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THE ρ BREMSSTRAHLUNG

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A B S T R A C T

We present a two-step Weizsäcker-Williams approximation for evaluating the cross-section for $e^+e^-\rho$ and related process. The validity of the approximation is checked by applying it to a different process where an exact computation can be carried out.

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1. INTRODUCTION

Hadron production in e^+e^- collisions proceeds at the lowest electromagnetic order through the one photon channel [1]. This mechanism is expected to give rise to cross-sections which decrease as $1/E^2$, E being the energy of each $e^\pm$ in the c.m. frame [2),3)].

Other mechanisms, although of higher order, lead to logarithmically increasing cross-sections, which are already comparable to that of the one photon channel when E is in the GeV region.

One such mechanism is that of the virtual $\gamma\gamma$ processes, on which an extensive literature already exists [4]. A second mechanism which leads to comparable cross-sections is the so-called ρ bremsstrahlung :

$$e^+ + e^- \rightarrow e^+ + e^- + \rho^0 \tag{1.1}$$

This process proceeds through the three graphs shown in Fig. 1 and three more graphs obtained by these through charge conjugation (i.e., exchange of electron and positron lines).

Although for simplicity we refer to this process as ρ bremsstrahlung, it is clear that what we say can also apply if we substitute for the ρ^0 any $J^{PC} = 1^{--}$ hadronic state, or a $\mu^+\mu^-$ pair. We note that, contrary to the $\gamma\gamma$ processes, this process is not in itself very interesting, as it explores the same matrix elements of the hadronic electromagnetic current which are already explored through the one-photon channel. However, in certain experimental situations it may give an appreciable background to either the one photon or the $\gamma\gamma$ processes.

In this paper we present an evaluation of the cross-section for process (1.1) based on a modified Weizsäcker-Williams approximation, which leads to the following result :

$$\sigma_{\rho\,br.} = \frac{8\alpha^2}{3\pi}\left[\left(\log\frac{E}{m_e}\right)^2 + \cdots\right]\frac{\Gamma_\rho\,\sigma_{peak}}{M_\rho} \tag{1.2}$$

where $\sigma_{peak} \approx 1.7\times10^{-30}$ cm^2 is the cross-section for $e^+ + e^- \rightarrow \pi^+ + \pi^-$ at the peak of the ρ^0 resonance [5]. The preferred kinematical configuration is one where the ρ meson is emitted along the direction of one of the incoming particles, and both members of the final pair are in the opposite direction. In this case, the ρ has a momentum

$$|p_\rho| = E - M_\rho^2/4E \tag{1.3}$$

In the next section we will discuss briefly the approximation scheme which leads to Eq. (1.2) [6]. It would be interesting to verify that this approximation scheme reproduces correctly, at least, the leading term of an exact computation, as $E \to \infty$. This verification exists for the kind of W-W approximation employed in the $\gamma\gamma$ processes [7]. Since an exact computation of ρ bremsstrahlung would be rather lengthy, we looked for a different process where the same approximation scheme can be used, and at the same time an exact computation can be done relatively easily. Such a process is the rather uninteresting one :

$$\gamma + \gamma \to e^+ + e^- + \pi^0 \tag{1.4}$$

This proceeds through the diagrams of Fig. 2.

The computation of the exact cross-section is simpler than in the case of the ρ bremsstrahlung, and in fact a good part of the work has been done for us by Jost, Luttinger and Slotnick [8].

2. THE TWO-STEP WEIZSÄCKER-WILLIAMS APPROXIMATION

Consider the three graphs in Fig. 1. Graph (a) is clearly depressed because the virtual e^- has $q^2 > (M_\rho+m_e)^2$. On the contrary, in graph (b), both virtual particles can be nearly real. This happens in a kinematical configuration where $\vec{p}_3$, $\vec{p}_4$, $\vec{k}$, $\vec{q}$ are all parallel to $\vec{p}_1$. Momentum conservation implies that we can write :

$$\begin{aligned} \vec{p}_3 &= (1-\eta)\,\vec{p}_1 \\ \vec{k} &= \eta\,\vec{p}_1 \\ \vec{p}_4 &= (1-\varepsilon)\,\eta\,\vec{p}_1 \\ \vec{q} &= \varepsilon\,\eta\,\vec{p}_1 \end{aligned} \qquad \text{parallel configuration} \tag{2.1}$$

One easily sees that, in this configuration, k^2 and q^2 are both of the order of m_e^2. Graph (b) can thus be pictured as follows (see Fig. 3) : the electron of momentum $\vec{p}_1$ splits in a quasi real e^--γ state. In turn, the photon materializes as an e^+e^- pair. Finally, the e^- of the pair annihilates with the incoming e^+ to give a ρ. The W-W approximation consists in assuming that a physical electron of high momentum has a certain probability of appearing as an $e^-+\gamma$ state. This probability, integrated over transverse momenta, can be computed by non-covariant perturbation theory to be :

$$\frac{d P_{e\to e\gamma}(\eta)}{d\eta} = \frac{\alpha}{\pi} \frac{1+(1-\eta)^2}{\eta} \left[\log\frac{E}{m_e}+\ldots\right] \tag{2.2}$$

where η is the fraction of longitudinal momentum carried by the photon and non-leading terms have been neglected. In our case we will define a probability for an electron to appear as an electron plus a pair. This probability will be taken to be the product of that in Eq. (2.2) and the probability for a photon to appear as an e^+e^- pair, for which we obtain (ε being the fraction of longitudinal momentum carried by the e^-) :

$$\frac{d P_{\gamma\to e^+e^-}(\varepsilon)}{d\varepsilon} = \frac{\alpha}{2\pi}\left[1+(1-2\varepsilon)^2\right]\left[\log\frac{E_\gamma}{m_e}+\ldots\right] \tag{2.3}$$

Finally, we note that the invariant mass of the hadronic state produced in the collision between the incoming positron and the electron of momentum $\varepsilon\eta\vec{p}_1$ is : $s = 4E^2\varepsilon\eta$. So that we will write :

$$\begin{aligned}\sigma_{\rho\ \text{brems.}}(4E^2) &= 2\int d\varepsilon\, d\eta\ \frac{dP_{\gamma\to e^+e^-}}{d\varepsilon}\ \frac{d P_{e^-\to e^-\gamma}}{d\eta}\ \sigma_{e^+e^-\to h}(4E^2\varepsilon\eta) \\ &= \frac{\alpha^2}{\pi^2}\ \frac{8}{3}\int\frac{ds}{s}\ \sigma_{e^+e^-\to h}(s)\ F\left(\frac{s}{4E^2}\right)\left[\log^2\left(\frac{E}{m_e}\right)+\ldots\right]\end{aligned} \tag{2.4}$$

where

$$F(x) = 1+\frac{3}{4}x-\frac{3}{4}x^2+\frac{3}{2}x(1+x)\log x - x^3$$

The factor two is introduced in Eq. (2.4) to take into account the situation where the roles of e^+ and e^- are exchanged. If in the narrow width approximation we write :

$$\sigma_{e^+e^-\to h}(s)\Big|_{\rho\ \text{contribution}} = \pi M_\rho \Gamma_\rho\, \delta(s-M_\rho^2)\ \sigma_{\text{peak}} \tag{2.5}$$

By substituting into Eq. (2.4) we obtain the result quoted in Eq. (1.2). Diagram (c) can also be interpreted as the annihilation of the incoming positron with the other quasi real electron, in the $e^+e^-e^-$ state, the one not belonging to the pair. Its contribution is, however, negligible, $O(\frac{1}{E^2}(\log\frac{E}{m_e})^2)$. The above evaluation of the ρ bremsstrahlung process involves a two step application of the W-W approximation. In the following sections we will apply the same idea to the process in Eq. (1.4), and compare the result with an exact computation of the same process.

3. AN UNINTERESTING PROCESS

Consider now the process in Eq. (1.4). Following the steps of the previous section, we can picture graph (a) in the following way (see Fig. 4). One of the incoming photons splits into an e^+e^- pair. The e^- splits into an e^- and a photon which finally interacts with the other incoming photon. We again have a two-step approximation scheme, which gives :

$$\sigma(4E^2) = 4\int d\varepsilon\, d\eta\, \frac{dP_{\gamma\to e^+e^-}}{d\varepsilon}\,\frac{dP_{e^-\to e^-\gamma}}{d\eta}\,\sigma_{\gamma\gamma\to\pi^0}(4\varepsilon\eta E^2)$$

$$\xrightarrow[E\to\infty]{} 32\,\alpha^2\,\Gamma_{\pi^0}\,M_{\pi^0}^{-3}\left[\left(\ln\frac{E}{m_e}\right)^2 + \dots\right] \tag{3.1}$$

The factor 4 takes into account the equal contributions of the other three graphs obtained by exchanging the final fermions and/or the initial photons. The leading behaviour of the cross-section coming from the graphs in Fig. 2 can be computed directly in the following way : let us write

$$\frac{d\sigma}{d\theta_\pi\, dE_\pi} = I_A(\cos\theta_\pi, E_\pi) + I_B(\cos\theta_\pi, E_\pi) + 2I_{AB}(\cos\theta_\pi, E_\pi) \tag{3.2}$$

where I_A is the contribution of graphs (a_1) and (a_2) (including their interference), I_B the contribution of (b_1) and (b_2), I_{AB} the interference term. Since $(b_{1,2})$ are obtained from $(a_{1,2})$ through the exchange of the incoming photons, one has :

$$I_B(\cos\theta_\pi, E_\pi) = I_A(-\cos\theta_\pi, E_\pi) \tag{3.3}$$

The I_A contribution can be represented by the graph in Fig. 5, where the bubble represents the absorptive part of the forward $\gamma\gamma$ amplitude, with one real and one virtual photon, to the lowest (α^2) order. This quantity has been computed in Ref. 8), where it enters the evaluation of pair production on nuclei. Summing the two contributions from I_A and I_B to the total cross-section one recovers the result in Eq. (3.1).

The interference term can be shown to be non-leading by the following majoration :

$$|I_{AB}(\theta, E_\pi)| \le \left[I_A(\theta, E_\pi)\, I_B(\theta, E_\pi)\right]^{1/2} \le \max_{(\theta, E_\pi)} \left[I_A(\theta, E_\pi)\, I_B(\theta, E_\pi)\right]^{1/2}$$

The first majoration follows from the Schwartz inequality. Working out the inequality one finds that the interference term has a bound of the form $K \ln E/m$ and is therefore non-leading.

In conclusion we find that the two-step W-W approximation reproduces correctly the high energy behaviour for this process.

4. FINAL REMARKS

The intuitive basis for the validity of the W-W approximation (both the normal one and the two-step one presented here) is that some virtual photons or electrons are very energetic and at the same time nearly real. It is interesting to note that in a process like the ρ bremsstrahlung considered here, the first requirement seems to lead to a higher as well as a lower bound to the range of energies where the approximation is applicable. In fact, looking back at Fig. 3, we see that the virtual e^- (in the parallel configuration) has an energy

$$q = \varepsilon \eta P_1 = \frac{M_\rho^2}{4E}$$

If we require this to be much higher than m_e, we find that

$$E \ll \frac{M_\rho^2}{4m_e} \approx 280 \text{ GeV}$$

This bound is amply satisfied by any foreseeable e^+e^- installation. It is, however, amusing to note that in the case of process (1.4) the behaviour obtained through the W-W approximation is confirmed by the exact result even when E exceeds the corresponding bound $(E \ll M_\pi^2/4m_e)$.

SOMMARIO :

Presentiamo una approssimazione di Weizsäcker-Williams per il calcolo di $e^+e^- \to e^+e^-\rho$ ed processi analoghi. Per verificare lo schemo di approssimazione questo viene applicato ad un altro processo per il quale è facile portare a termine un calcolo esatto.

REFERENCES

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4) For a review, see e.g. S.J. Brodsky in "Proceedings of the 1971 International Symposium on electron and photon interactions at high energies" ; Cornell University, 1971.

5) J.E. Augustin et al., Phys.Letters 28B, 503 (1969).

6) The approximation scheme has been applied by G. Parisi (unpublished) to evaluate certain contributions in $e^+e^- \to 2e^+2e^-$.

7) S.J. Brodsky, T. Kinoshita and H. Terazawa, Phys.Rev. D4, 1532 (1971).

8) R. Jost, F.H. Luttinger and M. Slotnick, Phys.Rev. 80, 189 (1950).

FIGURES CAPTIONS

Fig. 1 Three of the six diagrams for ρ bremsstrahlung. The arrows indicate the direction of flow of the different momenta.

Fig. 2 The diagrams for the process $\gamma\gamma \to e^+e^-\pi^o$.

Fig. 3 The splitting of an electron into an electron plus a pair.

Fig. 4 A two-step W-W interpretation of diagram $(2a_1)$.

Fig. 5 Contribution of graphs (a_1) and (a_2) to $\gamma\gamma \to e^+e^-\pi^o$.

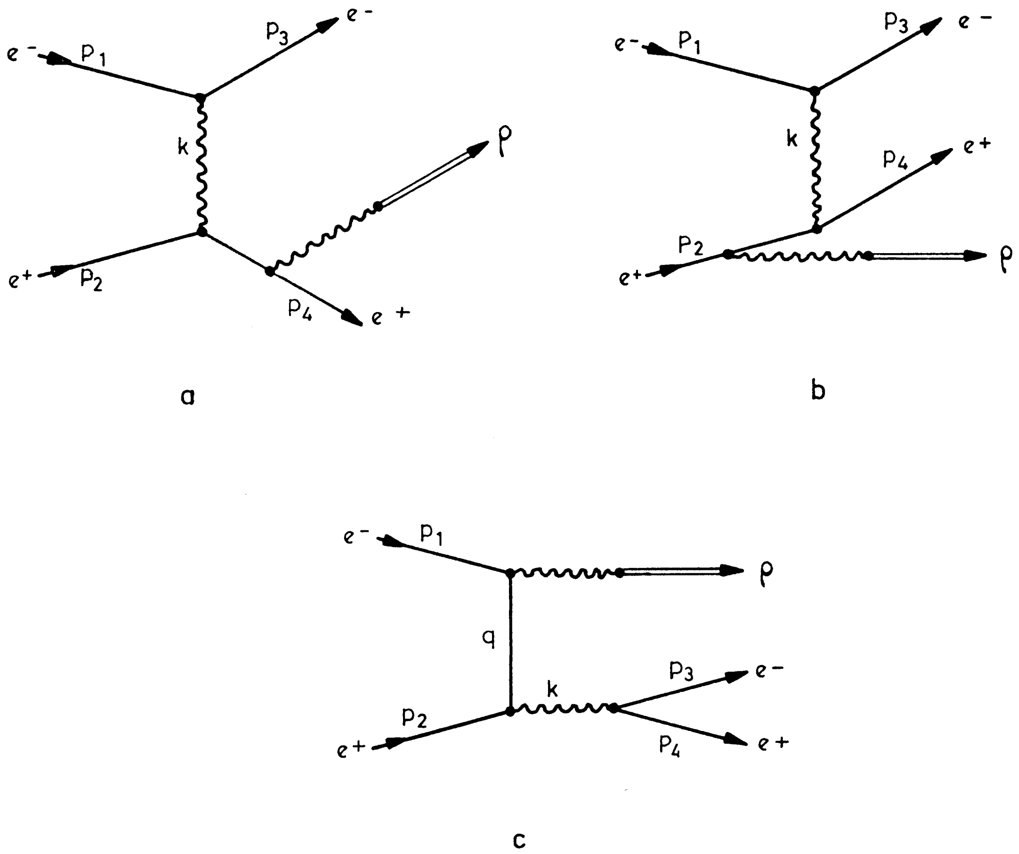
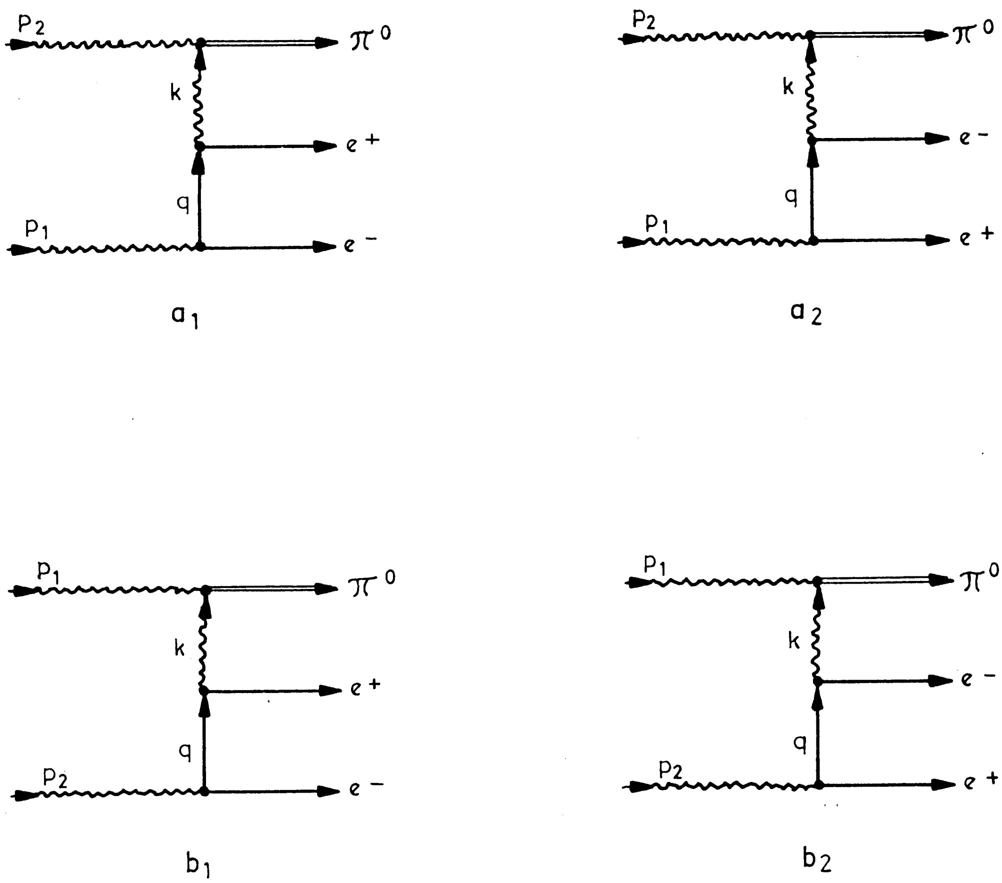


Fig . 1



p_2
π^0
k
e^+
q
p_1
e^-
a_1
p_2
π^0
k
e^-
q
p_1
e^+
a_2
p_1
π^0
k
e^+
q
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q
p_2
e^+
b_2

Fig. 2

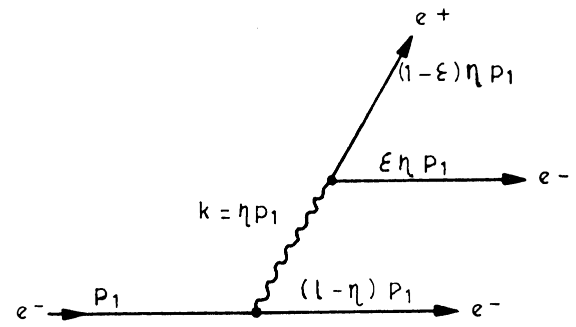

Fig . 3

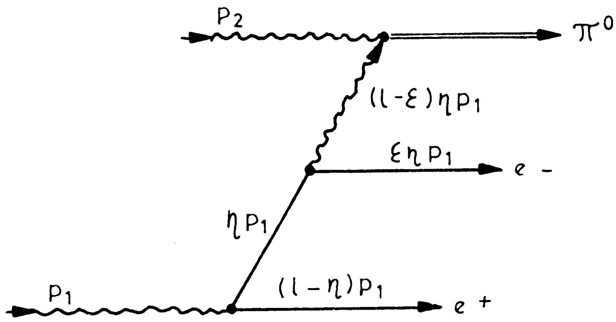

Fig . 4

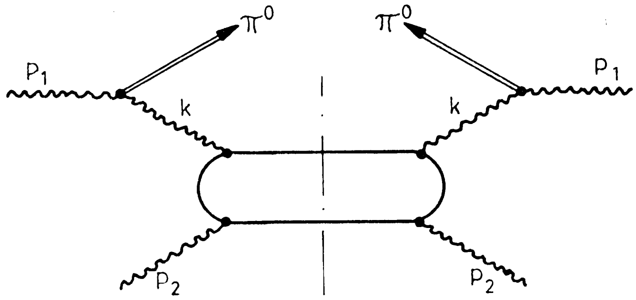

Fig . 5