

CP PHASES IN LEPTONIC FLAVOR VIOLATION

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We study the possibility to extract information on leptonic CP phases using a CP and T violating triple spin correlation in the process of muon to electron conversion in nuclei. We establish a negative result for all the three conventional seesaw types. Next, we focus our attention on the left-right symmetric theories and show that a signal is feasible. With a left-right scale below 10-30 TeV, a lepton flavor violating signal may be observed and the CP violating quantity of order one is generically expected. We also discuss the conditions which may lead to a conspiracy, suppressing such effects.

1 Introduction

Getting a handle on leptonic CP phases is one of the great challenges of neutrino physics. The aim of our work ¹ (where all the relevant references are found) is to establish whether it is possible to get some information by studying triple spin correlations in lepton flavor violating (LFV) processes, as recently suggested.

A textbook example of a quantity sensitive to CP phases is neutrinoless beta decay. More recently, determination of neutrino mass parameters at colliders, including the CP phases, has been discussed. These rates depend on the phases in an even way, they are not genuine CP-odd quantities. An example of a CP-odd observable have been neutrino-antineutrino oscillations or triple correlation functions, formed with either momenta or spin of particles. These correlations can be observed either at high energy colliders or at low energy experiments, looking for LFV.

We have focused on the LFV processes, in particular on the conversion of the muon to an electron inside a nucleus. The main motivation is a serious proposal at Fermilab and J-PARC to increase the present experimental sensitivity by four to six orders of magnitude, while other rare processes like $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ may be harder to improve (see also ²). Our analysis focused on the conventional seesaw mechanisms first. If a single additional representation is assumed only three distinct scenarios exist, called type I (a fermionic singlet), type II (hypercharge 2 bosonic triplet) and type III (fermionic triplet with zero hypercharge). Although these scenarios by themselves do not have any theoretical appeal, they do appear naturally in theories like Pati-Salam (type I + II) and minimal $SU(5)$ grand unified theories (type I+III), therefore we cover all three cases.

We present an intuitive argument why the CP-violating effects are suppressed beyond the planned sensitivity, regardless of the type. Afterwards, we turn to left-right symmetric theories, which originally led to the concept of seesaw. In this case, our findings are optimistic when the left-right symmetric scale M_R is below 10–30 TeV which may be within reach of the LHC.

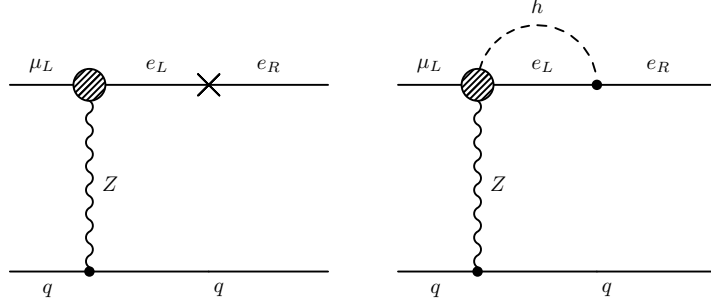


Figure 1: The right-handed electron contributes to $\mu \rightarrow e$ conversion only through the mass insertion on the left or through the Higgs loop on the right.

2 $\mu - e$ conversion and a no-go for seesaw

Presently, the best published experimental bound on the LFV processes comes from $\mu - e$ conversion in nuclei with

$$B(\mu \text{ Ti} \rightarrow e \text{ Ti}) \leq 4.3 \times 10^{-12}, \quad B(\mu \text{ Au} \rightarrow e \text{ Au}) \leq 7 \times 10^{-13}, \quad (1)$$

where

$$B(\mu \text{ N} \rightarrow e \text{ N}) \equiv \frac{\mu \text{ N} \rightarrow e \text{ N}}{\mu \text{ N} \rightarrow \text{capture}}. \quad (2)$$

The physics of $\mu - e$ conversion is quite rich, involving different nuclear effects, which affect the decay rate. A recently proposed T-odd quantity which probes the CP phases is obtained by forming a triple correlation of spins and electron momentum

$$(\vec{S}_\mu \times \vec{S}_e) \cdot \vec{P}_e.$$

To illustrate how the leptonic phases are being probed by this quantity, let us assume a simple vectorial Lorentz structure

$$\mathcal{L}_{\text{eff}} = G_F \sum_{q=u,d} (A_L \bar{e}_L \gamma^\mu \mu_L + A_R \bar{e}_R \gamma^\mu \mu_R) \times (V_L^q \bar{q}_L \gamma_\mu q_L + V_R^q \bar{q}_R \gamma_\mu q_R) + \text{h.c.} \quad (3)$$

With this effective interaction, the expression for the triple spin correlation turns out to be proportional to³

$$\delta_{CP} = \frac{\text{Im}(A_L^* A_R)}{|A_L|^2 + |A_R|^2}. \quad (4)$$

This result can be understood by physical intuition, since for a single helicity of the electron, both A_L and A_R are required in order to have the spin of an electron perpendicular to its motion. In turn, CP violation requires a relative phase between the two amplitudes. This reasoning can be generalized to other operators.

It is straightforward to see why the seesaw by itself cannot give a sizable contribution to δ_{CP} . The reason is a universal feature of all the seesaw types, which is that only left-handed charged leptons are coupled to the new messengers with the Yukawa couplings

- $\ell H F$, where F is either a fermionic singlet (right-handed neutrino) in case of type I or a fermionic $Y = 0$ triplet in type III;
- $\ell \ell \Delta$, where Δ is an $SU(2)$ scalar triplet with $Y = 2$ in case of type II.

In order to get δ_{CP} , we need A_R . Naïvely, we can put a mass insertion on the external electron leg as in Fig. 1 on the left which gives

$$A_R = \frac{m_e}{m_\mu} A_L. \quad (5)$$

This is not enough because the complex phases of A_R and A_L remain the same, therefore $\delta_{CP} = 0$. The only way to change the complexity of the amplitude is to couple the Higgs boson, which gives an additional loop suppression, coming from a diagram shown on the right of Fig. 1, which is at least

$$\delta_{CP} \approx \frac{\alpha}{\pi} \frac{m_e}{M_W} \approx 10^{-7}, \quad (6)$$

where m_e/M_W is due to the Yukawa coupling.

Even without any further suppression, this fact alone implies that the prospect of detecting a signal is hopeless, even if the experiments push to 10^{-18} for the branching ratio. Since we have not used any special features of a particular seesaw scenario, this argument is universal. Therefore it is valid for any theory which is left with only seesaw at the low energies, regardless of the type(s) and number of mediators. For any such theory, the T-odd correlations are suppressed beyond planned sensitivity. An example of such a theory is given by the minimal extension of the original $SU(5)$ theory, which can account for both gauge coupling unification and neutrino mass. It does so by adding a single adjoint fermionic representation, and leads to the hybrid type I and III. The triplet is predicted to be light and, as discussed above, δ_{CP} is vanishing.

3 The left-right symmetric model

We now turn to the minimal left-right symmetric theory with the seesaw of both, type I and type II. The minimal Higgs sector, which breaks spontaneously the left-right symmetric theory is defined by the following fields

$$\phi(2, 2, 0), \quad \Delta_L(3, 1, 2), \quad \Delta_R(1, 3, 2) \quad (7)$$

under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. These additional fields allow for new Yukawa couplings to the leptons, which mediate LFV processes

$$\mathcal{L}_\Delta = Y_\Delta(\ell_L \ell_L \Delta_L + \ell_R \ell_R \Delta_R) + \text{h.c.} \quad (8)$$

Parity is broken spontaneously by the vev

$$\langle \Delta_R \rangle \simeq M_{W_R} \quad \text{and} \quad \langle \Delta_L \rangle = 0, \quad (9)$$

to the SM group and the bi-doublet $\langle \phi \rangle = M_L$ completes the EW breaking. It induces an effective potential for the left-handed triplet

$$V_{\Delta_L} = M_{\Delta_L}^2 \Delta_L^2 + \alpha \Delta_L \phi^2 \Delta_R + \dots, \quad (10)$$

which leads to a small vev $\langle \Delta_L \rangle = \alpha M_L^2 \langle \Delta_R \rangle / M_{\Delta_L}^2$, responsible for the type II seesaw contribution to neutrino mass.

Parity is broken, therefore masses of Δ_L and Δ_R are different, with the mass split proportional to $\langle \Delta_R \rangle$. To simplify the analysis in the following, we can show that the mixing between the left and right-handed sector of the theory is small.

The mixing angle between the scalars triplets is

$$\theta_{\Delta_L \Delta_R} \simeq \frac{\langle \Delta_L \rangle}{\langle \Delta_R \rangle}, \quad (11)$$

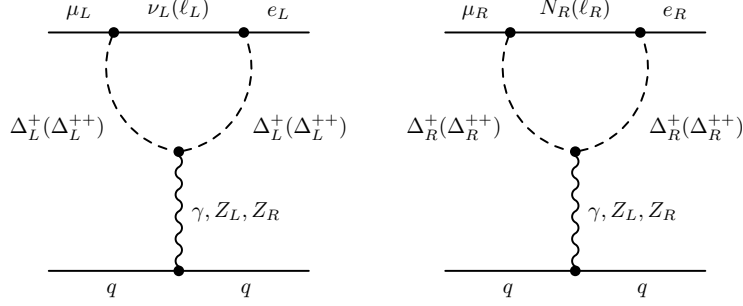


Figure 2: The contributions of the scalar triplets Δ_L and Δ_R to the typical penguin diagrams for $\mu \rightarrow e$ conversion.

and since $\langle \Delta_L \rangle \lesssim \text{GeV}$ and $\langle \Delta_L \rangle \gtrsim \text{TeV}$, the mixing is less than 10^{-3} . In fact, the limit is even stronger. Barring a miraculous cancellation of type I and II to get small neutrino masses, one has

$$\theta_{\Delta_L \Delta_R} \simeq \frac{m_\nu}{m_N}, \quad (12)$$

where the mass of the right-handed neutrino is

$$m_N = Y_\Delta \langle \Delta_R \rangle. \quad (13)$$

In order to get a signal from $\mu - e$ conversion, one needs $m_N \gtrsim (1 - 10)\text{GeV}$, which sets $\theta_{\Delta_L \Delta_R} \lesssim 10^{-9}$.

The gauge boson mixing is known to be very small $\theta_{W_L W_R} \lesssim 10^{-2}$ (see also⁴) and therefore any left-right mixing can safely be neglected as far as the discussion in the following is concerned.

4 LFV in L-R

The charged scalars, present in the minimal model, play a crucial role in what follows.

One might fear that the CP phase could be suppressed by M_L^2/M_R^2 , as typical of processes associated with a high scale M_R . Namely, the amplitude coming from W_L exchange is typically GIM suppressed

$$A_L(W_L) \propto \frac{\Delta m_\nu^2}{M_L^2} \leq 10^{-25}, \quad (14)$$

leading to a small δ_{CP} . Here, the contributions from the scalars come into the game. The exchange of Δ_L gives a contribution to A_L which is not GIM suppressed, since it is proportional to LFV Y_Δ couplings as seen in Fig. 2

$$A_L(\Delta_L) \approx A_R(\Delta_R) \approx \left(\frac{M_L}{M_R} \right)^2 \frac{\alpha}{\pi} Y_\Delta^2. \quad (15)$$

The question is whether the complex phases in A_L and A_R are the same, which again would render $\text{Im}(A_L^* A_R)$ small.

This is not the case, for the W_R contribution is also present and, unlike (14), not GIM suppressed. One can estimate the amplitude from boxes and penguins, similar to those in Fig. 2

$$A_R(W_R) \approx \left(\frac{M_L}{M_R} \right)^2 \frac{\alpha}{\pi} \left(\frac{m_N}{M_{W_R}} \right)^2, \quad (16)$$

where $(m_N/M_{W_R})^2$ stands symbolically for the right-handed GIM, which is not a priori small, therefore

$$\delta_{CP} = \mathcal{O}(1). \quad (17)$$

This result is encouraging and gives hope of probing leptonic CP for a not too high M_R scale. We can estimate the upper bound on M_R by requiring the branching ratio

$$B(\mu N \rightarrow e N) \approx \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{M_L}{M_R}\right)^4 \left(\frac{m_N}{M_{W_R}}\right)^4, \quad (18)$$

being a reasonably large number of 10^{-14} . With a lower limit of $M_R > 100$ GeV and an upper limit from perturbativity (following Eqs.(9) and (13)), one gets $3 \times 10^{-3} \leq m_N/m_{W_R} \leq 1$ and from the same equations $M_R \leq Y_\Delta 300 M_L$, which therefore implies an absolute upper limit of $M_R \leq 10$ TeV.

It is instructive to take a look at the flavor structure of the diagrams to get an idea which phases (or combinations thereof) a measurement of δ_{CP} would be probing. To illustrate this, we note that the diagram of Fig. 2 has the following flavor structure

$$U_R^\dagger m_N^2 f(m_N^2/m_R^2) U_R, \quad (19)$$

where U_R is the right-handed analog of the PMNS mixing matrix U_L , m_N is the diagonal mass matrix of the right-handed neutrinos and $f(x)$ is the loop function. This expression is a direct consequence of Eq.(13), written in the non-diagonal basis. On the other hand, the contribution of Δ_L^+ to A_L has a different flavor dependence, proportional to

$$U_L^\dagger U_{\nu N}^\dagger m_N^2 f(0) U_{\nu N} U_L, \quad (20)$$

where $U_{\nu N} = U_\nu^\dagger U_N$ is the mismatch between the unitary matrices which diagonalize ν and N mass matrices

$$U_\nu^T M_\nu U_\nu = m_\nu, \quad U_N^T M_N U_N = m_N. \quad (21)$$

The loop function $f(0)$ which arises is due to the fact that only light charged leptons are running inside the loop. The same flavor structure is obtained with the exchange of doubly charged triplets, except for the loop function, which is $f(0)$ in both cases.

In general, the matrix $U_{\nu N}$ is arbitrary since neutrino mass has two sources with both type I and II contributing. Only when type II dominates, the situation simplifies considerably. In this case $U_{\nu N} = I$ and this process probes the relative phases in the left and right sectors. As seen in (19) and (20) this requires the knowledge of the right-handed neutrino mass spectrum, which in principle can be achieved at colliders, as discussed below.

Knowing that there is a big potential to have sizable CP violation even if W_R is out of reach of the LHC, we discuss physical conditions (barring fine-tunings) under which δ_{CP} would vanish. First of all, it would require a suppression of the W_R loop due to the asymmetry between ν and N , effectively decoupling $m_{W_R} > 10$ TeV. If the scalars are heavy as well, this leads back to the SM with vanishing $\mu \rightarrow e$ so assume only W_R decouples. In this case we have two possibilities under which the phase would vanish

1. $M_{\Delta_L} \approx M_{\Delta_R} \ll M_{W_R}$ together with three additional conditions would cause δ_{CP} to naturally vanish. First of all, the mixings of the left and right-handed charged leptons should be equal, a consequence of hermitean mass matrices of charged leptons. While this does happen in the minimal model, it is by no means generic. Secondly, type II seesaw has to dominate over type I in order for $U_{\nu N} = I$ as discussed above and thirdly, right handed neutrinos should be much lighter than Δ_R . If all of the above is true, $U_L = U_R$ and $\text{Im}(A_L^* A_R) \rightarrow 0$.
2. $M_{\Delta_L} \ll M_{\Delta_R}$ (or vice-versa) implies $A_R/A_L \rightarrow 0$ ($A_L/A_R \rightarrow 0$) and with it $\delta_{CP} \rightarrow 0$.

While such conspiracies are possible, they seem quite unlikely.

4.1 Other manifestations of a low L-R scale

Colliders A fairly light W_R may be observed with striking signatures at the LHC. Since one can produce N through the gauge interactions, one can observe same-sign di-leptons which indicates lepton number violation due to the Majorana mass and probes both, parity restoration and the origin of neutrino mass. Studies of LFV channels may also probe CP violation, however a fairly light $M_{W_R} \leq (3-4)$ TeV is required. Although there have been claims that such a low scale is excluded in the minimal model by precision measurements, this may not be the case and a light W_R well within this region is still allowed⁵.

$\beta\beta 0\nu$ decay W_R with a mass of a few TeV and the right-handed N between 100 GeV – 1 TeV may easily dominate $\beta\beta 0\nu$ over the neutrino mass contribution. In this case, the signal depends on the right-handed PMNS analog U_R , which provides another source of information on the phases in U_R .

5 Conclusions and outlook

Measuring leptonic CP violation is a great challenge and LFV processes could play an important role. Muon conversion may lead the way with planned experimental improvement of many orders of magnitude.

At the same time, determining the origin of neutrino mass is as important and may be even more challenging. The prevailing view is that neutrino masses come from the seesaw mechanism. In the minimal setting only three realizations are possible and we discuss all cases. We find that the relevant CP phase is very small $\delta_{CP} < 10^{-7}$ and hence unobservable.

If instead of a simple seesaw scenario one considers a more complete theory, a L-R symmetric model is a natural candidate. For a moderately low scale $M_R \approx (10-30)$ TeV both LFV and large CP phase is generically predicted. If observed, it would discredit the simple seesaw and help to probe the neutrino mass origin in the minimal L-R model.

Measuring a single CP phase obviously is not enough to claim a L-R model is at work and not some other source of LFV. One way of establishing the operator(s) responsible for the decay is to compute both CP conserving and violating rates for different nuclei, which would help to determine the mediator(s). Obviously, most helpful in this regard would be the discovery of L-R symmetry at the LHC with measurements of the mass spectrum and mixings.

There is an important experimental caveat, under which all of the above holds true, decaying muons have to be polarized. Even if the muons in the beam are 100% polarized, they get depolarized when cascading to the ground state. This process is nucleus dependent and a small residual polarization remains. In order to re-polarize them, a polarized target should be used. The δ_{CP} estimated of order one, has to be weighted by the actual muon polarization.

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