

# On Synthesis of Electromechatronic Module of the Gearless Servo System for High-Precision Motion Control of the Loaded Axis

Yu. V. Grechushkin and O.K. Epifanov

Concern CSRI Elektropribor, St. Petersburg, Russian Federation

E-mail: oepifanov@eprib.ru

**Abstract.** The paper analyses the torque characteristics, angular errors, natural frequencies, and finite elasticities of electromechatronic module within gearless servo system with coaxially arranged multi-pole brushless DC torque motor and high-precision induction resolver. We provide analytical expressions and describe conditions for optimization with regard to the following criteria: sufficient torque, minimized angular errors, downsized construction, and absence of resonant frequencies as applied to principal and design solutions of the module. The results from calculation studies for module implementations with different types of torque motors are discussed.

## Introduction

Synthesis of the electromechatronic module (EM) [1, 2] within a gearless servo system (GSS) for high-precision control of angular motion of a loaded axis in various robotic information and executive devices for observation, orientation and navigation involves determining and achieving interrelated technical parameters providing the desired EM motion control accuracy [3, 4]. This synthesis has a number of specific features associated with the definition of motor torque developed by the multi-pole brushless DC torque motor (BDCM), GSS static and dynamic angular errors, and EM downsizing providing the control of loaded axis angular motion with the desired accuracy in the absence of resonant frequencies in the specified range. These features are discussed in the paper.

## Initial data and basic relations for estimating EM parameters

As a rule the EM is designed with coaxially arranged multi-pole high-precision induction resolver (IR) and BDCM with  $p$  pole pairs. The control object is fixed on EM shaft and serves as a load on the EM shaft.

Generally, the following parameters of angular motion of the loaded axis to be provided by EM are specified: maximum angular acceleration  $\alpha$ ; maximum angular rate  $\omega$ ; load moment of inertia  $J_{ext}$  and moment of unbalance  $T_{imb}$ ; weight of load  $m$  or moment of friction due to load in support assemblies  $T_f$ ; GSS static angular error  $\Delta K_{st}$ , and dynamic angular error  $\Delta K_{dyn}$ .

The design of EM with coaxially arranged BDCM, IR and support (bearing) assemblies is illustrated in Fig. 1.



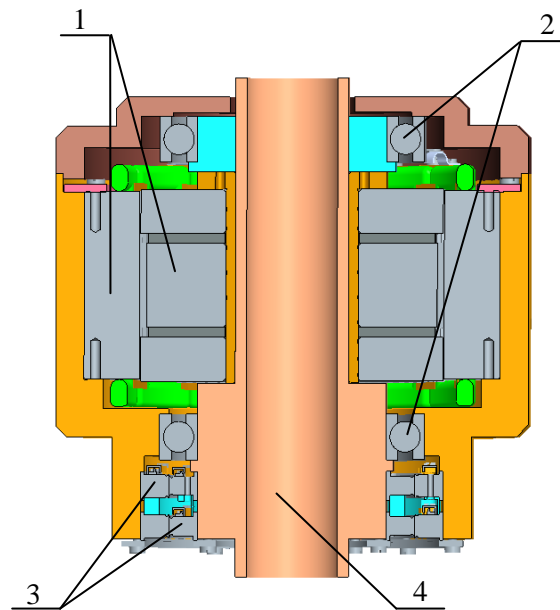


Fig. 1. The coaxial arrangement of BDCM, IR and bearings in EM  
1 – BDCM; 2 – bearings; 3 – IR; 4 – shaft.

The motor required maximum torque  $T_{pk}$  can be estimated using the moment equilibrium equation (according to Lagrange equation for a single-mass electric drive system [4, 5]):

$$T_{pk} = T_f + J \cdot \alpha + T'_{rsd} \cdot q_{pr} + q_i \cdot (T_f + T'_{rsd} + J \cdot \alpha), \quad (1)$$

$$T'_{rsd} = T_{rsd} + T_p + T_{unb}, \quad (2)$$

where  $T'_{rsd}$  – EM total residual torque;

$q_{pr} \approx 1-1.5$  – overshoot ratio;

$q_i \approx 0.1-0.2$  – torque ripple factor;  $T_p$  – BDCM torque ripple;  $T_{rsd}$  – BDCM residual torque.

The estimate of  $\Delta K_{st}$  caused by the sum of the constant components of the braking moments  $T_0$  from  $T_f$ ,  $T_{rsd}$ ,  $T_{tr}$  with respect to the motor torque can be obtained as follows with account for number of pole pairs  $p$ :

$$\Delta K_{st} = \frac{\pi \cdot T_0}{p \cdot T_{pk}}. \quad (3)$$

The estimate of  $\Delta K_{dyn}$  can be received as follows with account for the number of the dominating harmonic  $\nu$  of torque ripple in  $T'_{rsd}$ :

$$\Delta K_{dyn} = \frac{T'_{rsd} + T_f}{(\nu \cdot p \cdot \omega)^2 \cdot J_{ext}}. \quad (4)$$

However, an error due to the shaft twist angle  $\varphi$  is added to  $\Delta K_{dyn}$  [5, 6].

### Estimation of EM shaft twist angle and natural frequencies

From the point of view of EM downsizing, the substantiated determination of geometry of EM shaft with BDCM and IR fixed on it is of great importance.

It is known [5, 6] that EM hollow shaft with inner diameter  $d$  and outer diameter  $D$  is an effective means of reducing its weight and load resistance torque (braking moments). The shaft is characterized

by the hollow factor  $c=d/D \approx 0.4-0.8$ .

The shaft twist angle can be determined analytically [6] by the following expression:

$$\varphi = \frac{T_{pk}}{C} = \frac{T_{pk} \cdot l}{G \cdot J_p} = \frac{32 \cdot T_{pk} \cdot l}{G \cdot \pi \cdot D^4 \cdot (1-c^4)}; \quad (5)$$

where  $C$  – torsional stiffness factor;  $G$  – rigidity modulus;  $J_p$  – polar moment of inertia of hollow shaft;  $C=G \cdot J_p / l$ ;  $l$  – shaft length.

Polar moment of inertia of hollow shaft is determined by the following formula [6]:

$$J_p = \frac{\pi \cdot D^4}{32} \cdot (1-c^4); \quad (6)$$

The BDCM and IR rotors are installed and rigidly fixed on the EM shaft (see Figure 1), so to estimate  $D$  the outer diameters  $D_i$  of the installed rotors in  $i$ -th sections  $l_i$  of EM shaft length can be considered. Then EM shaft equivalent diameter  $D_{eq}$  is estimated with account for EM resonant natural frequency  $f_{rsn}$  [5, 6]:

$$D = \sqrt[4]{\frac{f_{rsn}^2 \cdot J_{ext} \cdot 128 \cdot \pi \cdot l}{G \cdot (1-c^4)}}, \quad (7)$$

$$D_{eq} = \frac{1}{\sqrt[4]{\sum \frac{l_i}{l} \cdot \frac{1}{D_i^4}}}. \quad (8)$$

EM natural frequency  $f_{rsn}$  is estimated as follows based on (7) and with account for (8):

$$f_{rsn} = \sqrt{\frac{D_{eq}^4 \cdot G \cdot (1-c^4)}{128 \cdot \pi \cdot J_{ext} \cdot l}}. \quad (9)$$

Note that the EM natural frequency should be beyond the specified EM resonant frequencies. Definition of  $\varphi$  and  $f_{rsn}$  and estimation of the impact of the shaft hollow factor  $c$  on  $f_{rsn}$  can also be done using Pro/Mechanica finite element analysis module from Pro/Engineer in Creo Elements. Figure 2 shows the design model of the EM shaft in Pro/Mechanica with the BDCM and IR rotors installed.

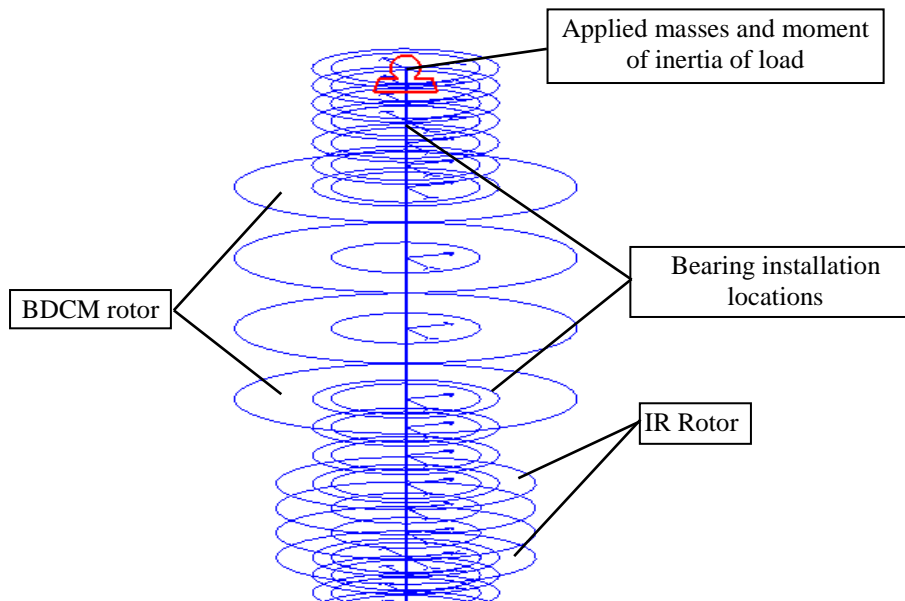


Fig. 2. Design model of EM shaft in Pro/Mechanica

To increase the natural frequency  $f_{rsn}$ , EM implementations with different IR were calculated and the outer diameter of the hollow shaft for the installation of bearings varied.

Iterative numerical modeling of the geometric parameters of the EM hollow shaft was carried out with account for the following. The minimum outer diameter  $D$  of the shaft is taken as the smallest of the inner diameters of BDCM and IR rotors. Since the parameters of the installed BDCM and IR rotors affect the strength and rigidity of EM hollow shaft, for calculation analysis the shaft is divided into  $i$ -th sections of length  $l_i$  equal in length to the axial lengths of the installed rotors, bearings and free sections with diameters  $D_i$ . At bearing installation points, the outer diameter of the shaft is selected depending on the obtained EM natural frequency  $f_{rsn}$ . If the  $f_{rsn}$  value is insufficient, the outer diameter of the shaft either at the location of the bearings or at the free portions of the shaft is increased. As a result, the values of the inner  $d$  and outer  $D$  diameters of EM hollow shaft,  $f_{rsn}$  for them and the shaft hollow factor  $c$  are determined.

Figure 3 shows the dependence of EM (implementation shown in Fig. 1) natural frequency  $f_{rsn}$  on the shaft hollow factor  $c$  based on the results of numerical modeling.

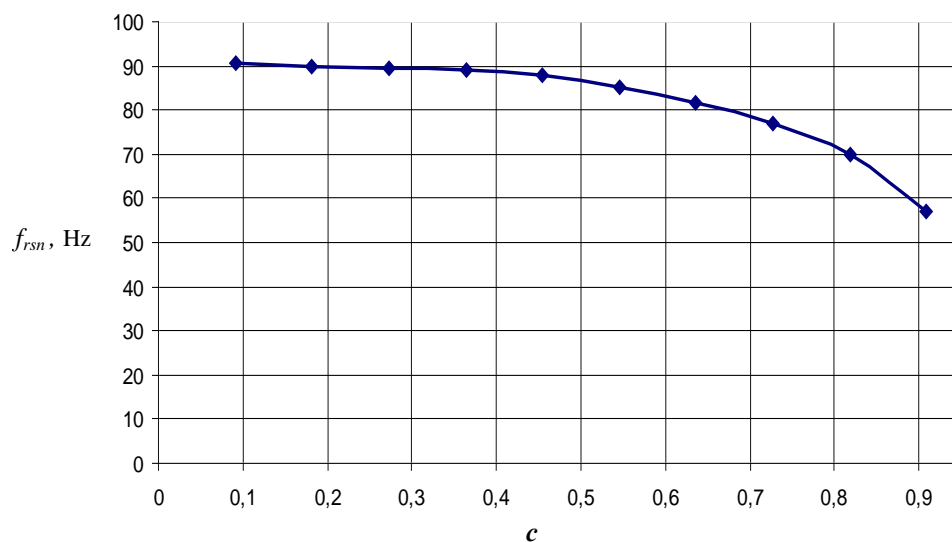


Fig. 3. Natural frequency  $f_{rsn}$  vs the shaft hollow factor  $c$ .

Using a hollow shaft in EM is a fairly effective means of reducing mass and material costs. In order to achieve the natural frequency obtained with the solid shaft using a hollow shaft EM design, the outer diameter of the hollow shaft should be increased by no more than 10% (at  $c = 0.8$ ). Note here that shaft weight decreases by more than 2 times. If it is not possible to increase the diameter of the hollow shaft, the shaft hollowness factor  $c$  should be reduced to no more than 0.4. As follows from the graph of the dependence of  $f_{rsn}$  on  $c$  in Fig. 3, further reduction of the factor  $c$  does not affect the value of the natural frequency  $f_{rsn}$ .

### Results from estimating EM version with different BDCMs

Below we provide the results of estimating EM implementations with different BDCMs: models MD220-30 [7] and DBMV240-100 [8]. The geometrical dimensions of the rotors of motors and IR, and diametrical surfaces for bearings on EM shaft are decisive for the EM design solution.

The initial data included, in addition to the above, the condition of the absence of EM resonant frequencies  $f_{rsn}$  within 60 Hz.

The calculated estimate of EM parameters is iterative with refinement of  $T_f$ ;  $J_{ext}$  and the BDCM desired  $T_{pk}$ , its results are shown in Table 1.

Table 1. Results from estimating the parameters of EM implementation

Parameter	DBMV240-100	MD220-30
Required maximum motor torque $T_{pk}$ , N·m	82.5	49.0
Number of pole pairs $p$	28	64
Required diameter of hollow shaft $D$ for BDCM installation, mm	169	60
Static angular error $\Delta K_{st}$ , "	23	0
Shaft twist angle, $\varphi$ , " (analytical/ ProMechanica)	3.22 / 3.35	1.2 / 1.28
Dynamic angular error $\Delta K_{dyn}$ , ":		
- for $v = p$ ;	216	0
- for $v = 3p$ ;	24	4.3
- for $v = 6p$ .	---	1.07
EM shaft weight, kg (with $c = 0 / c = 0.8$ )	34 / 13	17 / 7
EM natural frequency $f_{rsm}$ , Hz (with $c = 0.8$ )	52	63

It follows from the obtained results that with an increase in the number of BDCM pole pairs, GSS static and dynamic errors significantly decrease. At the same time, BDCMs with a large number of pole pairs develop a larger starting torque [7, 8] and thus provide an improvement in the weight and size parameters of EM. At the same time, determination and provision of optimal parameters of EM hollow shaft significantly reduce the weight and dimensions of EM shaft with the BDCM and IR rotors installed on it, and, accordingly, the moment of inertia. This, in turn, eliminates unreasonable redundancy in terms of technical and weight and size parameters planned to be used in EM BDCM and IR.

### Conclusions

The given analytical expressions and conditions of optimization with regard to the criteria of sufficient torque, minimized angular errors, downsized construction, and absence of resonant frequencies make it possible to evaluate the capabilities of the principal and design solutions of the EM design meeting the technical requirements.

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