{1}{(a-1)}e^2\bar{P}_2C_3 - C^2\bar{C}_2 + C^3\bar{C}_3'' + 2C^1\bar{C}_1. \end{aligned} \tag{291}$$

We are in a state to integrate out of the fields $\pi_0, \pi_1\eta, B_1, B_2, N^1$ $N^2, \bar{C}^1, \bar{P}_1, \bar{P}_3 and \bar{P}_2$ one by one in order to have the effective action in a desired shape. After integrating out of the said fields and choosing $N_3 = A_0$ the effective action reduces to

$$\begin{aligned} S_{eff} &= \int d^2x[\dot{\phi}\phi' - \phi'^2 + 2e\phi'(A_0 - A_1) \\ &- \frac{1}{2}e^2(A_0 - A_1)^2 + \frac{1}{2}ae^2(A_0^2 - A_1^2) \end{aligned}$$

$$
\begin{aligned}
&+ \frac{1}{2}(\dot{A}_1 - A_0')^2 + \frac{1}{(a-1)}\pi_\eta(\dot{A}_1 - A_0') \\
&+ (\pi_\eta' A_1 - \dot{\pi}_\eta A_0) + \frac{1}{2e^2(a-1)}(\dot{\pi_\eta}^2 - \pi_\eta'^2) \\
&+ B_3\dot{A}_0 - B_3 A_1{}' + \frac{\alpha}{2}B_3^2 + \partial_\mu C^3 \partial^\mu \bar{C}_3]
\end{aligned} \tag{292}
$$

If we now define $\pi_\eta = e(a-1)\eta$, $C_3 = C$ and $B_3 = B$ we get the desired BRST invariant action:

$$
\begin{aligned}
S_{BRST} &= \int d^2x[\dot{\phi}\phi' - \phi'^2 + 2e\phi'(A_0 - A_1) \\
&- \frac{1}{2}e^2(A_0 - A_1)^2 + \frac{1}{2}ae^2(A_0^2 - A_1^2) \\
&+ \frac{1}{2}(\dot{A}_1 - A_0')^2 + \frac{1}{2}(a-1)(\dot{\eta}^2 - \eta'^2) \\
&+ e(A_0\eta' - A_1\dot{\eta}) + e(a-1)(A_1\eta' - A_0\dot{\eta}) \\
&+ \partial_\mu C \partial^\mu \bar{C} + B\partial_\mu A^\mu + \frac{\alpha B^2}{2}].
\end{aligned} \tag{293}
$$

The action (293) is now found to remain invariant if the fields transform as follows.

$$\delta\phi = e\lambda C, \tag{294}$$

$$\delta N_0 = -\lambda\dot{C}, \tag{295}$$

$$\delta A_1 = -\lambda C', \tag{296}$$

$$\delta\eta = -\lambda e C, \tag{297}$$

$$\delta C = 0, \tag{298}$$

$$\delta\bar{C} = -\lambda B. \tag{299}$$

The above transformations are the very BRST transformation generated from the BRST charge (281). The Wess-Zumino term for the theory under consideration can easily be identified as

$$L_W = \int d^2x[\frac{1}{2}(a-1)(\dot{\eta}^2 - \eta'^2) + e(A_0\eta' - A_1\dot{\eta}) + e(a-1)(A_1\eta' - A_0\dot{\eta})] \tag{300}$$

This very action (300) is the appropriate Wess-Zumino term corresponding to the theory of our present consideration and it agrees with the Ref. [89]. We would like to reiterate that in [84] it was lacking for. In fact, in [84], the term which was demanded for by the author as the Wess-Zumino term was not the appropriate one and he tried to show the on shell BRST invariance with that inappropriate Wess-Zumino term. The term standing in equation (300) however establishes the off-shell BRST invariance. To achieve the appropriate Wess-Zumino term for this theory is a novel aspect of this reinvestigation. In the following section we will discuss the BRST invariant reformulation of the chiral Schwinger model with Faddeevian anomaly.

5.6 Study of BRST Quantization of Chiral Schwinger Model with Fadeevian Anomaly

Jackiw-Rajaraman version of chiral Schwinger model is an interesting field theoretical model which has been studied over the years for different purposes. Another parallel development of chiral Schwinger model with Fadeevian regularization was made few years later by Mitra [8, 9]. BRST invariant reformulation of Jackiw-Rajaraman version of chiral Schwinger model was done in [48]. However the BRST invariant reformulation was lacking for the chiral Schwinger model where anomaly is Faddeevian like. Quantization of this model has been done in [10], which suggests that the system may posses more degrees of freedom than the usual. With this in view and also as a pedagogical illustration of the BVF formalism effort has been made to obtain a BRST invariant effective action of this model. The work will certainly demonstrate the power of BFV formalism once more. This new study would be instrumental for future studies towards unitarity and renormalization of this model.

The lagrangian for chiral Schwinger model with Faddeevian anomaly is given by

$$L_{CH} = \int dx[\dot{\phi}\phi' - \phi'^2 + 2e\phi'(A_0 - A_1) - 2e^2 A_1^2]. \tag{301}$$

We find that the momenta corresponding to the fields ϕ, A_0, and A_1 are

$$\frac{\partial L_{CH}}{\partial \dot{\phi}} = \pi_\phi = \phi', \tag{302}$$

$$\frac{\partial L_{CH}}{\partial \dot{A}_0} = \pi_0 = 0, \tag{303}$$

$$\frac{\partial L_{CH}}{\partial \dot{A}_1} = \pi_1 = \dot{A}_1 - A_0'. \tag{304}$$

It is known that $\pi_0 \approx 0$ and $\pi_\phi = \phi' \approx 0$ are the primary constraints of the theory.

$$\omega_1 = \pi_\phi - \phi' \approx 0, \tag{305}$$

$$\omega_2 = \pi_0 \approx 0. \tag{306}$$

The effective Hamiltonian follows from the equations of motion is

$$H_p = \int d\,x[H_C + u\pi_0 + \nu(\pi_\phi - \phi')], \tag{307}$$

where H_c is

$$H_C = \int dx[\frac{1}{2}\pi_1^2 + \pi_1 A_0' + \phi^2 - 2e(A_0 - A_1)\phi' + 2e^2 A_1^2]. \tag{308}$$

Here u and v are two required Lagrange multipliers. The preservation of the constraints leads to two other constraints

$$G = \pi_1' + 2e\phi' \approx 0, \tag{309}$$

$$-2e^2(A_1 + A_0)' \approx 0. \tag{310}$$

The multipliers u and v are found out to be

$$u = -(\pi_1 + A_0'), \tag{311}$$

$$v = \phi - e(A_0 - A_1). \tag{312}$$

Therefore, the theory under consideration consists four constraints in its phase space. Precisely, the constraints are

$$\omega_1 = \pi_\phi - \phi' \approx 0, \tag{313}$$

$$\omega_2 = \pi_0 \approx 0, \tag{314}$$

$$\omega_3 = \pi_1' + 2e\phi' \approx 0, \tag{315}$$

$$\omega_4 = -2e^2(A_1 + A_0)' \approx 0. \tag{316}$$

These four constraints form a second class set and the closures of the constrains with respect to the Hamiltonian (307) are given by

$$\dot{\omega}_1 = \omega_1', \tag{317}$$

$$\dot{\omega}_2 = \omega_3 - \omega_2' + e\omega_1, \tag{318}$$

$$\dot{\omega}_3 = \omega_4 - e\omega_1', \tag{319}$$

$$\dot{\omega}_4 = 2e^2\omega_2'. \tag{320}$$

To obtain a BRST invariant reformulation we need to convert the second class set of constraints into a first class set. With this in view, we introduce four auxiliary fields ψ, η, π_ψ and π_η and fields are such that they satisfy the following canonical condition

$$[\eta(x), \pi_\eta(y)] = \delta(x-y), \tag{321}$$

$$[\psi(x), \pi_\psi(y)] = \delta(x-y). \tag{322}$$

The fields used here are known as Batalin-Fradkin (BF) fields. The constraints (313), (314),(315) and (316), with some suitable linear combination of the BF fields get converted into first class set as follows

$$\tilde{\omega}_1 = \pi_\phi - \phi' + \pi_\psi + \psi', \tag{323}$$

$$\tilde{\omega}_2 = \pi_0 - \pi_\eta, \tag{324}$$

$$\tilde{\omega}_3 = -2e\psi' + 2e\phi' + \pi_1' - \pi_\eta', \tag{325}$$

$$\tilde{\omega}_4 = -2e^2(A_0 + A_1)' - 2e^2\eta'. \tag{326}$$

First class Hamiltonian is obtained by the appropriate insertion of the BF fields within the Hamiltonian (307) and it is given by $\tilde{H} = H_P + H_{BF}$. Here H_{BF} is a polynomial of $\psi,\ \eta,\ \pi_\psi$ and π_η that extend the phase space respecting the closures (328), (329), (330) and (331). We find that H_{BF} for this system will be

$$H_{BF} = \int d\,x[-2e\eta\psi' + e(\pi_\psi + \psi')\eta + \frac{1}{2}(\pi_\eta^2 + \pi_\psi^2 + \psi'^2)]. \tag{327}$$

The above four first class constraints will be found consistent with the first class Hamiltonian if these new first class set satisfy the same closures as their ancestor did with the Hamiltonian (307). Precisely, the conditions are

$$\dot{\tilde{\omega}}_1 = \tilde{\omega}_1', \tag{328}$$

$$\dot{\tilde{\omega}}_2 = \tilde{\omega}_3 - \tilde{\omega}_2' + e\tilde{\omega}_1, \tag{329}$$

$$\dot{\tilde{\omega}}_3 = \tilde{\omega}_4 - e\tilde{\omega}_1', \tag{330}$$

$$\dot{\tilde{\omega}}_4 = 2e^2\tilde{\omega}_2'. \tag{331}$$

We now introduce four pairs of ghost $(C_i,\ \bar{P}^i)$ and four pairs of anti-ghost $(P_i,\ \bar{C}^i)$ fields. Four pairs of multiplier fields (N^i, B_i) are also needed. These fields need to satisfy the following canonical relations

$$[C_i,\ \bar{P}^j] = [P^i,\ \bar{C}_j] = [N^i, B_j] = i\delta^i_j\delta(x-y), \quad i = 1, 2, 3, 4. \tag{332}$$

From the definition we can write BRST invariant Hamiltonian

$$H_{BRST} = H_P + H_{BF} + \bar{P}_a V^a_b C^b + \int [Q,\ \psi] dx, \tag{333}$$

Q is the BRST charge and ψ's are the gauge fixing functions. The BRST charge Q is a nilpotent operator and it satisfies the equation

$$Q^2 = [Q,\ Q] = 0. \tag{334}$$

The definition of Q in this formalism is

$$Q = \int (B_i P^i + C_i \tilde{\omega}^i) d\,x, \tag{335}$$

and the definition of gauge fixing function ψ is

$$\psi = \int (\bar{C}_i X^i + P_i N^i) d\,x. \tag{336}$$

The BRST invariant Hamiltonian for the theory with which we are dealing with is

$$\begin{aligned} H_{BRST} &= H_P + H_{BF} + \int d\,x(-\bar{P}_1 C_1' + \bar{P}_3 C_2 \\ &+ \bar{P}_2 C_2' + e\bar{P}_1 C_2 + \bar{P}_4 C_3 \\ &- e\bar{P}_1' C_3 + 2e^2 \bar{P}_2' C_4). \end{aligned} \tag{337}$$

It would be helpful to write down the generating functional that ultimately leads to an effective action with the elimination of some fields by Gaussian integration. The generating functional reads

$$Z = \int [D\mu] e^{iS}. \tag{338}$$

Here the expression of S is

$$\begin{aligned} S &= \int d^2\,x[\pi_\phi\dot{\phi} + \pi_1\dot{A}_1 + \pi_0\dot{A}_0 + \pi_\psi\dot{\psi} + \pi_\eta\dot{\eta} + \bar{P}_i\dot{C}^i + \bar{C}_i\dot{P}^i \\ &+ B_i\dot{N}^i - H_{BRST}], \end{aligned} \tag{339}$$

where $[D\mu]$ is the Liouville measure in the extended phase space.

$$\begin{aligned} [D\mu] &= [d\phi][d\pi_\phi]\sum_{i=0}^{1}[dA_i][d\pi_i][d\eta][d\pi_\eta][d\psi][d\pi_\psi] \times \\ &\sum_{k=1}^{4}[dN^k][dB_k][dC^k],[d\bar{C}_k][dP^k],[d\bar{P}_k]. \end{aligned} \tag{340}$$

We are now in a position to fix up the gauge conditions.

$$\chi_1 = \pi_\phi - \phi', \tag{341}$$

$$\chi_2 = -\dot{N}^2 + A_0, \tag{342}$$

$$\chi_3 = \frac{B_3}{2} - A_1', \tag{343}$$

$$\chi_4 = \pi_\eta - \dot{N}^4. \tag{344}$$

When we substitute the simplified form of H_{BRST} obtained after plugging the gauge fixing conditions (341), (342), (343) and (344) in the action (339), we get the explicit expression of S:

$$\begin{aligned} S &= \int d^2\,x[\pi_\phi\dot{\phi} + \pi_\psi\dot{\psi} + \pi_\eta\dot{\eta} + \pi_1\dot{A}_1 \\ &+ \pi_0\dot{A}_0 + \bar{P}_i\dot{C}^i + \bar{C}_i\dot{P}^i + B_i\dot{N}^i \\ &- (\frac{\pi_1^2}{2} + \pi_1 A_0' - e\phi'(A_0 - A_1) + 2e^2A_1^2 \\ &- \pi_0(\pi_1 + A_0') + \pi_\phi\phi' - e\pi_\phi(A_0 - A_1) - e\eta\psi' + e\pi_\psi\eta \\ &+ \frac{1}{2}(\pi_\eta^2 + \pi_\psi^2 + \psi'^2) + B_i\chi^i + \tilde{\omega}_i N^i \\ &- \bar{P}_iP^i - \bar{P}_1C_1' + \bar{P}_3C_2 + \bar{P}_2C_2' \\ &+ e\bar{P}_1C_2 + \bar{P}_4C_3 - e\bar{P}_1'C_3 + 2e^2\bar{P}_2'C_4 \\ &- C_3\bar{C}'' - \bar{C}_2\dot{P}^2 - \bar{C}_4\dot{P}^4 \\ &+ 2e^2C^4\bar{C}_4 - 2C'\bar{C}_1 - e^2C^2\bar{C}_2. \end{aligned} \tag{345}$$

Here i runs from 1 to 4. Our next task is to simplify (338) through the elimination of some fields and that will lead us to our desired result. A careful look reveals that here exists a simplification

$$\int d^2x(B_1N^1 + \bar{C}_1\dot{P}_1) = -i[Q\int d^2x\bar{C}_1\dot{N}^1]. \tag{346}$$

with be Legendre transformation $B^i \to B^i + N^i$. However the simplification corresponding to i = 1 suffices in this situation. More simplification follows from the elimination of the fields $\pi_0,\ \pi_1,\ \pi_\eta,\ B_1,\ B_2,\ B_4,\ A_0,\ N^1,\ N^2,\ N^4,\ P_1,\ \bar{P}^1,\ P_2,\ \bar{P}^2,\ P4,\ \bar{P}^4,\ P_1,\ \bar{P}^1,\ C_1,\ \bar{C}^1,\ C_2$ and $\bar{C}^2$ by Gaussian integration. Ultimately we reach to a very simplified form of the generating functional (338) that contains the following effective action in its exponent.

$$\begin{aligned} S_{eff} &= \int d^2x(\dot{\phi}\phi' - \phi'^2 - 2e^2A_1^2 - \psi'^2 - \dot{\psi}\psi' + \frac{1}{2}(\dot{A}_1 - A_0')^2 \\ &\quad +2e\phi'(A_0 - A_1) + 2e\psi'(A_1 + A_0) + \partial_\mu BA^\mu + \frac{1}{2}\alpha B^2 \\ &\quad +\partial_\mu\bar{C}\partial^\mu C. \end{aligned} \tag{347}$$

We have used few redefinition of fields, e.g $N_3 = A_0$ and $P^3 = \dot{C}_3$ to reach to the result (347). Since after elimination there is no other B's and C's except B_3 and C_3 we are free to read them as B and C. It is now time to check the invariance of the action (347). The action is found invariant under the transformation

$$\begin{aligned} \delta A_1 &= -\lambda C', \qquad \delta A_0 = \delta N_3 = -\lambda\dot{C}, \\ \delta\phi = \lambda C, \quad \delta\psi &= -\lambda C, \qquad \delta\bar{C} = \lambda B, \qquad \delta C = 0. \end{aligned} \tag{348}$$

We can identify easily the Wess-Zumino term for this theory which is

$$L_{wz} = -\dot{\psi}\psi' - \psi'^2 + 2e\psi'(A_0 + A_1). \tag{349}$$

5.7 Discussion

We have described the BRST invariant reformulation of three different models using the improved version of BFV formulation due to Fujiwara, Igarishi and Kubo [46]. This improved version has helped us to obtain a BRST invariant reformulation along with the emergence of appropriate Wess-Zumino term of the GVQED where vector and axial vector interaction get mixed up with different weight, chiral Schwinger model with Fadeevian anomaly and gauged model of Floreanini-Jackiw type chiral Boson. In the BRST invariant reformulations of these models, extension of phase space have been needed because of the entry of the auxiliary fields in an essential way. The fields needed for the extension however keep themselve laid in the unphysical sector of the theory and the process keeps the physical content of the theory intact. Beauty as well as the advantage of this formalism is that the Wess Zumino terms appears automatically during the process of quantization. Note that the role of gauge fixing is very crucial to get the appropriate Wess-Zumino term in every case.

Though in [84], an attempt was made towards BRST quantization of the gauged version of FJ type chiral boson nevertheless in that work the part of the action which was demanded as the Wess-Zumino term was not the appropriate Wess-Zumino term for the corresponding model. The author with that inappropriate Wess-Zumino term tried to establish the on shell BRST invariance. The way we have made the BRST invariant reformulation leads to the appropriate Wess-Zumino term. It is interesting that the appropriate Wess-Zumino term has automatically appeared during the process and it has been found off shell BRST invariant. Note that equation (269) reveals the missing of a term in the involution relations corresponding to the constraint ω_3 in Ref. [84], and that may be considered as the reason behind obtaining an untrustworthy Wess-Zumino action.

We would like to end up this Chapter with the remarks that in the usual Hamiltonian formulation of a gauge invariant theory one some times need to destroy the gauge symmetry under the introduction of some gauge fixing terms. However, BRST invariant Hamiltonian which has been reformulated will help one to work in an extended phase space on which only a subspace corresponds to the state of physical interest.

Chapter 6

6 Alternative Quantization in the Extended Phase Space

6.1 Introduction

In the usual phase space of a theory Dirac's scheme of quantization is instrumental to determine the phase space structure of a theory. However in order to get a symmetric theory we need to extend the phase space. So quantization of a theory in the extended phase space is a natural extension. In [10, 12] a gauge symmetric versions of two models are made available by us. Extension of phase space has been occurred there by the advent of Wess Zumino field. In the extended phase space too an alternative quantization is found possible, which of course is based on the very Dirac theory of quantization of constrained system [25]. In presence of that Wess-Zumino term an extension towards an alternative quantization [11, 26] is made to determine the canonical pair of fields which describe the Fock-space. The Lorentz type gauge fixing term at the action level also has chosen for quantization in the alternative manner. A natural corollary, at this stage of course is to show that the physical contents of the theory remains identical, even after the extension of phase space. The use of the formalism available from the work of Falck and Kramer in [30] has come in use in this respect. It is shown here explicitly that an appropriate gauge fixing is capable of mapping the Wess-Zumino added action onto the initial gauge non-invariant effective action of the usual chiral Schwinger model.

The alternative quantization of GVQED and gauged FJ type chiral boson have been studied in [11, 26]. In this Chapter we are going to describe the alternative quantization of these two models. This Chapter also includes the discussion related to the study concerning equivalence between the physical contents of the actual gauge non invariant version and the gauge invariant version of the extended phase space for these two models.

6.2 Study of Alternative Quantization of GVQED

A known standard way of expressing a gauge non-invariant theory into its gauge invariant version is to extend the phase space with the inclusion of Wess-Zumino field [4]. So by adding the appropriate Wess-Zumino action to the action of the usual bosonized gauge non-invariant action of GVQED we get a gauge invariant theory of the same and the lagrangian of which is given by

$$\begin{aligned} L &= \int dx[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + e\epsilon^{\mu\nu}A_\mu\partial_\nu\phi + erg^{\mu\nu}A_\mu\partial_\nu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &+ \frac{ae^2}{2}A_\mu A^\mu + \frac{1}{2}(a-r^2)\partial_\mu\theta\partial^\mu\theta - er\epsilon^{\mu\nu}A_\mu\partial_\nu\theta \\ &+ e(a-r^2)g^{\mu\nu}A_\mu\partial_\nu\theta + B\partial^\mu A_\mu + \frac{\bar{\alpha}}{2}B^2]. \end{aligned} \tag{350}$$

Here we have included the admissible mass like term $\frac{ae^2}{2}A_\mu A^\mu$ which we have obtained from Poincar'e algebra in Chapter 4. The last two terms of the lagrangian (350), imply the Lorentz type gauge fixing term at the action level. It is needed for quantization in the alternative manner [25, 27, 28, 29]. The Euler-Lagrange equations of motion of the fields (of both the usual and extended phase space) are

$$\Box\phi = -e\tilde{\partial}_\mu A^\mu - er\partial_\mu A^\mu, \tag{351}$$

$$\Box\theta = \frac{er}{(a-r^2)}\tilde{\partial}_\mu A^\mu - e\partial_\mu A^\mu, \tag{352}$$

$$\partial_\mu A^\mu + \bar{\alpha}B = 0, \tag{353}$$

$$\partial_\mu F^{\mu\nu} - \partial^\nu B + J^\nu = 0. \tag{354}$$

Where J^ν is the electromagnetic current, which is defined by

$$J^\nu = e\epsilon^{\mu\nu}\partial_\nu\phi + erg^{\mu\nu}\partial_\nu\phi - er\epsilon^{\mu\nu}\partial_\nu\theta + e(a-r^2)g^{\mu\nu}\partial_\nu + e^2aA^\nu. \tag{355}$$

The equation (351),(352),(353) and (354), agrees with the following exact solution of the fields ϕ, θ and A_μ

$$\phi = \frac{(a-r^2)}{ea(1+a-r^2)}F + \frac{h}{a} + r\eta, \tag{356}$$

$$\theta = -\frac{r}{ea(1+a-r^2)}F - \frac{r}{a(a-r^2)}h + \eta, \tag{357}$$

$$A_\mu = \frac{1}{e^2 a}[\frac{(a-r^2)}{(1+a-r^2)}\tilde{\partial}_\mu F + \partial_\mu B + e\tilde{\partial}_\mu h - ea\partial_\mu \eta]. \tag{358}$$

where the h, B, η, F are fock space fields.
For Lorentz index $\mu = 0$ the equation (354) takes the following form

$$-A_0'' + \dot{A_1}' - \dot{B} + eJ^0 = 0. \tag{359}$$

Similarly for $\mu = 1$ the eqution (354) looks

$$-\ddot{A}_1 + \dot{A_0}' + B' + eJ^1 = 0 \tag{360}$$

Substituing the value of A_0 and A_1 in equation (353), we get the following relation

$$\Box\eta = \bar{\alpha}eB. \tag{361}$$

Putting the expression of A_0, A_1 and θ in (352) we have obtained the following condition

$$\Box B = 0, \tag{362}$$

Similarly putting the expression of A_0 and A_1, ϕ, θ in (354) we get

$$(\Box + m^2)\Box F = 0, \tag{363}$$

$$\Box h = 0, \tag{364}$$

where square of the mass m^2 is given by

$$m^2 = \frac{e^2 a(1+a-r^2)}{(a-r^2)}. \tag{365}$$

We have obtained the same mass during the quantization of the system in the usual phase space. The Fock-space fields has the following relation with canonical variables of the physical system.

$$\eta = \frac{r}{a}\phi + \frac{a - r^2}{a}\phi, \tag{366}$$

$$h = (a - r^2)(\phi - \theta r) - \frac{1}{e(1 + a - r^2)}\pi_1, \tag{367}$$

$$F = \Box^{-1}\pi_1. \tag{368}$$

Here F is the electric field and

$$B = \pi^0, \tag{369}$$

since $\pi_1 = -\epsilon^{\mu\nu}\partial_\nu A_\mu = -\tilde{\partial}^\mu A_\mu$. Now we find that the equal time commutator of the fock space fields are

$$[\eta(x), \dot{\eta}(y)] = i\frac{1}{a}\delta(x - y), \tag{370}$$

$$[F(x), \dot{F}(y)] = im^2\delta(x - y), \tag{371}$$

$$[h(x), \dot{h}(y)] = i\delta(x - y), \tag{372}$$

$$[B(x), \dot{\eta}(y)] = ie\delta(x - y). \tag{373}$$

This completes the quantization of the gauge invariant version of the theory in the extended phase space. The equation (363) and (364) represents a massive and a massless boson respectively. We have found that the appearance of massless and massive boson is identical as we have got in usual gauge non invariant version. Equation (361) and (362) appears because of the presence of the auxiliary field B in the Lorentz type gauge fixing term at the action level. When the phase space of a theory is extended in order to restore the gauge symmetry it is expected that the fields needed for the extension will allocate themselves in the un-physical sector the theory.

6.3 Study of Alternative Quantization of Gauged Floreanini-Jackiw Type Chiral Boson

The quantization of gauged FJ type chiral boson [51] was available in [35]. It was attempted there to quantize it in a gauge non-invariant manner. The gauge invariant version certainly can be quantized. Like the previous case some gauge fixing is needed in this situation indeed. We choose the Lorentz gauge and proceed to quantize the gauge symmetric version of the gauged FJ chiral boson. The gauge symmetric version of the said theory with Lorentz gauge is described by the lagrangian density:

$$\begin{aligned}\mathcal{L} &= \dot{\phi}\phi' - \phi'^2 + 2e\phi'(A_0 - A_1) - \frac{1}{2}e^2(A_0 - A_1)^2 + \frac{1}{2}ae^2(A_0^2 - A_1^2) \\ &+ \frac{1}{2}(\dot{A}_1 - A_0')^2 + \frac{1}{2}(a-1)(\dot{\eta}^2 - \eta'^2) \\ &+ e(A_0\eta' - A_1\dot{\eta}) + e(a-1)(A_1\eta' - A_0\dot{\eta}) \\ &+ B\partial_\mu A^\mu + \frac{\alpha B^2}{2}. \end{aligned} \quad (374)$$

Gauge fixing is needed in order to single out the real physical degrees of freedom from the gauge symmetric version of the extended phase space. The Euler-Lagrange equations of motion corresponding to the fields ϕ, A_0, A_1, B and η that follow from the lagrangian density (374) respectively are

$$\dot{\phi}' - \phi'' + e(A_0' - A_1') = 0, \quad (375)$$

$$A_0'' - \dot{A_1}' + e^2(1-a)A_0 - e^2A_1 + e(a-1)\dot{\eta} - e\eta' - 2e\phi' - \dot{B} = 0, \quad (376)$$

$$\ddot{A}_1 - \dot{A_0}' + ae^2A_1 + e^2A_1 - e^2A_0 - e(a-1)\eta' + e\dot{\eta} + 2e\phi' + B' = 0, \quad (377)$$

$$\partial_\mu A^\mu + \alpha B = 0, \quad (378)$$

$$(a-1)\ddot{\eta} - (a-1)\eta'' - e(a-1)\dot{A}_0 + e(a-1)A_1' + eA_0' - e\dot{A}_1 = 0. \quad (379)$$

It is found that the following expression of A_μ , ϕ and η represents the exact solution of the equations (375), (376), (377), (378) and (379)

$$A_\mu = \frac{1}{ae^2}[-\frac{(a-1)}{a}\tilde{\partial}_\mu F + \partial_\mu B - e\tilde{\partial}_\mu h - ea\partial_\mu\zeta], \quad (380)$$

$$\phi = -\frac{(a-1)}{ea^2}F - \frac{h}{a} + \zeta, \tag{381}$$

$$\eta = -\frac{F}{ea^2} - \zeta - \frac{h}{a(a-1)}. \tag{382}$$

If we put the expression of A_0, A_1, θ and ϕ in (375) , (376), (377), (378) and (379) we obtain some essential conditions which are given as follows

$$(\partial_0 - \partial_1)h = 0, \tag{383}$$

$$(\partial_0 - \partial_1)B = 0, \tag{384}$$

$$\Box\zeta = \alpha e B, \tag{385}$$

$$[\Box + m^2]F = 0, \tag{386}$$

where square of the mass is given as follows

$$m^2 = \frac{a^2e^2}{(a-1)}. \tag{387}$$

Therefore, the free fields in terms of which the system is completely described are

$$h = -(a-1)(\phi + \eta + \frac{1}{ea}F), \tag{388}$$

$$\zeta = \frac{1}{a}\phi - \frac{(a-1)}{a}\eta, \tag{389}$$

$$F = \pi_1, \tag{390}$$

$$B = \pi_0. \tag{391}$$

The equal time commutation relations corresponding to the free fields are found out to be

$$[F, \dot{F}] = im^2\delta(x-y), \tag{392}$$

$$[\zeta, \dot{\zeta}] = i\frac{1}{a}\delta(x-y), \tag{393}$$

$$[h, \dot{h}] = i\delta(x-y), \tag{394}$$

$$[B, \dot{\zeta}] = ie\delta(x-y). \tag{395}$$

Note that $F = \pi_1$ represents a massive field with mass m and h represents a massless chiral boson. These two are the replica of the spectrum as obtained in [35]. The equations involving B appear because of the presence of the auxiliary field in the Lorentz gauge fixing. Note that B has the vanishing commutation relation with the physical field F and h. The field ζ represents the zero mass dipole field playing the role of gauge degrees of freedom that can be eliminated by operator gauge transformation. So the spectrum agrees in an exact manner with the spectrum obtained in [35]. So it would be interesting if we get back the usual gauge non-invariant version from the gauge symmetric one of the extended phase space peeping the physical principles intact. We will now turn towards that.

6.4 Appropriate Gauge Fixing of GVQED and to get back the GNI from its GI form in presence of Wess-Zumino Term

In [30], we have found a technique how to get back the usual gauge non-invariant theory from a gauge symmetric theory of the extended phase space. We would like to make an extension for this GVQED following the guideline available in [30] towards getting back the original gauge non-invariant theory. Let us see how this technique responds to GVQED. Lagrangian of GVQED when added with the Wess-Zumino term in order to restore the local gauge symmetry turns into

$$\begin{aligned} L &= \int dx[\frac{1}{2}(\dot{\phi}^2 - \phi'^2) + \frac{1}{2}ae^2(A_0^2 - A_1^2) + \frac{1}{2}(\dot{A}_1 - A_0')^2 \\ &+ e(A_0\phi' - A_1\dot{\phi}) + er(A_0\dot{\phi} - A_1\phi') + \frac{1}{2}(a - r^2)(\dot{\theta}^2 - \theta'^2) \\ &- er(A_0\theta' - A_1\dot{\theta}) + e(a - r^2)(A_0\dot{\theta} - A_1\theta')]. \end{aligned} \tag{396}$$

Let us now proceed to calculate the momenta corresponding to the field A_0, A_1, ϕ and θ. From the standard definition, the momenta corresponding to the fields A_0, A_1, ϕ, and θ are found out:

$$\frac{\partial L}{\partial \dot{A}_0} = \pi_0 = 0, \tag{397}$$

$$\frac{\partial L}{\partial \dot{A}_1} = \pi_1 = \dot{A}_1 - A_0', \tag{398}$$

$$\frac{\partial L}{\partial \dot{\phi}} = \pi_\phi = \dot{\phi} - eA_1 + erA_0, \tag{399}$$

$$\frac{\partial L}{\partial \dot{\theta}} = \pi_\theta = (a - r^2)\dot{\theta} + e(a - r^2)A_0 + erA_1. \tag{400}$$

Using (397),(398),(399) and (400) canonical Hamiltonian is found out to be

$$\begin{aligned} H_c &= \int dx[\frac{1}{2}(\pi_\phi^2 + \pi_1^2 + \phi'^2) + \pi_1 A_0' + \frac{e^2 a(1 + a - r^2)}{2(a - r^2)} A_1^2 + e\phi'(rA_1 - A_0) \\ &+ e\pi_\phi(A_1 - rA_0) + \frac{1}{2}(a - r^2)\theta'^2 \\ &+ e\theta'((a - r^2)A_1 - A_0) + \frac{1}{2(a - r^2)}\pi_\theta^2 \\ &+ \frac{e}{(a - r^2)}(rA_1 + (a - r^2)A_0)\pi_\theta]. \end{aligned} \tag{401}$$

Equation (397) is independent of velocity. So it is the primary constraint of the theory as usual. The time evolution of the primary constraint (397) with respect to the Hamiltonian is

$$[\pi_0, H_c] = \pi_1' + e(\phi' + r\pi_\phi) - e(\pi_\theta - r\theta') \approx 0, \tag{402}$$

which gives the secondary constraint of the theory. It is found that the Poission bracket of the secondary constraint ω_2 with the Hamiltonian vanishes. So there lies only two constraints in the phase space of the theory. Following are those two.

$$\tilde{\omega_1} = \pi_0 \approx 0, \tag{403}$$

$$\tilde{\omega_2} = \pi_1' + e(\phi' + r\pi_\phi) - e\pi_\theta + er\theta' \approx 0. \tag{404}$$

As it has been found in [1], we too have introduced two gauge conditions to get back the gauge non-invariant theory. These two gauge fixing conditions are

$$\tilde{\omega_3} = \partial_1\theta \approx 0, \tag{405}$$

$$\tilde{\omega_4} = \pi_\theta + e((a - r^2)A_0 + rA_1) \approx 0. \tag{406}$$

Inserting the conditions (405) and (406) as strong condition into $\tilde{\omega_2}$ and H_c, we find that $\tilde{\omega_2}$ and H_c, reduce to the following

$$\tilde{\omega_{2R}} = \pi_1' + e^2((a - r^2)A_0 + rA_1) + e(r\pi_\phi + \phi') \approx 0, \tag{407}$$

$$\begin{aligned} \tilde{H_R} &= \frac{1}{2}(\pi_1^2 + \phi'^2) + \pi_1 A_0' + \frac{ae^2}{2}(A_1^2 - A_0^2) \\ &+ e\phi'(rA_1 - A_0) + \frac{1}{2}[\pi_\phi + e(A_1 - rA_0)]^2. \end{aligned} \tag{408}$$

Note that (407) and (408) is identical to the constraint and Hamiltonian in the usual phase space when $\alpha = -\gamma = a$ and $\beta = 0$. For $\tilde{H_R}$ the ordinary poisson brackets become inadequate [52]. So we need to evaluate the Dirac brackets among the fields. It necessities the computation of the matrix formed out of the Poission brackets between the constraints along with the gauge fixing conditions themselves. The constraints along with the gauge fixing conditions gives the following matrix when Poission brackets among themselve are evaluated.

$$C_{ij} = \begin{pmatrix} 0 & 0 & 0 & -e^2(a - r^2) \\ 0 & 0 & -e\partial_1 & 0 \\ 0 & -e\partial_1 & 0 & e\partial_1 \\ e^2(a - r^2) & 0 & e\partial_1 & 0 \end{pmatrix} \delta(x - y). \tag{409}$$

The determinant of C_{ij} is non vanishing. So it is invertible and its inverse is

$$C_{ij}^{-1} = \frac{1}{e^2(a - r^2)} \begin{pmatrix} 0 & \delta(x - y) & 0 & \delta(x - y) \\ \delta(x - y) & 0 & -\frac{C}{2e}\epsilon(x - y) & 0 \\ 0 & -\frac{C}{2e}\epsilon(x - y) & 0 & 0 \\ -\delta(x - y) & 0 & 0 & 0. \end{pmatrix} \tag{410}$$

where the factor $c = e^2(a - r^2)$. Therefore, from the definition of Dirac bracket, the Dirac brackets between the field variables can now be computed

$$[A_0(x), A_1(y)]^* = \frac{1}{e^2(a - r^2)}\delta'(x - y), \tag{411}$$

$$[A_0(x), \phi(y)]^* = \frac{r}{e(a - r^2)}\delta(x - y), \tag{412}$$

$$[A_0(x), \pi_\theta(y)]^* = \frac{1}{2e}\epsilon(x-y), \tag{413}$$

$$[A_0(x), \pi_\phi(y)]^* = \frac{1}{e(a-r^2)}\delta'(x-y), \tag{414}$$

$$[A_1(x), \pi_\theta(y)]^* = -\frac{1}{2e}\epsilon(x-y), \tag{415}$$

$$[A_0(x), \pi_1(y)]^* = \frac{r}{(a-r^2)}\delta(x-y), \tag{416}$$

$$[\pi_\phi(x), \pi_\theta(y)]^* = r\delta'(x-y), \tag{417}$$

$$[A_1(x), \pi_1(y)]^* = \delta(x-y), \tag{418}$$

$$[\phi(x), \pi_\phi(y)]^* = \delta(x-y). \tag{419}$$

Here also (*) stands to symbolize the Dirac bracket. Note that the role of gauge fixing is very crucial to gate back the usual theory since the other choice of valid gauge fixing certainly exists, but that will lead to a different effective theory which may not help to get back to the usual theory in a straightforward manner.

6.5 Appropriate Gauge Fixing of Gauged Floreanini-Jackiw type Chiral Boson and to Get Back the GNI model from its GI Form with the Weiss Zumino Term

An attempt is also made to show the equivalence between the gauge invariant version of the extended phase space and the gauge variant version of the usual phase space of the gauged model of FJ chiral boson. It is important because to make the model gauge invariant phase space is needed to extend introducing the Wess-Zumino fields. So, what service does the Wess-Zumino fields actually renders is a matter of utter curiosity. To meet it let us start with the lagrangian of the gauged FJ type chiral boson with the appropriate

Wess-Zumino term as is obtained from our investigation. The said lagrangian density reads

$$\begin{aligned}\mathcal{L} &= \dot{\phi}\phi' - \phi'^2 + 2e\phi'(A_0 - A_1) - \frac{1}{2}e^2(A_0 - A_1)^2 + \frac{1}{2}ae^2(A_0^2 - A_1^2) \\ &+ \frac{1}{2}(\dot{A}_1 - A_0')^2 + \frac{1}{2}(a-1)(\dot{\eta}^2 - \eta'^2) \\ &+ e(A_0\eta' - A_1\dot{\eta}) + e(a-1)(A_1\eta' - A_0\dot{\eta}). \end{aligned} \tag{420}$$

To show the equivalence between the gauge invariant and the gauge variant version of this model we proceed with computation of the canonical momenta corresponding to the fields ϕ, A_0, A_1, η:

$$\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \pi_\phi - \phi', \tag{421}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{A}_0} = \pi_0 = 0, \tag{422}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{A}_1} = \pi_1 = \dot{A}_1 - A_0'. \tag{423}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\eta}} = (a-1)\dot{\eta} - eA_1 - e(a-1)A_0 = \pi_\eta. \tag{424}$$

The equations (421) and (422) are independent of velocity so these two represent the two primary constraints. Explicitly these two are

$$\omega_1 = \pi_0 \approx 0, \tag{425}$$

$$\omega_2 = \pi_\phi - \phi' \approx 0. \tag{426}$$

Using the equations (421),(422),(423) and (424), a Legendre transformation leads to the canonical Hamiltonian H_c corresponding to the lagrangian density (420):

$$\begin{aligned} H_c &= \int dx[\phi'^2 + \frac{1}{2}\pi_1^2 + \pi_1 A_0' - 2e\phi'(A_0 - A_1) + \frac{1}{2}e^2(A_0 - A_1)^2 \\ &- \frac{1}{2}ae^2(A_0^2 - A_1^2) + \frac{1}{2}(a-1)\eta'^2 - eA_0\eta' - e(a-1)A_1\eta' \\ &+ \frac{1}{2(a-1)}\pi_\eta^2 + \frac{e^2}{2(a-1)}((a-1)A_0 + A_1)^2) \\ &+ \frac{e}{(a-1)}\pi_\eta((a-1)A_0 + eA_1)]. \end{aligned} \tag{427}$$

The preservation of the constraint of ω_1 leads to a new constraint

$$\omega_3 = \pi_1' + e\pi_\phi + e\phi' + e\eta' - e\pi_\eta \approx 0. \tag{428}$$

The ref.[30], suggests that we have to choose appropriate gauge fixing at this stage to meet our need and we find that gauge fixing conditions those which have been found suitable for this system are the following:

$$\omega_4 = e\eta' \approx 0, \tag{429}$$

$$\omega_5 = e^2(a-1)A_0 + e^2A_1 + e\pi_\eta \approx 0. \tag{430}$$

Under insertion of the conditions of (429) and (430), ω_3 and H_c turns into $\tilde{\omega_3}$ and $\tilde{H}_c$ those which are explicitly given by

$$\tilde{\omega_3} = \pi_1' + e\pi_\phi + e\phi' + e^2(a-1)A_0 + e^2A_1 \approx 0. \tag{431}$$

and

$$\begin{aligned} \tilde{H}_c &= \int dx[\phi'^2 + \frac{1}{2}\pi_1^2 + \pi_1 A_0' - 2e\phi'(A_0 - A_1) + \frac{1}{2}e^2(A_0 - A_1)^2 \\ &- \frac{1}{2}ae^2(A_0^2 - A_1^2)], \end{aligned} \tag{432}$$

respectively. Note that with the gauge fixing conditions (429) and (430) push back the constraint ω_3 into $\bar{\omega_3}$ which was the constraint of the usual phase space and as a result H_c lands onto $\bar{H}_c$ which was the Hamiltonian of the usual phase space. It has therefore become evident that physical contents remains the same in the gauge symmetric version of the theory in the extended phase space. The extra fields, therefore, renders their incredible service towards bring back of the symmetry without disturbing the physical sector. For completeness of the analysis we compute the Dirac brackets of the physical fields. The matrix C_{ij} in this situation is

$$C_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 & -e^2(a-1) \\ 0 & -2\partial & 0 & 0 & 0 \\ 0 & 0 & 0 & -e^2\partial & 0 \\ 0 & 0 & -e^2\partial & 0 & e^2\partial \\ e^2(a-1) & 0 & 0 & e^2\partial & 0 \end{pmatrix} \delta(x-y). \tag{433}$$

and the inverse of it is the following

$$C_{ij}^{-1} = \begin{pmatrix} 0 & 0 & \frac{1}{e^2(a-1)} & 0 & \frac{1}{e^2(a-1)} \\ 0 & -\frac{1}{2\partial_x} & 0 & 0 & 0 \\ -\frac{1}{e^2(a-1)} & 0 & 0 & -\frac{1}{e^2\partial_x} & 0 \\ 0 & 0 & -\frac{1}{e^2\partial_x} & 0 & 0 \\ -\frac{1}{e^2(a-1)} & 0 & 0 & 0 & 0 \end{pmatrix} \delta(x-y). \tag{434}$$

The Dirac brackets between the field variables are found to be

$$[A_0(x), A_1(y)]^* = \frac{1}{e^2(a-1)}\partial_x\delta(x-y), \tag{435}$$

$$[\phi(x), \phi(y)]^* = -\frac{1}{2\partial x}\delta(x-y), \tag{436}$$

$$[A_0(x), \phi(y)]^* = \frac{1}{e(a-1)}\delta(x-y), \tag{437}$$

$$[A_0(x), \pi_1(y)]^* = -\frac{1}{(a-1)}\delta(x-y), \tag{438}$$

$$[A_0(x), \pi_\phi(y)]^* = -\frac{1}{e(a-1)}\partial_x\delta(x-y), \tag{439}$$

$$[A_1(x), \pi_1(y)]^* = \delta(x-y), \tag{440}$$

$$[\phi(x), \pi_\phi(y)]^* = \delta(x-y), \tag{441}$$

Here also (*) stands to denote the Dirac bracket. It has been found that an appropriate gauge fixing helps us to land onto the the gauge non invariant version from gauge invariant version. The role of gauge fixing is crucial, it has been found once again.

6.6 Discussion

In this Chapter we have described the alternative quantization of two important field theoretical models which we have studied in [11] and [26]. The models are GVQED and the gauged model of FJ type chiral boson. In the usual phase space the quantization of the models were available in [11] and

[26]. The gauge symmetric version which arise from our study [11] and [12] have been quantized using Lorentz gauge. In both the cases physical content of the theory have been found identical. Because of the presence of the field B in Lorentz type gauge some extra equations occurred in both the cases but B has vanishingly Poission bracket with the physical fields.

Since the gauge symmetric part contains auxiliary fields an equivalence between the gauge symmetric and nonsymmetric version is of orth investigation which we have studied in [12] and [26] and in both the cases we have found the appropriate gauge fixing has got the success to map the gauge symmetric theory onto the respective gauge non symmetric version. The formalism developed by Falck Kramer [30] is found insrumental. We would like to mention that the role gauge fixing has been found crucial in both the cases.

Chapter 7

7 Study of Finite Field Dependent BRST and Finite Field Dependent Anti-BRST Quantization of GVQED

7.1 Introduction

The role of field dependent BRST (FFBRST) [13] is almost similar to the BRST so far symmetry is concerned. It does protect nilpotency and reflects the symmetry of the gauge fixed action [13] of a physically sensible theory. It can be considered as a generalization over the usual BRST formalism where transformation parameters becomes finite, field dependant and anti-commuting in nature [13]. Unlike BRST transformation, it fails to keep the measure of the generating functional unchanged [13]. However, the change appeared there renders several important services to make an equivalence between the different effective actions of a particular theory [13]. In this context, the services obtained through the exploitation of the change entered into the measure of the generating functional to relate the different gauge fixed actions of a particular theory is remarkable [13]. FFBRST is therefore important and interesting in its own right and FFBRST related studies has been carried out over the year [12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24]. So application of this formalism on any physically sensible theories would be of considerable interest and would certainly add a new contribution to the formal field theoretical regime.

We have studied a (1+1) dimensional generalized version of Quantum electrodynamics (GVQED) where axial and vector interaction get mixed up with different weight [41, 42, 43]. Application of FFBRST formalism on this model would also be instructive like its ancestor BRST formalism. So an extension using FFBRST formulation is also made in [12] to show how the contribution that enterers into the measure of the generating functional under FFBRST transformation helps to convert the BRST invariant effective action into its original gauge non-invariant version to ensures that the physical contents of these two effective actions are identical. The recent works [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24], indeed provides much insight into the way of approach towards this new endiviour. It reminds the work of Falck and Kramer [30], where they explicitly showed that physical content of chiral Schwinger model [32] remains identical both in the usual gauge non invariant action and the gauge symmetric action of the extended phase space. But it has to be kept in mind that in that situation the symmetry that was handled was the local gauge symmetry. Now FFBRST and anti-FFBRST formulation is applied to the BRST invariant effective action of GVQED to get back the original gauge non-invariant form of the action through the incredible service of the field dependent parameter of FFBRST and anti-FFBRST.

7.2 Brief Review of the Model

The model where we find the mixing of both vector and axial vector interaction with different weight is given by the following generating functional:

$$Z(A) = \int d\psi d\bar{\psi} exp[i \int d^2x L_F]. \tag{442}$$

with $L_F = \bar{\psi}\gamma^\mu[i\partial_\mu + e\sqrt{\pi}A_\mu(1-r\gamma_5)]\psi$. The phase space analysis of which is described in Chapter 4. The integration over the fermionic degrees of freedom leads to a determinant and if that fermionic determinant is expressed in terms of auxiliary scalar field ϕ, we get

$$Z(A) = \int d\phi exp[i \int d^2x \mathcal{L}_{\mathcal{B}}]. \tag{443}$$

where $\mathcal{L}_{\mathcal{B}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + eA^\mu(\tilde{\partial}_\mu + r\partial_\mu)\phi + \frac{1}{2}ae^2A_\mu A^\mu$. Here a is the regularization ambiguity emerged out during the process of regularization to remove the divergence of the fermionic determinant. If we now introduce the kinetic term of the back ground electromagnetic field we will get the lagrangian density:

$$\mathcal{L}_{\mathcal{B}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + eA_\mu(\epsilon_{\mu\nu}\partial^\nu + rg_{\mu\nu}\partial^\nu)\phi + \frac{e^2}{2}aA_\mu A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \quad (444)$$

The Euler-Lagrange equations for the fields describing the lagrangian density (444) are

$$\partial^\mu F_{\mu\nu} = -ae^2 A_\nu - e(\epsilon_{\mu\nu}\partial^\mu\phi + rg_{\mu\nu}\partial^\mu\phi), \quad (445)$$

$$\Box\phi = -e(rg_{\mu\nu}\partial^\nu + \epsilon_{\mu\nu}\partial^\nu)A^\mu. \quad (446)$$

It is known that the most general solution for A_μ is

$$A_\mu = \frac{1}{ae^2}[r\partial_\mu\phi + (a - r^2)\tilde{\partial_\mu}\phi + (1 + a - r^2)\tilde{\partial_\mu}h], \quad (447)$$

and the thoritical specturm are given by

$$(\Box + m^2)\sigma = 0, \quad (448)$$

$$\Box h = 0, \quad (449)$$

where

$$\phi + h = \sigma. \quad (450)$$

m^2 is given by

$$m^2 = \frac{e^2 a(1 + a - r^2)}{(a - r^2)}. \quad (451)$$

So the physical subspace of the model is constituted with a massive boson with square of the mass $m^2 = \frac{e^2a(1+a-r^2)}{(a-r^2)}$ and a massless boson. In short, this is the physical content of the model.

7.3 Application of FFBRST and Anti-FFBRST Formalism in the GVQED

An ingenious attempt was made in [13] to generalize the well celebrated BRST formulation. It was shown there that even making the BRST transformation field dependent the nilpotency can be protected and it is equally effective for anti-BRST formalism. Under finite field dependent transformation the path integral measure acquires a nontrivial change that though leads to a different effective theory, the physical contents of the theory remains unaffected. This generalization however is advantageous since that renders several important services. One of such advantage is that it helps to correlate the different gauge fixed versions of a particular theory [13]. The ability to relate a theory endowed with a set of first class constraint to an equivalent theory endowed with a set of second class constraint through appropriate choice of gauge fixing parameter is also an interesting extension of the field dependent BRST (FFBRST) [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24] formalism.

An illustration related to the calculation of Jacobian for this field dependent transformation is available in [13]. To make this Chapter self contained one, let us now proceed with the brief introduction concerning how FFBRST transformation brings a non trivial change in the integral measure of the generating functional and how this change adds a contribution to the effective action.

If the fields that describe a physically sensible theory are function of parameter η such that $\phi(x,\eta)$ is defined by $\phi(x,\eta=0)=\phi(x)$ and $\phi(x,\eta=1)=\tilde{\phi(x)}$ and the infinitesimal BRST transformation [4, 44, 45, 46, 47, 71] is given by

$$\frac{d}{d\eta}\phi(x)=\delta_{BRS}[\phi(\eta)\Theta'[\phi(\eta)]]. \tag{452}$$

The finite field dependence can be obtained through the integration over the infinitesimal transformation within the limit $\eta=0$ to $\eta=1$,

$$\tilde{\phi(x)}=\phi(x,\eta=1)=\phi(x,\eta=0)+\delta_{BRS}[\phi(x)\Theta[\phi(x)]] \tag{453}$$

where

$$\Theta[\phi(x)]=\int_0^1 d\eta'\Theta'[\phi(x,\eta')] \tag{454}$$

It should be mentioned here that the condition $\Theta^2=0$ is to be maintained in order to protect nilpotency. The Jacobian for the transformation can be evaluated from the field dependent function $\Theta[\phi(x)]$ by

$$\prod d\phi=J(\eta)\prod d\phi(\eta)=J(\eta+d\eta)\prod d\phi(\eta+d\eta). \tag{455}$$

The infinitesimal nature of transformation from $\phi(\eta)$ to $\phi(\eta + d\eta)$ leads to the following relations of the Jacobian $J(\eta)$:

$$\frac{J(\eta)}{J(\eta + d\eta)} = \sum \pm \frac{\delta\phi(x, \eta + d\eta)}{\delta\phi}. \qquad (456)$$

Here $\sum$ and $\prod$ singnifies the sum and product over all the fields involved within the theory respectively. In equation (456), (+) and (-) sign refers the boson and fermion fields respectively. Equation (456) renders the following in infinitesimal change in the Jacobian $J(\eta)$.

$$\frac{1}{J}\frac{dJ}{d\eta} = -\int d^2x[\pm\delta_b\phi(x,\eta)\frac{\partial\Theta'}{\partial\phi}]. \qquad (457)$$

The incredible characteristic of this extension is that within the functional integration $J(\eta)$ can be expressed as

$$J(\eta) = \exp iS_c(x, \eta), \qquad (458)$$

If and only if the condition

$$\int \prod d\phi(x)[\frac{1}{J}\frac{dJ}{d\eta} - i\frac{dS_c(x,\eta)}{d\eta}] \exp i(S_{eff} + S_c) = 0, \qquad (459)$$

is maintained within the phase space of the theory. The role of Θ though surprising, nevertheless plays a very crucial as well as intriguing role since the appropriate choice Θ leads to another equivalent effective action corresponding to the starting theory which is given by the generating functional:

$$\tilde{Z} = \int \prod d\phi(x) \exp i(S_{eff} + S_c), \qquad (460)$$

It indeed keeps the physical contents of the theory unchanged.

To see the transmutation between the gauge invariant and gauge non-invariant effective theory of GVQED we begin our analysis starting from the BRST invariant effective action of the theory which reads

$$\begin{aligned} S_{eff} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + e\epsilon_{\mu\nu}A^\mu\partial^\nu\phi + erg_{\mu\nu}A^\mu\partial^\nu\phi \\ &- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}ae^2A_\mu A^\mu + \frac{1}{2}(a - r^2)\partial_\mu\theta\partial^\mu\theta + e(a - r^2)g_{\mu\nu}A^\mu\partial^\nu\theta \\ &- er\epsilon_{\mu\nu}A^\mu\partial^\nu\theta + \partial_\mu C\partial^\mu\bar{C} + \alpha\frac{B^2}{2} + B\partial_\mu A^\mu. \\ &= S_{ORI} + S_{WZ} + S_{GHOST} + S_{GF} \end{aligned} \qquad (461)$$

The infinitesimal BRST transformations of the fields under which the above action (461) is found to remain invariant are

$$\delta\phi = r\lambda C, \delta A_\mu = -\frac{1}{e}\lambda\partial_\mu C, \delta\theta = \lambda C, \delta C = 0, \delta\bar{C} = -B, \delta B = 0. \quad (462)$$

and the FFBRST transformations of those fields describing the theory under consideration are

$$\delta\phi = rC\Theta, \delta A_\mu = -\frac{1}{e}\partial_\mu C\Theta, \delta\theta = -\Theta C, \delta C = 0, \delta\bar{C} = -B\Theta, \delta B = 0, \quad (463)$$

where Θ is an arbitrary finite field dependent function which is the transformation parameter corresponding to the FFBRST transformation. Our objective is to connect this BRST invariant effective action of the extended phase space to the original effective action of the usual physical phase space. To relate this we make a choice over the Θ in such a way that the change that would enter into the measure of the generating functional can be exploited to serve the desired purpose. To this end we define Θ as follows:

$$\Theta' = i\gamma\int d^2x[\bar{C}(\partial_\mu A^\mu + \frac{\alpha}{2}B)] \quad (464)$$

Here γ is the arbitrary parameter that would be fixed later. For the finite field dependent parameter the nontrivial infinitesimal change that would enter in the Jacobian can be computed using equation (457)

$$\frac{1}{J}\frac{dJ}{d\eta} = i\gamma\int d^2x[\delta\bar{C}\frac{d\Theta'}{d\bar{C}} + \delta A_\mu\frac{d\Theta'}{dA_\mu} + \delta B\frac{d\Theta'}{dB}] \quad (465)$$

$$= i\gamma\int d^2x[B(\partial_\mu A^\mu + \frac{\alpha}{2}B) + \frac{\lambda}{e}\partial^\mu C\partial_\mu\bar{C}]. \quad (466)$$

The Euler-Lagrange equation of motion for the ghost field simplifies the above equation to the following form:

$$\frac{1}{J}\frac{dJ}{d\eta} = i\gamma\int d^2x[-B(\partial_\mu A^\mu + \frac{\alpha}{2}B)]. \quad (467)$$

We are now in a state to choose an ansatz for S_c. The following ansatz for S_C suffices our need without violating any physical principle:

$$S_c = \int d^2x[\xi_1(k)B^2 + \xi_2(k)B\partial_\mu A^\mu]. \quad (468)$$

Here $\xi_1(\eta)$ and $\xi_2(\eta)$ are some functions of the parameter η. The differentiation of the action S_C with respect to η yields

$$\frac{\partial S_c}{\partial \eta} = \int d^2x[B^2\xi_1'(\eta) + B\partial_\mu A^\mu \xi_2'(\eta)]. \tag{469}$$

Here over prime denotes differentiation with respect to the parameter η. The contribution that enters into the measure of the generating function through the Jacobian under FFBRST transformation can be written down in the form of $\exp iS_c$, provided the following very equation:

$$\int d^2x \exp i(S_{eff} + S_c)[i(\xi_1' - \frac{\gamma\alpha}{2}B^2) + iB(\partial_\mu A^\mu(\xi_2' - \gamma)] = 0, \tag{470}$$

is satisfied. It fixes ξ_1 and ξ_2 and it can be expressed in terms of α and γ

$$\xi_1 = \frac{\alpha\gamma}{2}\eta, \tag{471}$$

$$\xi_2 = \gamma\eta. \tag{472}$$

Setting $\eta = 1$, we get S_c

$$S_c = \int d^2x[\gamma B\partial_\mu A^\mu + \frac{\alpha\gamma}{2}B^2], \tag{473}$$

And for $\gamma = -1$, S_c turns into

$$S_c = \int d^2x[-(B\partial_\mu A^\mu + \frac{\alpha}{2}B^2)]. \tag{474}$$

So through the exploitation of the change entered in to the measure of the generating functional through FFBRST transformation enables us to eliminate the gauge fixing part S_{GF} from the S_{eff} for the above setting of γ and η. It is the first step to proceed towards the effective action defined in the usual phase space. So the remaining part in the S_{eff} are

$$S_{ST} = S_{ORI} + S_{WZ} + S_{GHOST}. \tag{475}$$

Precisely, the parameter of FFBRST transformation (being field dependent) renders here a great job which is the elimination of the gauge fixing term through the contribution entered into the path integral measure of generating functional due to the finite field dependent nature of the transformation parameter of the FFBRST transformation. Our next task is to eliminate the ghost and the Wess-Zumino part one by one. The elimination of the ghost part is trivial because under integration the contribution that evolve out from this part can be absorbed within the normalization. However, the elimination of the Wess-Zumino term is not so trivial. It certainly needs the integrating out of the Wess-Zumino field but one has to keep it in mind that the theory has now converted into gauge invariant one and the constraints that embedded in its phase space are first class in nature. So proper gauge fixing is needed to land onto the theory of the usual phase pace [30]. This can be done in different ways. In [30], the authors did not use the path integral approach. However, since in the full body of the chapter path integral approach is followed, we use gauge fixing with the path integral formulation to which we now turn.

From the Hamiltonian analysis which is available in [26], it is known that the phase space of the theory contains two first class constraint. From [26], we find that the original action along with the Wess-Zumino part of the S_{eff} (461), leads to the following Hamiltonian:

$$
\begin{aligned}
H_{ce} &= \int dx[\frac{1}{2}(\pi_\phi^2 + \pi_1^2 + \phi^{'2}) + \pi_1 A_0^{'} + \frac{e^2 a(1+a-r^2)}{2(a-r^2)} A_1^2 + e\phi'(rA_1 - A_0) \\
&+ e\pi_\phi(A_1 - rA_0) + \frac{1}{2}(a-r^2)\theta^{'^2} \\
&+ e\theta'((a-r^2)A_1 + A_0) + \frac{1}{2(a-r^2)}\pi_\theta^2 \\
&+ \frac{e}{(a-r^2)}(rA_1 + (a-r^2)A_0)\pi_\theta],
\end{aligned}
\qquad (476)
$$

and there embeds the following two first class constraints in the phase space of the theory:

$$\tilde{\omega_1} = \pi_0 \approx 0, \qquad (477)$$

$$\tilde{\omega_2} = \pi_1' + e(\phi' + r\pi_\phi) + e\pi_\theta - er\theta' \approx 0. \qquad (478)$$

Therefore, two gauge fixing conditions are needed at this stage to get back the gauge non-invariant theory of the usual phase space. These two gauge fixing conditions that are chosen here are

$$\tilde{\omega_3} = \partial_1\theta \approx 0, \qquad (479)$$

$$\tilde{\omega_4} = \pi_\theta + e(a - r^2)A_0 + erA_1 \approx 0. \qquad (480)$$

With these inputs, the generating functional can be written down as

$$Z = \int [D\mu][det[\bar{w}_i, \bar{w}_j]^{\frac{1}{2}} e^{i\int d^2[\pi_1 \dot{A}_1 + \pi_0 \dot{A}_0 + \pi_\phi \dot{\phi} + \pi_\theta \dot{\theta} - H_{ce}}]\delta(\bar{w}_1)\delta(\bar{w}_2)\delta(\bar{w}_3)\delta(\bar{w}_4), \qquad (481)$$

where $[D\mu]$ is the Liouville measure. $[D\mu] = [d\pi_\phi][d\phi][d\pi_1][dA_1][d\pi_0][dA_0][d\pi_\theta][d\theta]$ and i and j runs from 1 to 4. After integrating out of the field θ and π_θ we find that equation (481) turns into

$$\begin{aligned} Z &= N \int [d\bar{\mu}] e^{i\int d^2x[\pi_1 \dot{A}_1 + \pi_0 \dot{A}_0 + \pi_\phi \dot{\phi} - \bar{H_{ce}}}] \times \\ & \quad \delta(\pi_0)\delta(\pi_1' + e(\phi' + r\pi_\phi) + e^2(a - r^2)A_0 + e^2 r A_1)], \end{aligned} \qquad (482)$$

where $[d\bar{\mu}] = [d\pi_\phi][d\phi][d\pi_1][dA_1][d\pi_0][dA_0]$ and N is a normalization constant having no significant physical importance and $\tilde{H_{ce}}$ is

$$\begin{aligned} \tilde{H_{ce}} &= \frac{1}{2}(\pi_1^2 + \phi'^2) + \pi_1 A_0' + \frac{1}{2} a e^2 (A_1^2 - A_0^2) \\ &+ e\phi'(rA_1 - A_0) + \frac{1}{2}[\pi_\phi + e(A_1 - rA_0)]^2. \end{aligned} \qquad (483)$$

Now after integrating out of the the fields π_ϕ, π_0 and π_1 we land on to the required result

$$Z = \int dA_1 dA_0 d\phi e^{iS_{ORI}}. \qquad (484)$$

Like BRST symmetry anti-BRST is also a symmetry of the effective action of a given theory and like the BRST transformations anti-BRST transformations do generate from a nilpotent charge. In the anti-BRST formulation the role of ghost and anti ghost fields interchanges. In addition to that there may be change in the coefficient depending upon the system. Therefore, study with anti-FFBRST is equally important like FFBRST. So we are intended to examine whether the anti-FFBRST formalism can be brought into the same service as it has been found to serve by the FFBRST formalism. The anti-BRST transformations for the fields describing the theory are given by

$$\delta\phi = r\lambda\bar{C}, \delta A_\mu = -\frac{1}{e}\lambda\partial_\mu\bar{C}, \delta\theta = -\lambda\bar{C}, \delta C = B\lambda, \delta\bar{C} = 0, \delta B = 0, \qquad (485)$$

and the corresponding anti-FFBRST transformation of the fields are

$$\delta\phi = r\bar{C}\Theta_A, \delta A_\mu = -\frac{1}{e}\partial_\mu\bar{C}\Theta_A, \delta\theta = -\bar{C}\Theta_A \qquad (486)$$

$$\delta\bar{C} = 0, \delta C = -B\Theta_A, \delta B = 0. \tag{487}$$

where Θ_A is an arbitrary finite field dependent function serving the role of transformation parameter corresponding to the anti-FFBRST transformation. Our objective is the same as we have done for the FFBRST. For that purpose here also we need to choose a Θ_A in such a way that the change that would enter into the generating functional can be exploited to serve the same purpose as it has been found to serve in the earlier situation. In an analogous manner, let us define Θ_A as

$$\Theta'_A = i\bar{\gamma}\int d^2x[C(\partial_\mu A^\mu + \frac{\beta}{2}B)], \tag{488}$$

where $\tilde{\gamma}$ is an arbitrary parameter that would be fixed later like the previous situation. For anti-FFBRST transformation also the nontrivial infinitesimal change in the Jacobian that would enter can be calculated using equation (457).

$$\frac{1}{J}\frac{dJ}{d\eta} = i\bar{\gamma}\int d^2x[B(\partial_\mu A^\mu + \frac{\beta}{2}B) + \frac{\lambda}{e}\partial_\mu C\partial^\mu\bar{C}]. \tag{489}$$

By the use of Euler-Lagrange equation of motion for the anti ghost field the above equation reduces to

$$\frac{1}{J}\frac{dJ}{d\eta} = i\bar{\gamma}\int d^2x[(-B(\partial_\mu A^\mu + \frac{\beta}{2}B)]. \tag{490}$$

In order to express the above in the form of $\exp i\bar{S}_C$, the following ansatz can be chosen for $\bar{S_C}$ without any loss of generic condition, and of course, without violating any physical principle:

$$\bar{S}_c = \int d^2x[\xi_1(k)B^2 + \xi_2(k)B\partial_\mu A^\mu]. \tag{491}$$

Here $\xi_1(\eta)$ and $\xi_2(\eta)$ are some functions of parameter η. If we now take the derivative of the action $\bar{S_C}$ with respect to η we get

$$\frac{\partial\bar{S}_c}{\partial\eta} = \int d^2x[B^2\xi'_1(\eta) + B\partial_\mu A^\mu\xi'_2(\eta)]. \tag{492}$$

The over prime denotes here the differentiation with respect to the parameter η as usual. The contribution that the path integral measure of the generating functional acquires under anti-FFBRST transformation can be written down in the form of $\exp^{i\bar{S_C}}$, if and only if the following relation

$$\int d^2x \exp i(S_{eff} + S_c)[i(\xi_1' - \frac{\bar{\gamma}\beta}{2}B^2) + iB(\partial_\mu A^\mu(\xi_2' - \bar{\gamma})] = 0, \qquad (493)$$

holds. Equation (493) fixes $\xi_1(\eta)$ and $\xi_2(\eta)$ so one can express these in terms of β and $\bar{\gamma}$:

$$\xi_1 = \frac{\bar{\gamma}\beta}{2}\eta, \qquad (494)$$

$$\xi_2 = \bar{\gamma}\eta. \qquad (495)$$

Thus setting the parameter $\eta = 1$ we get

$$\bar{S}_c = \int d^2x[\bar{\gamma}B\partial_\mu A^\mu + \frac{\beta\bar{\gamma}}{2}B^2]. \qquad (496)$$

and finally putting $\bar{\gamma} = -1$, we get the appropriate $\bar{S_C}$ in his situation:

$$\bar{S}_c = \int d^2x[-(B\partial_\mu A^\mu + \frac{\beta}{2}B^2)]. \qquad (497)$$

Therefore, we find that the exploitation of the change entered into path integral measure of the generating functional due to the anti-FFBRST transformation with the above choice of $\bar{\gamma}$ and η enables us to eliminate the gauge fixing term from our starting S_{eff} and it now reduces to

$$S_{ST} = S_{ORI} + S_{WZ} + S_{GHOST}. \qquad (498)$$

So the first step to reach towards the effective action defined in the usual phase space is successfully made in case of anti-FFBRST transformation too. Note that for this system the calculation may look similar to the FFBRST. After the first step, the task that is yet to be done is to make the S_{eff} part free from ghost as well as the Wess- Zumino part. The elimination of the ghost part is trivial like the previous case since under integration of the anti-ghost field the contribution that evolve out can again be absorbed within the normalization. However, the elimination of the Wess-Zumino term is not trivial but it is identical to the previous case as it has already been made for the FFBRST transformation. Explicit calculation in this situation therefore does not carry any new information. So, it is not shown here.

7.4 Discussion

Application of FFBRST and anti-FFFBRST are made here for interesting as well as important purpose. In fact, it has been used here to make the equivalence between the physical content of a model in the usual and in the extended phase space. Here extended phase space implies the presence of not only the Wess-Zumino field, but also the presence of ghost and the auxiliary B fields too. It has been found that both the FFBRST and anti-FFBRST formulation have successfully render their great services to show the equivalence. In both the cases, FFBRST and ant-FFBRST it has been found that the gauge fixing part, i.e., the part of the effective action involving the auxiliary B gets eliminated by the contribution entered into the effective action through the acquired contribution of the measure of the generating functional under FFBRST and anti-FFBRST transformations respectively. To eliminate rest of the part we have adopted here the formalism developed by Falck and Kramer [30]. So, the joint action of the two formalisms developed in [13] and [22] have done their novel services to show the equivalence. We can conclude that the joint action of these two formalisms would be instrumental to show the equivalence between the different effective actions of any field theoretical model if these two are employed in appropriate manner.

Chapter 8

8 Constraints Through Lagrangian Formulation: Few Case Studies

8.1 Introduction

The constraints structure of a theory can be studied through Lagrangian formulation in the velocity phase space too. In velocity phase space Lagrangian formulation is an useful instrument to study the gauge symmetric property of field theoretical models and it is also useful to find out gauge transformation generator. This formalism has developed by Shirzad [31]. Another approaches have been found in the literature to study the local symmetry of the gauge theories through the Hamiltonian formulation [32, 33, 34, 35, 36, 37] based on Dirac conjecture. Several authors have tried to find out the answer of several interesting questions related to the gauge symmetry using Hamiltonian formulation. The most general form of gauge transformation generator too can be determined with that Hamiltonian formulation [32, 33, 34, 35, 36, 37]. To study BRST symmetry, Hamiltonian approach also has been found to be instrumental. It is true that unitarity of a theory can not be well understood without Hamiltonian approach. However, Hamiltonian embedding of constrained system has some drawbacks. It does not always lead to Lorentz covariant generating functional. This drawback indeed has the remedy in the Lagrangian formulation. So the importance of the study of gauge symmetric property through the formalism based on Lagrangian formulation can not be ignored. Therefore, gauge symmetry related studies on dynamical theory should be extended with equal intensity in both the approaches. Few studies using Lagrangian approach are available in the literature [3, 31]. However, before Shirzad very little was achieved to understand the fundamental question related to the gauge symmetry in the Lagrangian formulation. In [31], Shirzad gave a systematic development of gauge symmetry related study for an arbitrary lagrangian and applied it to the so called generalized Schwinger model. So this formalism can be applied on the different field theoretical models in order to test whether a given model does have gauge symmetry or it is lacking in it and it is also useful to find out the gauge transformation generator. It becomes much more interesting if one apply this formalism in the extended phase space needed to restore the gauge invariance and to verify whether this formalism works there in an appropriate manner as it was found to work in the usual phase space in [31]. In [38] we have studied the different gauge symmetric and gauge non symmetric (anomalous) models with the prescription based on Lagrangian formulation developed by Shirzad [31]. We have investigated whether Shirzads formalism enables one to verify the presence or absence of gauge symmetry in a given theory. One reasons behind the consideration of anomalous model is to ensure whether this scheme is capable for testing the absence of gauge symmetry when it is lacking in a given model. The another reason is to study the power of this approach towards its applicability in the extended phase space.

8.2 A Brief Discussion of Shirzad's Formalism

It would be useful if we give a brief account of the formalism developed by Shirzad [31] in this section before going to apply it. If a dynamical system with N degrees of freedom is considered which is described by the lagrangian,

$$L = L(q_i, \dot{q}_i). \tag{499}$$

the Euler equations of motion for that lagrangian will be

$$L_i = w_{ij}\ddot{q}_j + \alpha_i. \tag{500}$$

Here $i = 1, 2.....N$. The matrix w stands for the Hessian matrix of the system. The Hessian matrix w_{ij} and α_i of equation (500) respectively are

$$w_{ij} = \frac{\partial^2 L}{\partial \dot{q}_i \partial \dot{q}_j}, \tag{501}$$

$$\alpha_i = \frac{\partial^2 L}{\partial q_j \partial \dot{q}_i}\dot{q}_j - \frac{\partial L}{\partial q_i}. \tag{502}$$

For a singular lagrangian $det[w_{ij}] = 0$. The equations of motion in this situation can not be solved for all accelerations. If the rank of w is $N - A_1$ then A_1 number of null eigen vector will be found for the matrix w_{ij}

$$\lambda_i^{a_1} w_{ij} = 0, \tag{503}$$

where $a_1 = 1.............A_1$. Here $\lambda_i^{a_1}$ indicates the null eigen vector. When equation (500) is multiplied by the null eigen vector $\lambda_i^{a_1}$ from the left it gives

$$\gamma^{a_1} = \lambda_i^{a_1} L_i = \lambda_i^{a_1} \alpha_i = 0. \tag{504}$$

This indicates the presence of A_1 number of lagrangian constraints of velocity and coordinates, but all of these constraints are not independent of each other in general. If it is assumed that the rank of equation (504) is $\bar{A}_1$, then $\bar{A}_1$ number of independent functions $\gamma^{\bar{a1}}$ can be written down as

$$\gamma^{\bar{a}_1}(q.\dot{q}) = \Sigma_{a_1=1}^{A_1} C_{a_1}^{\bar{a}_1}(q,\dot{q}) \gamma^{a_1}(q,\dot{q}), \tag{505}$$

where $\bar{a}_1 - 1..........\bar{A}_1$ and $C_{a_1}^{\bar{a}_1}$ represents the coefficients which may depend on q_i and $\dot{q}_i$. These set of lagrangian constraints are useful for determining the number of undetermined accelerations. Remaining constraints which vanish identically are

$$\Sigma_{a_1=1}^{A_1} C_{a_1}^{\hat{a}_1}(q,\dot{q}) \gamma^{a_1}(q,\dot{q}) = 0. \tag{506}$$

where $\hat{a}_1 = 1, ...A_1$. These are of course linear combinations of γ^{a_1}'s and their number will be $\hat{A}_1 = A_1 - \bar{A}_1$. This set of lagrangian constraints can be used to construct the form of the gauge transformation from lagrange equations of motion. Comparing equation (504) and (505) one gets,

$$\lambda^{\bar{a}_1}(q.\dot{q}) = \Sigma_{a_1=1}^{A_1} C_{a_1}^{\bar{a}_1}(q,\dot{q}) \lambda^{a_1}(q.\dot{q}) \tag{507}$$

Using equation (507), primary constraints can be calculated and these are given by

$$\gamma^{\bar{a}_1} = \lambda_i^{\bar{a}_1} L_i. \tag{508}$$

Equating equation (504) and (506) one can obtain

$$\lambda^{\hat{a}_1}(q.\dot{q}) = \Sigma_{a_1=1}^{A_1} C_{a_1}^{\hat{a}_1}(q,\dot{q}) \lambda^{a_1}(q,\dot{q}). \tag{509}$$

Equation (509), represents $\hat{A}_1$ number of null eigen vector. Now identities of the Euler derivatives appears as

$$\lambda_i^{\hat{a_1}} L_i = 0. \tag{510}$$

In order to get a consistent theory, the time derivatives of the primary constraint (508) is to be added to the equation of motion (500). Therefore, one gets $N + \bar{A}_1$ number of equations that contain accelerations which can be written in a combined manner as follows

$$L_{i_1} = w^1_{i_1 j}\ddot{q}_j + \alpha^1_{i_1} = 0, \tag{511}$$

where $i_1 = 1................N + \bar{A}_1$. Here $\bar{A}_1$ represents the rank of time derivatives of $\gamma^{\bar{a}_1}$. The matrix $w^1_{i_1 j}$ may also contain some other null eigen vector like the previous one. Following the previous process one gets new null eigen vector, and the expressions λ^{a_2} of it is

$$\gamma^{a_2} = \lambda^{a_2}_{i_1} L^1_{i_1} = 0, \tag{512}$$

where $a_2 = 1..........A_2$. So, one finds $\bar{A}_2$ number of independent functions $\gamma^{\bar{a_2}}$ and $\hat{A}_2$ number of identities $\hat{\gamma}^{a_2}$ for γ^{a_2} standing in equation (512). In the next step the time derivative of secondary constraint is to be added to the equation (511), as it is done for the former set of constraint in order to maintain consistency. This gives,

$$L_{i_2} = w^1_{i_2 j}\ddot{q_{ij}} + \alpha^2_{i_2} = 0, \tag{513}$$

where $i_2 = 1..........N+\bar{A}_1+\bar{A}_2$. Here $\bar{A}_2$ stands for the rank of time derivative of secondary constraint. In this way one needs to proceed step by step. In each step some identities along with some new constraints may results. Finally, in the n th stage the equations of motion for the system will be of the form

$$L_{i_n} = w^n_{i_n j}\ddot{q_{ij}} + \alpha^n_{i_n} = 0, \tag{514}$$

where $i_n = 1..........N + \bar{A}_1 + \bar{A}_n$. If in this case one finds new null eigen vector $\lambda^{a_{n+1}}$ for w^n and multiplication of $\lambda_i^{a_{n+1}}$ with equation (514) provides $\bar{A_{n+1}}$ number of lagrangian constraints and $\hat{A_{n+1}}$ number of identities and these with hold the following relation

$$\lambda_i^{\hat{a_{n+1}}} L_i + \lambda^{\hat{a_{n+1}}}_{\bar{a_1}} \frac{d\gamma^{\bar{a_1}}}{dt} + ... + \lambda^{\hat{a_{n+1}}}_{\bar{a_n}} \frac{d\gamma^{\bar{a_n}}}{dt} = 0. \tag{515}$$

Equation (515) can be written down in the form of a total derivative as follows

$$\Sigma_{s=0}^{n} \frac{d^s}{dt^s}(\phi_{si} L_i) = 0, \quad (516)$$

where ϕ_{si} are some functions of coordinate and their derivatives. That can be determined with a judicious choice. If w_n does not give any new eigen vector, it indicates that the process gets terminated. Another way of testing the termination of the procedure is to check whether the nth step gives any new constraint or not. The appearance of no new constraint too indicate the termination of the process. For the lagrangian $L(q_i, \dot{q}_i)$ the action is found to be invariant under the following transformation,

$$\delta q_i = \Sigma_{s=0}^{n} (-1)^s \frac{d^s f}{dt^s} \phi_{si} \quad (517)$$

if ϕ_{si} exists for that particular dynamical system represented by the $L(q_i, \dot{q}_i)$, where f (t) is an arbitrary function of time. The variation of lagrangian under the transformation (517) is given by

$$\delta L = -[\Sigma_{s=0}^{n} \frac{d^s}{dt^s}(\phi_{si} L_i)] f = 0. \quad (518)$$

If the lagrangian of a dynamical system is described by the set of fields $q_{i(x,t)}$ the general form of the lagrangian reads

$$L = \int dx L(q_i(x,t), \partial_x q_i(x,t) \partial_t q_i(x,t)). \quad (519)$$

The equation of motion in this situation becomes

$$L_i(x,t) = \int dy w_{ij}(x,y) \ddot{q}_j(y,t) + \alpha_i(x,t), \quad (520)$$

where $i = 1, ..., N$. N here represents the number of fields describing the dynamical system. $w(x,y,t)$ and $\alpha(x,t)$ in this situation takes the form

$$w_{ij}(x,y,t) = (\frac{\delta^2 L}{\delta \dot{q}_i(x,t) \delta \dot{q}_j(y,t)}), \quad (521)$$

$$\alpha_i(x,t) = \int dy (\frac{\delta^2 L}{\delta q_j(y,t) \delta \dot{q}_j(x,t)} - \frac{\delta L}{\delta q_i(x,t)}). \quad (522)$$

The null eigen vector of the Hessian matrix $w(x, y, t)$ here looks

$$\lambda^a_z(x) = \lambda^a \delta(z - x). \tag{523}$$

Multiplication of $\lambda^a_z(x)$ with equation of motion gives primary lagrangian constraint as follows

$$\gamma^a = \int dx \lambda^a_i \delta(z - x) L_i(x.t) = \lambda^a_i L_i(z, t). \tag{524}$$

If the process continues in the similar manner as it is described earlier keeping in mind that the system is described by field then one will arrive at the following gauge transformation formula

$$\delta q_i(x, t) = \Sigma^m_{\alpha=1} \Sigma^n_{s=0} (-1)^s \int dz \frac{\delta^s f_\alpha(z, t)}{\delta t^s} \phi^\alpha_{si}(z, x). \tag{525}$$

8.3 Free Maxwell's Lagrangian

Let us consider the lagrangian density of Free Maxwell field

$$\mathcal{L}_{\mathcal{FM}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \tag{526}$$

In $(1 + 1)$ dimension the lagrangian density reads

$$\mathcal{L}_{\mathcal{FM}} = \frac{1}{2}(\dot{A_1}^2 + A_0'^2 - 2A_0' \dot{A}_1). \tag{527}$$

The equations of motion for the field A_0 and A_1 that come out from equation (500) for the lagrangian $\mathcal{L}_{\mathcal{FM}}$ are

$$L_{A_0} = A_0'' - \dot{A_1}', \tag{528}$$

$$L_{A_1} = \ddot{A}_1 - \dot{A_0}'. \tag{529}$$

For the lagrangian (527) w and α respectively are

$$w = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \delta(z - x), \tag{530}$$

$$\alpha = \begin{pmatrix} A_0'' - \dot{A_1}' \\ -\dot{A_0}' \end{pmatrix}. \tag{531}$$

It is found that Hessian matrix w has a null eigen vector

$$\lambda^1(z) = (1,0)\delta(z-x). \tag{532}$$

Multiplying the equation (527) from the left by λ^1, we get the primary lagrangian constraint

$$\gamma^1(z,t) = \int dx\delta(z-x)L_{A_0}(x,t) = L_{A_0}(z,t) = (A_0'' - \dot{A_1}')(z,t). \tag{533}$$

Time derivatives of γ^1 yields

$$\frac{\partial}{\partial t}\gamma^1(x,t) = (\dot{A}_0'' - \ddot{A_1}')(x,t). \tag{534}$$

According to Shirzad's prescription equation (534) is to be added with (527) in order to maintain consistency condition of the primary constraint and that results

$$L_1(x,t) = (L_{FM} + \frac{\partial}{\partial t}\gamma^1)(x,t). \tag{535}$$

w^1 and α^1 as standing in equation (501) and (502) are found out to be

$$w^1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & -\frac{\partial}{\partial x} \end{pmatrix} \delta(z-x), \tag{536}$$

$$\alpha^1 = \begin{pmatrix} A_0'' - \dot{A_1}' \\ -\dot{A_0}' \\ \dot{A_0}'' \end{pmatrix}. \tag{537}$$

We find that w_1 also has a null eigen vector

$$\lambda^2(z) = (0, \frac{\partial}{\partial x}, 1)\delta(z-x). \tag{538}$$

Multiplying the equation (535) from left by λ^2 we find that γ^2 comes out to be zero. Explicitly

$$\gamma^2(z,t) = \int dx\lambda^2_{i_2}L_{i_1}(x,t) = \lambda^2\alpha_1(z,t) = 0. \tag{539}$$

So, λ^2 does not give rise to any new constraint. As a result we can not increase the the rank of equation for accelerations and γ^2 can be expressed in the following form

$$\gamma^2(x,t) = (\frac{\partial}{\partial t}L_{A_0} + \frac{\partial}{\partial x}L_{A_1})(x,t) = 0. \tag{540}$$

Comparing (540) with equation (516) we get the non vanishing $\phi's$:

$$\phi_{1,1}(z,x) = \delta(z-x), \tag{541}$$

$$\phi_{0,2}(z,x) = \frac{\partial}{\partial z}\delta(z-x). \tag{542}$$

The gauge transformation formula (517) gives the following gauge transformation for the field

$$\delta A_0 = -\int dz \frac{\partial}{\partial t}\delta(z-x)f(z,t) = -\frac{\partial}{\partial t}f(x,t), \tag{543}$$

$$\delta A_1 = -\int dz (\frac{\partial}{\partial z}\delta(z-x))f(z,t) = \frac{\partial}{\partial x}f(x,t). \tag{544}$$

The variation of the lagrangian density $\mathcal{L}_{\mathcal{FM}}$ lagrangian under the above transformation is

$$\delta\mathcal{L}_{\mathcal{FM}}(x,t) = -\Sigma_{s=0}^{n}\frac{\partial^s}{\partial t^s}(\phi_{si}L_i)(x,t) = -[\frac{\partial}{\partial t}L_{A0} + \frac{\partial}{\partial x}L_{A1}](x,t) = 0. \tag{545}$$

It is shows that the lagrangian (526) is invariant under the gauge transformation (543) and (544). It is the expected result since it is known that the lagrangian (526) is invariant under the transformation $A_\mu \rightarrow A_\mu - \partial_\mu f$.

8.4 Maxwell lagrangian with mass like term

Let us now add the mass like term $\frac{a^2}{2}A_\mu A^\mu$ with the Maxwell lagrangian and apply the formalism to test whether it has the gauge symmetry or not. So lagrangian with which we are going to start our analysis is

$$L_{MM} = \int[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{a^2}{2}A_\mu A^\mu]dx. \tag{546}$$

In (1 + 1) dimension the lagrangian density takes the form

$$\mathcal{L}_{\mathcal{MM}} = \frac{1}{2}(\dot{A_1}^2 + A_0'^2 - 2A_0'\dot{A}_1) + \frac{a^2}{2}(A_0^2 - A_1^2). \tag{547}$$

The equations of motion for the field A_0 and A_1 that come out from equation (500) for the lagrangian under consideration are

$$L_{A_0} = A_0'' - \dot{A_1}' - a^2 A_0, \tag{548}$$

$$L_{A_1} = \ddot{A}_1 - \dot{A_0}' + a^2 A_1. \tag{549}$$

The matrices w and α for this modified lagrangian come out to be

$$w = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \delta(z - x), \tag{550}$$

$$\alpha = \begin{pmatrix} A_0'' - \dot{A_1}' - a^2 A_0 \\ -\dot{A_0}' + a^2 A_1 \end{pmatrix}. \tag{551}$$

The Hessian matrix w has a null eigen vector

$$\lambda^1(z) = (1, 0)\delta(z - x). \tag{552}$$

Equation (547) when multiplied from left by λ^1, it results the primary lagrangian constraint

$$\gamma^1(x, t) = (A_0'' - \dot{A_1}' - a^2 A_0)(x, t) = L_{A_0}(x, t). \tag{553}$$

We need the time derivatives of γ^1 to calculate $L_1(x, t)$:

$$\frac{\partial}{\partial t}\gamma^1(x, t) = (\dot{A}_0'' - \ddot{A_1}' - a^2 \dot{A}_0)(x, t). \tag{554}$$

Now adding equation (554) with L_{MM} we get

$$L_1(x, t) = (L_{MM} + \frac{\partial}{\partial t}\gamma^1)(x, t). \tag{555}$$

This $L_1(x,t)$ is to be used for further analysis in order to maintain consistency of the primary constraint (553). Using equation (501) and (502) the matrices w_1 and α_1 for this system are calculated as follows.

$$w^1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & -\frac{\partial}{\partial x} \end{pmatrix} \delta(z-x), \tag{556}$$

$$\alpha^1 = \begin{pmatrix} A_0'' - \dot{A_1}' - a^2 A_0 \\ -\dot{A_0}' + a^2 A_1 \\ \dot{A_0}'' - a^2 \dot{A}_0 \end{pmatrix}. \tag{557}$$

We find that w_1 has another null eigen vector

$$\lambda^2(z) = (0, \frac{\partial}{\partial x}, 1)\delta(z-x). \tag{558}$$

Multiplying the equation (555) from left by λ^2, we get

$$\gamma^2(z,t) = \int dx \lambda^2_{i_2} L_1(x,t) = a^2(A_1' - \dot{A}_0)(x,t). \tag{559}$$

which is non vanishing one. This λ^2 is nothing but the s econdary constraint for this system. For maintaining consistency we have added the time derivative of secondary constraint to the equation (555) and obtain

$$L_2(x,t) = (L_1 + \frac{\partial}{\partial t}\gamma^2)(x,t). \tag{560}$$

The matrix w^2 and α^2 standing in equations (513) will be the following for this particular situation

$$w^2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & -\frac{\partial}{\partial x} \\ -a^2 & 0 \end{pmatrix} \delta(z-x), \tag{561}$$

$$\alpha^2 = \begin{pmatrix} A_0'' - \dot{A_1}' - a^2 A_0 \\ -\dot{A_0}' + a^2 A_1 \\ \dot{A_0}'' - a^2 \dot{A}_0 \\ a^2 \dot{A}_1' \end{pmatrix}. \tag{562}$$

We find that w^2 again gives a null eigen vector

$$\lambda^3(z) = (0, \frac{\partial}{\partial x}, 1, 0)\delta(z - x). \tag{563}$$

Multiplying the equation (560) from left by λ^3, we obtain

$$\gamma^3(z,t) = a^2(A_1' - \dot{A}_0)(x,t) = \gamma^2(x,t). \tag{564}$$

Note that $\gamma^3(x,t)$ and $\gamma^2(x,t)$ are identical. So multiplication of λ^3 with L_2 does not provide any new constraint. Therefore, we can not increase the the rank of equation for accelerations. Let us now try to write γ^3 in the form of equation (515)

$$\gamma^3(x,t) = (\frac{\partial}{\partial t} L_{A_0} + \frac{\partial}{\partial x} L_{A_1})(x,t). \tag{565}$$

Equating equation (565) with (516) we get $\phi' s$ and the non vanishing $\phi' s$ are found out to be

$$\phi_{1,1}(z,x) = \delta(z - x), \tag{566}$$

$$\phi_{0,2}(z,x) = \frac{\partial}{\partial z}\delta(z - x). \tag{567}$$

The gauge transformation formula (517) gives the following gauge transformation for the field

$$\delta A_0 = -\int dz \frac{\partial}{\partial t} f(z,t)\delta(z - x) = -\frac{\partial}{\partial t} f(x,t), \tag{568}$$

$$\delta A_1 = -\int dz (\frac{\partial}{\partial z}\delta(z - x)) f(z,t) = \frac{\partial}{\partial x} f(x,t). \tag{569}$$

The variation of the lagrangian density (547) under the variation of the fields (568) and (569) is

$$\begin{aligned} \delta L_{FM} f(x,t) &= -\Sigma_{s=0}^{n} \frac{d^s(\phi_{si} L_i)}{dt^s} f(x,t) \\ &= -[\frac{d}{dt} L_{A0} + \frac{d}{dx} L_{A1}] f(x,t) \\ &= -a^2(A_1' - \dot{A}_0) f(x,t). \end{aligned} \tag{570}$$

Since L_{MM} does not vanish, there is no gauge symmetry of the lagrangian density (547). The result here to does not go beyond our expectation since it is known that the presence of mass like term breaks the gauge invariance of the free Maxwell theory. In the following section we will proceed to study the application of the Shirzad's formalism in the extended phase space of this system.

8.5 Maxwell's Lagrangian with Masslike Term is Made Gauge Invariant with Auxiliary Field

We have seen in the previous Section that the lagrangian density (547) is not invariant under the Gauge transformation (568) and (569). So we add some terms involving auxiliary fields θ with the lagrangian (547) in order to make right hand side of the equation (570) zero. Lagrangian density under consideration along with the appropriate terms needed to make Eq. (570) zero is

$$L_{EM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{ae^2}{2}A_\mu A^\mu + \frac{a}{2}(\dot{\theta}^2 - \theta'^2) + ae(\dot{\theta}A_0 - \theta' A_1). \quad (571)$$

Note that the term is nothing but the Wess-Zunino term which we have needed to add to make equation (570) zero. The equations of motion for the field A_0, A_1 and θ are

$$L_{A_0} = A_0'' - \dot{A_1}' - ae^2 A_0 - ae\dot{\theta}, \quad (572)$$

$$L_{A_1} = \ddot{A_1} - \dot{A_0}' + ae^2 A_1 + ae\theta', \quad (573)$$

$$L_\theta = a\ddot{\theta} - a\theta'' + ae\dot{A}_0 - aeA_1'. \quad (574)$$

Here we repeat the same calculation as before and w and α for this lagrangian are found out to be

$$w = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & a \end{pmatrix} \delta(z-x) \quad (575)$$

$$\alpha = \begin{pmatrix} A_0'' - \dot{A_1}' - ae^2 A_0 - ae\dot{\theta} \\ -\dot{A_0}' + ae^2 A_1 + ae\theta' \\ -a\theta'' + ae\dot{A}_0 - aeA_1' \end{pmatrix} \quad (576)$$

We also find that the Hessian matrix w has the null eigen vector

$$\lambda^1(z) = (1, 0, 0)\delta(z - x). \tag{577}$$

Like the massless situation we calculate the primary lagrangian constraint in this situation too:

$$\gamma^1(x, t) = L_{A_0}(x, t) = (A_0'' - \dot{A_1}' - ae^2 A_0 - ae\dot{\theta})(x, t). \tag{578}$$

L_1 in this situation is obtained as

$$L_1(x, t) = (L_{EM} + \frac{\partial}{\partial t}\gamma^1)(x, t). \tag{579}$$

Here w^1 and α^1 too are found out using Eqs. (501) and (502).

$$w^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & a \\ 0 & -\frac{\partial}{\partial x} & -ae \end{pmatrix} \delta(z - x), \tag{580}$$

$$\alpha^1 = \begin{pmatrix} A_0'' - \dot{A_1}' - ae^2 A_0 - ae\dot{\theta} \\ -\dot{A_0}' + ae^2 A_1 + ae\theta' \\ -a\theta'' + ae\dot{A}_0 - aeA_1' \\ \dot{A_0}'' - ae^2 \dot{A}_0 \end{pmatrix}. \tag{581}$$

We see that w^1 has the following null eigen vector

$$\lambda^2(z) = (0, \frac{\partial}{\partial x}, +e, 1)\delta(z - x). \tag{582}$$

Multiplying equation (579) from left by λ^2, we find that

$$\gamma^2(x, t) = 0. \tag{583}$$

So, λ^2 does not give rise to any new constraint. Therefore, the process gets terminated. Thus the increase of the rank of equation for acceleration is not possible here. γ^2 here also can be written in the form of equation (515) as follows

$$\gamma^2(x, t) = (\frac{\partial}{\partial t} L_{A_0} + \frac{\partial}{\partial x} L_{A_1} + eL_\theta)(x, t). \tag{584}$$

Comparing equation (584) with (516) we get the non-vanishing $\phi' s$

$$\phi_{1,1} = \delta(z - x), \tag{585}$$

$$\phi_{0,2}(z,x) = \frac{\partial}{\partial z}\delta(z - x), \tag{586}$$

$$\phi_{0,3}(z,x) = e\delta(z - x). \tag{587}$$

Finally we find the gauge transformation of the field for this system with the help of (517)

$$\delta A_0 = -\int dz f(z,t)\delta(z - x) = -\frac{\partial}{\partial t} f(x,t), \tag{588}$$

$$\delta A_1 = -\int dz (\frac{\partial}{\partial z}\delta(z - x)) f(z,t) = -\frac{\partial}{\partial x} f(x,t), \tag{589}$$

$$\delta\theta = e f(x,t). \tag{590}$$

The variation of the lagrangian density (571) under the transformation (588),(589),(590) comes out to be

$$\begin{aligned} \delta L_{EM}(x,t) &= -\Sigma_{s=0}^{n} \frac{d^s}{dt^s}(\phi_{si} L_i)(x,t) \\ &= -[\frac{\partial}{\partial t} L_{A0} + \frac{\partial}{\partial x} L_{A1} + e L_{\theta}] f(x,t) = 0. \end{aligned} \tag{591}$$

Equation (591) confirms that the action is invariant under the above gauge transformation. Note that the formalism shows its successful application in the extended phase space of this simple non interacting system. In the following section the formalism is again applied to another noninteracting field theory, e.g., chiral boson which is known as a basic ingredient of heterotic string theory.

8.6 Free Chiral Boson

Free chiral boson [49, 50, 85, 86, 90, 91] though a very simple field theory the study of gauge symmetry for this system is very subtle and interesting because the lagrangian of chiral boson contains a second class constraint

$(\partial_0 + \partial_1)\phi = 0$. So it is studied here using Shirzad's formalism. Lagrangian density of free chiral Boson as described in [50, 91] is given by

$$\mathcal{L}_{\mathcal{CB}} = \frac{1}{2}(\dot{\phi}^2 - \phi'^2) + \eta(\dot{\phi} - \phi'). \tag{592}$$

Here η stands for the Lagrange multiplier field. The equations of motion for the field ϕ and η are

$$L_\phi = (\ddot{\phi} - \phi'' + \dot{\eta} - \eta'), \tag{593}$$

$$L_\eta = -(\dot{\phi} - \phi'). \tag{594}$$

In this case w and α are

$$w = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \delta(z - x), \tag{595}$$

$$\alpha = \begin{pmatrix} -\phi'' + \dot{\eta} - \eta' \\ -\dot{\phi} + \phi' \end{pmatrix}. \tag{596}$$

Hessian matrix w has the null eigen vector, $\lambda^1(z) = (0, 1)\delta(z-x)$. Multiplying λ^1 with equation (592) from left we obtain the primary constraint of the theory as usual

$$\gamma^1(x, t) = (-\dot{\phi} + \phi')(x, t). \tag{597}$$

The consistency of this primary constraint with time needs to be maintained which necessities to calculate L_1 as follows for further analysis

$$L_1(x, t) = (L_{CB} + \frac{\partial}{\partial t}\gamma^1)(x, t). \tag{598}$$

We find that w_1 and α_1 are :

$$w^1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ -1 & 0 \end{pmatrix} \delta(z - x), \tag{599}$$

$$\alpha^1 = \begin{pmatrix} -\phi'' + \dot{\eta} - \eta' \\ -\dot{\phi} + \phi' \\ \dot{\phi}' \end{pmatrix}. \tag{600}$$

We find that w_1 has another null eigen vector $\lambda^2 = (1,0,1)\delta(z-x)$. So Secondary lagrangian constraint is now obtained, multiplying λ^2 with (598).

$$\gamma^2(x,t) = (\dot{\phi}' - \phi'' - \eta' + \dot{\eta})(x,t). \tag{601}$$

Adding the time derivatives of γ^2 with L_1 , L_2 is obtained to maintain the consistency of the secondary constraint with time

$$L_2(x,t) = (L_1 + \frac{\partial}{\partial t}\gamma^2)(x,t). \tag{602}$$

For L_2 , the matrices w_2 and α_2 are found out as

$$w^2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ -1 & 0 \\ \frac{\partial}{\partial x} & 1 \end{pmatrix} \delta(z-x), \tag{603}$$

$$\alpha_2 = \begin{pmatrix} -\phi'' + \dot{\eta} - \eta' \\ -\dot{\phi} + \phi' \\ \dot{\phi}' \\ -\dot{\phi}'' - \dot{\eta}' \end{pmatrix}. \tag{604}$$

It is found that w_2 is also having a null eigen vector $\lambda^3(z) = (1,0,1,0)\delta(z-x)$. Multiplying the equation (602) from left by λ^3 we find

$$\gamma^3(x,t) = \gamma^2(x,t). \tag{605}$$

So from the previous step it can be concluded that there is no further constraint and we can not increase the rank of equations for accelerations. The process is thus terminated. We now write γ^3 in the following form.

$$\gamma^3(x,t) = (L_\phi + \frac{\partial}{\partial t}L_\eta)(x,t). \tag{606}$$

At this stage we need to compare (605) with equation (516) to compute the following non vanishing $\phi's$.

$$\phi_{0,1}(z,x) = \delta(z-x), \tag{607}$$

$$\phi_{1,2}(z,x) = \delta(z-x). \tag{608}$$

Finally we obtain the gauge transformation of the field ϕ and η for the system using equation (517).

$$\delta\phi = f(x,t), \tag{609}$$

$$\delta\eta = -\frac{\partial}{\partial t} f(x,t). \tag{610}$$

Let us now calculate the variation of L_{CB} under the above transformations of the fields

$$\delta\mathcal{L_{CB}} = -(L_\phi + \frac{\partial L_\eta}{\partial t})(x,t) = -(-\phi'' + \dot{\eta} - \eta' + \dot{\phi}')f(x,t). \tag{611}$$

This shows that the lagrangian (592) is not invariant under the above gauge transformations. The result of course have not gone beyond our expectation because chiral boson is known not to possess any gauge symmetry. This shows that the formalism is capable of testing the gauge symmetric property of this simple system having subtlety in many respects.

8.7 Free Chiral Boson in the Extended phase space

Let us add some appropriate terms involving auxiliary fields θ to the lagrangian density of free Chiral Boson that makes the right hand side of the equation (611) zero. It is found that Lagrangian density that satisfy the above requirement is

$$\begin{aligned} L_{ECB} &= \frac{1}{2}(\dot{\phi}^2 - \phi'^2) + \eta(\dot{\phi} - \phi') - \frac{1}{2}(\dot{\theta}^2 + \theta'^2) \\ &+ \phi'\theta' + \dot{\theta}\theta' - \dot{\theta}\phi' - \eta(\dot{\theta} - \theta') \end{aligned} \tag{612}$$

What follows next is to study the gauge symmetric property of the lagrangian (612) using the formalism given in previous section. To this end we calculate the equations of motion corresponding to the field ϕ, η, θ

$$L_\phi = (\ddot{\phi} - \phi'' + \dot{\eta} - \eta' + \theta'' - \dot{\theta}'), \tag{613}$$

$$L_\eta = -(\dot{\phi} - \phi') + \dot{\theta} - \theta', \tag{614}$$

$$L_\theta = -\ddot{\theta} - \theta'' + \phi'' + 2\dot{\theta}' - \dot{\eta} + \eta' - \dot{\phi}'. \tag{615}$$

In this situation w and α are

$$w = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \delta(z-x), \tag{616}$$

$$\alpha = \begin{pmatrix} -\phi'' + \dot{\eta} - \eta' + \theta'' - \dot{\theta}' \\ -\dot{\phi} + \phi' + \dot{\theta} - \theta' \\ -\theta'' + 2\dot{\theta}' - \dot{\eta} + \eta' - \dot{\phi}' + \phi'' \end{pmatrix}. \tag{617}$$

We find that w has the following null eigen vector

$$\lambda^1(z) = (0,1,0)\delta(z-x). \tag{618}$$

Multiplying λ^1 with equation (612) from left we obtain the primary constraint in this situation

$$\gamma^1(x,t) = (-\dot{\phi} + \phi' + \dot{\theta} - \theta')(x,t). \tag{619}$$

It is needed to add the time derivatives of γ^1 with L for further analysis otherwise we will fail to maintain consistency of the primary constraint

$$L_1(x,t) = (L_{ECB} + \frac{\partial}{\partial t}\gamma^1)(x,t). \tag{620}$$

w_1 and α_1 here are:

$$w^1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 1 \end{pmatrix} \delta(z-x), \tag{621}$$

$$\alpha^1 = \begin{pmatrix} -\phi'' + \dot{\eta} - \eta' + \theta'' - \dot{\theta}' \\ -\dot{\phi} + \phi' + \dot{\theta} - \theta' \\ -\theta'' + 2\dot{\theta}' - \dot{\eta} + \eta' - \dot{\phi}' + \phi'' \\ \dot{\phi}' - \dot{\theta}' \end{pmatrix}. \tag{622}$$

w_1 has a new null eigen vector

$$\lambda^2 = (1,0,1,1)\delta(z-x). \tag{623}$$

Multiplying λ^2 with equation (620) we get the following vanishing condition:

$$\gamma^2(x,t) = 0. \tag{624}$$

If we proceed to write γ^2 in the form of equation (515) we reach at

$$\gamma^2(x,t) = (L_\phi + \frac{\partial}{\partial t}L_\eta + L_\theta)(x,t). \tag{625}$$

Comparing the above equation (625) with equation (516) we get the non vanishing $\phi' s$

$$\phi_{0,1}(z,x) = \delta(z-x), \tag{626}$$

$$\phi_{1,2}(z,x) = \frac{\partial}{\partial t}\delta(z-x), \tag{627}$$

$$\phi_{0,3}(z,x) = \delta(z-x). \tag{628}$$

We are now in a position to compute the gauge transformation for the fields describing the system:

$$\delta\phi = f(x,t), \tag{629}$$

$$\delta\eta = -\frac{\partial}{\partial t}f(x,t), \tag{630}$$

$$\delta\theta = -f(x,t). \tag{631}$$

The variation of L_{ECB} under the above set of transformations (629), (630) and (631) gives.

$$\delta L_{ECB} = -(L_\phi + \frac{\partial L_\eta}{\partial t} + L_\theta)f(x,t) = 0. \tag{632}$$

It shows that the lagrangian (612) is invariant under the transformation (629), (630) and (631). It is the expected result because the terms which we are forced to add to make equation (611) zero is nothing but the Wess-Zumino term that has brought back the gauge symmetry in the system. Thus the formalism is found to work successfully in the extended phase space of this system too. So for free field theories the formalism is found to works equally well both in the usual and extended phase space. In the following sections we will consider some interacting system to test how well it woks there .

8.8 Chiral Schwinger Model with Fadeevian Anomaly

Let us consider the lagrangian of the so called chiral Schwinger model with Faddeevian anomaly [8, 9] and apply the same formalism to study its gauge symmetric property. The gauss law of this theory shows a special type of non vanishing commutation relation because of the presence of anomaly in the system. This is commonly known as Faddeevian type of anomaly [52, 53]. This model is interesting in different respect. So study of this model with this formalism would certainly be of interest.

$$\begin{aligned} L_{CSM} &= \int [(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + e(g_{\mu\nu} - \epsilon_{\mu\nu})\partial^\mu\phi A^\nu \\ &+ \frac{1}{2}e^2(A_0^2 - 2A_0A_1 - 3A_1^2)]dx. \end{aligned} \tag{633}$$

The equations of motions for the field ϕ, A_0 and A_1 are

$$L_\phi = (\ddot{\phi} - \phi'') + e(A_0' - A_1') + e(\dot{A}_0 - \dot{A}_1), \tag{634}$$

$$L_{A_0} = A_0'' - \dot{A}_1{}' - e^2A_0 + e^2A_1 - e(\phi' + \dot{\phi}), \tag{635}$$

$$L_{A_1} = \ddot{A}_1 - \dot{A}_0{}' + 3e^2A_1 + e(\dot{\phi} + \phi') + e^2A_0. \tag{636}$$

For this lagrangian the Hessian matrix w and α are

$$w = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \delta(z - x), \tag{637}$$

$$\alpha = \begin{pmatrix} -\phi'' + e(A_0' - A_1') + e(\dot{A}_0 - \dot{A}_1) \\ A_0'' - \dot{A}_1{}' - e^2A_0 + e^2A_1 - e(\phi' + \dot{\phi}) \\ -\dot{A}_0{}' + 3e^2A_1 + e(\dot{\phi} + \phi') + e^2A_0 \end{pmatrix}. \tag{638}$$

The Hessian matrix w bears the following null eigen vector

$$\lambda^1(z) = (0, 1, 0)\delta(z - x). \tag{639}$$

Here too multiplying equation (633) from left by λ^1 we get the primary constraint,

$$\gamma^1(x, t) = (A_0'' - \dot{A}_1{}' - e^2A_0 + e^2A_1 - e(\phi' + \dot{\phi}))(x, t). \tag{640}$$

The constraint has to be consistent with time. So we add time derivatives of γ^1 with L_{CSM} , and it results

$$L_1(x,t) = (L_{CSM} + \frac{\partial}{\partial t}\gamma^1)(x,t). \tag{641}$$

In matrix w^1 and α^1 that occurs in equation (633) are found out as

$$w^1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ -e & 0 & -\frac{\partial}{\partial x} \end{pmatrix} \delta(z-x), \tag{642}$$

$$\alpha^1 = \begin{pmatrix} -\phi'' + e(A_0' - A_1') + e(\dot{A}_0 - \dot{A}_1) \\ A_0'' - \dot{A_1}' - e^2 A_0 + e^2 A_1 - e(\phi' + \dot{\phi}) \\ -\dot{A_0}' + 3e^2 A_1 + e(\dot{\phi} + \phi') + e^2 A_0 \\ \dot{A}_0'' - e\dot{\phi}' + e^2 \dot{A}_1 - e^2 \dot{A}_0 \end{pmatrix}. \tag{643}$$

Let us now calculate null vector $\lambda^2(z)$ which the matrix w^1 is having.

$$\lambda^2(z) = (e, 0, \frac{\partial}{\partial x}, 1)\delta(z-x). \tag{644}$$

We get secondary lagrangian constraint multiplying equation (641) from left by λ^2 :

$$\gamma^2(z,t) = 2e^2(A_1' + A_0')(x,t). \tag{645}$$

In order to maintain consistency again the time derivatives of γ^2 is added with L_1 which results

$$L_2(x,t) = (L_1 + \frac{\partial}{\partial t}\gamma^2)(x,t). \tag{646}$$

In this situation the matrices w^2 and α^2 for $L_2(x,t)$ are found out as

$$w^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ -e & 0 & -\frac{\partial}{\partial x} \\ 0 & 0 & 0 \end{pmatrix} \delta(z-x), \tag{647}$$

$$\alpha^2 = \begin{pmatrix} -\phi'' + e(A_0' - A_1') + e(\dot{A}_0 - \dot{A}_1) \\ A_0'' - \dot{A_1}' - e^2 A_0 + e^2 A_1 - e(\phi' + \dot{\phi}) \\ -\dot{A_0}' + 3e^2 A_1 + e(\dot{\phi} + \phi') + e^2 A_0 \\ \dot{A}_0'' - e\dot{\phi}' + e^2 \dot{A}_1 - e^2 \dot{A}_0 \\ 2e^2(\dot{A}_1' + \dot{A}_0') \end{pmatrix}. \tag{648}$$

w^2 also has a new null eigen vector

$$\lambda^2(z) = (e, 0, \frac{\partial}{\partial x}, 1, 0)\delta(z - x). \tag{649}$$

Multiplying the equation (646) from left by λ^3 , we get

$$\gamma^3(z,t) = 2e^2(A_1' + A_0')(x,t) = \gamma^2(x,t). \tag{650}$$

The mapping of $\gamma^3(x,t)$ onto $\gamma^2(x,t)$ indicates that there is no other constraint. Thus the process is terminated. As it is done in this previous cases γ^3 here too is expressed in the form of equation (515).

$$\gamma^3(x,t) = (\frac{\partial}{\partial t} L_{A_0} + \frac{\partial}{\partial x} L_{A_1} + eL_{\phi})(x,t). \tag{651}$$

Comparing $\gamma^3(x,t)$ with equation (516) we get non vanishing $\phi's$ which will be useful to calculate gauge transformations of the fields describing the system.

$$\phi_{0,1}(z,x) = e\delta(z - x), \tag{652}$$

$$\phi_{1,2}(z,x) = \delta(z - x), \tag{653}$$

$$\phi_{0,3}(z,x) = \frac{\partial}{\partial x}\delta(z - x). \tag{654}$$

We are now in a state to find the gauge transformation of the field describing the system by using equation (517)

$$\delta\phi = ef(x,t), \tag{655}$$

$$\delta A_0 = -\frac{\partial}{\partial t} f(x,t), \tag{656}$$

$$\delta A_1 = -\frac{\partial}{\partial x} f(x,t). \tag{657}$$

The variation of L_{CSM} is under the transformation (655),(656) and (657)

$$\begin{aligned}
\delta L_{CSM}(x,t) &= -\Sigma_{s=0}^{n}\frac{\partial^s}{\partial t^s}(\phi_{si}L_i)f(x,t) \\
&= -[\frac{\partial}{\partial t}L_{A0}+\frac{\partial}{\partial x}L_{A1}+eL_{\phi}]f(x,t) \\
&= -2e^2(A_1'+\dot{A}_0)f(x,t). \qquad (658)
\end{aligned}$$

So it is found that the lagrangian (633) is not invariant under the above transformations. The formalism here too gives the expected result because it is an anomalous model with Faddeevian type of anomaly and the appearance of gauge non invariance for this model is obvious.

8.9 Chiral Schwinger Model with Faddeevian anomaly is Made Gauge Invariant in the Extended Phase space

The lagrangian of chiral Schwinger model with Fadeevian anomaly [8, 35] is found gauge non invariant under the transformation generated in the previous section by Shirzad's formalism. So we add some terms with the previous lagrangian (633) to bring back its symmetry and apply the prescription to verify whether we get the expected result in the extended phase space like the previous case. Lagrangian density with appropriate terms involving the auxiliary field θ that helps to make Eq. (658) zero reads

$$\begin{aligned}
L_{ESM} &= \int[(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+e(g_{\mu\nu}-\epsilon_{\mu\nu})\partial^\mu\phi A^\nu \\
&+ \frac{1}{2}e^2(A_0^2-2A_0A_1-3A_1^2) \\
&+ \frac{1}{2}(\dot{\theta}^2-2\theta'\dot{\theta}-3\theta'^2)-\frac{1}{2}(\dot{\theta}^2-\theta'^2) \\
&- e(A_0\dot{\theta}-A_0\theta'-A_1\dot{\theta}-3A_1\theta')+e(A_0\theta'-A_1\dot{\theta}) \\
&+ e(-A_1\theta'+A_0\dot{\theta})]dx. \qquad (659)
\end{aligned}$$

The equations of motions for the field ϕ, A_0, A_1 θ are

$$L_\phi = (\ddot{\phi} - \phi'') + e(A_0' - A_1') + e(\dot{A}_0 - \dot{A}_1), \tag{660}$$

$$L_{A_0} = A_0'' - \dot{A_1}' - e^2 A_0 + e^2 A_1 - e(\phi' + \dot{\phi}) - 2e\theta', \tag{661}$$

$$L_{A_1} = \ddot{A}_1 - \dot{A_0}' + 3e^2 A_1 + e(\dot{\phi} + \phi') + e^2 A_0 - 2e\theta', \tag{662}$$

$$L_\theta = (-2\theta'' - 2\dot{\theta}') + 2eA_0' + 2eA_1'. \tag{663}$$

The matrices w and α in this situation are

$$w = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \delta(x - y), \tag{664}$$

$$\alpha = \begin{pmatrix} -\phi'' + e(A_0' - A_1') + e(\dot{A}_0 - \dot{A}_1) \\ A_0'' - \dot{A_1}' - e^2 A_0 + e^2 A_1 - e(\phi' + \dot{\phi}) - 2e\theta' \\ -\dot{A_0}' + 3e^2 A_1 + e(\dot{\phi} + \phi') + e^2 A_0 - 2e\theta' \\ -2\theta'' - 2\dot{\theta}' + 2eA_0' + 2eA_1' \end{pmatrix}. \tag{665}$$

It is found that there exists a null vector within the the Hessian matrix w which is given by

$$\lambda^1(z) = (0, 1, 0, 0)\delta(z - x). \tag{666}$$

Multiplying the equation (659) with λ^1 from the left we obtain the following primary constraint

$$\gamma^1(x, t) = (A_0'' - \dot{A_1}' - e^2 A_0 + e^2 A_1 - e(\phi' + \dot{\phi}) - 2e\theta')(x, t). \tag{667}$$

When the time derivative of γ^1 is added with L_{ESM} , L_1 which is the requirement for the primary constraint to be consistent with time.

$$L_1(x, t) = (L_{ESM} + \frac{\partial}{\partial t}\gamma^1)(x, t). \tag{668}$$

It is now needed to find out w^1 and α^1 contained in (668)

$$w^1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -e & 0 & -\frac{\partial}{\partial x} & 0 \end{pmatrix} \delta(z - x), \tag{669}$$

$$\alpha^1 = \begin{pmatrix} -\phi'' + e(A_0' - A_1') + e(\dot{A}_0 - \dot{A}_1) \\ A_0'' - \dot{A}_1{}' - e^2 A_0 + e^2 A_1 - e(\phi' + \dot{\phi}) - 2e\theta' \\ -\dot{A}_0{}' + 3e^2 A_1 + e(\dot{\phi} + \phi') + e^2 A_0 - 2e\theta' \\ -2\theta'' - 2\dot{\theta}' + 2eA_0' + 2eA_1' \\ \dot{A}_0'' - e\dot{\phi}' - e^2\dot{A}_0 + e^2\dot{A}_1 - 2e\dot{\theta}' \end{pmatrix}. \tag{670}$$

Our next step is to find out whether the matrix do have any null eigen vector and it is seen that matrix w^1 has the following null eigen vector.

$$\lambda^2(z) = (e, 0, \frac{\partial}{\partial x}, -e, 1)\delta(z - x). \tag{671}$$

Now we multiply the equation (668) from left by λ^2 to find γ^2 which turns out to be zero here.

$$\gamma^2(z, t) = 0. \tag{672}$$

So it is not possible to increase the the rank of equation for accelerations. Since λ^2 does not give rise to any new constraint one needs to express γ^2 in the form of equation (515).

$$\gamma^2(x, t) = (\frac{\partial}{\partial t} L_{A_0} + \frac{\partial}{\partial x} L_{A_1} + eL_\phi - eL_\theta)(x, t). \tag{673}$$

Equating (673) with equation (516) we find that the non vanishing $\phi's$ in this situation are

$$\phi_{0,1}(z, x) = e\delta(z - x), \tag{674}$$

$$\phi_{1,2}(z, x) = \delta(z - x), \tag{675}$$

$$\phi_{0,3}(z, x) = \frac{\partial}{\partial z}\delta(z - x), \tag{676}$$

$$\phi_{0,4}(z, x) = -e\delta(z - x). \tag{677}$$

The gauge transformations (517) for the field ϕ, A_0, A_1 and θ are found out to be

$$\delta\phi = ef(x, t), \tag{678}$$

$$\delta A_0 = -\frac{\partial}{\partial t} f(x, t), \tag{679}$$

$$\delta A_1 = -\frac{\partial}{\partial x} f(x, t), \tag{680}$$

$$\delta\theta = -ef(x,t). \tag{681}$$

The variation of L_{ESM} is under the transformation (678),(679), (680),(681)

$$\begin{aligned} \delta L_{ESM}(x,t) &= -\Sigma_{s=0}^{n} \frac{d^s(\phi_{si} L_i)}{dt^s}(x,t) \\ &= -[\frac{\partial}{\partial t} L_{A0} + \frac{\partial}{\partial x} L_{A1} + eL_\phi - eL_\theta] f(x,t). \\ &= 0. \end{aligned} \tag{682}$$

It shows that the lagrangian (659) is invariant under the gauge transformation (678), (679), (680) and (681). Therefore, we again observe the real ability of this formalism for testing the gauge symmetric property in the extended phase space of the so called chiral Schwinger model with Faddeevian anomaly.

8.10 Discussion

An instrument for testing gauge symmetry as well as generating gauge transformation of a theory through Lagrangian formulation developed by Shirzad in [31] has been applied in different interacting and non interacting field theoretical model [38]. Some of the model had gauge symmetry to start with and in some model it was lacking. The formalism is found instrumental to study the gauge symmetric property for all the cases whatever subtleties are involved in these. Using Shirzad's formalism, we have successfully tested whether a given model does posses gauge symmetry or not. When a model is found gauge non-invariant it is made gauge invariant by adding some auxiliary fields. With the lagrangian of that model in the extended phase space investigation is carried out using Shirzad's prescription to test whether gauge symmetric gets restored in it. The process of adding auxiliary fields though extends the phase space the physical content of the theory remains unaltered because the fields required for the extension keep themselves allocated in the unphysical sector of the theory. More importantly, it has been possible to generate gauge transformation generator in the extended phase space too. So it is found that the formalism is not only useful in the usual phase space of the theory but also it is equally powerful in the extended phase space. In this context, we should mention that in [31], Shirzad kept himself confined within the usual phase space of the theory. One important aspect of this formalism which we have noticed here is that one can have a guess about the Weiss-Zumino term needed to bring back the symmetry of a gauge non invariant theory. In every cases of our studies we have noticed that the terms involving auxiliary field needed for making the variation of a particular gauge non invariant lagrangian zero under the respective transformations generated through Shirzads formalism leads to the Weiss- Zumino term for the respective theory. But it is fair to admit that the formalism is still lacking the mechanism to make a theory gauge invariant in a straightforward manner, i.e., the automatic generation of Wess-Zumino term as it has been found to be generated during the BRST invariant reformulation through Batalin-Fradkin- Vilkovisky formalism [44, 45, 47]. However there is enough room of improvement for the formalism towards this end. More serious and intense investigation is needed in that direction.

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List of publications

1. On the gauge and BRST invariance of the Chiral QED with Faddeevian Anomaly. A. Rahaman, S. Yasmin, S.Aziz: Int.J Theor. Phys. 49: 2607- 2620 (2010)

2. Study of Gauge symmetry Through the Lagrangian Formulation of some field Theoretical models. S. Yasmin, A. Rahaman: Int.J Theor. Phys. 52: 1539-1565 (2013)

3. On the Poincar'e and Gauge symmetry of a model where vector and axial vector interaction get mixed up with different weight. S. Yasmin, A. Rahaman: Int.J Theor. Phys 55: 5172-5185 (2016)

4. On the BRST and finite field dependent BRST of a model where vector and axial vector interaction get mixed up with different weight. S.Yasmin, A. Rahaman : Int.J Mod. Phys A 31 1650171 (2016).

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On the BRST and finite field-dependent BRST of a model where vector and axial vector interactions get mixed up with different weights

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The generalized version of a lower dimensional model where vector and axial vector interactions get mixed up with different weights is considered. The bosonized version of which does not possess the local gauge symmetry. An attempt has been made here to construct the BRST invariant reformulation of this model using Batalin–Fradlin and Vilkovisky formalism. It is found that the extra field needed to make it gauge invariant turns into Wess Zumino scalar with appropriate choice of gauge fixing. An application of finite field-dependent BRST and anti-BRST transformation is also made here in order to show the transmutation between the BRST symmetric and the usual nonsymmetric version of the model.

Keywords: BRST; finite field-dependent BRST; Wess–Zumino term; anomaly.

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1. Introduction

Dynamical equations of physical system cannot always be described in terms of observable physical degrees of freedom which pose problem to the straightforward physical interpretation of the solution of evaluation equations.[1–3] In some cases, few solutions need to be excluded since they do not describe the real physical situation or it may be the case that certain class of apparently different solutions appears to be physically indistinguishable. The BRST-formalism[4–6] has been developed precisely to deal with such systems. It is a technique to enlarge the phase space of a gauge theory and to restore the symmetry of the gauge fixed action in the extended phase

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On the Poincaré and Gauge Symmetry of a Model where Vector and Axial Vector Interaction get Mixed up with Different Weight

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Abstract A $(1+1)$ dimensional model where vector and axial vector interaction get mixed up with different weight is considered with a generalized masslike term for gauge field. Through Poincaré algebra it has been made confirm that only a Lorentz covariant masslike term leads to a physically sensible theory as long as the number of constraints in the phase space is two. With that admissible masslike term, phase space structure of this model with proper identification of physical canonical pair has been determined using Diracs' scheme of quantization of constrained system. The bosonized version of the model remains gauge non-invariant to start with. Therefore, with the inclusion of appropriate Wess-Zumino term it is made gauge symmetric. An alternative quantization has been carried out over this gauge symmetric version to determine the phase space structure in this situation. To establish that the Wess-Zumino fields allocates themselves in the un-physical sector of the theory an attempts has been made to get back the usual theory from the gauge symmetric theory of the extended phase-space without hampering any physical principle. It has been found that the role of gauge fixing is crucial for this transmutation.

Keywords Poincaré symmetry · Anomaly · Faddeevian regularization · Gauge symmetry

1 Introduction

In terms of fundamental interaction, Quantum Electrodynamics (QED) in (1+1) dimension can be categories in two different classes. The first way of description that came in the literature was originated from vector type of interaction between matter and gauge fields.

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Study of Gauge Symmetry Through the Lagrangian Formulation of Some Field Theoretical Models

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Abstract Study of gauge symmetry is carried over the different interacting and noninteracting field theoretical models through a prescription based on Lagrangian formulation. It is found that the prescription is capable of testing whether a given model posses a gauge symmetry or not. It can successfully formulate the gauge transformation generator in all the cases whatever subtleties are involved in it. It is found that the prescription has the ability to show a direction how to extend the phase space using auxiliary fields to restore the gauge invariance of a theory. Like the usual phase space the prescription is found to be equally powerful in the extended phase space of a theory.

Keywords Lagrangian formulation · Gauge theory · Constrained dynamics

1 Introduction

Every basic interaction is supposed to have their origin from the gauge principle and understanding of the gauge symmetry of a physical theory is a very important problem which has received much attention to the physicist from the long past. In a gauge theory, their exists some transformation that leaves physical content of the theory invariant. It even stands as a fundamental principle that determines the form of Lagrangian of a theory. Two main approaches have been followed in the literature to study the local symmetry of the Lagrangian of the gauge theories. The oldest one is the Hamiltonian formulation based on Dirac conjecture [1–3]. Several authors have tried to find out the answer of several interesting questions related to the gauge symmetry using Hamiltonian formulation [4–14]. The most general form of gauge transformation generator too can be determined with that Hamiltonian formulation. To study BRST symmetry, Hamiltonian approach also has been found to be instrumental [15–22].

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On the Gauge and BRST Invariance of the Chiral QED with Faddeevian Anomaly

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Abstract Chiral Schwinger model with the Faddeevian anomaly is considered. It is found that imposing a chiral constraint this model can be expressed in terms of chiral boson. The model when expressed in terms of chiral boson remains anomalous and the Gauss law of which gives anomalous Poisson brackets between itself. In spite of that a systematic BRST quantization is possible. The Wess-Zumino term corresponding to this theory appears automatically during the process of quantization. A gauge invariant reformulation of this model is also constructed. Unlike the former one gauge invariance is done here without any extension of phase space. This gauge invariant version maps onto the vector Schwinger model. The gauge invariant version of the chiral Schwinger model for $a = 2$ has a massive field with identical mass however gauge invariant version obtained here does not map on to that.

Keywords Chiral QED · BRST invariance · Faddeevian anomaly

1 Introduction

Symmetry plays a fundamental role in physics. Some times symmetry of a given theory may be broken and that has a profound consequences. Gauge symmetry of a theory is of particular interest in this context. Absence of gauge symmetry invites anomaly in a theory. There have been considerable efforts in the understanding of anomaly in quantum field theory [1–14]. The studies of chiral Schwinger model and anomalous Schwinger model [11] are worth mentionable in this respect. It is the anomaly that removed the long suffering of chiral Schwinger model from non-unitarity. Credit went to Jackiw and Rajaraman—those

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Gauged Floreanini-Jackiw type chiral boson and its BRST quantization

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The gauged Siegel type chiral boson is considered. It has been shown that the action of gauged Floreanini-Jackiw (FJ) type chiral boson is contained in it in an interesting manner. A BRST invariant effective action corresponding to the action of gauged FJ type chiral boson has been formulated using Batalin, Fradkin, Vilkovisky (BFV) based improved Fujiwara, Igarishi and Kubo (FIK) formalism. An alternative quantization of the gauge symmetric effective action has been made with a Lorentz gauge and an attempt has been made to establish the equivalence between the gauge symmetric version of the extended phase space and original gauge non-invariant version of the usual phase space.

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I. INTRODUCTION

The self-dual field in $(1+1)$ which is also known as chiral boson is the basic ingredient of heterotic string theory [1–4]. This very chiral boson plays a crucial role in the study of quantum hall effect too [5, 6]. Seigel initiated the study of chiral boson in his seminal work [7]. Another description of chiral boson came from the work of Srivastva [8]. In these two descriptions [7, 8], the lagrangian of chiral boson were constituted with the second order time derivative of the field. In the description of Seigel chiral constraint was in a quadratic form where as in the description of Srivastava it was in a linear form. One more ingenious description of chiral boson came from the description of Floreanini and Jackiw [9]. In this description the lagrangian of chiral boson was constituted with first order time derivative of the field. In Ref [10], we find an interesting description towards quantization of that free FJ type chiral boson. In a very resent work [8], we find an application of augmented super field approach to derive the off-shell nilpotent and absolutely anti-commuting (anti-)BRST and (anti-)co-BRST symmetry transformations for the BRST invariant Lagrangian density of a free chiral boson. Another recent important development towards the BFV quantization of the free chiral boson along with study of Hodge decomposition theorem in the context of conserved charges has came in [11]

The obvious generalization of free chiral boson is to take into account of the interaction of gauge field with that and this interacting field theoretical model is known as gauged chiral boson. The interacting theory of chiral boson was first described by Bellucci, Golterman and Petcher [13] with Seigel like kinetic term for chiral boson. So naturally the theory of interacting chiral boson with FJ type kinetic was wanted for as free FJ type chiral boson became available in [9] and that was successfully met up by Harada [14]. After the work of Harada [14], interacting chiral boson based on FJ type kinetic term attracted considerable attention [34] in spite of the fact that this theory of interacting chiral boson was not derived from any fundamental principle. Harada obtained it from Jackiw-Rajaraman (JR) version of chiral Schwinger model [15], imposing a chiral constraint into it by hand. So there is a missing link between the two types of interacting gauged chiral boson. An attempt towards search for a link is therefore a natural extension which we would like to explore. In fact, we want to show whether the gauged FJ type chiral boson is contained within the gauged chiral boson of Seigel type chiral boson which is available in [13]. The study of this model may be beneficial from another another point of view indeed; where anomaly is the central issue of investigation [14–21], since it is known from ref. [14] that the model took birth from the JR version of chiral Schwinger model this very chiral Schwinger model viz., chiral generation of Schwinger model [22] gets secured from unitarity problem when anomaly was taken into consideration [15]. In this respect, the recent chiral generation of Thirring model is of worth-mentioning [23, 24]. So when the issue search of desired link gets settled down a natural extension that comes automatically in mind is to study the symmetry underlying in the model and perform the quantization of the model. BRST quantization in this context scores over other.

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