

Nucleon-Hyperon (and YY) Scattering on the Lattice

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Abstract. Lattice QCD offers the chance to study the interactions of strange hadrons from the first principles of QCD. These NY (nucleon-hyperon) and YY (hyperon-hyperon) interactions are crucial to understanding the strange matter that may be created in extreme environments, such as the core of a neutron star. Since the fast decay of strange matter prevents experiments from providing strong constraints on the parameters of such interactions, direct theoretical calculations are especially valuable. In this presentation, I will report on the latest progress toward precision nucleon-hyperon and hyperon-hyperon scattering calculation in lattice QCD.

The strong interaction describes the fundamental physics of hadrons in terms of quarks and gluons. Recently, the application of strong-interaction phenomenology to cosmology, in areas such as supernovae, has started to catch the spotlight in these subfields. The study of microscopic systems, such as hadron-hadron interactions, can help us to understand the fate of a macroscopic astrophysical object, such as a neutron star. In normal environments, a hyperon is short-lived due to weak-interaction decay; thus, we have a periodic table filled with stable elements composed only by nucleons. However, in other parts (or times) of the universe, in extreme environments such as the core of a neutron star, strange hadrons (K , Λ , Σ) or even quark matter may persist. Nucleon-nucleon (NN) interactions are therefore not the sole input needed to understand what happens in the core of a neutron star; NY (nucleon-hyperon) and YY (hyperon-hyperon) interactions could become equally important.

However, experimentally, hyperon-related quantities are difficult to measure, due to the instability of the hyperons involved. Currently, there are a total of 35 cross-section measurements of hyperon-interaction processes, such as $\Lambda p \rightarrow \Lambda p$ and $\Sigma^- p \rightarrow \Sigma^0 n$, and the scattering parameters extracted from the data are highly model dependent. Other experimental information about these interactions can come from the study of hypernuclei, such as ${}^4_{\Lambda}\text{He}$ and ${}^{13}_{\Lambda\Lambda}\text{B}$. Yet we still cannot answer some basic questions with great confidence, such as: Is the Σn interaction attractive or repulsive?

There have been some theoretical attempts to study the YY and NY interactions. For example, the “realistic” potential uses phenomenological models based on meson exchange with a soft-core potential involving one-boson-exchange models of the NN interaction. The effective field theory approach suffers from a lack of information about a large number of couplings that need to be fit from data. Overall, most calculations describe the available NY cross-section data reasonably well but fail to predict consistent phase shifts.

Lattice QCD has been successfully employed to describe various phenomena involving strong interactions, such as the spectroscopy of heavy-quark hadrons, the tower of excited baryon states, flavor physics involving the CKM matrix, hadron decay constants and baryon axial couplings. In many cases, it can provide higher-precision data from the Standard Model than what can be

measured experimentally. One carries out the QCD path integral numerically in a finite four-dimensional Euclidean (rather than Minkowski) box with a shortest length scale, the lattice spacing. One can systematically study the effects due to having a finite volume and nonzero lattice-spacing and remove them by extrapolating using calculations with differing values of these quantities. Examining the dependence of quantities on the pion-mass can also help to determine many low-energy constants that appear in chiral perturbation theory.

Studying hadron scattering on the lattice, however, is not as straightforward as other physical quantities. The Maiani-Testa theorem states that the infinite-volume Euclidean-space Green functions cannot be used to extract S-matrix elements except at kinematic thresholds. Fortunately, one can apply Lüscher's method[1, 2] to study two-particle energy shifts in finite volumes:

$$\Delta E_n^{(AB)}(\mathbf{p} = \mathbf{0}) = \sqrt{q_n^2 + m_A^2} + \sqrt{q_n^2 + m_B^2} - m_A - m_B \quad (1)$$

(where q_n is the center-of-mass momentum) and extract the scattering phase shift up to the inelastic threshold. The function $p \cot \delta(p)$, which determines the low-energy elastic-scattering cross-section, $\mathcal{A}(p) \propto (p \cot \delta(p) - ip)^{-1}$, is determined at the energy $\Delta E_n^{(AB)}$ through q_n as

$$q_n \cot \delta(q_n) = \frac{1}{\pi L} S \left(q_n^2 \left(\frac{L}{2\pi} \right)^2 \right), \quad (2)$$

where

$$S(x) = \lim_{\Lambda \rightarrow \infty} \sum_{|\mathbf{j}| < \Lambda} \frac{1}{|\mathbf{j}|^2 - x} - 4\pi \Lambda. \quad (3)$$

In a certain kinematic regime, Eq. 2 can be series-expanded (known as the effective-range expansion) as

$$p \cot \delta(p) = -\frac{1}{a} + \frac{1}{2} r_0 |\mathbf{p}|^2 + \dots, \quad (4)$$

and the scattering length (with the nuclear-physics sign convention) a and the effective range r_0 can be extracted.

In this work, the baryon octet interpolating operators are of the form

$$\mathcal{B}_\alpha(\mathbf{x}, t) = \epsilon^{ijk} (q_1)_\alpha^i(\mathbf{x}, t) \left((q_2)^{j\top}(\mathbf{x}, t) C \gamma_5 (q_3)^k(\mathbf{x}, t) \right), \quad (5)$$

where C is the charge-conjugation matrix, ijk are color indices and $q_i \in \{u, d, s\}$. We then calculate single- and double-baryon correlators as

$$C_{\mathcal{B}}(t) = \sum_{\mathbf{x}} \Gamma \langle O_{\mathcal{B}}(\mathbf{0}, 0) O_{\mathcal{B}}^\dagger(\mathbf{x}, t) \rangle = \sum_n Z_{\mathcal{B},n} e^{-E_{\mathcal{B},n} t}, \quad (6)$$

$$C_{\mathcal{B}\mathcal{B}'}(t) = \sum_{\mathbf{x}} \Gamma' \langle O_{\mathcal{B}}(\mathbf{0}, 0) O_{\mathcal{B}'}(\mathbf{0}, 0) O_{\mathcal{B}}(\mathbf{x}, t)^\dagger O_{\mathcal{B}'}(\mathbf{x}, t)^\dagger \rangle = \sum_n Z_{\mathcal{B}\mathcal{B}',n} e^{-E_{\mathcal{B}\mathcal{B}',n} t}, \quad (7)$$

Γ and Γ' are spin tensors that typically project onto particular parity and/or angular momentum states. The energy shift of the two-baryon state can be extracted from a ratio of lattice correlators

$$\mathcal{R}(t) = \frac{C_{\mathcal{B}\mathcal{B}'}(t)}{C_{\mathcal{B}}(t) C_{\mathcal{B}'}(t)} e^{-\Delta E_{\mathcal{B}\mathcal{B}'} t}, \quad (8)$$

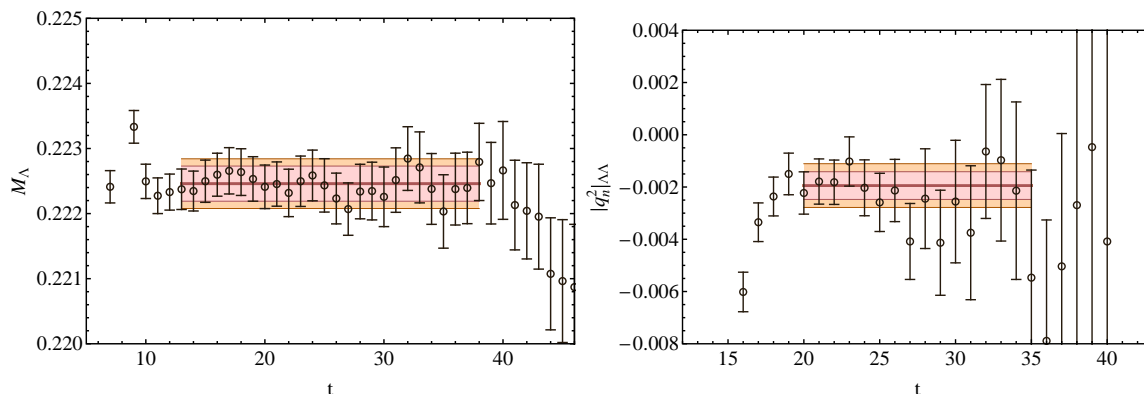


Figure 1. The Λ effective mass (left) and $\Lambda\Lambda$ effective- $|q_n^2|$ (right) plots in temporal and spatial lattice units as a function of Euclidean time away from the creation operator. The inner and outer bands indicate the statistical and systematic (due to the fit-range selection) errors.

and further fed into Eqs. 1–4 to extract scattering information.

In lattice QCD, we work in Euclidean space and often have to analyze time series of the form $C(t) = \sum_n Z_n e^{-E_n t}$. This makes it difficult to extract excited states ($n \geq 1$) or the ground state if there is only a small ground-excited gap (which often is the case when studying light nuclei). Reducing the lattice spacing would ameliorate the problem, since one has better resolution in the time direction and can carefully crosscheck different analysis techniques to avoid systematics caused by human bias. Working with finer lattice spacing would require a larger number of degrees of freedom in the spatial directions to avoid “squeezing” physics (finite-volume effects); this increases the cost of generating these lattices. We use an intermediate solution, “anisotropic” lattices[3, 4], where the spatial lattice spacing remains coarse (≈ 0.123 fm), but the temporal spacing is 3.5 times smaller (≈ 0.035 fm). This will allow us to properly distinguish any ground-state energy shift from potential nearby states. On top of fitting to the exponential form with multiple states, one can also try to solve for the exponents analytically. The most commonly used solution involves 2 time points of the correlators to extract the lowest-lying mass (or energy)

$$E_{\text{eff};\Delta t}(t) = \frac{1}{\Delta t} \log \left(\frac{C(t)}{C(t + \Delta t)} \right); \quad (9)$$

higher analytic solutions can extract the lowest four states without any fitting at all; for example, see Fig. 4 in Ref. [5]. This analytic technique can be further generalized to use multiple input correlators and add a “linear prediction” (to predict data at a later t' in terms of earlier time points); see Refs. [6, 7] for the details. One should carefully check the fitted energies in multi-hadron systems with these analytic solutions, assigning systematics that correspond to differences between different fit ranges or techniques. The left-hand side of Fig. 1 shows an example extraction of the Λ baryon mass (the band) and the effective masses (circles). We can process the correlators related to scattering in a similar manner. For example, the square of the effective center-of-mass momentum via Eq. 1 is shown on the right-hand-side of Fig. 1.

It is important to note that in order to apply lattice QCD to nuclear physics, we must go down from an energy scale of GeV on the lattice to nuclear physics, which usually presents MeV-level signals. This requires sub-percent precision, so working with multi-baryon systems mandates high statistics. To study the statistical effects, we concentrate on one ensemble of $N_f = 2 + 1$ anisotropic lattices with volume $(2.5 \text{ fm})^3$ and pion mass 390 MeV (with $\approx 0.435M$

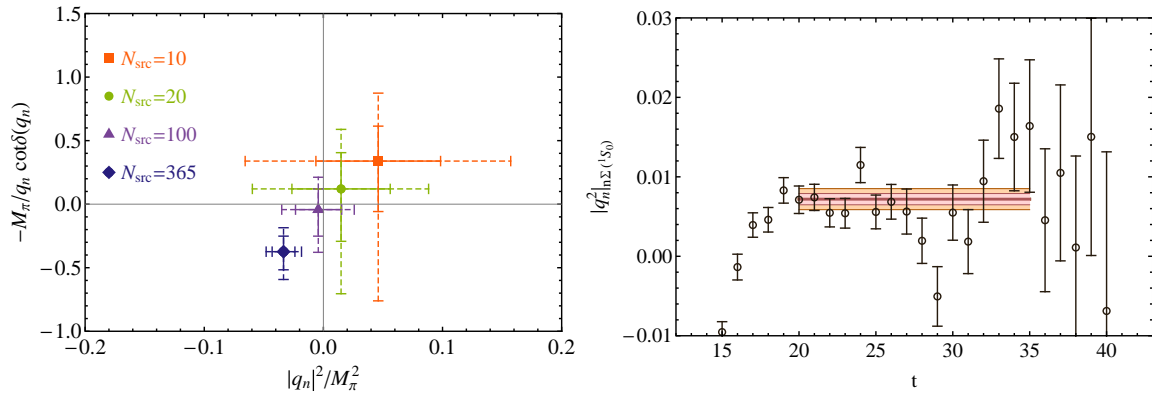


Figure 2. (Left) The extracted values of the inverse of the real part of the $\Lambda\Lambda$ scattering amplitude versus the extracted value of $|q_n|^2$ (normalized by the pion mass) with various numbers of measurements per configuration. The inner error bar is statistical error only and the outer one includes systematics. (Right) Effective- q_n plot (circles) for the $n\Sigma^-(^1S_0)$ interaction and its fitted value.

measurements in total). The single-hadron mass is one of the easiest observables on the lattice, and with high statistics around 0.4M measurements, one can reach better than 0.2% statistical uncertainty [8, 9]

$$\begin{aligned}
 M_N &= 1164.1(1.9)(1.7)(13.1) \text{ MeV} \\
 M_\Lambda &= 1252.1(1.5)(1.5)(14.1) \text{ MeV} \\
 M_\Sigma &= 1280.6(1.7)(1.6)(14.3) \text{ MeV} \\
 M_\Xi &= 1356.5(1.5)(1.5)(15.2) \text{ MeV},
 \end{aligned} \tag{10}$$

where the three uncertainties are statistical, systematic due to mass extraction and systematic from the combination of the lattice spacing and anisotropy uncertainty.

High statistics may not be necessary for the low-hanging fruit like ground-state measurements, but it is essential to reduce the uncertainties from extracting the scattering parameters on the lattice. To demonstrate the importance of high statistics to scattering processes, we break down our measurements into smaller subsets to see how the uncertainty evolves. The left-hand side of Fig. 2 shows the extracted values of $(q_n \cot \delta)^{-1}$ for $\Lambda\Lambda$ scattering versus the extracted value of $|q_n|^2$ with different numbers of measurements per configuration. With a small number of measurements (say, $N_{\text{src}} = 10$) per configuration, one could easily extract a $q_n \cot \delta$ that appears repulsive without a careful estimation of the systematic uncertainty. Even with $N_{\text{src}} = 100$ measurements per configuration, we are not yet able to conclusively determine whether the interaction is attractive or repulsive. It is only when the number of measurements per configuration approaches 400 that the interaction can be determined to be attractive, but only with $\approx 2\sigma$ significance.

In the nucleon-hyperon interactions ($S = -1$), we have calculated the energy eigenvalues of systems with the quantum numbers of $n\Lambda$ and $n\Sigma^-$ in both spin channels (1S_0 and 3S_1). The right-hand side of Fig. 2 shows the effective- $|q_n|^2$ plot for $n\Sigma^-(^1S_0)$ after processing the energy shift. We found both spin channels for $n\Sigma^-$ to have at least 5-sigma positive energy shift, and their interaction is strongly spin-dependent. The $n\Sigma^-(^1S_0)$ energy lies within the regime of applicability of the effective-range expansion, while $n\Sigma^-(^3S_1)$ is well outside the regime. This is consistent with previous results from NPLQCD using domain-wall fermions on staggered lattices. However, we cannot resolve with our current fixed-volume calculations whether $n\Sigma^-(^3S_1)$ is

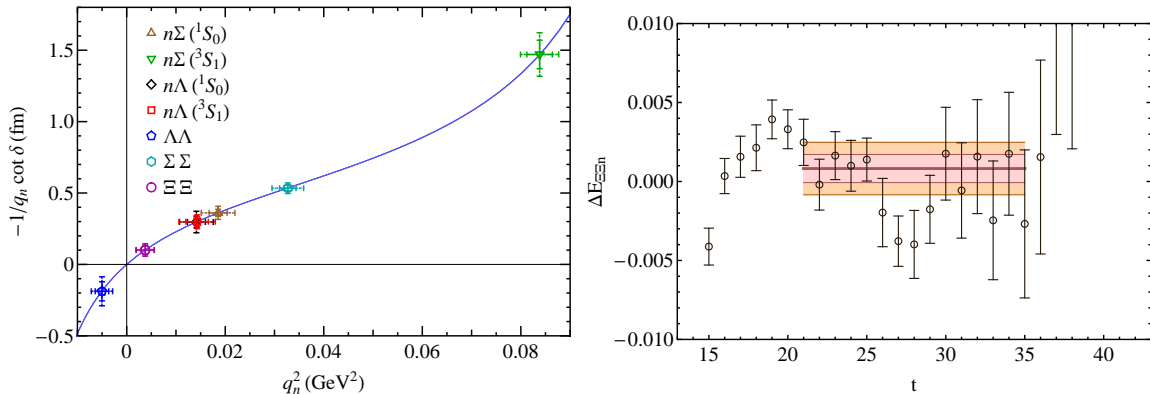


Figure 3. (Left) A summary plot of the inverse of the real part of the inverse scattering amplitude (in fm) versus the two-baryon center-of-mass energy for all of the NY and YY scattering channels calculated in this work. Note that the pion mass for this plot is 390 MeV. (Right) Effective energy-shift plot (in temporal lattice units) for the three-baryon system $\Xi^0 \Xi^0 n$

strongly interacting like a bound-state or a repulsive interaction of unnaturally large range. A multiple-volume calculation currently in progress will soon reveal how Σ^- interacts with neutron.

We also found both spin channels of $n\Lambda$ to have positive energy shift, but the interaction is rather independent of the spin. The energy-splittings are both found to be $a_t \Delta E \approx 0.002$, and are therefore smaller than the expected splitting between the energy eigenstates resulting from the $N\Lambda$ - $N\Sigma$ mixing, $a_t(M_\Sigma - M_\Lambda) = 0.0051$. It therefore seems likely a posteriori, that the single-channel analysis used here is applicable. The spin-independent interaction may suggest that one-pion exchange is absent (as the Λ is an isosinglet).

The hyperon-hyperon interactions are important, because they can provide guidance to experimental programs in hypernuclear physics and improve upon the current understanding of the stability of the core of supernovae if it becomes energetically favorable to have strange baryons present. The study of the $\Lambda\Lambda$ interactions is interesting to understand multistrange hypernuclei, such as ${}^6_{\Lambda\Lambda}\text{He}$, when combined with $N\Lambda$ interactions. The $\Lambda\Lambda$ interaction is also particularly interesting to determine whether two Λ 's could form a bound state (the infamous H-dibaryon[10]). We did observe a negatively-shifted energy splitting in this channel; however, we need measurements on additional lattice volumes to distinguish a bound state from an attractive scattering. $\Sigma^-\Sigma^-$ interactions ($I = 2$) would help us find out whether there can be Σ^- accumulation in neutron-star cores. We found it to have rather large positive energy shift. We also study a system involving greater strangeness, $I = 1 \Xi^-\Xi^-$, which might be more likely to become dominated hyperon as Λ in some neutron-star models. We find the result to be consistent with zero at the 2σ level; $\Xi^-\Xi^-$ interactions at this value of the pion mass are quite weak.

The left-hand side of Fig. 3 summarizes all the q_n and scattering amplitudes calculated for this ensemble.

We can take a step further toward the study of hypernuclei on the lattice. In fact, there already exists an exploratory study of the $\Xi^0 \Xi^0 n$ channel by NPLQCD from early 2009[11] (in parallel of the triton study). The slightly positive energy splitting is $\Delta E_{\Xi^0 \Xi^0 n} = 4.6(5.0)(7.9)(4.2)$ MeV. The energy-shift extrapolation and its effective plot are shown on the right-hand side of Fig. 3. It is very encouraging that the uncertainty of the energy shift per baryon is ≈ 3 MeV, which is smaller than the binding-energy per nucleon in typical nuclei, $B \approx 8$ MeV, and not significantly larger than the binding energy per nucleon in the deuteron

or triton at the physical values of the light-quark masses. However, the energy is consistent with zero within the uncertainties of the calculation. More statistics would be needed to make a better determination, since the quark-contraction cost increases significantly as one increases the number of the baryons in the calculation. Many groups are currently exploring smarter options that would extend the reach of lattice-QCD calculations to few- or many-body systems with currently available computational resources. The approach for systems with large numbers of mesons has been extended from 12 to many[12], and we expect more progress on baryon systems soon. A recent review[13] gives further details on the current standing of the ongoing effort of lattice QCD for light nuclei.

We have made a step toward precision nuclear physics with lattice QCD. With high-statistics calculations of nucleon-hyperon and hyperon-hyperon channels, we can extract their interactions with greater precision and less ambiguity, free from contamination by nearby unwanted states. We also demonstrated in the $\Lambda\Lambda$ channel that high statistics are necessary to extract reliable scattering parameters. We found $n\Sigma^-$ interactions are strongly spin dependent, while $n\Lambda$ is not. A negative energy shift was found in $\Lambda\Lambda$, which may suggest that the H-dibaryon exists. We are currently working on the calculations at smaller and larger volumes, and we will soon be able to identify bound states with certainty, and extend our work to lighter pion-mass ensembles. The precision of our calculation can be further improved with more computational resources. Investigation into light nuclei and few-body physics is on-going.

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