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RESEARCH

Primordial black holes and gravitational waves from inflation

Misao Sasaki[1,2,3,4]

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Abstract

Thanks to the rapid progress in gravitational wave astronomy/cosmology, primordial black holes (PBHs) have become one of the hot topics in cosmology. It has made projections of detecting signatures of PBHs feasible. In parallel, it has become clear that there exist a number of ways to produce PBHs from inflation. In this talk, I will review the PBH formation from inflation and the associated gravitational wave signatures.

Keywords Primordial black Holes · Inflation · Primordial curvature perturbation · Gravitational waves

Contents

1 Introduction

The primordial black holes (PBHs) are those formed in the very early universe, presumably when the universe was still radiation-dominated. This possibility was pointed

✉ Misao Sasaki
misao.sasaki@ipmu.jp

1 Kavli Institute for the Physics and Mathematics of the Universe (WPI), The University of Tokyo, Kashiwa, Chiba 277-8583, Japan

2 Asia Pacific Center for Theoretical Physics, Pohang 37673, Korea

3 Center for Gravitational Physics and Quantum Information, Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

4 Leung Center for Cosmology and Particle Astrophysics, National Taiwan University, Taipei 10617, Taiwan

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out more than half a century ago by Hawking [1]. The most notable character of PBHs is that there is practically no theoretical constraint in the values of their mass. It can be as small as the Planck mass $\sim 10^{-5}$g and it can be as large as a massive galaxy $\sim 10^{12} M_{\odot}$.

One may then ask the question: Since black holes themselves leave no trace of their origin due to the no-hair theorem, how can one discriminate PBHs from normal astrophysical black holes? One of the simplest answers to it is to look for a black hole with mass smaller than the solar mass. Since no subsolar mass black hole can be formed astrophysically, the discovery of a sub-solar mass black hole will prove the existence of PBHs. Accordingly, there have been some efforts to search for subsolar mass compact objects. But no compelling evidence has been found so far. Thus PBHs have been a minor topic in astrophysics/cosmology until recently.

This situation was changed by the discovery of gravitational waves (GWs) from a binary black hole merger in 2015, GW20150914 [2]. In addition to the fact that it was the first direct detection of GWs, another surprising fact was that the masses of these black holes were unexpectedly large, $\sim 30 M_{\odot}$, which was above the maximum mass expected in the gravitational collapse of massive stars in the standard theory of stellar evolution (at least until that time). This immediately lead to the idea that these black holes may be primordial [3, 4], thus triggered an exploding research efforts in the study of PBHs.

Since then numerous studies on the formation mechanisms, formation criteria, and the observational implications have been done, and we have significantly improved our understanding of PBHs (For a comprehensive review, see [5]). In this talk, we focus on the conventional formation mechanism and its implications to the recently blooming GW cosmology.

2 PBH formation

In linear theory, the intrinsic 3-curvature on a given hypersurface (time-slice) is given by ${}^{(3)}R = 4k^2\mathcal{R}/a^2$ where $\mathcal{R}$ is called the curvature perturbation, and k is the comoving wavenumber. When discussing the PBH formation, one usually employs the so-called comoving slicing, defined in such a way that the matter is at rest on these time-slices, and the curvature perturbation on this slicing is called the comoving curvature perturbation. Hereafter, we simply call it the curvature perturbation for simplicity. Conventionally, a PBH is formed from a rare and large curvature perturbation with the amplitude of $O(1)$ when the universe is radiation-dominated. Such large curvature perturbations are presumably generated during inflation. While the amplitude of the curvature perturbation is strongly constrained on large scales probed by observations of the cosmic microwave background (CMB) and of the large scale structure (LSS), on scales $\gtrsim 1\,\mathrm{Mpc}$ (or $k \lesssim 1\,\mathrm{Mpc}^{-1}$), there is virtually no constraint on much smaller scales $\lesssim 1\,\mathrm{kpc}$ (or $k \gtrsim \mathrm{kpc}^{-1}$). This motivates us to consider models of inflation that may give rise to large curvature perturbations on small scales.

As illustrated in Fig. 1, a region of positive spatial curvature may collapse to a PBH. Qualitatively, when the scale of the curvature perturbation becomes equal to the Hubble horizon size, we have $k^2 = a^2H^2$. Hence ${}^{(3)}R \sim 4\mathcal{R}$ at horizon crossing.

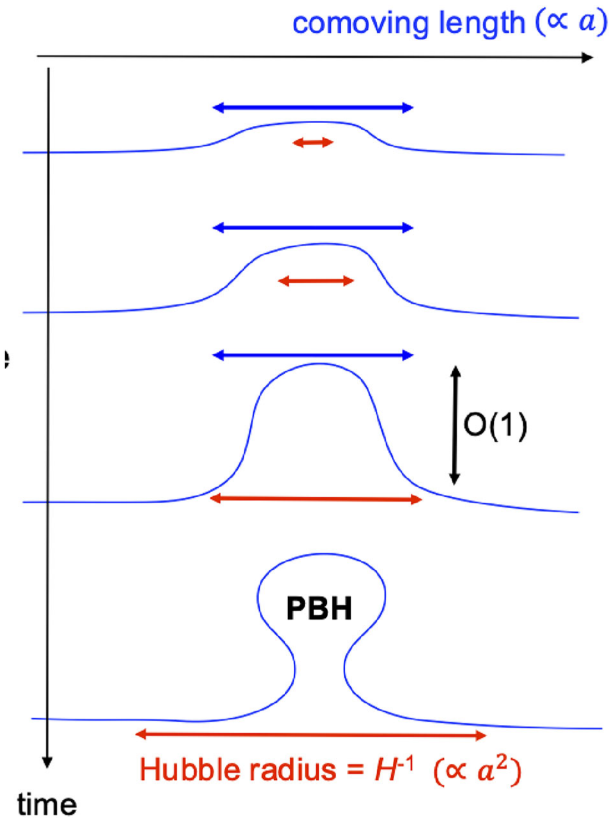

Fig. 1 An illustration of the PBH formation during radiation dominance. Taken from the conference proceedings, *Black Holes Inside Out 2024*, special collection in General Relativity and Gravitation [https://doi.org/10.1007/s10714-025-03400-6 (2025) 57:65]

Considering the Hamiltonian constraint, $3H^2 + {}^{(3)}R/2 \simeq 8\pi G\rho$, we see that the 3-curvature term becomes equal to the H^2 term for $\mathcal{R} = O(1)$ at the horizon crossing. Thus this Hubble size region starts to behave like a closed universe. As we know, a radiation-dominated closed universe eventually stops expanding and collapses. This is an intuitive picture of the PBH formation. Namely, the Hubble size region with a large positive 3-curvature behaves like a part of the closed universe, and collapses to a singularity.

The above picture tells us that the characteristic mass of PBHs is determined by the Hubble horizon size at the time when the PBH scale crosses it. Hence,

$$M_{PBH} \sim M_H \sim \left(\frac{100\,\mathrm{MeV}}{T}\right)^2 M_\odot \sim \left(\frac{\ell}{1\mathrm{pc}}\right)^2 M_\odot\,, \tag{1}$$

where T is the temperature when the comoving scale ℓ crosses the Hubble horizon.

Here we emphasize that the PBH formation must be a rare event, as the universe would be PBH dominated soon after their formation if it were not rare. Thus the root mean amplitude of $\mathcal{R}$ should not be too large, $\langle \mathcal{R}^2(k) \rangle = \mathcal{P}(k) \lesssim 10^{-2}$. This implies that PBH formation sensitively depends on the tail of the probability density function (PDF) of the curvature perturbation, which consequently implies that non-Gaussianities in the PDF have a strong impact on the PBH formation.

3 Non-minimal curvaton model

As mentioned in the previous section, observational data such as the Planck CMB data [6] strongly constrain the primordial spectrum of the curvature perturbation, and it is known that the data fit very well with a model of inflation in which a single scalar field ϕ slowly rolls down its potential, the so-called single-field slow-roll model of inflation. Nevertheless, the range of the potential probed by CMB and/or LSS observations is quite narrow, $\Delta\phi_{\rm CMB}/\Delta\phi_{\rm tot} \lesssim 0.1$, where $\Delta\phi_{\rm CMB}$ is the probed range of the inflaton field and $\Delta\phi_{\rm tot}$ is the total distance traveled by the inflaton field from the time when the comoving scale corresponding to the current Hubble radius crossed the Hubble horizon during inflation until the end of inflation. In terms of the duration of e-folds, $\Delta N_{\rm CMB} \lesssim 10$ while $N_{\rm tot} \sim 50$–60.

This gives rise to the possibility that certain interesting non-trivial features may exist in the potential that corresponds to 10–20 e-folds from the end of inflation. Such features may appear in single-field models or may be due to multi-field dynamics, and they can easily lead to a large enhancement of the curvature perturbation spectrum, $\mathcal{P}(k) \sim 10^{-2}$ on those small scales in comparison with $\mathcal{P}_{\rm CMB}(k) \sim 10^{-9}$ on the CMB scale.

A large number of such models have been proposed in the literature. Here we focus on the so-called curvaton model of inflation in which a spectator field during inflation χ turns out to dominate the universe after inflation, and hence determines the curvature perturbation from inflation rather than the one due to the inflaton field ϕ [7–9]. If the curvaton has a non-minimal coupling to the inflaton in the kinetic term, it may produce an enhanced curvature perturbation on small scales [10]. By tuning the model parameters, one can construct a scenario in which the dark matter of the universe is dominated by PBHs of the mass in the range $10^{18} - 10^{22}$ g, which is free from observational constraints [11]. An interesting feature of this model is that the PDF of the curvature perturbation can be highly non-Gaussian, and it may either enhance or suppress the PBH formation, depending on the parameters. The PDF of the curvature perturbation, $P(\mathcal{R})$, can be computed from the PDF of the curvaton fluctuations, $P_\chi(\delta)$,

$$P(\mathcal{R})d\mathcal{R} = P_\chi(\delta)d\delta\,;\;\; P_\chi(\delta) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{\delta^2}{2\sigma^2}\right], \tag{2}$$

where $\delta \equiv \delta\chi/\chi$ is assumed to be Gaussian with the dispersion σ. Using the relation between $\mathcal{R}$ and δ (see e.g., (19) with the replacement $\zeta \to \mathcal{R}$), one obtains in the limit

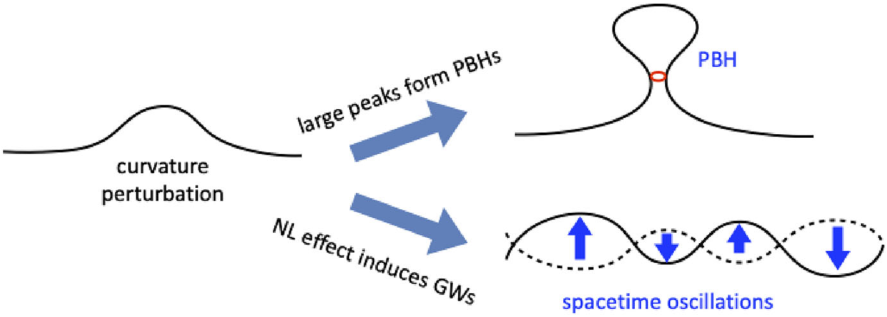

Fig. 2 An illustration of the relation between the PBH mass and the frequency of the induced GWs

$\delta \gg 1$,

$$P(\mathcal{R}) \approx \frac{1}{r^{1/2}\sigma} \exp\left[-\frac{3e^{3\mathcal{R}}}{8r\sigma^2} + \frac{3}{2}\mathcal{R}\right] \text{ at } r \ll 1, \tag{3}$$

where r is the ratio of the curvaton energy density to the total energy density at the time of the curvaton decay. Apparently, the probability to realize large positive curvature perturbations is enhanced if $r\sigma^2 \gg 1$, while it is strongly suppressed if $r\sigma^2 \ll 1$. Namely, depending on the choice of the parameters, we may enhance or suppress the PBH formation.

4 Induced gravitational waves

One of the intriguing implications of the PBH formation is that it is naturally associated with the enhanced power spectrum of the curvature perturbation, and hence will produce a non-negligible amount of gravitational waves (GWs) at second order in perturbation, the induced GWs (or the scalar-induced GWs to be more precise).

As the PBH formation is a rare event, the regions that collapse to PBHs occupy only a tiny fraction of space, the fraction $\beta \ll 1$. The most fraction of space, $1 - \beta \approx 1$, is dominated by fluctuations with the root mean amplitude given by the power spectrum $\mathcal{P}(k)$. These fluctuations behave as a sound wave after they enter the Hubble horizon. This implies that they produce space-time dependent energy density fluctuations at second order in perturbation, which results in generation of GWs.

The important point is that the PBH mass and the GW frequency are strongly correlated. This is because both the PBH mass and the GW frequency are determined by the same comoving scale when it enters the Hubble horizon. Furthermore, while the PBH formation is sensitive to the tail of the PDF, the amount of the induced GWs is predominantly determined by the power spectrum, as illustrated in Fig. 2. Hence, the existence of the induced GWs does not imply that of PBHs, but converse is almost always true. Thus if PBHs are the dark matter of the universe, the associated induced GWs must be found. In other words, this fact can be used to test the scenario of PBH as dark matter. This was first pointed out by Saito and Yokoyama in [12]. Namely, if

PBHs of mass $\sim 10^{20}$g are dark matter of the universe, the corresponding frequency of the induced GWs is $\sim 10^{-3}$ Hz, which is right in the range of target frequencies for future space-based GW detectors such as LISA, and the expected signal-to-noise ratio is a few orders of magnitude above the sensitivity curve of LISA, provided that $\mathcal{P}(k) \sim 10^{-2}$, virtually independent of the level of non-Gaussianities [13].

In passing, it may be worth pointing out that Saito and Yokoyama [12] also considered the PBH mass in the range 10–100 $M_\odot$, which corresponds to the frequency range of 10^{-8}– 10^{-9}Hz, the range that can be probed by pulsar timing array experiments. Many binary black hole mergers in this mass range have been discovered by the LKV collaboration [14] until today. If one assumes all of them, or a fraction of them are primordial, one can estimate the merger rate as well as the amount of induced GWs to test this scenario. Such an analysis was done in [15], and they already obtained an interesting constraint on the PBH scenario.

In the above, we considered PBHs that exist today. If we consider PBHs with sufficiently small mass, they may have evaporated completely through Hawking radiation before the epoch of big-bang nucleosynthesis. Such is the case if $M_{\rm PBH} \lesssim 10^9$g, for which the evaporation time is less than a second. Normally one might think that there is no way to probe PBHs with such small masses that have evaporated completely. However, recent studies have shown that if they once dominate the universe before they have disappeared, large spatial inhomogeneities are created due to the random nature of the spatial distribution of PBHs, and the induced GWs due to the inhomogeneities can leave a characteristic feature in the GW power spectrum, with the amplitude high enough to be detected in the near future [16–18].

5 Summary

We discussed the formation of PBHs from large positive curvature perturbations from inflation at small scales. In particular, we pointed out that if PBHs are dark matter of the universe, the near-future space-based GW observatories will detect the induced GWs from the enhanced curvature perturbations. Even if PBHs are not the dominant component of dark matter, the discovery of them will contribute to our understanding of the early universe significantly. Here we focused on the implications of PBHs to GW cosmology, which is definitely a blooming new area of research. However, there are plenty of other astrophysical consequences as well if PBHs exist. This makes PBHs a unique tool to explore the physics of the early universe. We may say that PBHs have become one of the main targets in cosmology, both theoretically and observationally.

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Author Contributions Misao Sasaki is the only author of this manuscript

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Data availability No datasets were generated or analysed during the current study.

Declarations

Conflict of interest The authors declare no Conflict of interest.

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