Stringent Constraints on Brans-Dicke Parameter using deci-Hz Gravitational Wave Interferometers

Kent Yagi[1], Takahiro Tanaka[2] and Naoki Seto[3]

[1,3] *Department of Physics, Kyoto University, Kyoto, 606–8502, Japan*
[2] *Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606–8502, Japan*

Abstract

We calculate how strong one can put constraints on Brans-Dicke parameter $\omega_{\rm BD}$ using 0.1Hz space laser interferometers such as DECIGO and BBO. We consider situations where neutron stars inspiral into small mass black holes whilst radiating gravitational waves. Compared to General Relativity, gravitational waves in Brans-Dicke theory have dipole radiation. For the amplitudes of the waveforms, we only take the leading quadrupole term and for the phases, we take subleading terms up to 2PN including spin-spin coupling term. For simplicity, we assume that the orbits are circular and we neglect the effect of spin precessions. We find that we can put 100 times stronger constraints on $\omega_{\rm BD}$ than the Cassini bound $\omega_{\rm BD} > 40000$, which is the current greatest constraint found by solar system experiments. This certainly gives a big scientific significance for DECIGO/BBO projects.

1 Introduction

One of the approaches to solve dark energy problem is to modify gravitational thoery from general relativity. The simplest modification is to add scalar degree of freedom to gravity. This theory is called scalar-tensor theory. This theory also appears in inflation problem and superstring theory. A prototype of scalar-tensor theory is Brans-Dicke theory. This theory is characterised by a parameter $\omega_{\rm BD}$ and by taking the limit $\omega_{\rm BD} \to \infty$, it reduces general relativity. The current strongest bound on $\omega_{\rm BD}$ is the Cassini bound obtained in the solar system experiment [1]; $\omega_{\rm BD,Cassini} > 40000$.

The aim of our work is to investigate how strongly we can constrain $\omega_{\rm BD}$ in the strong field regime by detecting gravitational waves from NS/BH binaries. Berti *et al.* [2] estimated this by using space interferometer LISA [3] and we estimated this by using deci-Hz interferometer DECIGO [4]. We take spin-spin coupling effect into account which Berti *et al.* [2] does not include. We find, by using DECIGO, we can put at least 100 times stronger constraint on $\omega_{\rm BD}$ than the Cassini bound.

2 Binary Gravitational Waveforms in Brans-Dicke Theory

Gravitational waveforms in general depend on the orientations of the binaries and the orbital angular momentum, but here, we average over these orientations. We adopt the restricted 2nd post-Newtonian (2PN) waveforms in which the amplitude is expressed to the leading order in a post-Newtonian expansion whilst the phase is taken up to 2PN order. (Post-Newtonian approximation is an expansion for slow-motion, weak-field systems in powers of binary velocity v.) For point masses, the phase evolution is calculated up to 3.5PN order but spin terms are known only up to 2PN order. Therefore to be consistent, we take the phase up to 2PN. Under the stationary phase approximation, the Fourier component of the waveform is [2]

$$\tilde{h}(f) = \frac{\sqrt{3}}{2}\mathcal{A} f^{-7/6} e^{i\Psi(f)}. \tag{1}$$

Here, f is the frequency of the gravitational waves. The amplitude is given by

[1] E-mail:kent@tap.scphys.kyoto-u.ac.jp
[2] E-mail:tanaka@yukawa.kyoto-u.ac.jp
[3] E-mail:seto@tap.scphys.kyoto-u.ac.jp

$$\mathcal{A} = \frac{1}{\sqrt{30}\pi^{2/3}}\frac{\mathcal{M}^{5/6}}{D}, \tag{2}$$

where $\mathcal{M} = \eta^{3/5}M$ is the chirp mass, with total mass $M = m_1 + m_2$ and dimensionless mass parameter $\eta = m_1 m_2/M^2$, and D is the luminosity distance to the source.

The phase is given by

$$\begin{aligned}\Psi(f) =& 2\pi f t_c - \phi_c + \frac{3}{128}(\pi\mathcal{M}f)^{-5/3}\left[1 - \frac{5}{84}\mathcal{S}^2\bar{\omega}x^{-1} + \left(\frac{3715}{756} + \frac{55}{9}\eta\right)x\right. \\ &\left. - 4(4\pi - \beta)x^{3/2} + \left(\frac{15293365}{508032} + \frac{27145}{504}\eta + \frac{3085}{72}\eta^2 - 10\sigma\right)x^2\right],\end{aligned} \tag{3}$$

where $x = v^2 = (\pi M f)^{2/3} = \eta^{-2/5}(\pi\mathcal{M}f)^{2/3}$. The first two terms are related to the time t_c and phase ϕ_c of coalescence. The first term ("1") inside the brackets corresponds to the leading quadrupole approximation of general relativity. The second term represents the dipole gravitational radiation in Brans-Dicke theory. $\bar{\omega} \equiv \omega_{\mathrm{BD}}^{-1}$ is the inverse of the Brans-Dicke parameter. $\mathcal{S} = s_2 - s_1$ where s_i is called the *sensitivity* of the i-th body defined as

$$s_i \equiv \left(\frac{\partial(\ln m_i)}{\partial(\ln G_{\mathrm{eff}})}\right)_0. \tag{4}$$

Here, G_{eff} is the gravitational constant at the location of the body and is proportional to the inverse of the Brans-Dicke scalar field there. The subscript 0 denotes that we evaluate s_i at infinity. This sensitivity roughly equals to the binding energy of the body per unit mass. For example, $s_{\mathrm{WD}} \sim 10^{-3}$ and $s_{\mathrm{NS}} \sim 0.2$. Because of No Hair Theorem, black holes cannot have scalar charges and $s_{\mathrm{BH}} = 0.5$. From Eq. (3), larger the $\mathcal{S}$, greater the contribution of dipole radiation. Binaries with large $\mathcal{S}$ are the ones with bodies of different types. Here, we consider NS/BH binaries. The event rate of NS/BH mergers is still uncertain, but it seems that it is considerably small for LISA, so only a lucky detection can constrain Brans-Dicke parameter. In contrast, for DECIGO, it is said to be around 10^4 merger events per year so NS/BH binaries should be the definite sources [5]. The rest of the terms in brackets are usual higher order PN terms in general relativity.

The quantities β and σ represent spin-orbit and spin-spin contributions to the phase respectively, given by

$$\beta = \frac{1}{12}\sum_{i=1}^{2}\chi_i\left(113\frac{m_i^2}{M^2} + 75\eta\right)\hat{\boldsymbol{L}}\cdot\hat{\boldsymbol{S}}_i, \tag{5}$$

$$\sigma = \frac{\eta}{48}\chi_1\chi_2(-247\hat{\boldsymbol{S}}_1\cdot\hat{\boldsymbol{S}}_2 + 721(\hat{\boldsymbol{L}}\cdot\hat{\boldsymbol{S}}_1)(\hat{\boldsymbol{L}}\cdot\hat{\boldsymbol{S}}_2)), \tag{6}$$

where $\hat{\boldsymbol{L}}$ and $\hat{\boldsymbol{S}}_i$ are unit vectors in the direction of the orbital angular momentum and spin angular momenta respectively. The spin angular momenta are given by $\boldsymbol{S}_i = \chi_i m_i^2\hat{\boldsymbol{S}}_i$ where χ_i are the dimensionless spin parameters. For black holes, they must be smaller than unity, and for neutron stars, they are generally much smaller than unity. It follows that $|\beta| \lesssim 9.4$ and $|\sigma| \lesssim 2.5$.

3 Parameter Estimation

The detected signal $s(t)$ is the sum of the gravitational wave signal $h(t;\boldsymbol{\theta})$ and the noise $n(t)$. We use the matched filtering analysis to estimate the binary parameters $\boldsymbol{\theta}$. We assume that the noise is stationary and Gaussian. Then, the probability that the GW parameters are $\boldsymbol{\theta}$ given by [2]

$$p(\boldsymbol{\theta}|s) \propto p^{(0)}(\boldsymbol{\theta})\exp\left[-\frac{1}{2}\Gamma_{ij}\Delta\theta^i\Delta\theta^j\right], \tag{7}$$

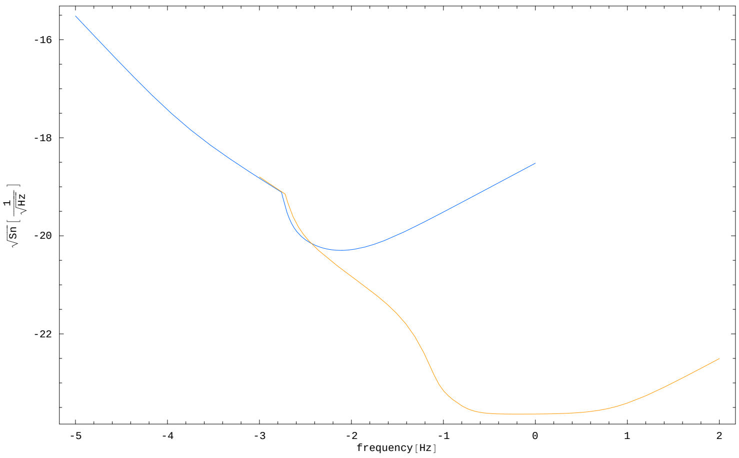

Figure 1: Noise curves for LISA(blue) [2] and DECIGO(red). Both horizontal and vertical axes are in log scales. Horizontal axis represents frequency[Hz] and vertical axis shows noise spectral density[$\mathrm{Hz}^{-1/2}$].

where the Fisher matrix Γ_{ij} is given by

$$\Gamma_{ij} = \left(\frac{\partial h}{\partial \theta^i} \middle| \frac{\partial h}{\partial \theta^i} \right). \tag{8}$$

Here, we define the inner product as

$$(A|B) = 4\mathrm{Re} \int_0^{\infty} df \, \frac{\tilde{A}^*(f)\tilde{B}(f)}{S_n(f)}, \tag{9}$$

where $S_n(f)$ is the noise spectral density. LISA and DECIGO noise strain sensitivity $\sqrt{S_n(f)}$ are shown in Fig.1. We denote estimates of rms errors as $\Delta\theta^i = \theta^i - \hat{\theta}^i$ where $\hat{\theta}^i$ is the fitted parameters. Then, $\Delta\theta^i$ can be calculated by taking the square root of the diagonal elements of the covariant matrix Σ_{ij}, which is the inverse of the Fisher matrix Γ_{ij};

$$\left\langle \Delta\theta^i \Delta\theta^j \right\rangle = \Sigma^{ij}, \qquad \Sigma^{ij} \equiv (\Gamma^{-1})^{ij}. \tag{10}$$

We take into account our prior information on the maximum spin by assuming

$$p^{(0)}(\boldsymbol{\theta}) \propto \exp\left[-\frac{1}{2}\left(\frac{\beta}{9.4}\right)^2 - \frac{1}{2}\left(\frac{\sigma}{2.5}\right)^2 \right]. \tag{11}$$

The signal to noise ratio(SNR) for a given h is given by $\rho[h] \equiv \sqrt{(h|h)}$.

4 Numerical Calculations and Results

4.1 Set Up

We think of detecting NS/BH inspiralling gravitational waves by LISA and DECIGO, and calculate how accurately we can deterimine the binary parameters, especially ω_{BD}. We assume that the orbit is circular and observation lasts 1 year. Also, we neglect spin precessions for simplicity.

The binary parameters are as follows; chirp mass $\ln\mathcal{M}$, dimensionless mass parameter $\ln\eta$, coalescence time t_c, coalescence phase ϕ_c, distance to the source $\ln D$, spin-orbit coupling β, spin-spin coupling σ, and reciprocal of Brans-Dicke parameter $\bar{\omega}$. We assume $t_c = 0, \phi_c = 0, \beta = 0, \sigma = 0, \bar{\omega} = 0$ and $\mathcal{S} = 0.3$. We fix $m_{\mathrm{NS}} = 1.4M_{\odot}$. We also fix the distance to be the one that gives SNR $\rho = 10$. We change m_{BH} and see how the constraints on ω_{BD} change. Berti *et al.* did not take σ into binary parameters. We

Table 1: Constraints on $\omega_{\rm BD}/10^4$ with different $m_{\rm BH}$ by using DECIGO and LISA. 1st row shows the constraints with σ taken into parameters, and 2nd row shows the ones without σ.

	DECIGO				LISA			
	$3M_\odot$	$10M_\odot$	$50M_\odot$	$400M_\odot$	$400M_\odot$	$1000M_\odot$	$5000M_\odot$	$10^4M_\odot$
parameters without σ	539.5	269.8	81.72	10.22	3.891	2.110	0.6432	0.3048
parameters with σ	429.4	162.9	35.94	4.254	2.472	0.8154	0.1916	0.0854

evaluate the constraints on $\omega_{\rm BD}$ in both cases where σ is not taken into binary parameters and where σ is taken into parameters, and compare both results.

4.2 Results

Table 1 shows the estimated constraints on $\omega_{\rm BD}/10^4$ with different $m_{\rm BH}$ by using DECIGO and LISA. The 1st row shows the constraints in the case where we do not take σ into binary parameters, and the 2nd row shows the ones where we do take σ into binary parameters.

From the table, including σ into parameters reduces the constraint by a factor of a few. Generally, the more the number of parameters increases, the worse the parameter determination accuracies are. You can also see that DECIGO can put about 200 times stronger constraint than LISA. There are mainly 2 reasons for this. First reason is because the number of GW cycles $\mathcal{N}_{\rm GW} = \int_{f_{\rm in}}^{f_{\rm fin}} df\ (f/\dot{f})$ are larger for DECIGO sources than LISA sources. Another reason is that the sensitivity of DECIGO is much better than that of LISA. Again from the table, you can see the constraint becomes more stringent as the BH mass decreases. This is because the bodies of the binaries become slower, which makes the dipole contribution greater. Even if we include σ as binary parameters, DECIGO can put at least 100 times stronger constraint than the current strongest one ($\omega_{\rm BD,Cassini} > 40000$).

5 Conclusions

We estimate how strongly we can put constraint on $\omega_{\rm BD}$ by detecting gravitational waves from inspiralling NS/BH binaries using LISA and DECIGO. We found that including σ as binary parameters reduces the constraint by a factor of a few. We also found that DECIGO can put at least 100 times stronger constraint than the current strongest one.

We have also calculated the constraint including eccentricity of the orbit and the effect of spin precessions [6]. We took the source orientation dependence into account. We performed following Monte Carlo simulation. We randomly distribute 10^4 binaries, evaluate the parameter estimation accuracies for each binary, and take the average. We found that for binaries with $\rho = 10$, DECIGO can put at least 10 times stronger constraint than the Cassini bound. For binaries with $D = 200$Mpc, whose event rate is thought to be roughly 1 merger per year, DECIGO can put 1000 times stronger constraint than Cassini bound. This certainly gives a big scientific significance to DECIGO project.

References

[1] B. Bertotti, L. Iess and P. Tortora, Nature **425**, 374 (2003)

[2] E.Berti, A.Buonanno and C.M.Will, Phys.Rev.**D71**, 084025 (2005)

[3] K.Danzmann, Class.Quant.Grav.**14**, 1399 (1997)

[4] S.Kawamura *et al.*, Class. Quantum Grav. **23**, S125 (2006)

[5] R.Voss and T.M.Tauris, Mon.Not.Roy.Astron.Soc.**342**,1169 (2003)

[6] K.Yagi, T.Tanaka and N.Seto (to be published)