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LIMITS ON SYSTEMATIC SKEW MULTIPOLE COMPONENTS IN SSC DIPOLES FROM LINEAR APERTURE REQUIREMENTS

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ABSTRACT

Limits on the systematic skew multipole components in the SSC dipoles were derived by applying the linear aperture criteria to an analysis of the clustered-IR $(2,4)_b$ test lattice.

INTRODUCTION

Limits on the allowable systematic b_n in the SSC dipoles have been calculated from the tune shift criterion (which requires that the tune shift be less than 0.005) for the Conceptual Design Report. Treating the skew multipole errors is not quite as straightforward, since only a_1 causes an on-momentum tune shift and that goes like a_1^2. However, they do produce coupling of horizontal and vertical motion (on-momentum in the case of a_1 and off-momentum for the higher order errors) which distorts the β functions, and they produce tune shifts which are second order in their strength. They are also a source of non-linearity, and thus result in a "smearing" of the linear invariants and amplitude dependent tune shifts, which are restricted by the linear aperture criteria of Reference 1. We set out to determine the limits on the systematic skew multipoles from both the linear aperture criteria and the demand that the coupling constant (defined below) be less than 0.005.

ANALYSIS AND RESULTS

For this study, the clustered-IR $(2,4)_6$ test lattice was adjusted to have tunes of 89.265 in x, and 88.285 in y. The integer tune split was necessary to reduce the effects of coupling, and, indeed this reduced the coupling constant by more than an order of magnitude. In all other respects the lattice used was the same as that used for previous studies (Reference 2). The lattice was studied with TEAPOT (Reference 3) without random multipole errors in the dipoles.

First the coupling constant and tune shifts at zero amplitude were computed for systematic a_n. The coupling constant is a measure of the coupling strength which is comparable to the tune shift and is defined

$$|C| = \left| \frac{1}{4\pi} \sum q_s \sqrt{\beta_x \beta_y} e^{-i(\phi_x - \phi_y)} e^{-i\theta(k - \nu_x + \nu_y)} \right|$$

$$\approx \left| \frac{2det^{1/2}m}{4\pi(sin\mu_x + sin\mu_y)} \right|$$

where m is the lower 2×2 block of the usual 4×4 transfer matrix for the machine (Reference 4), and the sum is over all the skew elements in the machine. k is the integer closest to $\nu_x - \nu_y$ and $\theta = \int \frac{ds}{R}$. $2|C|$ is the distance of closest approach of the perturbed tunes in a coupled machine. It is assumed that the x-y coupling is approximately the same as the y-x coupling.

The results are shown in Table 1. With this data we can set an upper limit on the systematic a_n, from the requirements that the tune shift be less than 0.005 and the coupling constant be less than 0.005 (at $\delta = 0.001$ for $n \geq 2$). Later we will set a more stringent limit using the linear aperture criteria. After the scan in a_1 the values for a_2, a_3 and a_4 for which the tune shifts were the maximum allowed could be predicted using the fact that the coupling constant is linear in the strength and the tune shift goes like the strength squared. The limits are set by the tune shift criterion, not the coupling constant, and are $a_1 \leq 0.070\times10^{-2}m^{-1}$, $a_2 \leq 0.11m^{-2}$, $a_3 \leq 0.25\times10^{2}m^{-3}$, and $a_4 \leq 0.625\times10^{4}m^{-4}$.

		C	ΔQ_x	ΔQ_y
$a_1(m^{-1})$	0.050×10^{-2}	0.0019	0.003	-0.003
	0.070×10^{-2}	0.0027	0.005	-0.005
	0.100×10^{-2}	0.0041	0.010	-0.010
$a_2(m^{-2})$	0.100	0.0028	0.003	-0.004
	0.200	0.0058	0.012	-0.013
$a_3(m^{-3})$	0.250×10^{2}	0.0033	0.004	-0.005
$a_4(m^{-4})$	0.625×10^{4}	0.0033	0.003	-0.005

Table 1. Coupling constant and zero amplitude tune shifts at $\delta = 0.001$ ($\delta = 0.000$ for a_1) for various values of a_n.

Next we consider the linear aperture criteria, which require that the tune shift be 0.005 or less for an amplitude of $7mm$ at $\widehat{\beta}$ in the arcs on-momentum, and $5mm$ at $\delta = 0.001$. Under the same conditions, the smear is required to be 10% or less. These conditions give the limits on the systematic a_{1-4} in Table 2. These limits are set by the off-momentum amplitude dependent tune shift.

		δ	smear(%)	ΔQ_x	ΔQ_y
$a_1(m^{-1})$	0.055×10^{-2}	0.001	0.5	0.003	-0.005
		0.000	0.4	0.003	-0.004
$a_2(m^{-2})$	0.10	0.001	0.6	0.003	-0.005
		0.000	0.5	-0.001	-0.001
$a_3(m^{-3})$	0.18×10^2	0.001	1.7	-0.001	0.005
		0.000	2.1	-0.002	-0.001
$a_4(m^{-4})$	0.35×10^4	0.001	2.8	-0.005	0.005
		0.000	0.5	-0.001	-0.001

Table 2. Smear and tune shifts for $7mm$ (at $\delta = 0.0$) and $5mm$ (at $\delta = 0.001$) particles for maximum permissible values of a_n.

CONCLUSIONS

Limits on the systematic skew multipole errors in the SSC dipoles have been arrived at from linear aperture considerations. Those limits are $a_1 \leq 0.055 \times 10^{-2}m^{-1}$, $a_2 \leq 0.10m^{-2}$, $a_3 \leq 0.18 \times 10^2 m^{-3}$, $a_4 \leq 0.35 \times 10^4 m^{-4}$. These limits when expressed in the units used by the magnet errors group (parts per 10^4 at $1cm$) are 0.055, 0.10, 0.18, and 0.35.

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